Surv Geophys (2009) 30:39–49
DOI 10.1007/s10712-009-9057-3

ORIGINAL PAPER

Free Wobble of the Triaxial Earth: Theory and Comparisons with International Earth Rotation Service (IERS) Data

Wei Chen · WenBin Shen · Jiancheng Han · Jin Li

Received: 1 July 2008 / Accepted: 6 January 2009 / Published online: 5 March 2009

Abstract Earth's free wobble is often referred to as the Euler wobble (for the rigid case) or the Chandler wobble for the real case. In this study, we investigate the theory of the free wobble of the triaxial Earth and demonstrate that: (1) the Euler period should actually be expressed by the complete elliptic integral of first kind, and (2) the trace of the free polar motion is elliptic, with the orientations of its semi-minor and major axes being approximately parallel to the Earth's principal axes A and B, respectively. Numerical calculations show that, due to the triaxiality of the Earth, the spin rate ω_3 fluctuates with the semi-Euler/Chandler period, although its amplitude (about 10^{-19} rad/s) is rather small and beyond the present measurement accuracy; the tilt of the instantaneous spin axis (or the amplitude of the free wobble), θ, has a fluctuation whose amplitude is around 0.34 milli-arcsecond (mas), which could be detected by present observations. Thus, we conclude that the Earth's triaxial nature has little impact on ω_3, but has an influence on the polar motion which should not be ignored. On the other hand, our study shows that there is a mechanism of frequency–amplitude modulation in the Chandler wobble which might be a candidate to explain the correlation between the amplitude and period of the Chandler wobble. We compare the theoretical polar parameters (m_1, m_2) with the observed values for the Chandler components obtained from the data EOP (IERS) C 04, and find that they coincide with each other quite well, especially for recent years. In addition, a polar wander towards 76.7°W, which is in agreement with previous results given by other scientists, is also obtained.

Keywords Triaxial Earth · Elliptic free polar motion · Variation of Chandler period · Frequency–amplitude modulation

W. Chen · W. Shen (✉) · J. Han · J. Li
Department of Geophysics, School of Geodesy and Geomatics, Wuhan University, Wuhan, China
e-mail: wbshen@sgg.whu.edu.cn

W. Shen
Key Laboratory of Geospace Environment and geodesy, Wuhan University, Wuhan 430079, China

1 Introduction

Due to the mass migration and coupling effects within the Earth as well as the interactions between the Earth and astronomical objects, both the spin rate and the orientation of the spin axis of the Earth vary on many time scales. In other words, the Earth's rotation is a process of great complexity. Theoretical studies of the Earth's rotation are based on the application of Euler's dynamic equations (e.g., Lambeck 1980). Conventionally, the Earth's equatorial principal moments of inertia A and B are assumed to be equivalent to simplify Euler's dynamic equations (e.g., Landau 1975; Lambeck 1980; Goldstein et al. 2002). However, recent measurements have demonstrated that all the Earth's principal moments of inertia, A, B and C, are different from each other (e.g., Burša and Śima 1984; Liu and Chao 1991; Marchenko and Abrikosov 2001; Groten 2004). The hypothesis that $A = B$, although simplifying the solutions to the traditional analytical ones (e.g., Lambeck 1980), makes the actual rotation state of the Earth theoretically unclear (although we can obtain it at a certain accuracy by making various kinds of observations).

Van Hoolst and Dehant (2002) pointed out that the triaxiality of the Earth (i.e. $A \neq B \neq C$) could reduce the values of the Chandler and FCN (Free Core Nutation) frequencies. Wang (2004) suggested that the triaxial nature might be responsible for the decadal polar motion. Folgueira and Souchay (2005) discussed the free polar motion of the triaxial and elastic Earth in a Hamiltonian formalism, and found that both the longitude and latitude of the pole oscillate with the semi-Chandler period. However, all of these authors applied linear approximations which will hide some important aspects of the free wobble. Shen et al. (2007) did an elementary study on the free Euler motion of a triaxial rigid Earth without making any linear approximation, and found that the triaxial nature could give rise to a small fluctuation in the length of day (LOD). The study of Shen et al. (2007) has partly conquered some shortcomings of the linear approximations adopted by the above studies.

