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ORIGINAL PAPER

Effect of Particles Shape on the Hydraulic Conductivity of Stokesian Flow in Granular Materials

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Abstract The hydraulic conductivity of a granular porous medium depends on parameters such as the porosity, particles shape and fluid viscosity. Although other factors may affect the permeability of the granular matrix, their effects are often insignificant while their measurements are sometimes very difficult. There are many experimental studies on the hydraulic conductivity of granular media suggesting empirical equations for the coefficient of hydraulic conductivity. In most of these equations, factors such as the effective grain size and the void ratio have been focused more than the others. In the present study, the hydraulic conductivity under the same condition for a Stokesian flow has been studied with particles of the same size, but different shape. Here, the shape is mainly referred to the roundness of particles. Results indicated that the hydraulic conductivity of a clean and nearly uniformly graded sand, shows a linear relationship with the percentage of round particles.

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1 Introduction

The geometry of the grains, i.e. the Grain Size Distribution (GSD) and the particles shape, in a porous granular material has been found to have a direct influence on the hydraulic conductivity of the material. In most coarse granular media, the ability of the domain to conduct the fluid flow is mainly governed only by the geometry of the grains and the characteristics of the fluid. For instance, if the shape of the particles is perfectly spherical and the fluid has relatively low velocity and/or very high viscosity (resulting in a small Reynolds number, e.g. $R_e \ll 1$), the arrangement and size of the particles are adequate characteristics to study the fluid transport in the medium. In this case, the permeability of the porous medium can be practically studied by solving the equations governing the motion of a viscous fluid. The problem will be more simplified if the fluid could be considered incompressible which is the case for the water.

The notion hydraulic conductivity of porous media finds its roots in historically important works of Darcy (1856). He found that the velocity of a fluid, such as water, passing through a porous medium, is linearly proportional to the gradient of the hydraulic head. Afterwards, a variety of methods and procedures were developed to study the hydraulic conductivity of granular porous media based on this important finding. For instance, Slichter (1898) suggested that the hydraulic conductivity is related to the mean grain size, volumetric porosity and some shape factors. Later, Kozeny (1927) and Carman (1938) worked further on the subject and developed a more elaborative equation for the hydraulic conductivity which relied on some factors such as the dynamic viscosity, void ratio, specific surface area and the unit weight of the permeant. Latest studies such as Moulton (1980), Kenny et al. (1984) or Chapuis (2004) have proposed the hydraulic conductivity as a function of the grain size distribution and the percentage of fine grained materials. Moreover, Fujikura (2019) has also developed the effect of the dynamic viscosity on the hydraulic conductivity in his proposed equation; although this is not the first study on the effect of dynamic viscosity.

Recently, Roshanali (2021) investigated the hydraulic conductivity of anisotropic porous media using a microstructure tensor as a geometric measure of the anisotropy of the soil matrix. This geometric measure was based on the directional porosity which is meaningful for the hydraulic conductivity problem. Using this geometric measure to characterize the topology of the pore space re-opened a historical question of the effect of the particles shape, as well as the size distribution, on the hydraulic conductivity. In fact, the geometric measure reduces to the bulk porosity for isotropic materials but does not involve the particle shapes which is the subject of the current study for isotropic materials. Therefore, the present study is intended to inspect the effect of particles shape on the hydraulic conductivity in granular media under Stokesian flow regime. There are few studies known to the authors made on this topic (Garcia et al. 2009; Gerke et al. 2018). In the present work, by fixing all factors but the shape of particles, the fluid flow has been modeled and the governing differential equations have been numerically solved for some assemblages of particles with different percentage of round particles. The flow has been modeled in two-dimensions and to allow an independent study on the effect of particles shape, the materials have been assumed to be clean (free of fines) and comprise of relatively uniform sized particles. Details of the methodology and the approach followed to arrive at the desired goals are presented in the subsequent sections.

2 Empirical Equations for the Hydraulic Conductivity

As stated before, the coefficient of hydraulic conductivity is often related to some factors depending on the geometry of the grains and dynamical properties of the permeant. A number of selected equations for the hydraulic conductivity have been presented in Table 1. Though there are numerous equations available in literature, it is tried to present those with the simplest forms and least number of parameters which are basically functions of porosity or the void ratio.

2.1 Governing Hydro-Mechanical (Navier–Stokes/Potential Flow) Equations

2.1.1 Stokesian Flow

The constitutive equation of a fluid is often expressed as a functional relationship between the stress tensor $\boldsymbol{T}$, and the rate of deformation tensor, $\boldsymbol{D}$ (and also the density, ρ) among other parameters. The linear Reiner-Rivlin fluid is often universally accepted to describe most fluids, in particular, the water. The governing hydro-mechanical equations, comprising the three universal conservation laws require the constitutive equation to be also prescribed and the combination of all these equations eventually leads to a system of nonlinear partial differential equations, i.e. the Navier–Stokes equations. Conservation of mass for incompressible materials ($\dot{\rho} = 0$) will yield:

