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ESSAY

Heavy ion collisions and black hole dynamics

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Abstract Relativistic heavy ion collisions create a strongly coupled quark-gluon plasma. Some of the plasma's properties can be approximately understood in terms of a dual black hole. These properties include shear viscosity, thermalization time, and drag force on heavy quarks. They are hard to calculate from first principles in QCD. Extracting predictions about quark-gluon plasmas from dual black holes mostly involves solving Einstein's equations and classical string equations of motion. AdS/CFT provides a translation from gravitational calculations to gauge theory predictions. The gauge theory to which the predictions apply is $\mathcal{N} = 4$ super-Yang-Mills theory. QCD is different in many respects from super-Yang-Mills, but it seems that its high temperature properties are similar enough to make some meaningful comparisons.

The Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory collides gold nuclei at a total center of mass energy of 39 TeV. When the nuclei collide, a quark-gluon plasma (QGP) is formed. The QGP probably thermalizes before it blows itself apart. Its peak temperature is about 300 MeV. This is hotter than the QCD transition temperature, which is about 170 MeV. The physics of the QGP is described by quantum chromodynamics (QCD). But QCD is hard to solve. The reliability of perturbative methods is questionable because α_s may be as large as $1/2$ at the scale of RHIC physics. Lattice methods are well-suited for computing static quantities like pressure and entropy, but transport coefficients like shear viscocity and diffusion constants are hard to extract from the lattice because they relate to real-time processes rather than periodic Euclidean time.

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The experimental summaries [1–4] provide a point of entry into the enormous literature on RHIC physics. A review of lattice results can be found in [5], and reviews of theoretical developments include [6–10].

The world of heavy ion physics was shaken by a result combining black hole physics and string theory [11,12]:

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}\,, \tag{1}$$

where η is the shear viscosity of a wide class of spatially extended black hole horizons and s is the entropy density. (Usually I will set $\hbar = c = k_B = 1$.) The result (1) is smaller by more than an order of magnitude than predicted for the QGP by perturbative QCD [13–16]. Experimental constraints from RHIC may be roughly summarized as $\eta/s \lesssim 0.3$ [15,17].

We are thus led to inquire [18,19]: Can the QGP be described in terms of a dual black hole?

What hope there is for a positive answer comes from the AdS/CFT[1] correspondence [20–22]. It says that $\mathcal{N} = 4$ super-Yang-Mills (SYM) theory in four dimensions is dual to type IIB string theory on $AdS_5 \times S^5$. (There will be no reason to keep track of the S^5. It relates to a global flavor symmetry in SYM.) Einstein gravity is a good description of the dynamics in AdS_5—and, indirectly, the dual gauge theory—to the extent that the number of colors N and the 't Hooft coupling $g^2_{\rm YM}N$ are both large. The success of the relation (1) suggests that, at least in one important respect, the QGP is more similar to strongly coupled SYM than to a weakly coupled plasma of quarks and gluons.

The result (1) is meaningful only when a hydrodynamical description of the QGP is valid, which is to say for times later than the thermalization time $\tau_{\rm therm}$. Successful hydrodynamical models of RHIC physics assume $\tau_{\rm therm} \approx 0.6 - 1.0\,{\rm fm}/c$ [2,23,24]. But perturbative QCD calculations [24–26] predict $\tau_{\rm therm} \gtrsim 2.5\,{\rm fm}/c$. AdS/CFT encodes in black holes everything there is to know about thermal SYM. So it must make a prediction about $\tau_{\rm therm}$. To extract this prediction, my students and I studied perturbations of the global AdS_5-Schwarzschild solution [27]. A conformal transformation shows that the unperturbed solution is dual to a radially expanding flow of thermal SYM matter. Comparing to the radial component of the collective flows in the QGP requires us to take the horizon radius ρ_H of the black hole much bigger than the curvature scale L of AdS_5: $\rho_H/L \sim 13$. The reason is that ρ_H/L is roughly equated with the peak temperature in the gauge theory (about 300 MeV) times the extent of the thermal matter at maximal compression (about 10 fm). When $\rho_H/L \gg 1$, the perturbations split into two groups [27,28]: fast and slow quasi-normal modes. The slow modes can be approximately described in terms of solutions of the linearized Navier-Stokes equations in the boundary theory. They damp out slowly because the viscosity is small. The fast modes damp out to $1/e$ of their original amplitude in a time no greater than

$$\tau_{\rm fast} = \frac{1}{8.6T_{\rm peak}} \approx 0.08\,{\rm fm}/c\,. \tag{2}$$

[1] AdS stands for anti-de Sitter space, and CFT stands for conformal field theory.

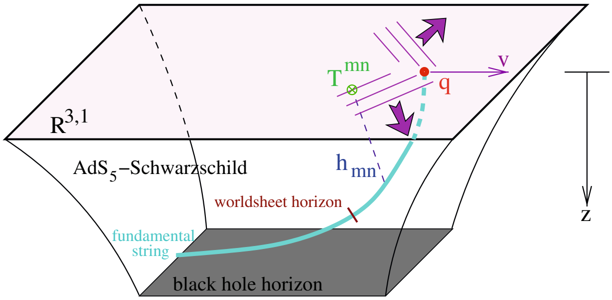

Fig. 1 A string trails behind a heavy quark q into AdS_5-Schwarzschild. The quark creates a wake of stress-energy $\langle T^{mn}\rangle$ which is dual to gravitons h_{mn} sourced by the string [29]

