

Dark energy interacting with neutrinos and dark matter: a phenomenological theory

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Abstract A model for a flat homogeneous and isotropic Universe composed of dark energy, dark matter, neutrinos, radiation and baryons is analyzed. The fields of dark matter and neutrinos are supposed to interact with the dark energy. The dark energy is considered to obey either the van der Waals or the Chaplygin equations of state. The ratio between the pressure and the energy density of the neutrinos varies with the red-shift simulating massive and non-relativistic neutrinos at small red-shifts and non-massive relativistic neutrinos at high red-shifts. The model can reproduce the expected red-shift behaviors of the deceleration parameter and of the density parameters of each constituent.

The recent astronomical measurements of type-IA supernovae [1–4] and the analysis of the power spectrum of the CMBR [5–9] provided strong evidence for a present accelerated expansion of the Universe [3, 10–14]; the nature of the responsible entity, called dark energy, still remains unknown. Furthermore, the measurements of the rotation curves of spiral galaxies [15] as well as other astronomical experiments suggest that the luminous matter represents only a small amount of the massive particles of the Universe, and that the more significant amount is related to dark matter. That offered a new setting for cosmological models with dark energy and dark matter and in these contexts many interesting phenomenological models appear in the literature analyzing the interaction of neutrinos [16–18] and dark matter [19–24] with dark energy. With respect to dark energy some exotic equations of state were proposed in the literature and among others we quote the van der Waals [25–29] and the Chaplygin [30–33] equations of state.

Dedicated to Professor Ingo Müller on the occasion of his seventieth birthday.

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In the present work a very simple cosmological model—for a homogeneous, isotropic and flat Universe composed by dark matter, dark energy, baryons, radiation and neutrinos—is investigated where the dark energy is modeled either by the van der Waals or the Chaplygin equations of state and interact with neutrinos and dark matter. Units have been chosen so that $8\pi G/3 = c = 1$, whereas the metric tensor has signature $(+, -, -, -)$.

Let a homogeneous, isotropic and spatially flat Universe be characterized by the Robertson Walker metric $ds^2 = dt^2 - a(t)^2\delta_{ij}dx^i dx^j$, where $a(t)$ denotes the cosmic scale factor. The sources of the gravitational field are related to a mixture of five constituents described by the fields of dark energy, dark matter, baryons, neutrinos and radiation. The components of the energy-momentum tensor of the sources is written as

$$(T^\mu_\nu) = \text{diag}(\rho, -p, -p, -p), \tag{1}$$

where ρ and p denote the total energy density and pressure of the sources, respectively. In terms of the energy densities and pressures of the constituents it follows

$$\rho = \rho_b + \rho_{dm} + \rho_r + \rho_\nu + \rho_{de}, \quad p = p_b + p_{dm} + p_r + p_\nu + p_{de}. \tag{2}$$

Above the indexes (b, dm, r, ν, de) refer to the baryons, dark matter, radiation, neutrinos and dark energy, respectively.

The conservation law of the energy-momentum tensor $T^{\mu\nu}{}_{;\nu} = 0$ leads to the evolution equation for the total energy density of the sources, namely

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0, \tag{3}$$

where the dot refers to a differentiation with respect to time.

The baryons and radiation are considered as non-interacting fields so that the evolution equations for their energy densities read

$$\dot{\rho}_b + 3\frac{\dot{a}}{a}\rho_b = 0, \quad \dot{\rho}_r + 4\frac{\dot{a}}{a}\rho_r = 0, \tag{4}$$

once the baryons represent a pressureless fluid, i.e., $p_b = 0$, and the radiation pressure is given in terms of its energy density by $p_r = \rho_r/3$.

According to a model proposed by Wetterich [19] the evolution equation for the energy density of a pressureless ($p_{dm} = 0$) dark matter field which interacts with a scalar field ϕ is given by

$$\dot{\rho}_{dm} + 3\frac{\dot{a}}{a}\rho_{dm} = \beta\rho_{dm}\dot{\phi}. \tag{5}$$

Here the scalar field plays the role of the dark energy and β is a constant which couples the fields of dark matter and dark energy.