$$\mathrm{div}\boldsymbol{v} = 0 \tag{1a}$$

$$v_{i,i} = 0 \tag{1b}$$

In this equation, $\boldsymbol{v}$ is the velocity vector and $(.)_{i,i}$ is the indicial form of the divergence operator. The linear Reiner-Rivlin fluid has the following general form of the governing equation:

$$\boldsymbol{T} = -p\boldsymbol{I} + \lambda \mathrm{div}\boldsymbol{v}\boldsymbol{I} + 2\mu\boldsymbol{D} \tag{2a}$$

$$t_{ij} = -p\delta_{ij} + \lambda v_{k,k}\delta_{ij} + 2\mu d_{ij} \tag{2b}$$

where $\boldsymbol{T}$ is the Cauchy stress tensor, λ and μ are the coefficient of dynamic viscosity, p serves as the [confining] pressure and $\boldsymbol{D}$ shows the rate of deformation tensor. It is noteworthy that the relationship between the stress tensor and the rate of deformation tensor

Table 1 Selected empirical equations for the hydraulic conductivity

References	Equation	Remarks
Hazen (1892)	$k\left[\frac{cm}{s}\right]=C_H D_{10}^2$	C_H: Hazen's empirical factor; D_{10}: Effective grain size (cm) Common range of C_H: 100 (Hazen 1892) 1–42 (Lambe and Whitman 1969) 80–120 (Holtz and Kovacs 1981)
Kozeny (1927) and Carman (1938)	$k=\left(\frac{\gamma}{\mu}\right)\left(\frac{1}{C_{K-C}}\right)\left(\frac{1}{S_0^2}\right)\left[\frac{e^3}{1+e}\right]$	γ and μ: Unit weight and dynamic viscosity of the permeant; C_{K-C}: Kozeny-Carman empirical factor; S_0: Specific surface area per unit volume of particles $(1/cm)$; e: Void ratio
Saffman (1959)	$k=\frac{nd^2}{96\tau^2}$	n: Porosity; d: Mean diameter of flow channels (estimated based on the pore size; distribution); τ: Tortuosity
Zamarin (Based on Lu et al. 2012)	$k\left[\frac{m}{s}\right]=\beta_z\frac{g}{v}\frac{n^3}{(1-n)^2}d_e^2$	g: Gravity acceleration (LT^{-2}); v : Kinematic viscosity; d_e Particle diameter (mm); Δg_1: Weight of the material of the finest fraction in parts of the total weight. d_1: The largest diameter of the finest fraction; d_i^g and d_i^d: The maximum and the minimum grain diameters of the fraction; Δg_i: Weight of the fraction
	$\frac{1}{d_e}=\frac{3}{2}\frac{\Delta g_1}{d_1}+\sum_{i=2}^{i=n}\Delta g_i\left((\ln\frac{d_i^g}{d_i^d})/d_i^g-d_i^d\right)$	
Ranaivomanana et al. (2016)	$k=\frac{nd^2}{32}$	n: Porosity; d: Mean diameter of channels estimated from the pore-size distribution
	$k=\frac{nd^2}{8}\frac{\alpha^4}{(1+\alpha^4)(1+\alpha^2)}$	*Note* For $\alpha=1$, $k=nd^2/32$ (parallel capillary pores) and for $\alpha=0$ (clogged channels), the permeability is null

is linear, but non-homogeneous. As stated earlier the combination of these equations with the universal equations of the conservation laws, leads to the well-known Navier–Stokes nonlinear partial differential equations, which are often expressed as follows:

$$\rho\dot{\boldsymbol{v}}=\rho\left(\frac{\partial \boldsymbol{v}}{\partial t}+\boldsymbol{v}.\nabla\boldsymbol{v}\right)=-\nabla p+\rho\boldsymbol{b}+(\lambda+\mu)\nabla(\nabla.\boldsymbol{v})+\mu\nabla^2\boldsymbol{v} \tag{3a}$$

$$\rho\dot{v}_i=\rho\left(\frac{\partial v_i}{\partial t}+v_j\frac{\partial v_i}{\partial x_j}\right)=-p_{,i}+\rho b_i+(\lambda+\mu)v_{k,ki}+\mu v_{i,jj} \tag{3b}$$

Famous Euler equations are also obtained by applying the condition of volume incompressibility, $v_{k,k}=0$ (also $\mu=0$) which makes the dynamic viscosity, λ, ineffective. In addition, volume incompressibility is required for the general form of Darcy's law in porous media. One may refer to Zienkiewicz and Shiomi (1984) for details. Euler equations are as follows:

$$\rho\dot{\boldsymbol{v}}=\rho\left(\frac{\partial \boldsymbol{v}}{\partial t}+\boldsymbol{v}.\nabla\boldsymbol{v}\right)=-\nabla p+\rho\boldsymbol{b} \tag{4a}$$

$$\rho\dot{v}_i=\rho\left(\frac{\partial v_i}{\partial t}+v_j\frac{\partial v_i}{\partial x_j}\right)=-p_{,i}+\rho b_i \tag{4b}$$

These equations are very difficult (if not impossible at all) to be analytically solved and numerical solutions have been developed to solve them for simple and complex domains, such as the flow in a pipe, in a ventury or in a matrix of grains with interconnected pores.

