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Controlled Line Smoothing by Snakes

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Abstract

A major focus of research in recent years has been the development of algorithms for automated line smoothing. However, combination of the algorithms with other generalization operators is a challenging problem. In this research a key aim was to extend a snakes optimization approach, allowing displacement of lines, to also be used for line smoothing. Furthermore, automated selection of control parameters is important for fully automated solutions. An existing approach based on line segmentation was used to control the selection of smoothing parameters dependent on object characteristics. Additionally a new typification routine is presented, which uses the same preprocessed analysis for the segmentation of lines to find suitable candidates from curve bends. The typification is realized by deleting undersized bends and emphasizing the remaining curve bends. The main results of this research are two new algorithms for line generalization, where the importance of the line smoothing algorithm lies in the usage of a optimization approach which can also be used for line displacement.

Keywords: automated generalization, smoothing, typification, optimization, energy minimization, snakes

1. Introduction

Line generalization applies to different types of cartographic objects, such as angular lines (e.g., canals, buildings), sinuous lines (e.g., hydrography), as well as the depiction of 2.5-D continua (e.g., relief) by contours. Line generalization, just like any other generalization operation, has to observe and preserve the particular characteristics of cartographic objects in the generalization process. Hence, different operators are available for the overall task of line simplification. Angular lines are dealt with by line *simplification* operators that often rely on a procedure that reduces the number of original points on the line. Line *smoothing*, on the other hand, is often used for one-dimensional generalization of sinuous lines by 'ironing' away small crenulations [17].

The approach presented here focuses on the generalization of sinuous lines. As Figure 1 shows, an approach that was purely based on simplification (i.e., weeding) and smoothing operators, respectively, would not be sufficient, for two major reasons. First, the transitions between angular and sinuous parts of a line are often not distinct. For instance, a border line might contain both angular and sinuous sections, depending on whether it follows survey markers or the centerline of a river. Second, mere point weeding or smoothing simply focuses on single vertices of a line, rather than identifying

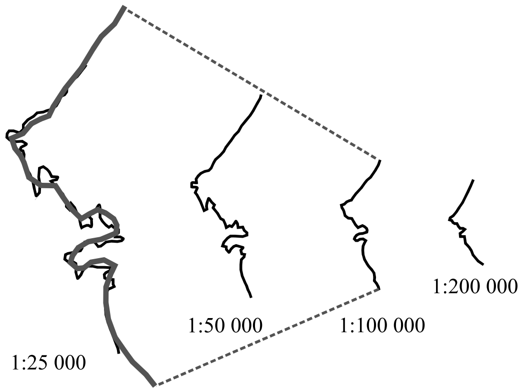


Figure 1. Levels of detail for a line object at different scales, after Töpfer [29].

compound shapes on the line (e.g., bends) and generalizing these shapes [19]. Plazanet et al. [22] have presented several algorithms to deal with shape-based generalization of roads. In this article, a method is proposed that is based on an energy minimizing optimization technique, called snakes, that allows to apply controlled line smoothing, while at the same time taking into account the overall shape of the line as well as allowing to integrate different generalization operations.

Because the generalization process follows as a combination of different basic generalization operations, it is always a compromise. In this sense the cartographic solution is generally not unique, but satisfies the different cartographic requirements in a better or worse way. As shown earlier, the use of optimization techniques seems to be suitable for both the combination of different basic generalization operations as well as the control of varying constraints of one generalization operation [7].

The use of optimization techniques in the field of automated generalization has been proposed by a number of authors (e.g., [5]; Højholt 1998). Such techniques are primarily applied to the displacement of line objects by means of different approaches—*energy minimization methods* such as snakes [4], [7] and beams [2] and *least squares adjustment* [13], [27]. Sester shows also how to apply optimization techniques for other generalization operations, such as simplification of buildings.

Harrie [12] extended the least square adjustment approach by inclusion of additional constraint types. The aim of his *simultaneous graphic generalization* was to have one method for solving different generalization operations in a single optimization step. The new constraint types were concerned with simplification, smoothing and exaggeration. These constraints were based upon pre-computed point movements. For smoothing, he used a Gaussian smoothing approach.

Even though different optimization techniques from several disciplines have been adapted for tasks of cartographic generalization, the resulting linear vector equations after variation or Taylor expansion are all quite similar. Following that, similar methods are used for solving such equations and existing difficulties also are comparable. On the one hand, there is the computational effort required for solving the large equation systems. On the other hand, there are the difficulties to find suitable weights and parameters. Harrie [12] investigates different strategies (empiricism, machine learning, constraint violation and variance component estimation) with one important restriction: weights have to be determined independently of the shape of objects. This assumption is acceptable only for certain types of constraints, for others, such as curvature constraints, it is important to have different weights for objects of the same type, depending on their shape.

In this paper, a line smoothing approach that is fully integrated with the energy minimization method is first presented. Following that, it is shown how the analysis of shape can help in selecting suitable parameter values for smoothing. An automated controlled line smoothing algorithm is then derived. The same preprocessed analysis is also used to find suitable candidates for typification of curve bends. To improve the recognition of generalized lines a typification procedure with a geometrical basis is suggested.

