RESEARCH ARTICLE

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$f(R)$ theories of gravity in the Palatini approach matched with observations

Abstract We investigate the viability of $f(R)$ theories in the framework of the Palatini approach as solutions to the problem of the observed accelerated expansion of the universe. Two physically motivated popular choices for $f(R)$ are considered : power law, $f(R) = \beta R^n$, and logarithmic, $f(R) = \alpha \ln R$. Under the Palatini approach, both Lagrangians give rise to cosmological models comprising only standard matter and undergoing a present phase of accelerated expansion. We use the Hubble diagram of type Ia Supernovae and the data on the gas mass fraction in relaxed galaxy clusters to see whether these models are able to reproduce what is observed and to constrain their parameters. It turns out that they are indeed able to fit the data with values of the Hubble constant and of the matter density parameter in agreement with some model independent estimates, but the today deceleration parameter is higher than what is measured in the concordance ΛCDM model.

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1 Introduction

The Hubble diagram of type Ia supernovae (hereafter SNeIa) [1, 2], the anisotropy spectrum of the cosmic microwave background radiation (hereafter CMBR) [3–5], the matter power spectrum determined by the large scale distribution of galaxies [6, 7] and by the data on the Lyα clouds [8] are all convincing evidences in favour of a new picture of the universe, unexpected only few years ago. According to this nowadays standard scenario, the universe is flat and undergoing an accelerated expansion driven by a mysterious fluid with negative pressure nearly homogeneously distributed and making up to ∼70% of the energy content. This exotic component is what is called *dark energy*, while the model we have just depicted is usually referred to as the *concordance model*.

Even if strongly supported by the bulk of the available astrophysical data, this new picture is not free of problems. Actually, while it is clear how dark energy works, its nature remains an unsolved problem. The simplest explanation claims for the cosmological constant Λ thus leading to the so called ΛCDM model[1] [9]. Although being the best fit to most of the available astrophysical data [4, 7], the ΛCDM model is also plagued by many problems on different scales. If interpreted as vacuum energy, Λ is up to 120 orders of magnitudes smaller than the predicted value. Furthermore, one should also solve the *coincidence problem*, i.e. the nearly equivalence of the matter and Λ contribution to the total energy density. As a response to these problems, much interest has been devoted to models with dynamical vacuum energy, dubbed *quintessence* [10]. These models typically involve a scalar field rolling down its self interaction potential thus allowing the vacuum energy to become dominant only recently (see [11, 12] for good reviews). Although quintessence by a scalar field is the most studied candidate for dark energy, it generally does not avoid *ad hoc* fine tuning to solve the coincidence problem. Moreover, it is not clear where this scalar field comes from and how to choose the self interaction potential.

On the other hand, it is worth noting that, despite the broad interest in dark matter and dark energy, their physical properties are still poorly understood at a fundamental level and, indeed, it has never been shown that they are in fact two different ingredients. This observation motivated the great interest recently devoted to a completely different approach to quintessence. Rather than fine tuning a scalar field potential, it is also possible to explain the acceleration of the universe by introducing a cosmic fluid with an exotic equation of state causing it to act like dark matter at high density and dark energy at low density. An attractive feature of these models is that they can explain both dark energy and dark matter with a single component (thus automatically solving the coincidence problem) and have therefore been referred to as *unified dark energy* (UDE) or *unified dark matter* (UDM). Some interesting examples are the generalized Chaplygin gas [13], the tachyonic field [14], the condensate cosmology [15] and the Hobbit model [16]. It is worth noting, however, that such models are seriously affected by problems with structure formation [17] so that some work is still needed before they can be considered as reliable alternatives to dark energy.

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[1] It is common in literature to make no distinction between the concordance and the ΛCDM model even if, strictly speaking, in the concordance model the dark energy may also be provided by a different mechanism.

Actually, there is still a different way to face the problem of cosmic acceleration. As stressed in Lue et al. [18], it is possible that the observed acceleration is not the manifestation of another ingredient in the cosmic pie, but rather the first signal of a breakdown of our understanding of the laws of gravitation. From this point of view, it is thus tempting to modify the Friedmann equations to see whether it is possible to fit the astrophysical data with a model comprising only the standard matter. Interesting examples of this kind are the Cardassian expansion [19] and the DGP gravity [20].

In this same framework, there is also the attractive possibility to consider the Einsteinian general relativity as a particular case of a more fundamental theory. This is the underlying philosophy of what are referred to as $f(R)$ theories [21–25, 28]. In this case, the Friedmann equations have to be given away in favour of a modified set of cosmological equations that are obtained by varying a generalized gravity Lagrangian where the scalar curvature R has been replaced by a generic function $f(R)$. The usual general relativity is recovered in the limit $f(R) = R$, while completely different results may be obtained for other choices of $f(R)$. While in the weak field limit the theory should give the usual newtonian gravity, at cosmological scales there is an almost complete freedom in the choice of $f(R)$ thus leaving open the way to a wide range of models.

The key point of $f(R)$ theories is the presence of modified Friedmann equations obtained by varying the generalized Lagrangian. However, here lies also the main problem of this approach since it is not clear how the variation has to be performed. Actually, once the Robertson–Walker metric has been assumed, the equations governing the dynamics of the universe are different depending on whether one varies with respect to the metric only or with respect to the metric components and the connections. It is usual to refer to these two possibilities as the *metric* and the *Palatini approach* respectively. The two methods give the same result only in the case $f(R) = R$, while lead to significantly different dynamical equations for every other choice of $f(R)$ (see [26–29] and references therein).

It is worth noting $f(R)$ theories were initially investigated using the metric approach [21–23]. Even if some interesting and successful results have been obtained [30], this way to $f(R)$ theories is plagued by serious mathematical difficulties. Actually, even for the simplest $f(R)$, the metric approach leads to a fourth order nonlinear differential equation for the scale factor that is impossible to solve analytically and is affected by several problems that greatly complicate the search for numerical solutions. Moreover, some doubts have been cast on the consistency among the weak field limit of the theory and the newtonian gravity as tested at the Solar system scale [31] even if some interesting different results have also been obtained [32].

On the other hand, theoretical considerations about the stability of the equations and the newtonian limit argue in favor of the Palatini approach to $f(R)$ theories. Moreover, the dynamics of the universe may be analytically determined from the cosmological equations obtained with this method for some interesting cases. To this aim, a clear mathematical machinery has been presented in Ref. [28] (hereafter ABF04) that allows to determine analytic expressions for the Hubble parameter as function of the redshift. As we will see later, this is all what is needed to test a given cosmological model.

The Palatini approach to $f(R)$ theories has been widely studied in literature [24–29] and the dynamics of the cosmological models obtained by applying this method to different choices of $f(R)$ has been investigated in detail. Here we adopt an observational point of view on the Palatini approach. Assuming that this is the correct way to treat $f(R)$ theories, we investigate the viability of two classes of models obtained by two popular choices for $f(R)$, namely the power law $f(R) = \beta R^n$ and the logarithmic $f(R) = \alpha \ln R$. To this aim, we compare the model predictions against the SNeIa Hubble diagram and the data on the gas mass fraction in relaxed galaxy clusters. This analysis will allow us to constrain the model parameters and to see whether $f(R)$ theories are indeed reliable alternatives to dark energy. Moreover, this will be an observational validation of the theoretically motivated Palatini approach.

The paper is organized as follows. Section 2 details the method we employ to constrain the models and present the dataset we will use. The two classes of models we consider are briefly discussed in Sect. 3 where we also individuate the parameters that are better suited to both assign the model and be constrained by the data. A detailed discussion of the results is the subject of Sect. 4, while we summarize and conclude in Sect. 5.