On the other hand, long time observations show that the Chandler period, T_C, and the amplitude of Chandler wobble, θ, are time dependent and might be positively correlated with each other. Chandler (1891) first suggested the possible existence of this phenomenon. Iijima (1965) analyzed the International Latitude Service (ILS) data for the period 1900.0–1963.2 with a 0.1-year sampling, and found that the Chandler period varies from about 1.1 to 1.2 years and the smaller period happens when the Chandler component has a smaller amplitude, and vice versa. Later, Proverbio et al. (1971) confirmed the correlation between the amplitudes and periods of the Chandler motion. Carter (1981) obtained the Chandler period with a variation of 10 days within 3 years based on the analysis of polar motion data. Gao (1997) concluded that the Chandler period T_C might have a 10-day fluctuation in correlation with θ during the last several decades. Höpfner (2003) found that the Chandler wobble has a period variation between 422 and 438 days with an estimated standard deviation of only 0.48 days, while its amplitude varies from 0.15 to 0.20 mas (milli-arcsecond) with a temporal dependence similar to the period. From the traditional theory, Jochmann (2003) inferred that the mass redistribution does contribute to the variation of the Chandler period, but he found that its effect is too small to excite the period variations observed. Thus, the mechanism of the correlation between the amplitude and period of the Chandler wobble is still open to be explained.

In this study, we develop the theory of Shen et al. (2007) to obtain the rigorous solutions to Euler's dynamic equations as well as a rigorous expression for the Euler period. We find that the trace of the free polar motion is an ellipse with the following features: (1) the length of its semi-major axis is around 0.67 mas larger than that of the semi-minor axis, and (2) the orientations of its semi-major and minor axes are related to the orientations of the Earth's principal axes A and B. Further, a mechanism of frequency–amplitude modulation in the Chandler wobble is found, and the variation of the Chandler period could be partly explained.

2 The Euler Wobble of the Triaxial Rigid Earth

Setting $\alpha = (C - B)/A, \beta = (C - A)/B, \gamma = (B - A)/C$ (where A, B, C are the principal moments of inertia of the Earth), and concerning the free rotation of the rigid Earth, in the Earth's principal axial coordinate system, the Earth's angular velocity could be written as (Landau 1975; Goldstein et al. 2002; Shen et al. 2007)

$$\begin{cases} \omega_1 = \Omega a \ \mathrm{cn}(u) \\ \omega_2 = \Omega b \ \mathrm{sn}(u) \\ \omega_3 = \Omega \mathrm{dn}(u) \end{cases} \quad \text{or} \quad \begin{cases} m_1 = a \ \mathrm{cn}(u) \\ m_2 = b \ \mathrm{sn}(u) \\ m_3 = \mathrm{dn}(u) - 1 \end{cases} \tag{1}$$

where $a/b = \sqrt{\alpha/\beta}$, $\omega_i = \Omega(\delta_{3i} + m_i)$, $i = 1, 2, 3$, δ_{ij} is the Kronecker symbol ($\delta_{ij} = 1$ if $i = j$; in other cases $\delta_{ij} = 0$), Ω is the mean rotation rate of the Earth, cn, sn and dn are the Jacobian elliptic functions, and u is defined by

$$u = \frac{1}{\Omega\sqrt{\alpha\beta}} \int_0^{\varphi} \frac{d\varphi}{\sqrt{1 - m \sin^2 \varphi}} \tag{2}$$

where $m = \gamma C_{12}/\alpha C_{23}$, $\varphi = \arcsin\left[(\alpha/C_{12})^{\frac{1}{2}}\omega_2\right]$, $C_{12} = \beta\omega_1^2(0) = \beta\Omega^2 a^2$, $C_{23} = \beta\omega_3^2(0) = \beta\Omega^2$.

According to Eq. 1, the polar motion is elliptic, defined by $m_1^2/a^2 + m_2^2/b^2 = 1$. Since $a/b = \sqrt{\alpha/\beta} \approx 0.996663$, $\sqrt{ab} = m_0$, where $m_0 \approx 200$ mas is the mean amplitude of the free polar motion, one can show that $a = (\alpha/\beta)^{1/4} m_0$ and $b = (\beta/\alpha)^{1/4} m_0$.

It is noted that ω_1 and ω_2 have a common period, namely the Euler period

$$T_E = \frac{4}{\Omega\sqrt{\alpha\beta}} \int_0^{\frac{\pi}{2}} \frac{d\varphi}{\sqrt{1 - m \sin^2 \varphi}}, \quad m = \frac{\gamma}{\sqrt{\alpha\beta}} m_0^2. \tag{3}$$

Taking into account that (cn, sn) tend to (cos, sin) as m tends to zero (Abramowitz and Stegun 1965), (cn, sn) can be regarded as equivalent to (cos, sin) due to the fact that $m \approx 3.941901 \times 10^{-14}$ in the present case. Hence, Eq. 1 reduces to

$$\begin{cases} m_1 = a \cos \sigma_E t \\ m_2 = b \sin \sigma_E t \\ m_3 = \sqrt{1 - m \sin^2 \sigma_E t} - 1 = \sqrt{1 + \frac{m}{2}(\cos 2\sigma_E t - 1)} - 1 \end{cases} \tag{4}$$

where $\sigma_E = 2\pi/T_E$ is the Euler frequency (noting that T_E is rigorously defined by Eq. 3). Equation 1 as well as Eq. 4 strongly suggests that, for the real Earth with $A < B < C$, the trace of the free polar motion is no longer a circle but an ellipse, of which the length of the semi-minor axis is $\sqrt{\alpha/\beta} \approx 0.996663$ times that of the semi-major axis. The elliptic polar motion obviously leads to fluctuations of both the longitude and latitude of the pole with the semi-Euler period while the circular polar motion only leads to an Euler-period fluctuation of the longitude and a constant latitude. This result coincides well with the study of Folgueira and Souchay (2005), if it is noted that the Euler period for the rigid Earth changes to the Chandler period for the real Earth.