2. Energy minimization for smoothing of line objects

The snakes optimization technique used here allows the consideration of different, partly contradictory, generalization constraints. In automated displacement such constraints are the maintenance of minimal distances between objects with correctly represented relative positions and the preservation of typical shapes. Snakes can model these constraints with the help of an energy function consisting of internal and external energies. The internal energy is used to describe the cartographic object's shape and structure. Conflicts, such as distances to other objects that are too small, are calculated by the external energy. In this paper, the snakes model is extended from that originally developed for cartographic object displacement by Burghardt and Meier [7] by smoothing the shape of line objects, in addition to the internal energy. In this case energy correlates to the degree of detail of the line: the smoother the line the less energy it contains. In combination with automated displacement it would be possible to have one common solution for the generalization of line objects their selection and symbolization. The control of optimization is intuitive, because the external energy refers to other objects, whereas the internal energy describes the line itself.

In this paper the snakes approach will be used in the original form, proposed by Kass et al. [15]. Snakes are energy minimizing splines which adapt their shape and position under the influence of an energy functional. Representing the position of snake parametrically by $\mathbf{d}(s) = (x(s), y(s))^T$ with arc length s, $s \in [0,l]$, the energy functional can be written as

$$E(\mathbf{d}) := \int \left(\mathbf{E}^{int} + \mathbf{E}^{ext}\right) \cdot ds \qquad (1)$$

$$= \int \left(\frac{1}{2}\left(\alpha(s) \cdot |\mathbf{d}'(s)|^2 + \beta(s) \cdot |\mathbf{d}''(s)|^2\right) + \mathbf{E}^{ext}\right) \cdot ds$$

The snake represents a line l which has to be generalized and the energies are used to model the constraints for generalization. While internal energy E^{int} has an influence on the shape of the line, the E^{ext} describes conflicts with objects of the neighborhood. Such conflicts could arise as a result of symbolization, when distances between signatures fall below a minimal distance threshold or overlap each other. There are different requirements for line shape preservation during generalization operations. While displacement should not change too much the shape of the line, smoothing operations may intentionally modify the shape of lines for better visualization quality. With internal energy these different aspects can both be taken into consideration.

For displacement internal energy calculates differences in shape between the original and the displaced line, so minimal internal energy implies minimal deformation of the line. The shape modification is measured by changes of the differences of the first and second derivatives of the original and the displaced line. In case of smoothing internal energy is used to simplify the line. Therefore, first and second derivatives of the line are minimized, resulting in shorter distances between points of vector **d** and minimized curvature of the spline curve. The internal energy terms used for displacement and smoothing, respectively, are formally equal. The differences depend on the definition of vector d, which contains the coordinates of the altered line in the case of smoothing (2a). For displacement (2b), the differences between the initial and the derived line are used.

$$\mathbf{d}_{smooth}(s) = (x(s), y(s))^T \tag{2a}$$

$$\mathbf{d}_{displ}(s) = \left(x(s) - x^0(s), y(s) - y^0(s)\right)^T \tag{2b}$$

To find the stable state of the snake the functional $E(\mathbf{d})$ has to be minimized (see Appendix). The variation of $E(\mathbf{d})$ with constant user-definable parameters α and β leads to the Eulerian equations, which are solved by discretization in time [2], [6]. The differences in the final formulae between displacement (3a,b) and smoothing (4a,b) are straightforward. For smoothing of line objects no external forces are modeled, so the derivatives of external energy ($\mathbf{E}^{ext}$) in x- and y-direction are zero.

$$(\mathbf{A} + \lambda\mathbf{I}) \cdot \left(\mathbf{x}^t - \mathbf{x}^0\right) = \lambda\left(\mathbf{x}^{t-1} - \mathbf{x}^0\right) - \mathbf{E}_x^{ext}\left(\mathbf{x}^{t-1}, \mathbf{y}^{t-1}\right) \tag{3a}$$

$$(\mathbf{A} + \lambda\mathbf{I}) \cdot \left(\mathbf{y}^t - \mathbf{y}^0\right) = \lambda\left(\mathbf{y}^{t-1} - \mathbf{y}^0\right) - \mathbf{E}_y^{ext}\left(\mathbf{x}^{t-1}, \mathbf{y}^{t-1}\right) \tag{3b}$$

$$(\mathbf{A} + \lambda\mathbf{I}) \cdot \mathbf{x}^t = \lambda\mathbf{x}^{t-1} \tag{4a}$$

$$(\mathbf{A} + \lambda\mathbf{I}) \cdot \mathbf{y}^t = \lambda\mathbf{y}^{t-1} \tag{4b}$$

$$\mathbf{A} = \begin{bmatrix} 2\alpha + 6\beta & -\alpha - 4\beta & \beta & 0 & 0 & 0 \\ -\alpha - 4\beta & 2\alpha + 6\beta & -\alpha - 4\beta & \beta & 0 & 0 \\ \beta & -\alpha - 4\beta & 2\alpha + 6\beta & -\alpha - 4\beta & \beta & 0 \\ 0 & \beta & -\alpha - 4\beta & 2\alpha + 6\beta & -\alpha - 4\beta & \beta \\ 0 & 0 & \beta & -\alpha - 4\beta & 2\alpha + 6\beta & -\alpha - 4\beta \\ 0 & 0 & 0 & \beta & -\alpha - 4\beta & 2\alpha - 6\beta \end{bmatrix} \tag{5}$$