2 Constraining a model

Considered for a long time a purely theoretical science, cosmology has today entered the realm of observations since it is now possible to test cosmological predictions against a meaningful set of astrophysical data. Much attention, in this sense, has been devoted to standard candles, i.e. astrophysical objects whose absolute magnitude M is known (or may be exactly predicted) *a priori* so that a measurement of its apparent magnitude m immediately gives the distance modulus $\mu = m - M$. The distance to the object is then estimated as:

$$\mu(z) = 5 \log D_L(z) + 25 \tag{1}$$

with $D_L(z)$ the luminosity distance (in Mpc) and z the redshift of the object. The relation between μ and z is what is referred to as Hubble diagram and is an open window on the cosmography of the universe. Furthermore, the Hubble diagram is a powerful cosmological test since the luminosity distance is determined by the expansion rate as :

$$D_L(z) = \frac{c}{H_0}(1 + z) \int_0^z \frac{d\zeta}{E(\zeta)} \tag{2}$$

with $E(z) = H(z)/H_0$, $H = \dot{a}/a$ the Hubble parameter and $a(t)$ the scale factor. Note that an overdot means differentiation with respect to cosmic time, while an underscript 0 denotes the present day value of a quantity.

Being the Hubble diagram related to the luminosity distance and being D_L determined by the expansion rate $H(z)$, it is clear why it may be used as an efficient tool to test cosmological models and constrain their parameters. To this aim, however, it is mandatory that the relation $\mu = \mu(z)$ is measured up to high enough redshift since, for low z, D_L reduces to a linear function of the redshift (thus recovering the Hubble law) whatever the background cosmological model is.

This necessity claims for standard candles that are bright enough to be visible at such high redshift that the Hubble diagram may discriminate among different rival theories. SNeIa are, up to now, the objects that best match these requirements. It is thus not surprising that the first evidences of an accelerating universe came from the SNeIa Hubble diagram [1] and dedicated survey (like the SNAP satellite [33]) have been planned in order to increase the number of SNeIa observed and the redshift range probed.

The most updated and reliable compilation of SNeIa is the *Gold* dataset recently released by Riess et al. [2]. The authors have compiled a catalog containing 157 SNeIa with z in the range (0.01, 1.70) and visual absorption $A_V < 0.5$. The distance modulus of each object has been evaluated by using a set of calibrated methods so that the sample is homogenous in the sense that all the SNeIa have been re-analyzed using the same technique in such a way that the resulting Hubble diagram is indeed reliable and accurate. Given a cosmological model assigned by a set of parameters $\mathbf{p} = (p_1, \ldots, p_n)$, the luminosity distance may be evaluated with Eq. (2) and the predicted Hubble diagram contrasted with the observed SNeIa one. Constraints on the model parameters may then be extracted by mean of a χ^2-based analysis defining the χ^2 as:

$$\chi^2_{SNeIa} = \sum_{i=1}^{N_{SNeIa}} \left[\frac{\mu(z_i, \mathbf{p}) - \mu_{obs}(z_i)}{\sigma_i} \right]^2 \tag{3}$$

where σ_i is the error on the distance modulus at redshift z_i and the sum is over the N_{SNeIa} SNeIa observed. It is worth stressing that the uncertainty on each measurement also takes into account the error on the redshift and are not gaussian distributed. As a consequence, the reduced χ^2 (i.e., χ^2_{SNeIa} divided by the number of degrees of freedom) for the best fit model is not forced to be close to unity. Nonetheless, different models may still be compared on the basis of the χ^2 value : the lower is χ^2_{SNeIa}, the better the model fits the SNeIa Hubble diagram.

The method outlined above is a simple and quite efficient way to test whether a given model is a viable candidate to describe the late time evolution of the universe. Nonetheless, it is affected by some degeneracies that could be only partially broken by increasing the sample size and extending the redshift range probed. A straightforward example may help in elucidating this point. Let us consider the flat concordance cosmological model with matter and cosmological constant. It is:

$$E^2(z) = \Omega_M(1 + z)^3 + (1 - \Omega_M)$$

so that χ^2_{SNeIa} will only depend on the Hubble constant H_0 and the matter density parameter Ω_M. Actually, we could split the matter term in a baryonic and a non baryonic part denoting with Ω_b the baryon density parameter. Since both baryons and non baryonic dark matter scales as $(1 + z)^3$, $E(z)$ and thus the luminosity distance will depend only on the total matter density parameter and we could never constrain Ω_b by fitting the SNeIa Hubble diagram. Similar degeneracies could also happen with other cosmological models thus stressing the need for complementary probes to be combined with the SNeIa data.

To this aim, we consider a recently proposed test based on the gas mass fraction in galaxy clusters. We briefly outline here the method referring the interested reader to the literature for further details [34, 35]. Both theoretical arguments and

numerical simulations predict that the baryonic mass fraction in the largest relaxed galaxy clusters should be invariant with the redshift (see, e.g., Ref. [37]). However, this will only appear to be the case when the reference cosmology in making the baryonic mass fraction measurements matches the true underlying cosmology. From the observational point of view, it is worth noting that the baryonic content in galaxy clusters is dominated by the hot X‑ray emitting intra-cluster gas so that what is actually measured is the gas mass fraction f_{gas} and it is this quantity that should be invariant with the redshift within the caveat quoted above. Moreover, it is expected that the baryonic fraction in clusters equals the universal ratio Ω_b/Ω_M so that f_{gas} should indeed be given by $b \times \Omega_b/\Omega_M$ where the multiplicative factor b is motivated by simulations that suggest that the gas fraction is slightly lower than the universal ratio because of processes that convert part of the gas into stars or eject it outside the cluster itself.

Following Ref. [36] (hereafter A04), we adopt the SCDM model (i.e., a flat universe with $\Omega_M = 1$ and $h = 0.5$, being h the Hubble constant in units of $100 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$) as reference cosmology in making the measurements so that the theoretical expectation for the apparent variation of f_{gas} with the redshift is [36]:

$$f_{gas}(z) = \frac{b\Omega_b}{(1 + 0.19\sqrt{h})\Omega_M} \left[\frac{D_A^{SCDM}(z)}{D_A^{mod}(z)} \right]^{1.5} \tag{4}$$

where D_A^{SCDM} and D_A^{mod} is the angular diameter distance for the SCDM and the model to be tested respectively. $D_A(z)$ may be evaluated from the luminosity distance $D_L(z)$ as:

$$D_A(z) = (1 + z)^{-2} D_L(z) \tag{5}$$

with $D_L(z)$ given by Eq. (2) above.

A04 have extensively analyzed the set of simulations in Ref. [37] to get $b = 0.824 \pm 0.089$. In our analysis below, we will set $b = 0.824$ in order to not increase the number of parameters to be constrained. Actually, we have checked that, for values in the 1σ range quoted above, the main results are independent on b. It is worth noting that, while the angular diameter distance depends on $E(z)$ and thus on h and Ω_M, the prefactor in Eq. (4) makes f_{gas} explicitly depending on Ω_b/Ω_M so that a direct estimate of Ω_b is (in principle) possible. Actually, we will see later that, for the models we will consider, the quantity that is constrained by the data is the ratio Ω_b/Ω_M rather than Ω_b itself.

To simultaneously take into account both the fit to the SNeIa Hubble diagram and the test on the f_{gas} data, it is convenient to perform a likelihood analysis defining the following likelihood function:

$$\mathcal{L}(\mathbf{p}) \propto \exp\left[-\frac{\chi^2(\mathbf{p})}{2} \right] \tag{6}$$

with

$$\chi^2 = \chi^2_{SNeIa} + \chi^2_{gas} + \left(\frac{h - 0.72}{0.08} \right)^2 + \left(\frac{\Omega_b/\Omega_M - 0.16}{0.06} \right)^2 \tag{7}$$

where we have defined

$$\chi^2_{\text{gas}} = \sum_{i=1}^{N_{\text{gas}}} \left[\frac{f_{\text{gas}}(z_i, \mathbf{p}) - f_{\text{gas}}^{obs}(z_i)}{\sigma_{gi}} \right]^2 \tag{8}$$

being $f_{\text{gas}}^{obs}(z_i)$ the measured gas fraction in a galaxy clusters at redshift z_i with an error σ_{gi} and the sum is over the N_{gas} clusters considered. In order to avoid possible systematic errors in the f_{gas} measurement, it is desirable that the cluster is both highly luminous (so that the S/N ratio is high) and relaxed so that both merging processes and cooling flows are absent. A04 [36] have recently released a catalog comprising 26 large relaxed clusters with a precise measurement of both the gas mass fraction f_{gas} and the redshift z (not presented in the quoted paper). We use these data to perform our likelihood analysis in the following.

