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LETTER

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On the general solution for a class of charged fluid spheres

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Abstract We have derived all the charged fluid spheres described by a space-time with its hypersurfaces $t=$ const. as spheroids subject to a particular form of electric field intensity. Only one out of the four solutions so obtained is reducible to its uncharged counterpart in the absence of the charge.

Keywords Charged fluids · Astrophysics · Superdense stars

1 Introduction

Recently Sharma et al. [1] have obtained a class of solutions to the Einstein–Maxwell equations for charged spheres with a particular choice of electric field intensity by considering a space-time with its hypersurfaces t = const. as spheroids [2]. In the present article the authors want to inform about the remaining three classes of solutions, which seems to have been left out in the above process. The data for the model of maximum mass M has also been provided by considering the surface density ρ_a is equal to 2×10^{14} gm cm^{-3}.

2 Basic equations and various solutions

The space-time with its hypersurfaces $t=$ constant as spheroids is given by [1, 2]

$$ds^2 = -\frac{(1+\lambda r^2/R^2)}{(1-r^2/R^2)}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2) + \psi^2(r)\, dt^2 \tag{1}$$

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and satisfies the Einstein–Maxwell equations

$$R_j^i - \frac{1}{2}R\delta_j^i = -\frac{8\pi G}{c^4}\Bigg[(c^2\rho + p)v^i v_j - p\delta_j^i + \frac{1}{4\pi}\left(-F^{in}F_{jn} + \frac{1}{4}\delta_j^i F_{mn}F^{mn}\right)\Bigg], \tag{2}$$

with all the symbols have their usual meaning. (2) together with (1) demand the following equations to be satisfied by ψ,

$$(1 - z^2)\psi_{zz} + z\psi_z + \left(1 + \lambda - \frac{2\alpha^2}{\lambda}\right)\psi = 0, \tag{3}$$

where

$$z = \sqrt{\frac{\lambda}{\lambda + 1}}\sqrt{1 - \frac{r^2}{R^2}},$$

while the electric field intensity is assumed to be [1]

$$-F_{14}F^{14} = \frac{(\alpha^2 r^2)}{R^4(1 + \lambda r^2/R^2)^2} = E^2 \quad \text{and} \quad \frac{\partial}{\partial x^k}(\sqrt{-g}F^{ik}) = -4\pi\sqrt{-g}j^i, \tag{4}$$

F_{14} being the only non-vanishing component of electromagnetic tensor F_{ij} while j^4 is the non-vanishing component of 4-current vector j^i. Differentiating (3) with respect to z we get the equations No. (12) of the reference [1] as

$$(1 - z^2)\psi_{zzz} - z\psi_{zz} + \left(2 + \lambda - \frac{2\alpha^2}{\lambda}\right)\psi_z = 0. \tag{5}$$

Now instead solving (5) by the method mentioned in [1] we set

$$z = \sin\theta \quad \text{and} \quad d\psi/dz = G, \text{ we get}$$

$$\frac{d^2G}{d\theta^2} + \left(2 + \lambda - \frac{2\alpha^2}{\lambda}\right)G = 0. \tag{6}$$

Equation (5) and ultimately (3) can possess four solutions according to the expression $2 + \lambda - \frac{2\alpha^2}{\lambda}$ is (1) negative, (2) zero, (3) positive and (4) unity.

Case (1) For $2 + \lambda - \frac{2\alpha^2}{\lambda} = -\beta^2$,

$$\psi = \frac{A}{(\beta^2 + 1)}\Big[\beta\cos\theta\sinh(\beta\theta) + \sin\theta\cosh(\beta\theta) + B\{\beta\cos\theta\cosh(\beta\theta) + \sin\theta\sinh(\beta\theta)\}\Big]. \tag{7}$$

Case (2) For $2 + \lambda - \frac{2\alpha^2}{\lambda} = 0$,

$$\psi = A(\cos\theta + \theta\sin\theta + B\sin\theta). \tag{8}$$

Case (3) For $2+\lambda-\frac{2\alpha^2}{\lambda}=\beta^2(\neq 1)$,

$$\psi=\frac{A}{(1-\beta^2)}\,[\beta\cos\theta\cos(\beta\theta+B)+\sin\theta\sin(\beta\theta+B)]. \tag{9}$$