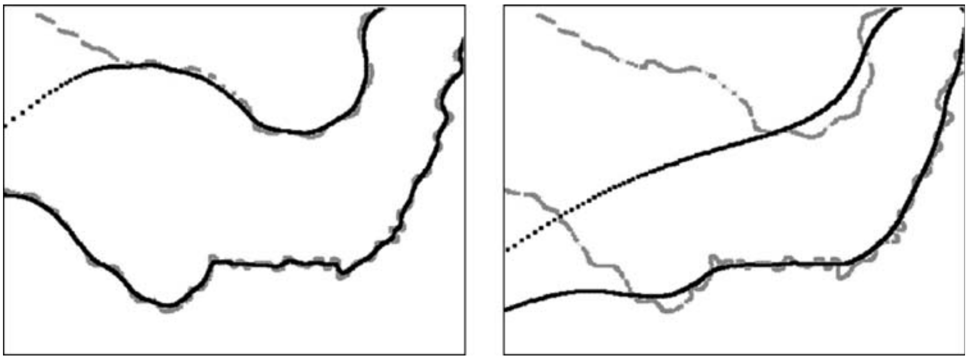


Figure 2. Excessive shifting of end points of line objects depending on the degree of smoothing.

3. Automated control of parameter selection

Depending on the degree of smoothing and the distance between points, the end points of the lines are shifted away from their initial position (see Figure 2). Higher values of α produce a stronger minimization of the first term of the internal energy and hence a stronger smoothing and end point shifting (cf. Equation 1).

To overcome the problem of end point shift there are several counter-measures. One possibility is to include the first and last point of the line multiple times in vector **d**. The points which are added before will be deleted after smoothing. This procedure results in forcing the smoothed line through the end points (see Figure 3). As a result of internal energy also the curvature at both ends of line are influced. The line becomes quite straight at their ends because of multiple adding of the same point.

That's why a second version of extending the original line was investigated. The idea is to duplicate the segments at either end of the line rotated by 180°. Instead of using the

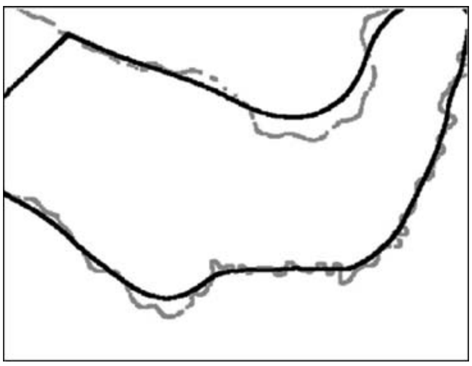


Figure 3. Forced fixed boundary points as a result of introducing multiple end points.

Figure 4. Forced fixed boundary points as a result of using a number of duplicated points at the line end rotated by 180° around the crossroads (left). Resulting lines after smoothing (right).

last point multiple times, now some more points at the line end used twice. Thus, the character of the lines is better represented (Figure 4).

An important question for any automated solution is the number of additional duplicated points required for bounding the solution. One possible approach which was used here consists in smoothing the line without additional end points and subsequently analyzing the shifted points. After counting the points for which the distance between the original and smoothed line falls below a threshold value the remaining points up to the end of the line then determine the number of additional end points. Figure 5 shows the distance between the original and the smoothed line for every point on a sample line.

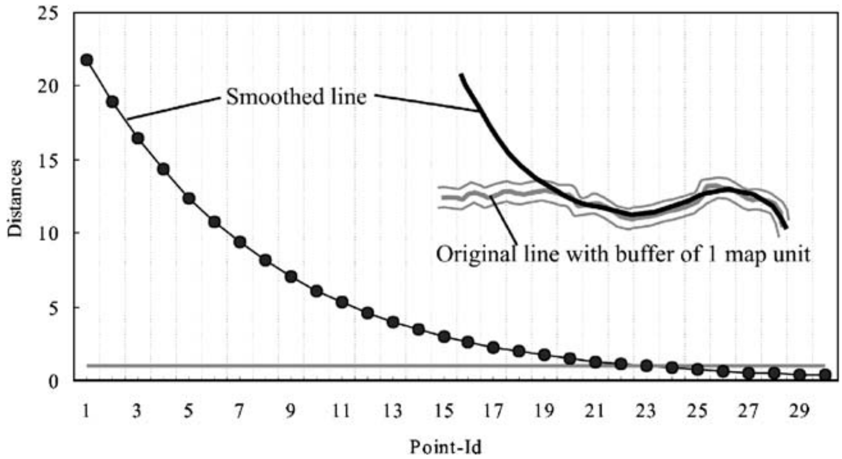


Figure 5. Calculation of number of duplicated points depending on distances between original and smoothed line to achieve fixed boundaries.

In the diagram the distance for the point with Id 24 falls below the threshold value 1.0. That means 23 vertices up to the end of the line were shifted too far. This value will then be increased by 20%, to make sure that the shift affects only the additional points. So, it is recomputed with 23 plus 5 additional end point coordinates. Note that only a fraction of the original line is displayed in the diagram of Figure 5; most of the points stay within the threshold distance, as can be seen from the buffer display in the top part of Figure 5.

Before smoothing the line objects a segmentation is necessary to determine the smoothing parameters α and β of $\mathbf{E}^{int}$. With the help of the segmentation the lines can be subdivided into smaller segments of different sinuosity. One possibility for segmentation is to smooth the original lines twice, first to determine the characteristics and second with adapted parameter values. The intersection points between the original and the smoothed line correlate with the degree of sinuosity of the original curve. In Figure 6(left) the arrows show the intersection points between original and smoothed line. A measure is obtained by counting the intersection points with reference to segment length (Figure 6(right)). In case the distance between two points of intersection is less than a given threshold value the segment is defined as being sinuous. Note that Plazanet et al. [22] proposed a similar approach based on the detection of vertices and the subsequent analysis of the distances between these points to segment the original line. The advantage of the snakes smoothing approach is that it can stay within the same methodological context of energy minimizing optimization techniques. Hence, it does not have to 'piece together' different methodological approaches into one framework (see also [28]).