Note that, in Eq. (7), we have explicitly introduced two gaussian priors to better constrain the model parameters. First, there is a prior on the Hubble constant h determined by the results of the HST Key project [38] from an accurate calibration of a set of different local distance estimators. Further, we impose a constraint on the ratio Ω_b/Ω_M by considering the estimates of $\Omega_b h^2$ and $\Omega_M h^2$ obtained by Tegmark et al. [7] from a combined fit to the SNeIa Hubble diagram, the CMBR anisotropy spectrum measured by WMAP and the matter power spectrum extracted from over 200000 galaxies observed by the SDSS collaboration. It is worth noting that, while our prior on h is the same as that used by many authors when applying the f_{gas} test [35, 36], it is common to put a second prior on Ω_b rather than Ω_b/Ω_M. Actually, this choice is motivated by the peculiar features of the models we will consider that lead us to choose this unusual prior for reasons that will be clear later.

With the definition (6) of the likelihood function, the best fit model parameters are those that maximize $\mathcal{L}(\mathbf{p})$. However, to constrain a given parameter p_i, one resorts to the marginalized likelihood function defined as:

$$\mathcal{L}_{p_i}(p_i) \propto \int dp_1 \ldots \int dp_{i-1} \int dp_{i+1} \ldots \int dp_n \mathcal{L}(\mathbf{p}) \tag{9}$$

that is normalized at unity at maximum. The 1σ confidence regions are determined by $\delta\chi^2 = \chi^2 - \chi_0^2 = 1$, while the condition $\delta\chi^2 = 4$ delimited the 2σ confidence regions. Here, χ_0^2 is the value of the χ^2 for the best fit model. Projections of the likelihood function allow to show eventual correlations among the model parameters. In these two dimensional plots, the 1σ and 2σ regions are formally defined by $\Delta\chi^2 = 2.30$ and 6.17 respectively so that these contours are not necessarily equivalent to the same confidence level for single parameter estimates.

3 The $f(R)$ models

The observed cosmic acceleration is currently explained by invoking the presence of a new fluid with negative pressure which smoothly fills the universe driving its expansion. However, the nature and the nurture of this fluid are yet unknown so that other radically different proposals, such as unified dark energy models [13–16] or brane world inspired theories [18, 20], are still viable and worth exploring.

A quite interesting and fascinating scenario predicts that standard matter is the only ingredient of the cosmic pie as it is indeed observed, but the Einsteinian general relativity breaks down at the present small curvature scale. As a result, one should generalize the action as:

$$\mathcal{A} = \int [\sqrt{g} f(R) + 2\kappa L_{\text{mat}}] d^4 x$$

with $\kappa = 8\pi G$ and L_{mat} the matter Lagrangian. Varying with respect to the metric components and adopting then the Robertson - Walker metric, one obtains modified Friedmann equations that, by rearranging suitably the different terms, may still be formally written in the same way as the usual ones provided that a new fictitious component is added. For instance, the Hubble parameter is now given as:

$$H^2 = \frac{\kappa}{3} \left(\rho_m + \rho_{\text{curv}} \right) \tag{10}$$

with ρ_m the standard matter energy density and ρ_{curv} the energy density of a *curvature fluid* whose density and pressure are given in terms of $f(R)$ and its derivatives (see [21, 23] for details). Although intriguing, this approach leads to a mathematically untractable problem. Indeed, it turns out that the scale factor $a(t)$ should be obtained by solving a nonlinear fourth order differential equation. Not surprisingly, it is not possible to analytically solve this equation even for the simplest choices of $f(R)$. Moreover, some conceptual difficulties make it a daunting task to look for numerical solutions.

An attractive way to escape these problems is to resort to the so called *Palatini approach* in which the field equations are obtained by varying with respect to both the metric components and the connections considered as independent variables. A consistency condition is then imposed to complement the system thus giving a set of first order differential equations for the scale factor $a(t)$ and the scalar curvature R. The modified Friedmann equations are finally obtained by imposing that the metric is the Robertson–Walker one (see, e.g., [28] for a clear illustration of the procedure).

The Palatini approach is physically well motivated and has the attractive feature that the Hubble parameter $H(z)$, that is all what is needed for constraining the model, may be expressed analytically for some choices of the function $f(R)$. It is thus quite interesting to constrain the cosmological models obtained by applying the Palatini approach with two different choices of the function $f(R)$. The main characteristics of these models are briefly presented below. We follow Ref. [28] (hereafter ABF04) which the interested reader is referred to for further details.

3.1 The power law Lagrangian

We first consider the class of Lagrangians that are linear in an arbitrary power of the scalar curvature R:

$$f(R) = \beta R^n \tag{11}$$

with $\beta \neq 0$ and $n \neq 0, 2$ real parameters to be constrained. Note that β has the same units of R^n so that $f(R)$ is adimensional. This model has been already discussed by many authors [21–23] using the standard way of varying the Lagrangian. In particular, in Ref. [30], some of us have also successfully tested a simplified version of this model (with no matter term) against the SNeIa Hubble diagram. Moreover, this kind of Lagrangian has also been investigated in the framework of the Palatini approach [24, 28]. It is thus particularly interesting to see whether the Palatini approach leads to results that are in agreement with the observed data. Using the same notation as in ABF04, the scale factor $a(t)$ and the Hubble parameter $H(z)$ for a flat universe are given as:

$$a(t) = \left[\frac{3\epsilon}{2n(3-n)} \right]^{n/3} \left[\frac{\kappa\eta}{\beta(2-n)} \right]^{1/3} t^{2n/3}, \tag{12}$$

$$H^2(z) = \frac{2\epsilon n(\kappa\eta)^{1/n}}{3(3-n)[\beta(2-n)(1+z)^{-3}]^{1/n}}, \tag{13}$$

with $\eta = \rho_m(z = 0)$ the present day value of the matter density and $\epsilon = \pm 1$ depending on n in such a way that both $a(t)$ and $H(z)$ are correctly defined. For the applications, it is better to use the following relation:

$$\kappa\eta = 3\Omega_M H_0^2$$

with Ω_M the usual matter density parameter. It is worth stressing that, even if we assume a flat model, Ω_M is not forced to be unity since the critical density for closure is now different from the usual value $\rho_c = 3H_0^2/8\pi G$. The present day age of the universe may be obtained by evaluating Eq. (12) at the present day and then solving with respect to t_0 thus obtaining:

$$t_0 = \left[\frac{3\epsilon}{2n(3-n)} \right]^{-1/2} \left[\frac{3\Omega_M H_0^2}{\beta(2-n)} \right]^{-\frac{1}{2n}}. \tag{14}$$

Being the scale factor a power law function of the time, the deceleration parameter is constant and given as:

$$q \equiv -\frac{a\ddot{a}}{\dot{a}^2} = \frac{3-2n}{2n} \tag{15}$$

so that we may exclude all the Lagrangians with $n \leq 3/2$ since they give rise to non accelerating models ($q_0 \geq 0$).