Case (4) For $2+\lambda-\frac{2\alpha^2}{\lambda}=1$,

$$\psi=\frac{A}{2}(\theta+\sin\theta\cos\theta+B). \tag{10}$$

x	D	P	$(D-3P)$	$(dp/dr)\times 10^6$	$(d\rho/dr)\times 10^6$	$dp/d\rho$	E^2R^2	J^4R^2
Case (A1)								
$\alpha=1.2869$		$\lambda=1$	$s=.33$	Radius = 20.3260	$M/M_o=$ 8.2239			
0	6	1.3614	1.9156	0	0	.1045	0	2.804
.2	5.6528	1.3208	1.6903	$-.2118$	-1.632	.1298	.0486	2.516
.4	4.7774	1.1752	1.2519	$-.5205$	-2.522	.2064	.1632	1.828
.6	3.7180	0.8932	1.0382	$-.8595$	-2.575	.3338	.2833	1.088
.8	2.7436	0.4897	1.2785	-1.106	-2.155	.5130	.3677	.5229
1	1.9800	0.0	1.98	-1.339	-1.623	.8249	.4077	.1729
Case (A2)								
$\alpha=1.6202$		$\lambda=1.5$	$s=.25$	Radius = 19.2572	$M/M_o=$ 7.8221			
0	7.5	1.5029	2.9912	0	0	.0531	0	3.781
.2	6.9075	1.4636	2.5166	$-.2318$	-2.886	.0803	.0711	3.292
.4	5.5075	1.2970	1.6163	$-.6592$	-4.057	.1625	.2238	2.213
.6	3.9858	0.9560	1.1179	-1.085	-3.669	.2958	.3567	1.192
.8	2.7498	0.4932	1.2704	-1.272	-2.726	.4667	.4248	.5230
1	1.8570	0.0	1.8570	-1.290	-1.853	.6964	.4363	.1687
Case (A3)								
$\alpha=1.2233$		$\lambda=1$	$s=.35$	Radius = 20.4290	$M/M_o=$ 7.7808			
0	6	1.4911	1.5267	0	0	.1782	0	2.312
.2	5.6731	1.4292	1.3856	$-.3082$	-1.532	.2011	.0419	2.083
.4	4.8417	1.2347	1.1376	$-.6457$	-2.397	.2692	.1420	1.533
.6	3.8204	0.9067	1.1003	$-.9444$	-2.492	.3790	.2490	1.088
.8	2.8679	0.4802	1.4273	-1.121	-2.126	.5274	.3267	.9327
1	2.1	0.0	2.1	-1.246	-1.631	.7640	.3659	.1697
Case (A4)								
$\alpha=1.7321$		$\lambda=2$	$s=.24$	Radius = 18.9576	$M/M_o=$ 6.8211			
0	9	2.6239	1.1282	0	0	.2225	0	3.400
.2	8.2355	2.4439	.9037	$-.9352$	-3.757	.2489	.0677	2.920
.4	6.4697	1.9422	.6430	-1.650	-5.113	.3227	.2075	1.901
.6	4.6165	1.2538	.8550	-1.894	-4.455	.4251	.3203	.9971
.8	3.1624	0.5646	1.4686	-1.685	-3.207	.5254	.3701	.4428
1	2.16	0.0	2.16	-1.288	-2.134	.6034	.3706	.1644

Solar mass $M_\Theta=1.475$ km, $G=6.673\times 10^{-8}$ cm^3/gm s^2, $c=2.997\times 10^{10}$ cm/s.
Also $P=\frac{8\pi G}{c^4}\,pR^2$ and $D=\frac{8\pi G}{c^2}\,\rho R^2$.

Also the expression of density and its gradients can be written as

$$\frac{8\pi G}{c^2}\rho = \frac{\{2(\lambda+1)(3R^2+\lambda r^2) - \alpha^2 r^2\}}{2(R^2+\lambda r^2)^2}$$

$$\frac{8\pi G}{c4}\frac{d\rho}{dr} = -\frac{r\{2\lambda(\lambda+1)(5R^2+\lambda r^2) + \alpha^2(R^2-\lambda r^2)\}}{(R^2+\lambda r^2)^3}.$$

Clearly when $\beta = 0$ or positive integer ψ need not be polynomial at all as was mentioned in [1]. Moreover α (or charge) can vanish only in the case (3) as negativeness of $\frac{d\rho}{dr}$ and positiveness of ρ requires $\lambda > o$.

The authors in [1] have analysed the case (3) in details. The present authors assure that the new solutions in case (1), (2) and (4) are also of some physically use and the fact is demonstrated through the data for some specific values of the parameter λ, α and $s(=\frac{\rho_a}{\rho_0})$, $x(=\frac{r}{a})$ provided the model joins the Nordström metric at the pressure free interface $r = a$.

For $0 \leq 3p \leq c^2\rho \,\&\, dp/d\rho \leq c^2$ (strong energy conditions).

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