We note that the spin rate ω_3 fluctuates with the semi-Euler period $T_E/2$, just as illustrated by the last equation of Eq. 4. Namely, the triaxiality could give rise to a semi-Euler period fluctuation in the length of day (LOD).

Based on Eqs. 1–3 and the parameters listed in Table 1, we obtain the Euler period $T_E = 304.461118$ sidereal days, as well as m_i ($i = 1, 2, 3$), and the amplitude of free wobble, θ (see Fig. 1). One may recognize that the triaxiality could lengthen the Euler period by about 0.0017 sidereal day, or 146.606144 s (see the following text for the Euler period of the biaxial Earth), which is in agreement with the conclusion of Van Hoolst and Dehant (2002). The amplitude of m_2 is about 0.67 mas larger than that of m_1 and this difference could be detected by the Very Long Baseline Interferometry (VLBI), which can reach an accuracy better than 0.1 mas at present (McCarthy and Petit 2003). However, m_3 is in the order of 10^{-15}, which is equivalent to a variation of 10^{-10} s in the LOD, and it is 4–5 orders of magnitude smaller than the present measurement accuracy (McCarthy and Petit 2003). Hence, the variation of ω_3 is so small that it can be regarded as time independent in the present study.

If we set $A = B$, then $a = b$, and Eq. 4 reduces to the solutions for the conventional case. Obviously, $A = B$ leads to a circular free wobble of the Earth. In this case, the Euler frequency and period are defined by $\sigma_E^0 = \frac{C-A}{A}\Omega = \alpha\Omega$ and $T_E^0 = \frac{2\pi}{\sigma_E^0}$, respectively. Adopting these definitions, one can obtain that $T_E^0 = 304.459417$ sidereal days, which is a little smaller than the period corresponding to the triaxial case.

3 The Euler Wobble in the Polar Coordinate

It is worth noting that all the above investigations are in the Earth's principal axial coordinate system. Since the A axis points to $\lambda_A = 14.9291°$W (Groten 2004) and the C axis has a tilt $\theta_C \approx 0.1$–0.2 arcsecond, ω_i or m_i ($i = 1, 2, 3$) need to be transformed to the usual Earth-fixed coordinate o-xyz by the rotation matrix

$$R = \begin{bmatrix} \cos\lambda_A \cos\theta_C & \sin\lambda_A \cos\theta_C & -\sin\theta_C \\ -\sin\lambda_A & \cos\lambda_A & 0 \\ \cos\lambda_A \sin\theta_C & \sin\lambda_A \sin\theta_C & \cos\theta_C \end{bmatrix}. \tag{5}$$

However, m_i ($i = 1, 2, 3$) are hardly influenced by θ_C. Then by setting $\cos\theta_C = 1$ and $\sin\theta_C = 0$(noting that $\theta_C \approx 0.1$–0.2 arcsecond), one obtains

Table 1 Values of the relevant parameters

Parameters	Values
α	$(327{,}353 \pm 6) \times 10^{-8}$
β	$(329{,}549 \pm 6) \times 10^{-8}$
γ	$(2{,}196 \pm 6) \times 10^{-8}$
Ω	7.292115×10^{-5} rad/s
m_0	200 mas

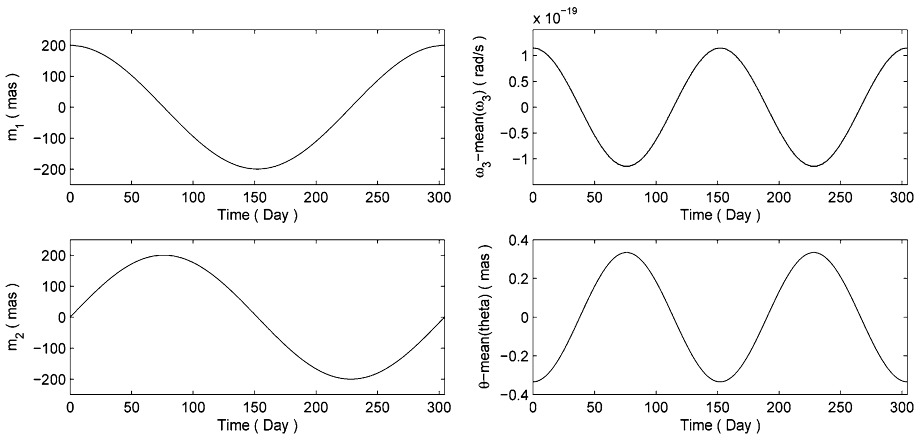

Fig. 1 Parameters of the Euler wobble in the Earth's principal axial coordinate system. Both ω_3 and θ (with mean $(\theta) = 200$ mas) fluctuate with the semi-Euler period, which coincides with the results of Folgueira and Souchay (2005) and Shen et al. (2007), respectively