The first equality in (2) comes from solving the linearized Einstein equations for the slowest of the fast (non-hydrodynamic) modes. T_{peak} is the peak temperature in the gauge theory. It is related to the Hawking temperature of the black hole. To obtain the last expression in (2) we used $T_{\text{peak}} \approx 300\,\text{MeV}$. Given the highly anisotropic momentum space distribution expected in the early stages of a RHIC collision, it is reasonable to expect that several e-folding times of the relevant thermalization processes must elapse before hydrodynamic approximations can be used. Thus one may estimate

$$\tau_{\text{therm}} \sim 4\tau_{\text{fast}} \approx 0.3\,\text{fm}/c\,. \tag{3}$$

This estimate is risky because quasi-normal modes describe only the late-time stages of thermalization. However, when matched against estimates of τ_{therm} from plasma instabilities [24], the estimate (3) compares reasonably well, both in terms of precision and tenability of the final answer in the face of data.

$\mathcal{N} = 4$ SYM does not have light fundamental quarks like QCD does. However, one can introduce infinitely massive quarks by dangling a string into translationally-invariant AdS_5-Schwarzschild, see Fig. 1.

The drag force on such a quark moving at a velocity $\vec{v}$ relative to the QGP is [30,31]

$$\vec{F} = -\frac{\pi\sqrt{g_{\text{YM}}^2 N}}{2} T^2 \frac{\vec{v}}{\sqrt{1-v^2}}\,. \tag{4}$$

To understand the origin of (4), consider first the AdS_5-Schwarzschild geometry:

$$ds^2 = \frac{L^2}{z^2}\left(-h(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{h(z)}\right) \quad \text{where } h = 1 - \frac{z^4}{z_H^4}. \tag{5}$$

A steady-state configuration of the string must move with the quark:

$$x^1 = vt + \xi(z) \quad \text{if } \vec{v} = (v, 0, 0). \tag{6}$$

$\xi(z)$ measures how much the string trails behind the quark at a depth z. It can be determined explicitly by solving the string equations of motion, subject to a boundary condition at a causal horizon on the string worldsheet at $z_* = z_H \sqrt[4]{1-v^2}$. No signal can propagate classically along the string from $z > z_*$ to $z < z_*$. The drag force (4) can be computed from the flow of energy-momentum down the string.

To compare with data, consider b and c quarks, whose masses are well above characteristic RHIC temperatures. In [32] I argued that, with a physically motivated choice of $g^2_{\mathrm{YM}}N$, the result (4) translates (for energy densities characteristic of RHIC physics) into a time $t_c \approx 2.1\,\mathrm{fm}/c$ for a charm quark's velocity relative to the QGP to fall by $1/e$. Perturbative QCD estimates vary, but according to [33], $t_c \gtrsim 10\,\mathrm{fm}/c$ is a representative range at energy densities typical of RHIC. Comparisons with data appear to favor models whose values for t_c are closer to $4.5\,\mathrm{fm}/c$ [32,34–36]. Determination of t_c is complicated, however, by lack of an experimental tag to distinguish c's from b's, and by competing models of hadronization. In summary: perturbative predictions led to the incorrect expectation that charm quarks do not thermalize in RHIC collisions, whereas calculations from dual black holes are at least in the right ballpark in predicting that they do.

More physics can be extracted from the trailing string:

- The shape of the string encodes a configuration of color fields which interpolates smoothly between Coulombic near-field behavior and a sonic boom [29]. The stress tensor in the gauge theory is computed by solving linearized Einstein equations sourced by the string. Its Fourier-space components show rough agreement with jet-splitting data (see for example [37]), except that the Mach angle is about 15° too small.
- The leading near-field correction to Coulombic behavior shows that energy density piles up in front of the quark for sufficiently relativistic velocities v [38,39].
- There are stochastic fluctuations in the force on a heavy quark. These fluctuations can be related to properties of the worldsheet horizon [40–42]. They are an analog of Hawking radiation. The fluctuations are stronger than the Einstein relation permits away from the non-relativistic regime $v \ll 1$, signalling that a Langevin description is inadequate. To capture the true stochastic dynamics, the quark and its near field should probably be treated as a composite object.

The dynamics of horizons is at the heart of every prediction made using AdS/CFT about quark-gluon plasmas. The Bekenstein–Hawking entropy—normalized against free field content, augmented by the leading stringy corrections, and evaluated at a reasonable value of $g^2_{\mathrm{YM}}N$—approximately matches lattice QCD calculations at RHIC energy scales [5,32,43,44]. Horizon dynamics interfaces spectacularly with hydrodynamics by predicting low viscosity, rapid thermalization, and sonic booms. The drag force calculation hinges on both the black hole horizon and the worldsheet horizon. But these horizons are not in our spacetime: they peak coyly at us from a fifth dimension employed by string theory to describe different energy scales.

To claim that the QGP is described by a dual black hole, we must be able to compute most (ideally all) of its measurable properties using AdS/CFT. This is a high bar. To clear it we probably have to use the holographic dual of a theory more closely resembling QCD than does $\mathcal{N} = 4$ SYM. Computations to date, however, give reason to hope. Agreement with data, while not spot-on, is approaching a level that is as good as I would expect given the significant differences between $\mathcal{N} = 4$ SYM and QCD.

While methods owing nothing to black hole physics continue—rightly—to dominate the theoretical literature on heavy ion collisions, I suspect we have only begun to learn what black holes can teach us about the ongoing experimental program at RHIC.

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