Because the sinuousity attribute can change quite often between segments, the next step is to concatenate segments until a user-defined minimal length of line parts is reached. It is necessary to start the procedure with the shortest segments (Figure 7b). If there are adjacent segments with different sinuosity, the longer one determines the value of the sinuosity attribute (Figure 7c). At the last step in Figure 7d the scattered line part keeps the attribute "not sinuous" if the length is longer then the minimal length of line

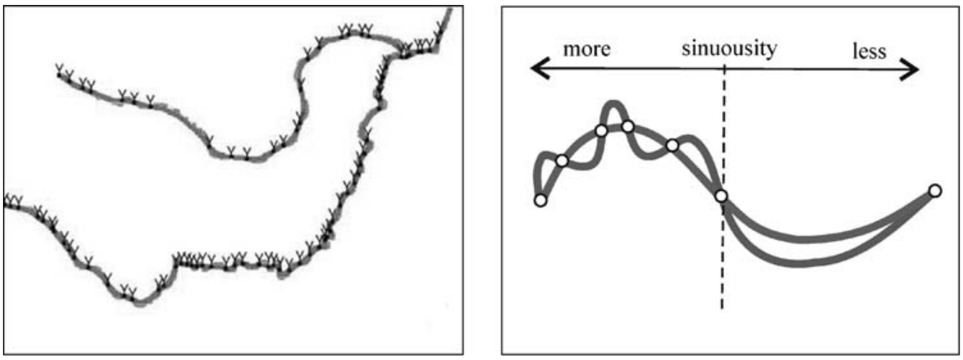


Figure 6. Segmentation based on intersection points between original and smoothed line, arrows show the intersection points (left). The number of intersection points related to segment length is used as measure of sinuosity (right).

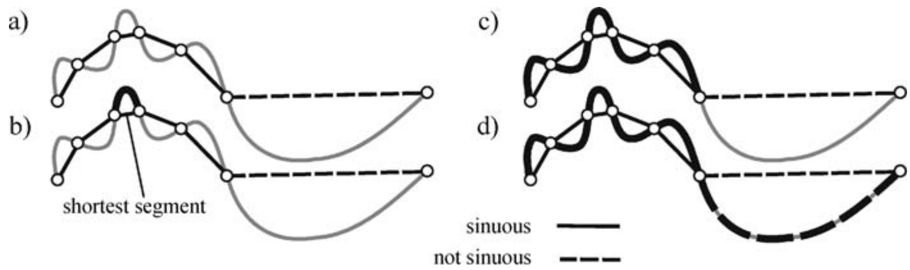


Figure 7. Concatenation of segments with different sinuousity attribute. **a** Determination of intersection points between the original and the smoothed line. **b** Identification of the shortest segment. **c** Concatenation of segments until a user-defined minimal length of line parts is reached. **d** Final result of a segmented line in sinuous and not sinuous line parts.

parts defined by the user. In cases where the minimal length is not reached, the scattered line parts take the attribute "sinuous," because the concatenated segments with attribute "sinuous" are longer.

After segmentation detection the lines are subdivided into the segments and each of them is smoothed with different parameter values for α and β of the internal energy. Figure 8 shows an example in which different parameter values were applied to the segmented lines. Hence, lines which are sinuous were smoothed more to eliminate the high frequency bends. The less sinuous lines, on the other hand, were less strongly smoothed and the large bends were preserved. However, from a cartographic point of view, this approach can be improved. In order to maintain the more sinuous parts it is necessary to apply typification rules which are described in the next section.

The presented algorithm is integrated in a cartographic production system. The system is used by cartographic experts to create high quality topographic maps. Through the

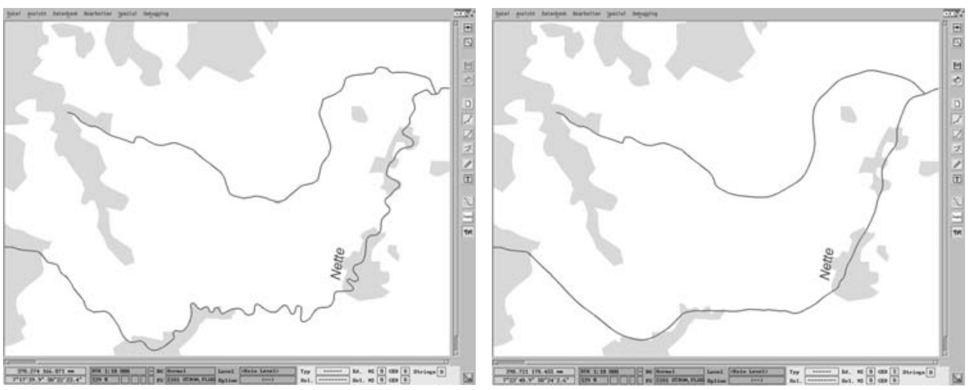


Figure 8. An example of the use of snakes for line smoothing. Smoothing was applied after line segmentation.

Table 1. Methodical overview of smoothing approaches.