$$\begin{cases} m_1^{\mathrm{E}} \approx a\cos\lambda_A \mathrm{cn}(u) + b\sin\lambda_A \mathrm{sn}(u) \approx a\cos\lambda_A\cos\sigma_E t + b\sin\lambda_A\sin\sigma_E t \\ m_2^{\mathrm{E}} \approx -a\sin\lambda_A \mathrm{cn}(u) + b\cos\lambda_A \mathrm{sn}(u) \approx -a\sin\lambda_A\cos\sigma_E t + b\cos\lambda_A\sin\sigma_E t \end{cases} \tag{6}$$

where m_1^{E} and m_2^{E} are parameters in the Earth-fixed coordinate system *o-xyz*. If the small difference between a and b is neglected, and $a = b = m_0$, one has

$$\begin{cases} m_1^{\mathrm{E}} \approx m_0\cos(\sigma_E t - \lambda_A) \\ m_2^{\mathrm{E}} \approx m_0\sin(\sigma_E t - \lambda_A) \end{cases}. \tag{7}$$

Equation 7 shows that both m_1^{E} and m_2^{E} approximately have the initial phase $\lambda_A \approx 15°$W.

Figure 2 provides us a schematic description of the free polar motion of the Earth. Traditionally, the free wobble is regarded as being circular (denoted by the dashed line). In fact, it is elliptic (denoted by the solid line). In addition, the semi-minor axis of the elliptic polar trace points approximately to 15°W.

4 The Chandler Wobble of the Triaxial Earth

For the real Earth, the Euler frequency σ_E should be replaced by the Chandler frequency σ_C, and Eq. 6 changes to (the superscript E is omitted for convenience, and it is noted that all the following discussions are in the Earth-fixed coordinate system)

$$\begin{cases} m_1 \approx a\cos\lambda_A\cos\sigma_C t + b\sin\lambda_A\sin\sigma_C t \\ m_2 \approx -a\sin\lambda_A\cos\sigma_C t + b\cos\lambda_A\sin\sigma_C t \end{cases}, \quad \frac{a}{b} = \sqrt{\frac{\alpha}{\beta}} \tag{8}$$

where (e.g., Lambeck 1980)

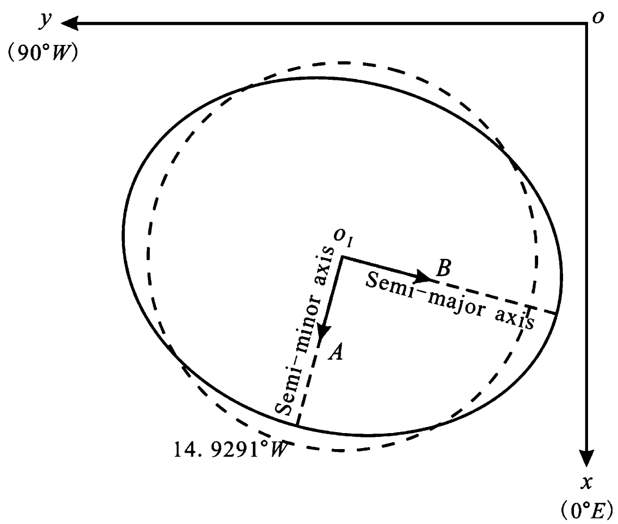

Fig. 2 The elliptic free polar motion of the Earth. The semi-minor axis of the elliptic polar trace is parallel to the principal axis A and points approximately to 15°W. On the other hand, the dashed-line circle denotes the circular free wobble of the biaxial Earth, as the traditional theory predicts

$$\sigma_C \approx \frac{k_s - k}{k_s} \sigma_E \approx 0.7\sigma_E \tag{9}$$

and $k \approx 0.30$ and $k_S \approx 0.94$ are the second order and secular Love numbers, respectively. From Eq. 8, one gets

$$\begin{bmatrix} a \\ b \end{bmatrix} = P^{-1} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}, \quad P = \begin{bmatrix} \cos\lambda_A \cos\sigma_C t & \sin\lambda_A \sin\sigma_C t \\ -\sin\lambda_A \cos\sigma_C t & \cos\lambda_A \sin\sigma_C t \end{bmatrix}. \tag{10}$$