A nice feature of this model is that the dimensionless Hubble parameter is simply:

$$E^2(z) = (1+z)^{3/n} \tag{16}$$

so that the luminosity distance turns out to be:

$$D_L(z) = \frac{c}{H_0} \frac{2n}{2n-3} (1+z) \left[(1+z)^{\frac{2n-3}{2n}} - 1 \right]. \tag{17}$$

Both D_L and $D_A = (1+z)^{-2} D_L$ depend only on the two parameters n and H_0 so that fitting to the SNeIa Hubble diagram is unable to put any constraint neither

on β or Ω_M. Adding the test on the f_{gas} data described in the previous section partially alleviates this problem since $f_{\text{gas}}(z)$ depends also on Ω_b/Ω_M. It is then possible to get an estimate of Ω_M combining the constraint on Ω_b/Ω_M with an independent knowledge of Ω_b from the measured abundance of light elements or primordial nucleosynthesis. Finally, the coupling parameter could be derived inverting Eq. (14) with respect to β itself provided that t_0 has been somehow evaluated (possibly from a model independent method).

As a general remark, let us observe that, without a knowledge of t_0, the parameter that can be constrained is Ω_M/β. Qualitatively, this could be explained by noting that all the tests we are considering are related to the *cosmography* of the universe. This is determined by the balance between the matter content and the *exotic* geometrical effects due to the replacement of R with $f(R)$ in the gravity Lagrangian. Actually, this feature is common to all $f(R)$ theories and could be expected since now geometry plays the same role as the scalar field in the usual dark energy models.

3.2 The logarithmic Lagrangian

Quantum effects in curved spacetimes may induce logarithmic terms in the gravity Lagrangian [25]. It is thus interesting to consider the choice:

$$f(R) = \alpha \ln R \tag{18}$$

where the dimensions of α are such that $f(R)$ is dimensionless.[2] This model is more complicated than the power law one so that, as a result, it is not possible to derive an analytical expression for the scale factor. However, the dimensionless Hubble parameter may still be expressed analytically as:

$$E^2(z) = \left[\frac{1 + (9/4)\Omega_M H_0^2 \alpha^{-1}}{1 + (9/4)\Omega_M H_0^2 \alpha^{-1}(1+z)^3} \right]^2 \frac{1 + 9\Omega_M H_0^2 \alpha^{-1}(1+z)^3}{1 + 9\Omega_M H_0^2 \alpha^{-1}}$$

$$\times \exp\{(3/2)\Omega_M H_0^2 \alpha^{-1}[(1+z)^3 - 1]\}. \tag{19}$$

The luminosity density is obtained inserting Eq. (19) into the definition (2). There is not an analytic expression for D_L, but the integral is straightforward to evaluate numerically for a given value of $\Omega_M H_0 \alpha^{-1}$. As a consequence, the likelihood function for this model depend on the Hubble constant H_0, the ratio Ω_b/Ω_M between the baryonic and total matter density and the combined parameter $\Omega_M H_0^2 \alpha^{-1}$. It is worth stressing that, even if in principle possible, constraining separately the three parameters (Ω_M, H_0, α) is not correct since both $D_L(z)$ and $f_{\text{gas}}(z)$ depend on α only through the combination $\Omega_M H_0^2 \alpha^{-1}$. Henceforth, it is this quantity that is constrained by the data. Actually, this degeneracy may be broken by an independent estimate of Ω_b that can be combined with the constraint on Ω_b/Ω_M to evaluate Ω_M and then α from the constrained $\Omega_M H_0^2 \alpha^{-1}$. Note that, without an estimate of Ω_b the only quantities estimated from the fit to the SNeIa

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[2] Note that, in literature, it is sometimes adopted the choice $f(R) = \alpha \ln \beta_l R$. We follow ABF04 and set $\beta_l = 1$ with no loss of generality.

Hubble diagram are H_0 and $\Omega_M H_0^2 \alpha^{-1}$ so that only the parameter Ω_m/α may be constrained as a result of the above mentioned degeneracy between matter and geometry.

There is no explicit analytic expression for the age of the universe so that one has to resort to numerical integration of the following relation :

$$t_0 = 9.78\, h^{-1} \int_0^\infty \frac{d\zeta}{(1+\zeta)H(\zeta)} \tag{20}$$

giving t_0 expressed in Gyr. Let us remark that, while for power law Lagrangians t_0 and Ω_b are needed to break the degeneracy Ω_M/β, now Ω_b and the likelihood analysis are sufficient to estimate both Ω_M and α so that t_0 may be used to check the results against an independent quantity.

Another striking difference with the case of power law $f(R)$ is the fact that the deceleration parameter is no longer constant. Even if we do not have an analytic expression for $a(t)$, we may still evaluate q as follows:

$$q = -1 + \frac{1+z}{H}\frac{dH}{dz}.$$

Inserting Eq. (19) into the above relation and evaluating the result at the present day ($z = 0$), we get:

$$q_0 = -1 + \frac{\Omega_M H_0^2 \alpha^{-1}}{4}\left(9 + \frac{54}{1+9\Omega_M H_0^2 \alpha^{-1}} - \frac{108}{4+9\Omega_M H_0^2 \alpha^{-1}}\right). \tag{21}$$

Eq. (21) shows that q_0 depends only on the parameter $\Omega_M H_0^2 \alpha^{-1}$ that is therefore what determines whether the universe is today accelerating or decelerating. It is also worth noting that $q(z)$ (not explicitly reported here for sake of shortness) changes sign during the evolution of the universe so that it is possible to estimate a transition redshift z_T as $q(z_T) = 0$ that only depends on $\Omega_M H_0^2 \alpha^{-1}$. It should be possible to estimate somewhat z_T, this could give an independent check of the results. Actually, we will see that this is not possible since all the estimates of z_T are model dependent. However, it is interesting to compare the transition redshift predicted for the logarithmic $f(R)$ with that of other dark energy models.

4 Results

We have applied the method described in Sect. 2 to investigate whether the cosmological models obtained by applying the Palatini approach to $f(R)$ theories for the two choices in Eqs. (11) and (18) are in agreement with both the SNeIa Hubble diagram and the data on the gas mass fraction in relaxed galaxy clusters. This also allows us to constrain the model parameters and compare the estimated values of some of them (as the Hubble constant h and the matter density Ω_M) with the recent results in literature in order to see whether they are reliable or not.
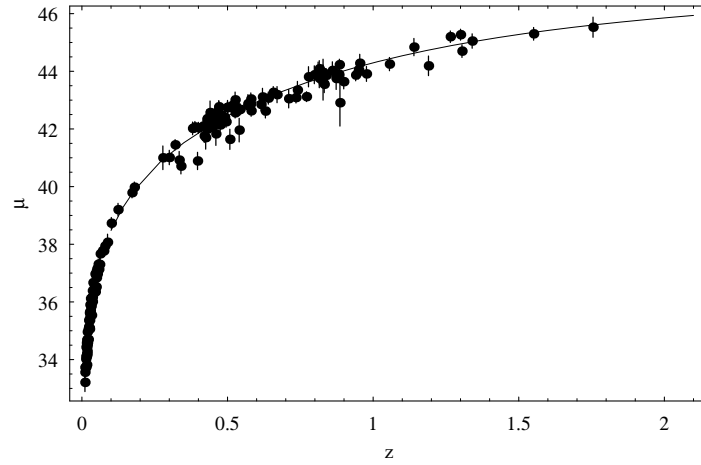
Fig. 1 Best fit curve to the SNeIa Hubble diagram for the power law Lagrangian model