	Spatial domain	Frequency domain
Local	· Epsilon filtering—Perkal [20]	Wavelets
	· Gaussian smoothing—Badaud et al. [1]	· Fritsch and Lagrange [11]
	· Sliding average—McMaster [16]	· Balboa and Lopez [3]
	· Plaster—Fritsch [10]	· Saux [24]
Global	Raster:	Fourier series
	· Morphological operations—Schweinfurth [26]	· Clarke et al. [8]
	Vector:	· Schwarzbach [25]
	· Snakes (as presented in this paper)	· Fritsch and Lagrange [11]

extension of automated parameterization the line smoothing could be carried out also by non-experts, nevertheless the operator would need to decide which classes the smoothing should be applied to and which other generalization operators were needed. Runtime for line smoothing with snakes is much faster than the line displacement, particularly after subdividing the line in several segments. A disadvantage of the presented approach might be the effort required to implement the matrix equations (2a,b), which can be justified, if a line displacement is also carried out, with the same optimization approach.

The differences between snakes smoothing and other smoothing approaches can be shown on a methodical level (Table 1). One classification is made by the filter theory, which uses transformations between spatial and frequency domain. Smoothing after transforming into the frequency domain is realized by frequency filters, e.g., low-pass filters which allows low frequencies to pass. Smoothing within the spatial domain can also be interpreted as filter operations, applied on coordinates instead of frequencies. Detailed analyses of filter characterization for snakes is published by Meier [18]. A second way of classification is by distinguishing between local and global effects of the smoothing algorithms. In the frequency domain algorithms using a Fourier series approach influence the whole line, while Wavelets have a localising component based on their restricted basis functions. Fritsch and Lagrange [11] have shown that wavelet coefficients are appropriate to characterise the local shape of a curve.

In the spatial domain several smoothing algorithms suggested, which have a local focus. For instance the smoothing approach suggested by McMaster [16] considers two or four surrounding points and calculates a straight arithmetic average. In a second step, the actual point is displaced towards the calculated coordinates. Until now smoothing algorithm in the spatial domain with a global focus have only been available for raster data, e.g. the morphological operations [26]. Snakes smoothing fills this gap with their global matrix calculation. The advantage of a global approach is it better preserves the main characteristic of a line, while local adjustment is missing. To obtain a compromise between global and local approaches the automated line segmentation was suggested for controlling the selection of smoothing parameters dependent on object characteristics. While the lines are subdivided into segments of comparable characteristics the approach shifts its focus from a global to a local perspective.

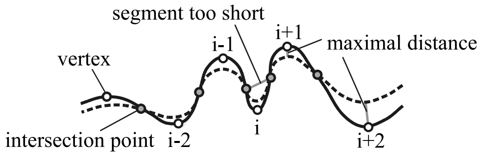


Figure 9. Calculation of vertices (white) and intersection points (gray).

4. Typification

The aim of typification is to visualize line characteristics even though there are limitations on resolution. For strong sinuous parts this could be achieved by deleting undersized bends and emphasizing the retained or reconstructed ones. To find small bends smoothing is applied the same way as in the first step for segmentation (cf. preceding section). Short segments between intersection points of original and smoothed line indicate undersized bends. For typification the intersection points and vertices of the original line have to be calculated. There is one vertex (white) for every bend and every bend is delimited by two intersection points (gray), see Figure 9. The vertex of one bend is the point with the maximal distance between the original (black) and the smoothed line (dashed).

Starting with the vertex (i) of the undersized bend, all line vertices between the previous (i − 1) and following vertex (i + 1) of the original line are deleted (Figure 10a). The vertices of the adjacent bend sides, between vertex i − 1 and i − 2 as well as i + 1 and i + 2, respectively, are moved for the construction of the new, typified bend. Its direction and length are calculated from the connection of intersection points between the original and the smoothed line. The translation leads in the direction of the deleted bend with a value of half the intersection point distance. The dotted gray lines in Figure 10b show the intermediate step.

An exaggeration of the newly constructed bends is also possible with a distance dependent stretching of line vertices perpendicular to the connecting segment between intersection points. The solid black lines in Figure 10b show the exaggerated bends. Finally, the original line with its constructed new bends is smoothed by energy minimization. The second term of the internal energy guarantees the continuity of the smoothed line. Also, the distance between vertices of the line becomes approximately equidistant as a result of the first term of internal energy.

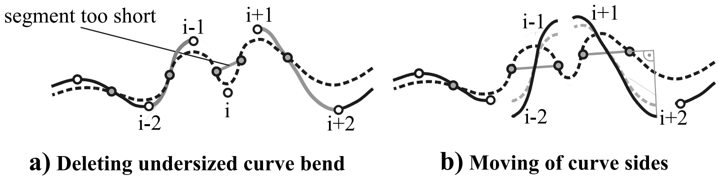


Figure 10. Construction of new curve bends through translation and exaggeration.