Noting $m_0 = \sqrt{ab}$, the Chandler period reads

$$T_C = \frac{2\pi}{\sigma_C} \approx \frac{T_E}{0.7} \approx \frac{40}{7\Omega\sqrt{\alpha\beta}} \int_0^{\frac{\pi}{2}} \frac{d\varphi}{\sqrt{1 - m\sin^2\varphi}}, \quad m = \frac{\gamma}{\sqrt{\alpha\beta}} m_0^2. \tag{11}$$

Equation 11 shows that $T_C = T_C(m_0)$, and inversely, $m_0 = m_0(T_C)$ or $m_0 = m_0(\sigma_C)$.

Long time observations show that the Chandler period and the amplitude of Chandler wobble are time dependent and might be positively correlated with each other (e.g., Chandler 1891; Iijima 1965; Proverbio et al. 1971; Carter 1981; Gao 1997; Höpfner 2003). However, the mechanism of the correlation remains unexplained. It is most likely that the correlation between T_C and m_0 is due to the triaxial nature of the Earth. Examining Eqs. 8 and 11 (or Eqs. 3 and 4), one could find a frequency–amplitude modulation mechanism in the Chandler wobble if a and b are regarded as the instantaneous semi-minor and major axes, respectively, noting that $m_0(t) = \sqrt{a(t)b(t)}$ will give rise to the variation of T_C or σ_C (variations of a and b correspond to the amplitude modulation while the variation of σ_C corresponds to the frequency modulation). Since the range of the variation of m_0 is around 80–240 mas, m varies in an interval around (1–8) $\times 10^{-14}$, which ensures Eqs. 3 and 11 being increasing functions of m. Thus a theoretical model for the positive correlation between T_C and m_0 is established.

In order to verify our model, here we adopt the data EOP (IERS) C 04, which contains the Earth orientation parameters (EOP) ranging from January 1962 to August 2007 (see Fig. 3). The mentioned data provide the polar motion of the Celestial Intermediate Pole (CIP), $\tilde{p} = x - iy$. According to Gross (1992), $\tilde{p}$ and $\tilde{m}$ are connected by

$$\tilde{m} = \tilde{p} - \frac{i}{\Omega}\dot{\tilde{p}}\left(i = \sqrt{-1}\right). \tag{12}$$

Thus the complex coordinate $\tilde{m} = m_1 + im_2$ could be obtained from the data EOP (IERS) C 04 by using Eq. 12 (see the red lines in Fig. 3), and then the Chandler component of $\tilde{m}$ might be separated by the following steps:

First of all, coif5 wavelet is adopted to analyze the secular part of $\tilde{m}$. We find that it is sufficient to decompose $\tilde{m}$ to the 10th order and then adopt the 10th-order approximation as the secular wobble $\tilde{m}_S$ (denoted by the green lines in Fig. 3).

Subsequently, we must remove the annual component from $\tilde{m}$. However, as a significant component with a frequency close to the Chandler wobble, the annual wobble cannot be perfectly eliminated simply by traditional filtration methods, which usually give only 7-year-averaged values for the variations of two wobbles and no details might be presented. However, the annual wobble is rather regular compared to the Chandler one and could be modeled by $\tilde{m}_A = m_A e^{i(\sigma_A t + \chi_A)}$, where m_A ($\approx$80 mas), σ_A (=1 cycle/year) and χ_A are the amplitude, frequency and initial phase of the annual wobble, respectively. Then, together with step one, one could obtain the residual $\tilde{m}_R = \tilde{m} - \tilde{m}_S - \tilde{m}_A$.

Finally, narrow bandpass filtering (the third-order Butterworth filter is adopted) is applied to the residual $\tilde{m}_R$ to obtain the Chandler component (see the blue lines in Fig. 3). The passbands are 1/400–1/460 cycle/day and thus other components are also removed.