For interacting neutrinos with dark energy it is supposed that the evolution equation of the energy density is given by (see [17, 18])

$$\dot{\rho}_\nu + 3\frac{\dot{a}}{a}(\rho_\nu + p_\nu) = \alpha(\rho_\nu - 3p_\nu)\dot{\phi}. \tag{6}$$

The coefficient α is connected with the mass of the neutrinos and for more details one is referred to [17, 18] and to the references therein. Here α will be consider a phenomenological coefficient that couples the dark energy field with the neutrinos. Note that if $p_\nu = \rho_\nu/3$, there is no coupling between the fields of dark energy and neutrinos. Moreover, it is also important to note that the neutrinos in the past must behave as massless particles where the relationship between the pressure and the energy density is $p_\nu = \rho_\nu/3$. Due to the coupling of the neutrinos with the scalar field they become massive and non-relativistic. For these reasons a barotropic equation of state for the neutrinos is proposed where the ratio between the pressure and the energy density $w_\nu = p_\nu/\rho_\nu$, given in terms of the red-shift z , reads

$$w_\nu = \left[\frac{1}{z^2} + \frac{5}{z} \frac{K_3(1/z)}{K_2(1/z)} - \frac{1}{z^2} \left(\frac{K_3(1/z)}{K_2(1/z)} \right)^2 - 1 \right]^{-1}. \tag{7}$$

Above $K_2(1/z)$ and $K_3(1/z)$ are modified Bessel functions of second kind. For small values of z , w_ν tends to the non-relativistic limit equal to 2/3, whereas for large values of z , w_ν tends to the relativistic limit equal to 1/3. It is noteworthy that for red-shifts $z \approx 10$ this ratio reaches the value $w_\nu \approx 1/3$ and the coupling between the neutrinos and the dark energy is negligible. The expression given in (7) is motivated by the equation of the specific heat of a relativistic gas (see e.g. [34]).

The evolution equation for the energy density of the dark energy field is obtained from Eqs. (2) through (6), yielding

$$\dot{\rho}_{de} + 3\frac{\dot{a}}{a}(\rho_{de} + p_{de}) = -\alpha\dot{\phi}(\rho_\nu - 3p_\nu) - \beta\rho_{dm}\dot{\phi}. \tag{8}$$

The energy density and pressure of the dark energy are connected with the scalar field by $\dot{\phi} = \sqrt{\rho_{de} + p_{de}}$. Since the purpose of this work is to develop a phenomenological theory, it is assumed that the dark energy field behaves either as a van der Waals or a Chaplygin fluid with an equation of state given by [28–33]

$$p_{vw} = \frac{8w_{vw}\rho_{vw}}{3 - \rho_{vw}} - 3\rho_{vw}^2, \quad p_{ch} = -\frac{A}{\rho_{ch}}, \tag{9}$$

where w_{vw} and A are positive free parameters in the van der Waals and Chaplygin equations of state, respectively.

For the determination of the time evolution of the energy densities one has to close the system of differential equations by introducing the Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \rho. \tag{10}$$

From now on the red-shift will be used as a variable instead of time thanks to the following relationships

$$z = \frac{1}{a} - 1, \quad \frac{d}{dt} = -\sqrt{\rho}(1+z)\frac{d}{dz}. \tag{11}$$

Equations (4) can be easily integrated leading to the well-known dependence of the energy densities of the baryons and radiation with the red-shift

$$\rho_r(z) = \rho_r(0)(1+z)^4, \quad \rho_b(z) = \rho_b(0)(1+z)^3, \tag{12}$$

whereas Eqs. (5), (6) and (8) become a system of coupled differential equations for the energy densities ρ_{dm} , ρ_v and ρ_{de} , namely,

$$\frac{(1+z)\rho'_{dm} - 3\rho_{dm}}{\sqrt{(\rho_{de} + p_{de})/\rho}} = -\beta\rho_{dm}, \tag{13}$$

$$\frac{(1+z)\rho'_v - 3(\rho_v + p_v)}{\sqrt{(\rho_{de} + p_{de})/\rho}} = -\alpha(\rho_v - 3p_v), \tag{14}$$

$$\frac{(1+z)\rho'_{de} - 3(\rho_{de} + p_{de})}{\sqrt{(\rho_{de} + p_{de})/\rho}} = \beta\rho_{dm} + \alpha(\rho_v - 3p_v). \tag{15}$$

In the above equations the prime refers to a differentiation with respect to the red-shift.

In order to solve the coupled system of differential equations (13)–(15) one has to specify initial values for the energy densities at $z = 0$. The following initial values for the density parameters $\Omega_i(z) = \rho_i(z)/\rho(z)$ taken from the literature (see [35] for a review) were chosen: $\Omega_{de}(0) = 0.72$, $\Omega_{dm}(0) = 0.229916$, $\Omega_b(0) = 5 \times 10^{-2}$, $\Omega_r(0) = 5 \times 10^{-5}$, $\Omega_v(0) = 3.4 \times 10^{-5}$. Moreover, one has to specify values for the coupling parameters α and β and for the parameters w_{vw} and A which appear in the van der Waals and Chaplygin equations of state (9). One way to fix the two last parameters is through the use of the value of the deceleration parameter $q = 1/2 + 3p/2\rho$ at $z = 0$. Indeed, by considering $q(0) = -0.55$ it follows $w_{vw} = 0.33851$ and $A = 0.50403$. For the coupling parameters two sets of values were chosen, namely, (a) $\alpha = 5 \times 10^{-5}$ and $\beta = -5 \times 10^{-5}$ for the van der Waals equation of state and (b) $\alpha = 10^{-1}$ and $\beta = -10^{-2}$ for the Chaplygin equation of state. Its is also important to note that by increasing the value of the coupling parameter α (and/ or β) the transfer of energy between the dark energy and neutrinos (and/or dark matter) becomes more efficient.