4.1 $f(R) = \beta R^n$

Let us first discuss the case of the power law Lagrangian. The best fit parameters turn out to be:

$$n = 2.25 \ , \ \ h = 0.641 \ , \ \ \Omega_b/\Omega_M = 0.181 \tag{22}$$

that gives the best fit curves shown in Figs. 1 and 2. The agreement with the data (in particular, with the SNeIa Hubble diagram) is quite good which should be considered a strong evidence in favor of the model. However, Fig. 1 shows that the model slightly overpredicts the distance modulus for two highest redshift $SNeIa$, but, given the paucity of the data in this redshift range, the discrepancy is hardly
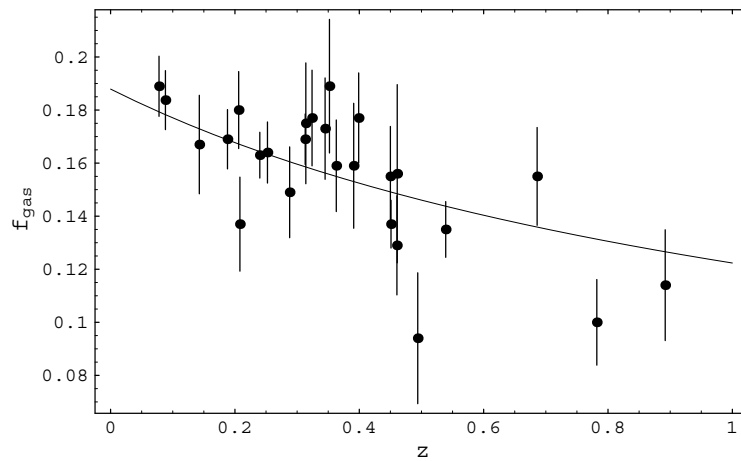
Fig. 2 Best fit curve to the f_{gas} data for the power law Lagrangian model

significant. Should this trend be confirmed by future data (observable, e.g., with the SNAP satellite mission that will detect SNeIa up to $z \sim 2$), we should exclude the choice (11) for $f(R)$. Actually, such a result could be expected since the deceleration parameter is constant, while Riess et al. [2] claimed to have detected a change in the sign of q at a transition redshift $z_T \sim 0.5$. We will return later to the problems connected with the result of Riess et al. that lead us to consider (at least) premature to deem as unreliable a model with a constant q. Therefore, we still retain $f(R)$ theories with power law Lagrangian.

It is interesting to look at the confidence contours in the projected two parameters space. Figures. 3 and 4 show the confidence regions for the parameters (n, h) and $(n, \Omega_b/\Omega_M)$ respectively. It turns out that n is positively correlated with both h and Ω_b/Ω_M so that the higher is n, the higher is the expansion rate and the lower is the matter content Ω_M. As a consequence, to fit the available data, models with steeper (higher n) power law Lagrangians should contain less matter which is a result disfavoring values of n much larger than our best fit.

Using the method detailed in Sect. 2, we have obtained the following constraint on the model parameters:
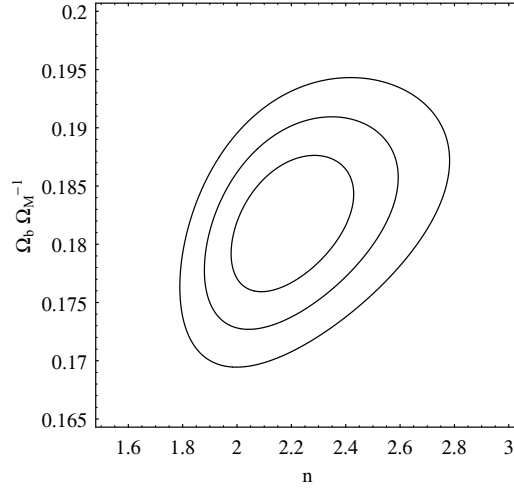
$$n \in \begin{cases} (2.06, 2.46) \text{ at } 1\sigma \\ (1.91, 2.61) \text{ at } 2\sigma \end{cases} ; \tag{23}$$

$$h \in \begin{cases} (0.637, 0.648) \text{ at } 1\sigma \\ (0.633, 0.654) \text{ at } 2\sigma \end{cases} ; \tag{24}$$

$$\Omega_b/\Omega_M \in \begin{cases} (0.177, 0.185) \text{ at } 1\sigma \\ (0.173, 0.189) \text{ at } 2\sigma \end{cases} . \tag{25}$$
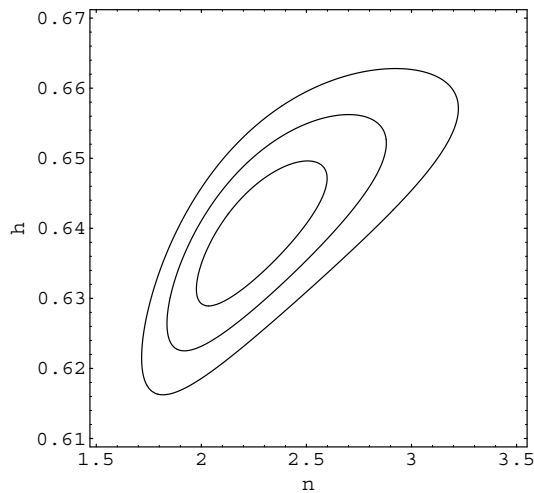
Fig. 3 1, 2, and 3σ confidence regions in the two dimensional parameter space (n, h)

Fig. 4 1, 2, and 3σ confidence regions in the two dimensional parameter space $(n, \Omega_b/\Omega_M)$

The cosmological model originating from power law $f(R)$ has been already considered by different authors in literature under the metric approach to the variation of the Lagrangian [21–23]. However, the lack of analytic solutions for the scale factor or the Hubble parameter has prevented any attempt to constrain the value of n against the observed data. Actually, only the model without matter has been investigated giving $n \in (-0.450, -0.370)$ or $n \in (1.366, 1.376)$ [30] in clear disagreement with our estimate (23). However, such a comparison is meaningless because of the presence of the matter term in the present case and the absence in the other one.

Actually, using Eq. (15), it is possible to convert the estimate of n in a constraint on the present day value of the deceleration parameter. The best fit value for n thus translates into $q_0 = -0.33$, while, combining Eqs. (15) and (23), we get:

$$
q_0 \in
\begin{cases}
(-0.39, -0.27) \text{ at } 1\sigma \\
(-0.43, -0.21) \text{ at } 2\sigma
\end{cases}.
\tag{26}
$$

While consistent with the picture of an accelerating universe, our estimates for q_0 disagree with other recent results. Let us consider what is obtained for the ΛCDM model.[3] Using a flat geometry prior and fitting to the SNeIa Hubble diagram only,

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[3] We limit our attention to the ΛCDM model only since the cosmological constant is the simplest and most efficient way to fit most of the astrophysical data [7]. Moreover, the constraints on the equation of state parameter $w = p/\rho$ are still consistent with the cosmological constant value $w = -1$ [39]. This conclusion is further strengthened by the methods that aim at recovering the evolution of the dark energy density from the data in model independent way (see, e.g., [40] and references therein).

Riess et al. [2] have found $\Omega_M = 0.29^{+0.05}_{-0.03}$ that gives[4] $q_0 = -0.56 \pm 0.07$ that is not consistent with our estimate. Adding the data on the CMBR anisotropy and the power spectrum of SDSS galaxies, Tegmark et al. [7] give $\Omega_M = 0.30 \pm 0.04$ so that the estimated q_0 is in agreement with Riess et al. and hence in contrast with our value. A similar result has also been obtained by A04 only using the same f_{gas} data we have considered here with a prior on h and $\Omega_b h^2$. For a flat ΛCDM model, their analysis gives $\Omega_M = 0.24 \pm 0.04$ and hence $q_0 = -0.64 \pm 0.06$ still in disagreement with our Eq. (26). As a general remark, we notice that our models turn out to be *less accelerating* (i.e., the predicted q_0 is higher) than is observed for the standard concordance model. From a different point of view, lower values of q_0 correspond to higher n, i.e. to steeper power law Lagrangians that are, however, disfavoured by the lower matter content of the corresponding best fit model.