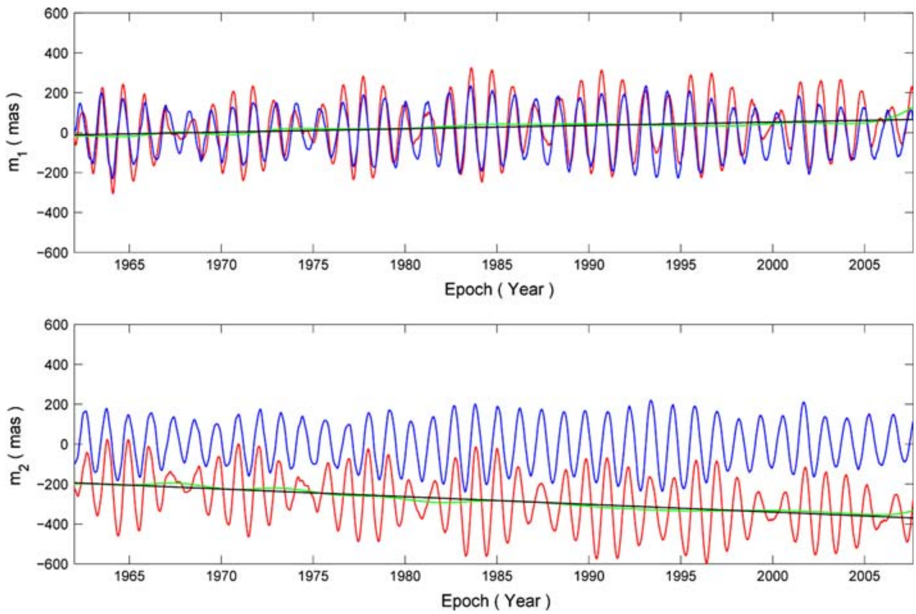

Fig. 3 The observed polar parameters as well as their Chandler and secular components. The original data are plotted with the *red lines*; the Chandler components are denoted by the *blue lines* while the secular components are denoted by the *green lines*. In addition, we have made a linear fitting with the secular components and the fitting results are described by the *black lines*

Once the Chandler component is obtained, the time-dependent values of $a(t)$, $b(t)$ and $m_0(t) = \sqrt{a(t)b(t)}$ could be obtained according to Eq. 10 (see Fig. 4, where $m_0(t)$ is denoted by θ). It is clear that m_0 has complex fluctuations while the Chandler period T_C, obtained by using m_0 according to Eq. 11, increases or decreases simultaneously. This is a key clue to the frequency–amplitude modulation of the Chandler wobble (see Fig. 4). Besides, a 160-mas variation in the Chandler amplitude m_0 could approximately give rise to a 0.04-day variation in the Chandler period T_C. This result is too small compared to the fluctuation in the Chandler period about 10 days. However, it is noted that no effects of the ocean and the an elasticity of the mantle, etc., which play significant roles in extending the Euler period to the Chandler one, are considered in our model. These factors might amplify the contribution of m_0 to the variation of T_C by several orders in magnitude to match the actual observations.

On the other hand, the secular trends of m_1 and m_2 could be fitted by two black straight lines with k_1 and k_2 as their slopes (see Fig. 3). This is the secular motion of the pole, known as the polar wander. One could conclude that the pole moves toward the direction near $\psi = 76.7°$W, determined by

$$\tan\psi = \frac{m_2^s}{m_1^s} = \frac{k_2}{k_1} \tag{13}$$

where $\tilde{m}_S = m_1^s + im_2^s$ is the secular components of $\tilde{m}$ (see Fig. 3).

Vondrák (1999) declared that the pole has a secular wander in the direction 77.1°W. Gross and Vondrák (1999) provided a similar result for the secular wander of the pole at

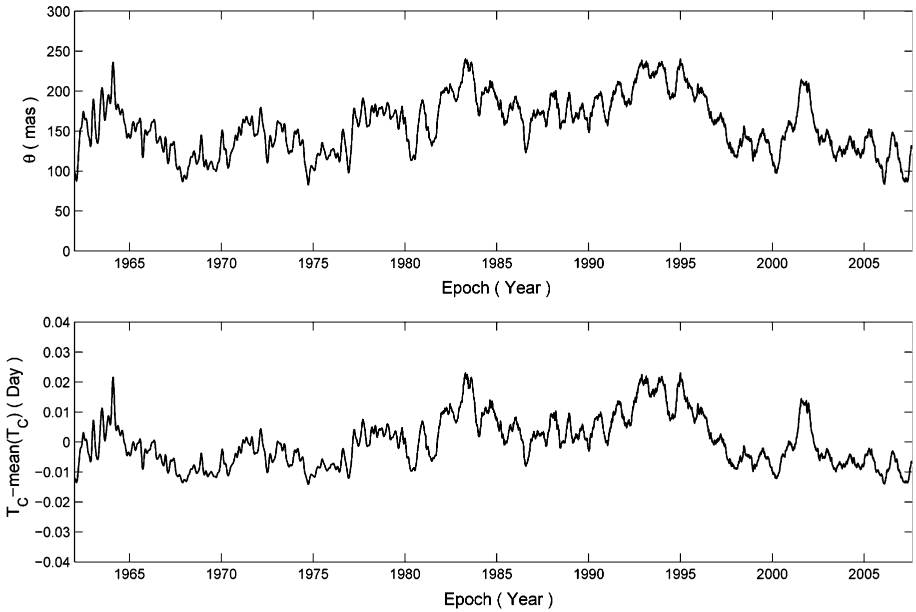

Fig. 4 The correlation between the amplitude and period of the Chandler wobble. m_0 (denoted by θ) and T_C fluctuate synchronously since the triaxiality will cause a mechanism of frequency–amplitude modulation in the free wobble. Here, mean (T_C) = 435 sidereal days. One can see only a weak fluctuation with a roughly 7-year period in the θ data, which should correspond to the beat frequency of the Chandler and annual wobble. This implies that the annual wobble is almost completely filtered