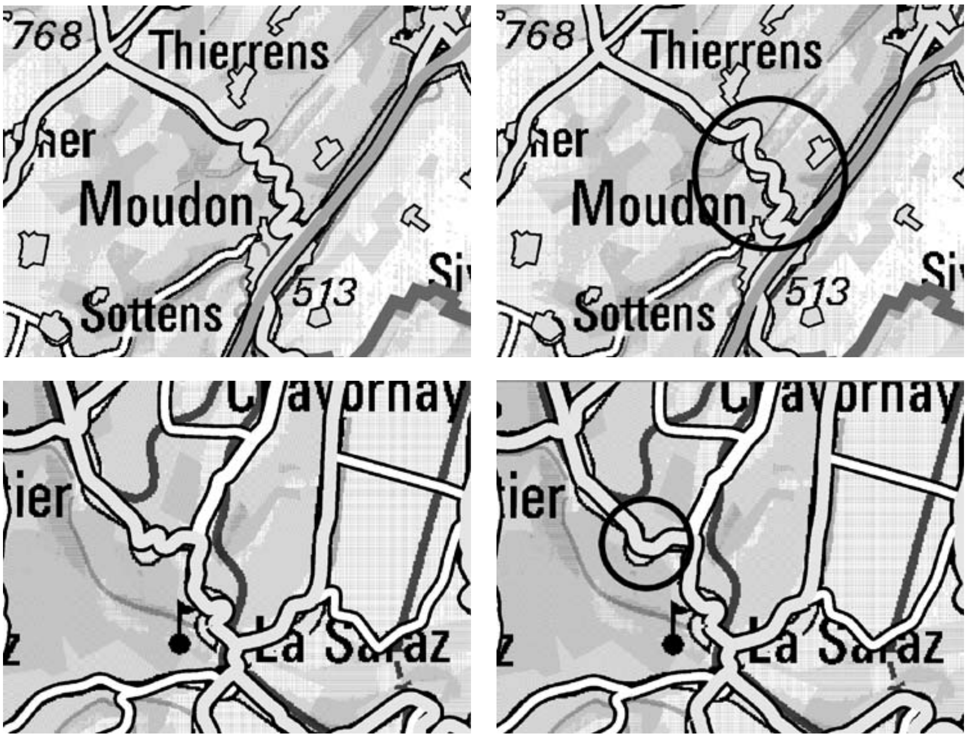


Figure 11. Typification examples of roads (to compare the manual generalized topographic maps are shown in the background—© 2004 swisstopo (BA046257)).

Figure 11 shows two examples of typification of line objects from VECTOR25[1] road network. The road network of VECTOR25 was digitized on the basis of topographic maps with scale 1:25,000. The aim for this example was to generalize the road network for a smaller scale 1:500,000. In the background of road objects the corresponding manual generlized topographic map with scale 1:500,000 is shown. On the left side you can see the situation before on the ride side after typification.

The circles bring out the results of typification of object class “Main road,” which are in the second example very similar to the manual solution. More examples are shown in Figure 12 for another object class “Road open to traffic” of road network. It can be seen that not all sinuous line segments would be typified (double encircled curve sequence). The main reason is that calculation of vertices and intersection points is dependent on preprocessed smoothing. In this example the preprocessed smoothing was stronger, so in the double encircled area no intersection points are calculated between original and smoothed line.

To overcome this problem a strategy could be used which implies a frequency dependent typification. If preprocessed smoothing is not so strong, only smaller bend sequences are typified (Figure 13b), if a stronger preprocessed smoothing is used the

Figure 12. Typification and smoothing of road network.

longer bends (relating to long wavelength) will be typified (Figure 13c). In general an iterative typification with different amounts of preprocessed smoothing could be applied.

An alternative to this typification approach is the Accordion algorithm suggested by Plazanet [21]. This algorithm aims to enlarge a bend or bend series to remove the bends that coalesce. The central inflexion point of the line has to be fixed and all the others points are moved away from it, specific to every bend, in the orthogonal direction of each bend axis. The main difficulty of this approach is to avoid creating new conflicts when solving the initial ones. Further research [9], [23] has been introduced micro and meso Agents to overcome this side effects.

Compared with the Accordion algorithm, the typification presented her approach has the advantage that no side effects were produced, because bends will be removed instead

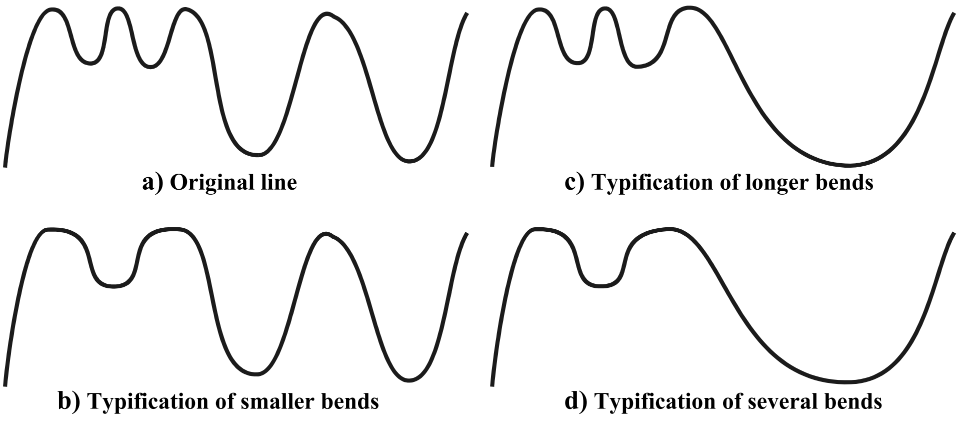


Figure 13. Frequency dependent typification with different amounts of preprocessed smoothing.

of displaced. A negative consequence is that less information will be visualized in the map. In this sense the typification approach suggested here can be used as a complementary procedure to the Accordion algorithm. If the local situation allows visualizing of all curve bends, the Accordion algorithm makes sure that the bends do not overlap and if to much side effect occurs then our typification approach can be used to reduce the number of curve bends.