However, one could deem as unreliable the comparison among q_0 constraints obtained under different underlying cosmological models and look for model independent estimates of the deceleration parameter. For instance, Riess et al. have tried to constrain the deceleration parameter by using the simple ansatz $q(z) = q_0 + (dq/dz)_{z=0} z$ or resorting to a fourth order expansion of the scale factor thus estimating also the jerk and snap parameters [41]. While the (quite large) constraints on q_0 shown in their Fig. 6 agree with our own in Eq. (26), the vanishing of $(dq/dz)_{z=0}$ is clearly ruled out. It is interesting to notice, however, that a similar analysis performed in Ref. [42] expanding the scale factor up to the fifth order and using no priors at all gives different results. A glance at Fig. 2 in that paper shows that our ranges for q_0 are indeed acceptable even if the best fit value quoted there ($q_0 = -0.76$) is outside our 2σ interval. Moreover, Fig. 3 of the same paper suggests that the jerk parameter is only weakly constrained and may be also consistent with a null value so that it is not possible to reject models with constant $q(z)$.

Actually, there is some evidence in favor of the model. First, the estimated Hubble constant is in good agreement with recent values quoted in literature. In the framework of the concordance model, a combined analysis of the CMBR anisotropy spectrum measured by WMAP, the power spectrum of SDSS galaxies, the SNeIa Gold dataset, the dependence of the bias on luminosity and the Lyα power spectrum lead Seljak et al. to finally estimate $h = 0.710^{+0.075}_{-0.067}$ (at 99% CL) [43] consistent with our range in Eq. (24). Results in agreement with those of Seljak et al. (but with larger uncertainties) have also been obtained by applying the same method to less complete dataset and are not reported here for sake of shortness (see, e.g., [4, 7] and references therein). It is even more appealing the agreement among our estimate of h and those coming from model independent methods. For instance, by combining different calibrated local distance indicators, the HST Key project finally furnish $h = 0.72 \pm 0.08$ [38] in quite good agreement with our results. This conclusion is further strenghtened when comparing to the results from time delays in lensed quasars [44] and Sunyaev - Zel'dovich effect in galaxy clusters [45].

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[4] Hereafter, we will compute the error on q_0 propagating the maximum 1σ uncertainty on Ω_M. Although not statistically correct, this method gives a quick order of magnitude estimate of the error which is enough for our aims.
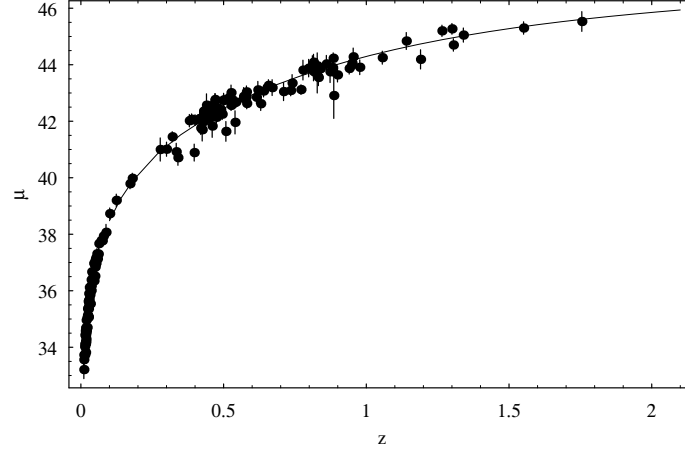
Fig. 5 Best fit curve to the SNeIa Hubble diagram for the logarithmic Lagrangian model

Having constrained with the likelihood analysis both h and Ω_b/Ω_M, we may derive Ω_M by using an independent estimate of Ω_b. Following Kirkman et al. [46], we adopt:

$$\Omega_b h^2 = 0.0214 \pm 0.0020$$

that, combined with Eqs. (24) and (25), gives:

$$\Omega_M = 0.28 \pm 0.02 \tag{27}$$

where the error has been roughly evaluated by propagating the 1σ uncertainties on h, Ω_b/Ω_M and $\Omega_b h^2$ approximated as symmetric around the best fit values.[5] Equation (27) is in very good agreement with recent results. As already quoted above, using only the SNeIa Gold dataset, Riess et al. have found $0.29^{+0.05}_{-0.03}$ for a flat ΛCDM model, while the analysis of Seljak et al. gives $\Omega_M = 0.284^{0.079}_{-0.060}$ (at 99% CL). Finally, fitting to the f_{gas} data only with priors on both h and $\Omega_B h^2$, but not imposing the flatness condition *ab initio*, A04 estimates $\Omega_M = 0.245^{+0.040}_{-0.037}$, while including the CMB data, they get $\Omega_M = 0.26^{+0.06}_{-0.04}$. All these results are in almost perfect agreement with our estimate of Ω_M which is indeed a remarkable success.

Finally, we could use the estimated values of n, h and Ω_M and the age of the universe t_0 to put constraints on the coupling constant β through Eq. (14). However, this does not give us any useful information since we have no theoretical motivation that may suggest us what is the value of β. On the other hand, the freedom we have in the choice of β leaves us open the possibility to find a R^n model which fits both the SNeIa Hubble diagram and the f_{gas} data and also predicts the right value of t_0.

[5] It is likely that this method underestimates the true error thus only giving an order of magnitude estimate.
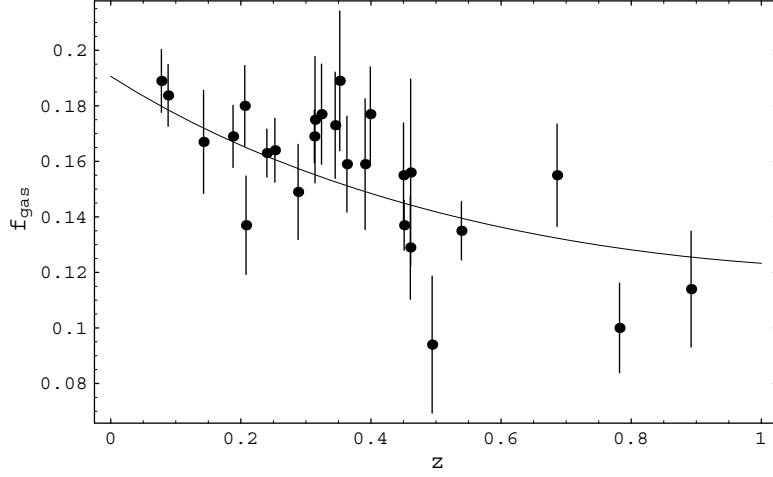
Fig. 6 Best fit curve to the f_{gas} data for the power law Lagrangian model

4.2 $f(R) = \alpha \log R$

Let us now discuss briefly the results for models with the logarithmic Lagrangian in Eq. (18). With the following choice of the model parameters:

$$\Omega_M H_0^2 \alpha^{-1} = 0.162, \quad h = 0.650, \quad \Omega_b / \Omega_M = 0.184 \qquad (28)$$

we get the best fit curves shown in Figs. 5 and 6. While both fits are indeed very good, it is interesting to note that the SNeIa Hubble diagram is now reproduced with great accuracy also for the two SNeIa with the highest redshift in contrast with what is observed for the power law Lagrangian models. This is likely a
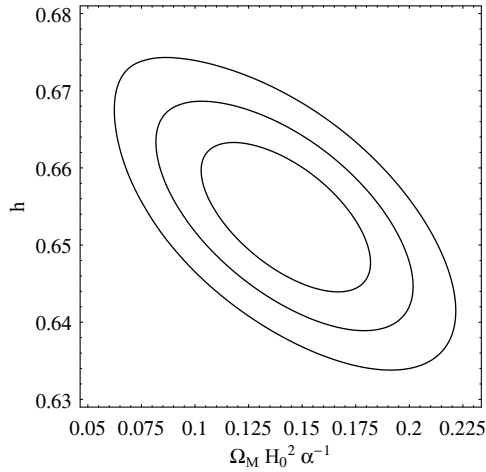
Fig. 7 1, 2, and 3σ confidence regions in the two dimensional parameter space $(\Omega_M H_0^2 \alpha^{-1}, h)$
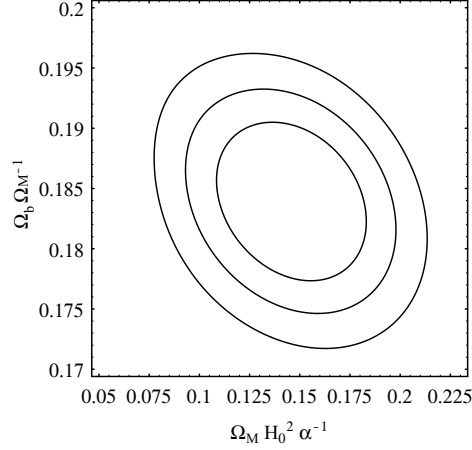
Fig. 8 1, 2, and 3σ confidence regions in the two dimensional parameter space $(\Omega_M H_0^2 \alpha^{-1}, \Omega_b/\Omega_M)$

consequence of having this class of model a non constant deceleration parameter in agreement with what is suggested by Riess et al. (within the caveat noted above).