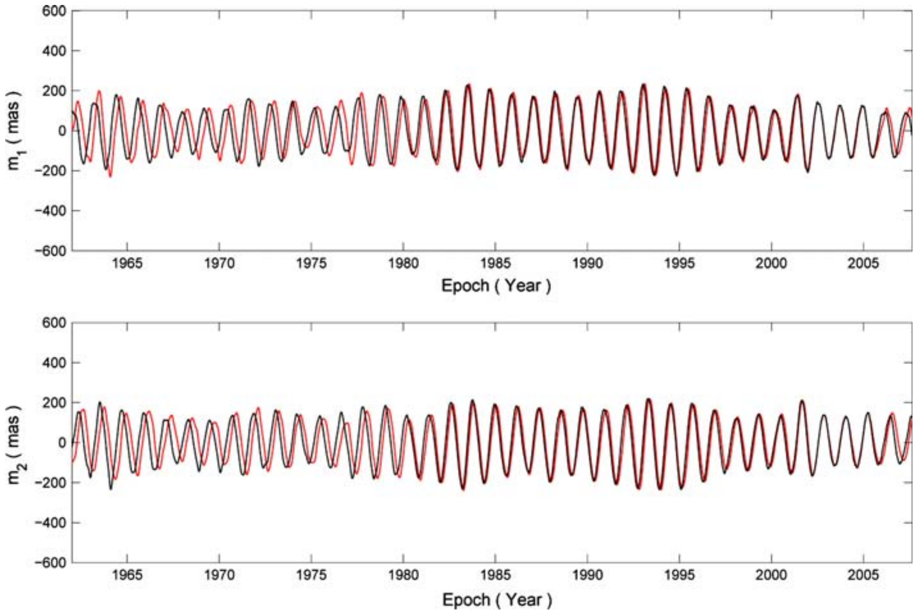

Fig. 5 Comparisons between the theoretical (denoted by *black lines*) and observed (denoted by *red lines*) polar parameters. Their phases coincide with each other quite well except for some small differences in both earlier and recent years

the speed rate 3.51 mas/year in the direction 79.2°W. Schuh et al. (2001) applied the pole series OA97 and obtained the polar wander in the direction 76.1°W. Obviously, our results are quite coincident with the earlier studies.

Further, using $a(t)$ and $b(t)$ obtained by Eq. 10, we calculate the polar motion parameters m_1 and m_2 according to Eq. 8 to test our prediction that the frequency–amplitude modulation of the Chandler wobble as well as that both m_1 and m_2 have initial phases around 15°W. The computed results (denoted by black lines) and the Chandler component of data EOP (IERS) C 04 (denoted by red lines) are compared in Fig. 5. One can see that their phases agree quite well with each other, especially in recent years. The obvious phase discrepancies in earlier years are probably due to the variation of the ratio γ/α, which reflects the relative magnitudes of A, B and C. Since A, B and C are determined from the gravity model EGM96 obtained by recent observations (see Groten 2004), it is not strange that there are discrepancies between our theoretical predictions and the observations in the earlier years. Due to a similar reason, the theoretical predictions could not completely coincide with the observations in recent years either.

5 Conclusion and Discussion

The Earth rotates in a way that depends on its shape, its internal structure, and its initial rotating state (neglecting external torques). As a consequence, the dynamic shape of the Earth (biaxial or triaxial) might play a significant role in understanding and modelling the rotation of the Earth.

Our study shows that the conventional treatment, i.e. setting $A = B$, will inevitably bring a misunderstanding (i.e. the trace of pole is circular) and a discrepancy about 0.67 mas in the free polar motion. Further, the hypothesis $A = B$ hides the mechanism of the frequency–amplitude modulation of the Chandler wobble, which might be the nature of a rotating triaxial body and has been long observed (see discussions in Sect. 1). Here we demonstrate a set of elliptic functions (with new parameters) as the solution to Euler's dynamic equations, give new expressions for the Euler and Chandler periods as well as the free motion trace of the pole, and obtain a theoretical model for the frequency–amplitude modulation of the free wobble, which might be a candidate to explain the positive correlation between the amplitude and period of the Chandler wobble. Obviously, further investigations, taking into account the viscoelasticity of the mantle, the effects of the ocean and of the Earth's core, are needed.