5. Conclusions

The results of the research presented here are two new algorithms for line smoothing and typification. Advantages of line smoothing with snakes are discussed from a methodical and a practical point of view. Snakes smoothing fills the gap of smoothing operators with global focus in the spatial domain. The advantage of a global approach is the better preservation of the main characteristic of a line. From a practical point of view the snakes approach uses an optimization approach which can also be used for line displacement. The advantage of the snakes model is the simple combination of several constraints with the help of different energies. External energy describes conflicts with other map objects, while internal energy models the shape constraints of cartographic lines. The main difference of using snakes for displacement or smoothing, respectively, depends on the definition of the internal energy. While the shapes of lines should be preserved during displacement, smoothing implies more considerable deformations.

To improve the results of smoothing, the lines can be smoothed twice. A first pass is executed with default parameters for α and β, independently of line characteristics. The resulting line crosses the original line and the density of intersection points between the two lines provides a measure of sinuosity. A subsequent segmentation helps to determine parts of the line with similar characteristics. For the second smoothing pass the parameters α and β can then be selected in relation to the line shape established previously. For instance, lines which are more sinuous can be smoothed more strongly to eliminate high frequency bends. The reason for using one common approach for smoothing and displacement is the easier control of the interaction between displacement and smoothing. Additionally, it is faster to apply smoothing and displacement together then one after another. Further work should investigate smoothing with position dependent parameter $\alpha = \alpha(s)$ and $\beta = \beta(s)$, to achieve a more local control of smoothing.

Additionally, a typification routine with a geometrical basis is suggested. Comparing with other algorithm the approach has the advantage that no side effects were produced. It is well suited for greater scale transitions and can be used as supplementation for Accordion algorithm. In general the algorithm works well, but depending on the degree of the preprocessed smoothing different curve bends for typification are selected. The consequence is that not all curve bends are typified in one step and an iterative process has to be applied, which leads to a frequency dependent typification.

Appendix

To find a minimum of the following energy integral

$$I[x(s), y(s)] = \int_0^1 E^{ges}(x, x_s, x_{ss}, y, y_s, y_{ss})ds = \int_0^1 \left(E^{ext} + E^{int}\right)ds \tag{6}$$

the variation in x- and y-direction should be zero

$$\delta I[x + \delta x, y] = 0 \tag{7a}$$

$$\delta I[x, y + \delta y] = 0. \tag{7b}$$

The variation in x-direction

$$\delta I[x + \delta x, y] = \int_0^1 ds \cdot \delta E(x, x_s, x_{ss}, y, y_s, y_{ss}) \tag{8a}$$

$$= \int_0^1 ds \cdot \left(\frac{\partial E}{\partial x}\delta x + \frac{\partial E}{\partial x_s}\delta x_s + \frac{\partial E}{\partial x_{ss}}\delta x_{ss}\right) \tag{8b}$$

$$= \int_0^1 ds \cdot (E_x \delta x + E_{x_s}\delta x_s + E_{x_{ss}}\delta x_{ss}) \tag{8c}$$

$$= \int_0^1 ds \cdot \left(E_x \delta x + E_{x_s}\frac{d}{ds}\delta x + E_{x_{ss}}\frac{d^2}{ds^2}\delta x\right) \tag{8d}$$

$$= \int_0^1 ds \cdot \left(E_x - \frac{dE_{x_s}}{ds} + \frac{d^2 E_{x_{ss}}}{ds^2}\right)\delta x = 0 \tag{8e}$$

leads to two independent Eulerian equations

$$\frac{\partial E_{ext}}{\partial x} - \alpha x_{ss} + \beta x_{ssss} = 0 \tag{9a}$$

$$\frac{\partial E_{ext}}{\partial y} - \alpha y_{ss} + \beta y_{ssss} = 0. \tag{9b}$$

Approximating the derivatives with finite differences

$$\begin{aligned}&\frac{\partial E_{ext}}{\partial x_i} - \alpha\{(x_{i-1} - x_i) - (x_i - x_{i+1})\}\\ &\quad + \beta\big\{(x_{i-2} - 2x_{i-1} + x_i) - 2(x_{i-1} - 2x_i + x_{i+1})\\ &\quad + (x_i - 2x_{i+1} + x_{i+2})\big\} = 0\end{aligned} \tag{10}$$

and converting to matrix notation

$$\begin{pmatrix} & & \vdots & & \\ \cdots\beta & -\alpha-4\beta & 2\alpha+6\beta & -\alpha-4\beta & \beta\cdots \\ & & \vdots & & \end{pmatrix}\begin{pmatrix} \vdots \\ x_{i-2} \\ x_{i-1} \\ x_i \\ x_{i+1} \\ x_{i+2} \\ \vdots \end{pmatrix} + \frac{\partial E_{ext}}{\partial x_i} = 0 \qquad (11)$$

gives final equations, which can be solved iteratively.