In Fig. 1 the density parameters are plotted as functions of the red-shift for values in the range $0 \leq z \leq 10$. The straight lines refer to the case where the van der Waals equation of state is used to describe the dark energy field whereas the dashed lines correspond to the Chaplygin equation of state. The two density parameters that represent the dark energy field are denoted by Ω_{vw} and Ω_{ch} . One can infer from this figure that the dark energy density parameter tends to zero for high red-shifts when the van der Waals equation of state is used, whereas it tends to a constant value for the Chaplygin equation of state. While for high red-shifts the van der Waals equation of state

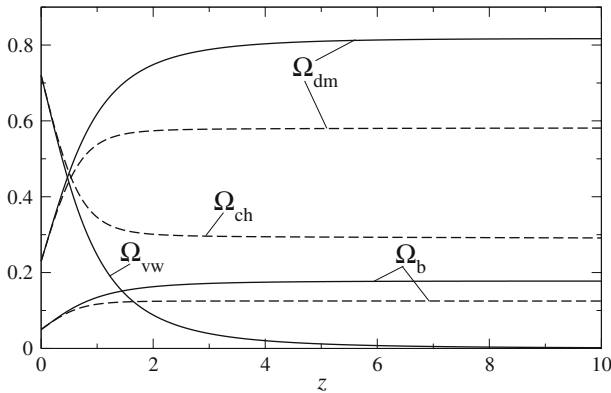


Fig. 1 Density parameters as functions of red-shift: van der Waals fluid (*solid lines*) and Chaplygin fluid (*dashed lines*)

simulates a cosmological constant with $p_{vw} = -\rho_{vw}$, the pressure of the Chaplygin fluid vanishes indicating that it becomes another component of the dark matter field (see also the behavior of the pressures indicated in Fig. 4). It is also important to note that the density parameters of the baryons and of the dark matter increase more with the red-shift for the van der Waals equation of state, since there is an accentuated decrease in the density parameter of the dark energy for this case. Note that the density parameters of the radiation and neutrinos are very small in this range of the red-shift and are not represented in this figure.

The behavior of the density parameters for the cases of the van der Waals and Chaplygin equations of state are shown in Figs. 2 and 3, respectively, for red-shifts in the range $0 \leq z \leq 3000$. One can conclude from these figures, as expected, that the density parameters of the neutrinos and radiation increase with the red-shift whereas those of the baryons and dark matter decrease. Furthermore, the equality between the “matter” and “radiation” fields occurs when $z \approx 3000$ for the case where the dark matter field is modeled as a van der Waals fluid and $z \approx 4200$ for the case of a Chaplygin fluid. This can be easily understood, since in the latter case the dark energy becomes dark matter for high red-shifts contributing for the density parameter of the “matter” field.

In Fig. 4 are plotted the deceleration parameter and the ratio between the pressure and the energy density for both cases, the large frame corresponding to the van der Waals fluid whereas the small frame to the Chaplygin fluid. For both cases the deceleration parameter at $z = 0$ is equal to $q(0) = -0.55$, since this value was fixed in order to find the parameters w_{vw} and A in the equations of state (9). The transition from the decelerated to the accelerated phase of the Universe occurs at $z_T = 0.73$ and $z_T = 0.53$ for the van der Waals and Chaplygin equations of state, respectively. It is interesting to note that while the Chaplygin equation of state simulates a cosmological constant with $p_{ch} = -\rho_{ch}$ for negative red-shifts which implies an accelerated phase of the Universe in the future, the van der Waals equation of state leads to a positive pressure and brings the Universe to another decelerated phase in the future. It is noteworthy to call attention that for positive values of the red-shift, the solution of the coupled differential equations (13) through (15) predicts that the van der Waals fluid

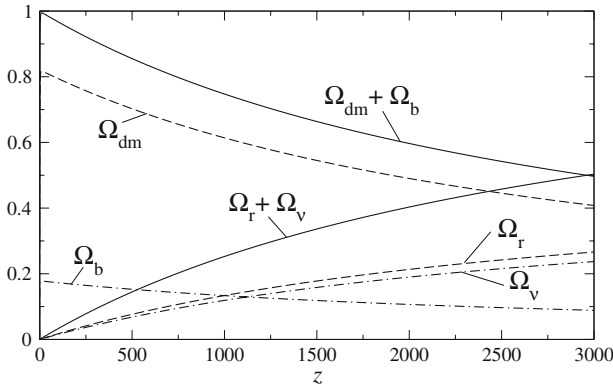


Fig. 2 Density parameters as functions of red-shift for a van der Waals fluid as dark matter