The triaxial nature of the Earth extends the Euler period about 0.0017 sidereal day, forces the trace of the free polar motion into an ellipse with an ellipticity about 1/298.67 ($= (b - a)/b$), leads to tiny fluctuations in ω_3 (with an amplitude about 10^{-19} rad/s in one Chandler period) and LOD (with an amplitude about 10^{-10} s in one Chandler period), and gives rise to the frequency–amplitude modulation of the Chandler wobble. Thus, presently, one should consider the Earth's triaxial nature in the polar motion (including the Chandler wobble), but neglect its impact on ω_3 or LOD. One should note that this conclusion might be only valid in the case of free wobble, and the variation of ω_3 of the triaxial Earth might be very important if the external torques are taken into account. Based on the above reasons (also see Van Hoolst and Dehant 2002; Wang 2004; Folgueira and Souchay 2005; Shen et al. 2007), one may conclude that the hypothesis $A = B$ hides many important aspects of the Earth's rotation, and therefore that the Earth's triaxiality should not be ignored due to the rapid development of the measurement technologies and the successive improvement of the observation accuracy.

Acknowledgments M.J. Rycroft and an anonymous reviewer are highly appreciated for their constructive suggestions and very valuable comments on the original manuscript. We also thank MJR, who improved the manuscript in English writing. This study is funded in parts by Special Project Fund of State Key Laboratory of Information Engineering in Surveying, Mapping and Remote Sensing (China); National Natural Science Foundation of China (Grant No. 40574004, 40637034), and National 863 Project of China (Grant No. 2006AA12Z211).

References

Abramowitz M, Stegun IA (1965) Handbook of mathematical functions, with formulas, graphs, and mathematical tables. Dover Publications, New York

Burša M, Śima Z (1984) Equatorial flattening and principal moments of inertia of the Earth. Studia Geoph Geod 28:9–10

Carter WE (1981) Frequency modulation of the Chandlerian component of polar motion. J Geophys Res 86:1653–1658

Chandler SC (1891) On the variation of the latitude. Astron J 11:83–86

Folgueira M, Souchay J (2005) Free polar motion of a triaxial and elastic body in Hamiltonian formalism: application to the Earth and Mars. Astron Astrophys 432:1101–1113

Gao BX (1997) Principles of astro-geodynamics. Scientific and technological Press of China, Beijing

Goldstein H, Poole C, Safko J (2002) Classical mechanics, 3rd edn. Addison-Wesley Press, Massachusetts

Gross RS (1992) Correspondence between theory and observations of polar motion. Geophys J Int 109:162–170

Gross RS, Vondrák J (1999) Astrometric and space-geodetic observations of polar wander. Geophys Res Lett 26:2085–2088

Groten E (2004) Fundamental parameters and current (2004) best estimates of the parameters of common relevance to astronomy, geodesy, and geodynamics. J Geodesy 77:724–797

Höpfner J (2003) Chandler and annual wobbles based on space-geodetic measurements. J Geodyn 36:369–381

Iijima S (1965) On the yearly trend of the periodic components of polar motion. Ann Tokyo Astron Obs 9:155–181

Jochmann H (2003) Period variations of the Chandler wobble. J Geodesy 77:454–458

Lambeck K (1980) The Earth's variable rotation: geophysical causes and consequences. Cambridge University Press, Cambridge

Landau L (1975) Mechanics. Nauka, Moscow

Liu HS, Chao BF (1991) The Earth's equatorial principal axes and moments of inertia. J Geophys Res 106:699–702

Marchenko AN, Abrikosov OA (2001) Evolution of the Earth's principal axes and moments of inertia: the canonical from of solution. J Geodesy 74:655–669

McCarthy DD, Petit G (eds) (2003) IERS conventions (2003). IERS technical notes 32, Observatoire de Paris

Proverbio E, Carta F, Mazzoleni F (1971) Analysis of the Chandler period of polar coordinates calculated with Orlov's method, Pubblicazioni Della Stazione Astronomica Internazionale Di Latitudine Carloforte-Cagliari. Nuova Serie N. 18

Schuh H, Nagel S, Seitz T (2001) Linear drift and periodic variations observed in long time series in polar motion. J Geodesy 74:701–710

Shen WB, Chen W, Wang WJ, Liang Y (2007) Rotation of the Earth as a triaxial rigid body. Geospat Inf Sci 10:85–90

Van Hoolst T, Dehant V (2002) Influence of triaxiality and second-order terms in flattenings on the rotation of terrestrial planets I. Formalism and rotational normal modes. Phys Earth Planet Inter 134:17–33

Vondrák J (1999) Earth rotation parameters 1899.7–1992.0 after reanalysis within the HIPPARCOS frame. Surv Geophys 20:169–195

Wang WJ (2004) Decadal free polar motion of triaxial Earth. Geophys J Int 158:1–15