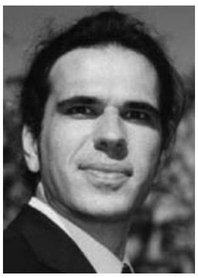
Note

1. VECTOR25 defines the digital landscape model by Swiss Federal Office of Topography

References

1. J. Badaud, A.P. Witkin, and R.O. Duda, “Uniqueness of the Gaussian Kernel for scale-space filtering,” *IEEE Transactions on Pattern Analysis and Image Processing*, Vol. 8(1):183–193, 1986.
2. M. Bader. Energy Minimization Methods for Feature Displacement in Map Generalization. Ph.D. thesis, Department of Geography, University of Zurich, 2001.
3. J. Balboa and F. Lopez “Frequency filtering of linear features by means of wavelets: A method and an example,” *Cartographic Journal*, Vol. 37(1):39–49, 2000.
4. M. Barrault, M. Bader and R. Weibel. *Topology Preserving Conflict Removal between Symbolized Roads in Cartographic Generalization: Extending Snakes Methods*. in Abstract for GIScience2000, 2000.
5. J. Bobrich. Ein neuer Ansatz zur kartographischen Verdrängung auf der Grundlage eines mechanischen Federmodells. Ph.D. thesis, Deutsche Geodätische Kommission, München Reihe C, H. 455, 1996.
6. D. Burghardt. *Automatisierung der kartographischen Verdrängung mittels Energieminimierung*. Ph.D. thesis, Deutsche Geodätische Kommission, München Reihe C, H. 536, 2000.
7. D. Burghardt and S. Meier. “Cartographic displacement using the snakes concept,” in W. Foerstner and L. Pluemer (Eds.), *Semantic Modeling for the Acquisition of Topografic Information from Images and Maps*, Birkhaeuser-Verlag: Basel, 1997.
8. K.C. Clarke, P. Cippoletti and G. Olsen. “Empirical comparison of two line enhancement methods,” *Proceedings of AUTO-CARTO 11*, 1993.
9. C. Duchêne. “Road generalisation using agents,” *Proceedings of the GIS Research UK, 9th Annual Conference*, University of Glamorgan Wales, pp. 325–328, 2001.
10. E. Fritsch. “Utilisation de la coubure pour la généralisation du linéaire routier,” *Bulletin d’Information n.66, IGN*, France, 1997.
11. E. Fritsch and J.P. Lagrange. “Spectral representation of linear features for generalisation,” *Proceedings of COSIT’95*, pp. 157–171, Austria, Springer Verlag, 1995.
12. L. Harrie. An Optimisation Approach to Cartographic Generalisation. Ph.D. thesis, Department of Technology and Society, Lund University, 2001.
13. L. Harrie and T. Sarjakoski. “Generalisation of vector data sets by simultaneous least squares adjustment,” *International Archives of Photogrammetry and Remote Sensing*, Vol. XXXIII, Part A4, Amsterdam, pp. 340–347, 2000.
14. P. Højholt. “Solving local and global space conflicts in map generalization using a finite element method adapted from structural mechanics,” in *Proceedings 8th International Symposium on Spatial Data Handling*, pp. 679–689. Vancouver, Canada, 1998.

15. M. Kass, A. Witkin and D. Terzopoulos. "Snakes: Active contour models," in *Proceedings of the First International Conference on Computer Vision*, pp. 259–268, 1987.
16. R.B. McMaster. "The integration of simplification and smoothing algorithms in line generalization," *Cartographica*, Vol. 26:100–121, 1989.
17. R.B. McMaster and K.S. Shea. "Generalization in Digital Cartography," (*Resource Publications in Geography*). Washington, D.C.: Association of American Geographers, 1992.
18. S. Meier. "Zur qualität snakes-approximierter höhenprofile mit diskontinuitäten," *Photogrammetrie Fernerkundung Geoinformation*, Vol. 6:399–409, 2000.
19. B. Nakos and V. Miropoulos. "Local length ratio as a measure of critical points detection for line simplification," *Fifth Workshop on Progress in Automated Map Generalization*, Paris, France, 2003.
20. Perkal. "An attempt at objective generalization," in *Michigan Inter-University Community of Mathematical Geographers*, Discussion Paper 10, University of Michigan, 1966.
21. C. Plazanet. *Enrichissement des bases données géeographiques: Analyse dela géometrie des objets linéaires pour la généralisation cartographique (application aux routes)*. Ph.D. thesis of Marne la Vallée University, 1996.
22. C. Plazanet, N.M. Bigolin and A. Ruas. "Experiments with Learning Techniques for Spatial Model Enrichment and Line Generalization," *GeoInformatica*, Vol. 2(4):315–333, 1998.
23. A. Ruas. "The role of meso objects for generalisation," *Proceedings of the International Symposium on Spatial Data Handling*, Beijing, 3b.50, 2000.
24. E. Saux. "B-spline Functions and Wavelets for Cartographic Line Generalization," *Cartography and Geographic Information Science*, Vol. 30(1):33–50, 2003.
25. F. Schwarzbach. "Untersuchungen zur rechnergestützten Linienglättung," Ph.D. thesis, Kartographische Bausteine 10, Department of Cartography, University of Dresden, 1995.
26. G. Schweinfurth. *Höhenliniengeneralisierung mit Methoden der digitalen Bildverarbeitung*. Deutsche Geodätische Kommission, Reihe C, No. 291, 1984.
27. M. Sester. "Generalization based on least squares adjustment," *International Archives of Photogrammetry and Remote Sensing*, Vol. XXXIII, Part B4, Amsterdam, pp. 931–938, 2000.
28. S. Steiniger and S. Meier. "Snakes: A technique for line smoothing and displacement in map generalisation," *Sixth Workshop on Progress in Automated Map Generalization*, Leicester, UK, 2004.
29. F. Töpfer. *Kartographische Generalisierung, Ergänzungsheft Nr. 276 zu Geographische Mitteilungen*. VEB Hermann Haack, Geographisch-Kartographische Anstalt Gotha/Leipzig, 1974.



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