Figures 7 and 8 show the two dimensional projections of the 1, 2, and 3σ confidence regions on the subset parameter space $(\Omega_M H_0^2 \alpha^{-1}, h)$ and $(\Omega_M H_0^2 \alpha^{-1}, \Omega_b/\Omega_M)$ respectively. It is clear that $\Omega_M H_0^2 \alpha^{-1}$ anticorrelates with both h and Ω_b/Ω_M. From the projection on the $(h, \Omega_b/\Omega_M)$ plane (not shown here), we see that these parameters are negatively correlated. Combining these plots, we may argue that the Hubble constant is positively correlated with both Ω_M and α so that the anticorrelation with $\Omega_M H_0^2 \alpha^{-1}$ is due to the degeneracy between h and α that turns out to be stronger than that between h and Ω_M.

Let us now consider the constraints on the single parameters. We get:

$$\Omega_M H_0^2 \alpha^{-1} \in \begin{cases} (0.148, 0.174) \text{ at } 1\sigma \\ (0.129, 0.194) \text{ at } 2\sigma \end{cases} ; \tag{29}$$

$$h \in \begin{cases} (0.644, 0.657) \text{ at } 1\sigma \\ (0.637, 0.664) \text{ at } 2\sigma \end{cases} ; \tag{30}$$

$$\Omega_b/\Omega_M \in \begin{cases} (0.180, 0.188) \text{ at } 1\sigma \\ (0.176, 0.192) \text{ at } 2\sigma \end{cases} . \tag{31}$$

It is more useful to translate the constraint on the combined parameter $\Omega_M H_0^2 \alpha^{-1}$ (whose physical meaning is not immediate) in a range for the present day value of

the deceleration parameter. Using Eq. (21), we get $q_0 = -0.55$ as best fit value, while the confidence regions turn out to be:

$$q_0 \in \begin{cases} (-0.56, -0.54) \text{ at } 1\sigma \\ (-0.58, -0.52) \text{ at } 2\sigma \end{cases}. \tag{32}$$

Moreover, being $q(z)$ no longer constant for this class of models, we may also estimate the transition redshift obtaining $z_T = 0.61$ as best fit value and the following confidence regions:

$$z_T \in \begin{cases} (0.57, 0.66) \text{ at } 1\sigma \\ (0.52, 0.74) \text{ at } 2\sigma \end{cases}. \tag{33}$$

Even if the deceleration parameter is varying with the redshift z, our estimate of q_0 is still in disagreement with the estimates discussed in the previous subsection. As a general remark, we notice that, as for the class of models with power law Lagrangian, the estimated q_0 is higher than what is predicted by the best fit ΛCDM model. However, the disagreement is now less severe and, indeed, a marginal agreement may be sometimes obtained by considering the 3σ confidence regions.

We may also compare the transition redshift that, for a flat ΛCDM model, is given by: $z_T = [2(1 - \Omega_M)/\Omega_M]^{1/3} - 1$. Using, for instance, the estimate of Ω_M given by Seljak et al., we get $z_T \in (0.52, 0.91)$ with $z_T = 0.71$ as best fit in quite a good agreement with our Eq. (33). Moreover, it is encouraging that our 1σ confidence region has a non null overlap with that estimated by Riess et al., $z_T = 0.46 \pm 0.13$, using the model independent parametrization of $q(z) = q_0 + (dq/dz)_{z=0}z$.

Regarding the Hubble constant, the confidence regions for h are almost the same as those obtained for the power law Lagrangian case. Hence, we are still in agreement with previous results in literature. This is not very surprising since h is essentially determined by the fit to the low redshift SNeIa and, in this range, both D_L and D_A are almost model independent. As a consequence, the estimated h turns out to be the same whatever is the underlying cosmology and in agreement with what one should obtain by a linear fit to the $z \leq 0.1$ SNeIa data.

From the constraints (30) and (31) and the value of $\Omega_b h^2$ in Ref. [46], we estimate :

$$\Omega_M = 0.27 \pm 0.03 \tag{34}$$

with the error evaluated as for that in Eq.(27). This is in perfect agreement with both the result for the power law Lagrangian case and the other estimates quoted above.

One could use Eq. (34) and the constraints on $\Omega_M H_0^2 \alpha^{-1}$ and h to narrow the range for the coupling parameter α. However, this does not give any useful information since there is no way to theoretically predict the value of α. It is, on the contrary, more interesting to evaluate the present age of the universe using Eq. (20) and the constraints in Eqs. (29) and (30). For the best fit model, it is $t_0 = 10.3$ Gyr, while t_0 ranges between 10 and 11 Gyr for the parameters running in their 1σ confidence regions. These values are too low when compared to the

estimated t_0 for the best fit ΛCDM model. For instance, the best fit vanilla model of Tegmark et al. [7] predicts $t_0 = 13.54^{+0.23}_{-0.27}$ Gyr more than 9σ higher than our estimated upper value. Notice, however, that the disagreement is less severe (but still of high significance) if compared to $t_0 = 14.4^{+1.4}_{-1.3}$ Gyr determined by Rebolo et al. [5] by fitting the ΛCDM model to the anisotropy spectrum measured by WMAP and VSA and to the clustering properties of 2dFGRS galaxies. However, even if in agreement with those obtained by completely different methods, these estimates are model dependent. Actually, our predicted t_0 is not unreasonably low if we consider that globular clusters data lead to $t_0 = 12.6^{+3.4}_{-2.6}$ Gyr [47], while a lower limit $t_0 > 12.5\pm3.5$ Gyr is obtained by nucleochronology [48]. Considering the 2σ confidence regions for the parameters $\Omega_M H_0^2 \alpha^{-1}$ and h, it is therefore possible to find models that are able to successfully fit the astrophysical data we are considering (even if they are not the preferred ones) and also predict a present age of the universe that is not in disagreement with cosmology independent estimates of t_0.

5 Discussion and conclusions

Assuming that the Palatini (first order) approach is the correct way to treat $f(R)$ theories, we have investigated the viability of two different class of cosmological models corresponding to two popular choices of $f(R)$, namely a power law in the scalar curvature and a logarithmic function of R. The expansion rate $H = \dot{a}/a$ may be analytically expressed as a function of the redshift z for both classes of models so that it is possible to contrast the model predictions against the observations. In particular, we have used the SNeIa Hubble diagram and the data on the gas mass fraction in relaxed galaxy clusters to investigate the viability of each class as dark energy alternative and to constrain their parameters. The main results are sketched below.

1. Both classes of models provide very good fits to the data even if the choice $f(R) = \alpha \ln R$ leads to a Hubble diagram that is in better agreement with the trend shown by the highest redshift SNeIa. However, the paucity of the data does not allow us to eventually prefer one model to the other.