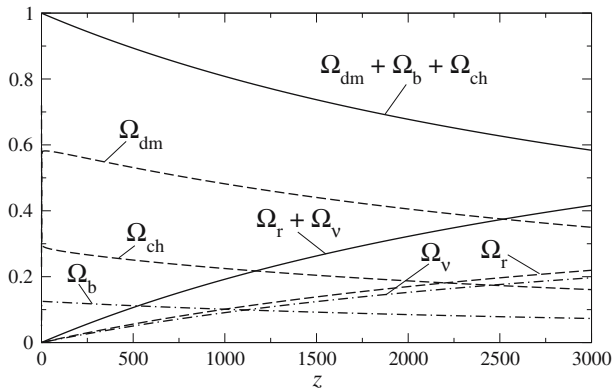


Fig. 3 Density parameters as functions of red-shift for a Chaplygin fluid as dark matter

behaves close to a cosmological constant with $p_{vw} \approx -\rho_{vw}$. This behavior does not lead to a new transition from a decelerated to an accelerated phase in the very early Universe, since the energy density of the radiation field increases so that the radiation pressure becomes larger than that of the van der Waals fluid. For high red-shifts the Universe first becomes dominated by the baryon and dark matter fields and for higher red-shifts by the radiation field. This model does not attempt to model the inflationary period, where the inflaton field dominates a short and rapid evolution of the Universe.

As final remarks we call attention to the fact that one expects that the coupling between dark energy, dark matter and neutrinos should be weak so that the parameters α and β are restricted to small values. The difference between the parameters adopted for the van der Waals and Chaplygin equations of state is due to stability conditions of the non-linear coupled system of differential equations (13)–(15), the van der Waals equation of state being more unstable for large values of these parameters than the Chaplygin equation of state. In Fig. 5 we have plotted the density parameters as functions of the red-shift for the case where a Chaplygin equation of state is used as dark energy. One can infer from this figure that the decay of the dark energy density

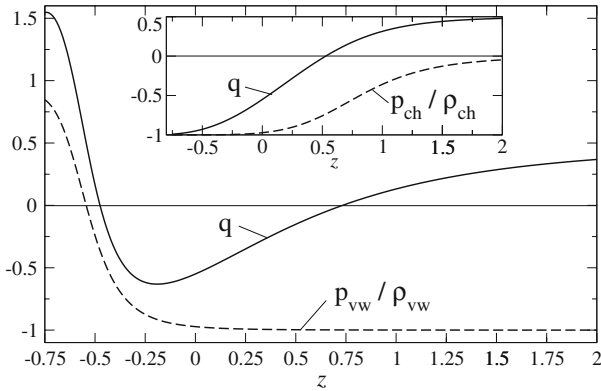


Fig. 4 Deceleration parameter and ratio between the pressure and the energy density as functions of red-shift: *large frame* (van der Waals), *small frame* (Chaplygin)

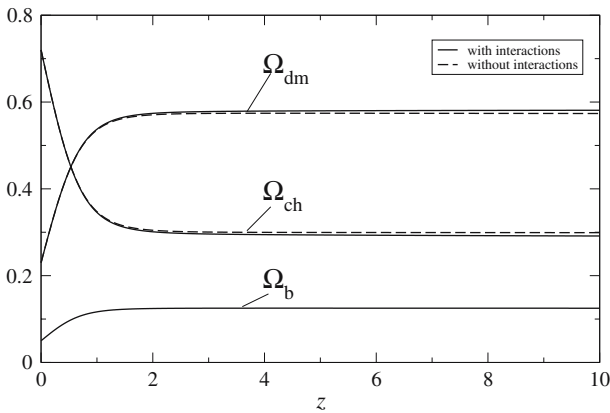


Fig. 5 Density parameters as functions of red-shift for a Chaplygin fluid with and without interactions

parameter and the increase of the dark matter density parameter with the red-shift are more pronounced when there exists a coupling between the fields. The density parameter of the baryons remains unchanged since the baryons are uncoupled.

As final comment it is important to note that even without couplings between the fields of dark energy, dark matter and neutrinos, this phenomenological model—with the equations of state of van der Waals and Chaplygin as dark energy—can describe satisfactorily the evolution of a Universe whose constituents are dark energy, dark matter, baryons, neutrinos and radiation.

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