2. Equations(22) and (28) give the best fit parameters for the power law and logarithmic Lagrangian models respectively. The confidence regions have been determined from the marginalized likelihoods and are reported in Eqs. (23) - (25) for the models with $f(R) = \beta R^n$ and in Eqs. (29)–(31) for those with $f(R) = \alpha \ln R$. To better compare the model predictions with previous results in literature, we have evaluated the present day deceleration parameter q_0 and the matter density parameter Ω_M (assuming the estimate of $\Omega_b h^2$ in Ref. [46]). For both classes of models, q_0 turns out to be higher than what is predicted for the concordance ΛCDM model, i.e. $f(R)$ theories lead to models that accelerate less than what is observed. This result is however somewhat weakened by comparing with model independent estimates of q_0 even if these latter may be affected by systematic errors. As far as the matter content is concerned, for both classes of models Ω_M is in very good agreement with what is inferred from galaxy clusters and estimated by fitting the ΛCDM model to the available astrophysical data.

3. To ameliorate the agreement with the observed q_0, one should increase the value of n for the models with power law Lagrangians or decrease that of $\Omega_M H_0^2 \alpha^{-1}$ for models with $f(R) = \alpha \ln R$. In this case, a good fit to the data may still be obtained provided that both h and Ω_b/Ω_M are increased. While higher values of h could still be compatible with the local estimates of the Hubble constant, increasing Ω_b/Ω_M leads to lower values of Ω_M. Actually, the very good agreement among the estimated Ω_M in Eqs. (27) and (34) and the results in literature is a strong evidence against this choice. Therefore, we conclude that it is not possible to recover the same value of q_0 in the concordance model by using power law or logarithmic Lagrangians.

4. A model independent estimate of the present day age of the universe t_0 allows one to break the matter/geometry degeneracy inherent in $f(R)$ theories recovering the value of the coupling constant. For power law Lagrangians, this is indeed the only way to determine β thus offering the possibility to always recover a model that both fits the SNeIa Hubble diagram and the data on the gas mass fraction in relaxed galaxy clusters and also has the correct age. On the other hand, t_0 is an independent check for models with logarithmic Lagrangian since, in this case, it may be evaluated as a function of the two parameters $\Omega_M H_0^2 \alpha^{-1}$ and h and compared with previous results in literature. It turns out that the predicted t_0 is lower than the value estimated for the ΛCDM model and only marginally consistent with what is inferred from globular clusters and nucleochronology.

The results summarized above may pave the way to the solution of an intriguing dilemma : is Einsteinian general relativity the correct theory of gravity? If yes, then dark energy is absolutely needed to explain the accelerated expansion of the universe and hence all the theoretical efforts of cosmologists have to be dedicated to understanding its nature. On the contrary, if $f(R)$ theories are indeed able to explain the accelerated expansion, then it is time to investigate in more detail what is the right choice for the function $f(R)$ and how the variation has to be performed (higher order metric or first order Palatini approach).

From the observational point of view we have adopted here, there are no strong evidences against models with power law or logarithmic Lagrangians in the framework of the Palatini approach. On the contrary, we have seen that both classes of models successfully fit the data with values of the Hubble constant and matter content in good agreement with some model independent estimates. However, there are some hints that could lead to reject both choices for $f(R)$. Models with power law Lagrangians have a constant $q(z)$ so that they are always accelerating. This is not consistent with the (tentatively) observed transition from acceleration to deceleration at $z_T \simeq 0.5$. Moreover, a constant $q(z)$ could give rise to problems with nucleosynthesis and structure formation. On the contrary, models with a logarithmic Lagrangian are not affected by such problems and indeed they predicts a transition redshift which is in good agreement with the estimates for the ΛCDM model. On the other hand, these models turn out to be too young, i.e. t_0 is lower than what is expected.

Actually, a more general remark is in order here. Let us suppose we have found that a given choice for $f(R)$ leads to models that are in agreement with the data so that we should conclude that this class of models correctly describe the present day universe. What about the early universe? One could expect that the functional

expression of $f(R)$ is not changing during the evolution of the universe, even if R may evolve with cosmic time. If this were the case, then the correct choice for $f(R)$ should be the one that leads to models that are not only able to reproduce the phenomenology we observe today, but also give rise to an inflationary period in the early universe. Therefore, we should reject logarithmic Lagrangians since it is well known they do not predict any inflationary period. On the other hand, the choice $f(R) = \beta R^n$ is able to explain inflation provided one sets $n = 2$, not too far from our estimate in Eq. (23). From this point of view, it is worth noticing that the astrophysical data we have considered probe only the present day universe, while t_0 depends on the full evolutionary history. Indeed, logarithmic Lagrangians fail to reproduce the correct t_0 in the same way as they fail to give rise to inflation, while both inflation and t_0 are correctly predicted by models with power law $f(R)$. This may argue in favour of this choice for $f(R)$, but actually there is no reason to exclude the possibility that also the functional expression of $f(R)$ changes with time so that neither class of models may be definitively rejected or deemed as the correct one from this point of view.

Summarizing, the likelihood analysis presented here allows us to conclude that the Palatini approach to $f(R)$ theories leads to models that are able to reproduce both the SNeIa Hubble diagram and the data on the gas mass fraction in galaxy clusters. From an observational point of view, this means that both power law and logarithmic $f(R)$ are viable candidates to explain the observed accelerated expansion without the need of any kind of dark energy. However, open questions are still on the ground.

First, we have not yet been able to discriminate between the two classes of models. Theoretical considerations and some hints from the age of the universe could argue in favour of the power law $f(R)$, while the observed transition from acceleration to deceleration in the past disfavors this choice. To solve this issue, one has to resort to high redshift probes such as the CMBR anisotropy spectrum. While the data are of superb quality, the underlying theory is still to be developed so that fitting the CMBR anisotropy temperature and polarization spectra with $f(R)$ theories will be quite a demanding task.

Second, we have only considered two physically motivated and popular choices for $f(R)$. Several other models are possible and are worth of being tested against the data. In particular the $R \ln R$ Lagrangian which is related to the Straobinsky inflationary model [49] and to the limit $R^n \to R$ for $n \simeq 1$ being [50]

$$R^{1+\epsilon} = R R^\epsilon = R(e^{\epsilon \ln R}) \simeq R + \epsilon R \ln R + \mathcal{O}(\epsilon^2). \tag{35}$$

However, rather than being confused by a plethora of successful models, it is desiderable to develop a method that allows to directly reconstruct $f(R)$ from the data with as less as possible aprioristic assumptions. This will be the subject of a forthcoming paper [51].

Last but not least, whether the Palatini approach is indeed the correct method to treat $f(R)$ theories or the metric approach should be preferred is still an unsolved problem. We have shown here that the Palatini approach is not rejected by the data, but a similar analysis for the same models considered in the framework of the metric approach is still lacking. However, it is worth noticing that even this test will not be conclusive. Let us consider, for instance, two choices $f_1(R)$ and $f_2(R)$ and let us suppose that $f_1(R)$ fit the data if considered in the framework

of the metric approach, but not if the Palatini approach is used. Let us further assume that the opposite holds for $f_2(R)$. From an observational point of view, it is impossible to select between $f_1(R)$ and $f_2(R)$. Hence, observations could never suggest what is the correct way of performing the variation of a $f(R)$ Lagrangian. The answer to this question is outside the possibilities of an astronomer and lies fully in the field of a theoretician.

As a final comment, we would like to stress the need for synergy between theory and observations. While it is possible to build a physically motivated and mathematically elegant theory, it is not so easy to fit the significant amount of astrophysical data now available. Since the words *observational* and *cosmology* may today be joined together in a single meaningful term (*observational cosmology*), it is time to look at every theoretician's proposal from an observational point of view before drawing any conclusion about the validity of a whatever model. Even if not always conclusive, in our opinion, this is the only way to shed light on the dark side of the universe.

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