2.1.2 Potential Flow

Another type of equation, governing the flow of an incompressible fluid through the soil matrix can be set by assuming that the flow obeys some potential.

This type of flow must satisfy the condition of irrotationality, i.e. $\nabla \times \boldsymbol{v} = 0$. If this condition holds, then the velocity can be obtained by the gradient of some scalar potential field, φ, i.e. $\boldsymbol{v} = \nabla\varphi$. This particular type of flow will be reduced to the Laplace equation, i.e. $\nabla^2\varphi = 0$ which is easily solved even in complex domains such as matrix of grains with interconnected pores. There are also some analytical solutions for this type of flow in simple problems and hence, it can be often used to verify the form of the flow for simple problems. Here, our focus is on the more general type of flow, that is, the Stokesian flow.

3 Numerical Analysis and Finite Element Formulation

Solving the hydro-mechanical equations using the finite element method is a standard procedure which is very suitable for Stokesian flow or potential flow. In Stokesian flow with relatively small Reynolds Number, the equations can be simplified to linear equations which require simple procedures to obtain the finite element solution. A typical case of small Reynolds Number is the case of water flow soils. Here, we present only the final forms of the finite element equations for the Stokes flow:

$$\begin{pmatrix} \boldsymbol{k}_{11} & 0 & \boldsymbol{k}_{13} \\ 0 & \boldsymbol{k}_{22} & \boldsymbol{k}_{23} \\ \boldsymbol{k}_{31} & \boldsymbol{k}_{32} & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{v}_x^e \\ \boldsymbol{v}_y^e \\ \boldsymbol{p}^e \end{pmatrix} = \begin{pmatrix} \boldsymbol{b}_x^e \\ \boldsymbol{b}_y^e \\ 0 \end{pmatrix} + \begin{pmatrix} \boldsymbol{t}_x^e \\ \boldsymbol{t}_y^e \\ 0 \end{pmatrix} \quad \text{Stokesian Flow} \tag{5}$$

with matrix coefficients defined as follows:

$$\boldsymbol{k}_{11} = \int_P \mu \begin{pmatrix} \partial\overline{\boldsymbol{N}}/\partial x & 0 \\ 0 & \partial\overline{\boldsymbol{N}}/\partial y \end{pmatrix} \begin{pmatrix} \partial\overline{\boldsymbol{N}}/\partial x & 0 \\ 0 & \partial\overline{\boldsymbol{N}}/\partial y \end{pmatrix} dv = \boldsymbol{k}_{22},$$

$$\boldsymbol{k}_{13} = \int_P \overline{\boldsymbol{N}}^T \left(\partial\overline{\boldsymbol{N}}/\partial x\right) dv = \boldsymbol{k}_{31}, \boldsymbol{k}_{23} = \int_P \overline{\boldsymbol{N}}^T \left(\partial\overline{\boldsymbol{N}}/\partial y\right) dv = \boldsymbol{k}_{32},$$

$$\begin{aligned} \boldsymbol{b}_x^e &= -\rho b_x \int_P \overline{\boldsymbol{N}}^T dv, \boldsymbol{b}_y^e = -\rho b_y \int_P \overline{\boldsymbol{N}}^T dv, \\ \boldsymbol{t}_x^e &= +\mu t_{xx} \int_{\partial P} \overline{\boldsymbol{N}}^T da, \boldsymbol{t}_y^e = +\mu t_{yy} \int_{\partial P} \overline{\boldsymbol{N}}^T da \end{aligned} \tag{6}$$

In these equations, $\boldsymbol{v}_x^e$ and $\boldsymbol{v}_y^e$ are components of the nodal velocity vectors, $\boldsymbol{p}^e$ is vector of nodal pressures, $\boldsymbol{b}_x^e$ and $\boldsymbol{b}_y^e$ are components of nodal body forces (which can be assumed to be zero in two-dimensional problems), $\boldsymbol{t}_x^e$ and $\boldsymbol{t}_y^e$ are nodal components of boundary tractions. In addition, $\overline{\boldsymbol{N}}$ is the matrix of the element interpolation functions. The form of this matrix depends on the type of the element, number of nodes and the degree of interpolation. For the present study, we have used simple triangular elements with three nodes which can be efficiently used to cover the irregular void spaces.

The research method is theoretical-numerical in which matrices of a granular soil with a fixed porosity and GSD curve, but different particle shapes, are generated. In these matrices, the hydraulic conductivity is calculated by solving the Navier–Stokes equations and using the average of fluid velocity passing through the porous medium. These matrices are called Representative Elementary Volume (REV) as they formally express the local–global relationship between an infinitesimal domain of the porous medium (relating to the material property) and the characteristics of the medium (relating to macroscopic properties, such as the grain size distribution, etc.) Also, such type of modeling is called a multiscale (or meso-scale) modeling. This is important to keep a balance between the size of the REV in order to (a) be a good representative of soil matrix, (b) be a representative of the material element, satisfying continuum theory (not too much small) and (c) prevent very high computational effort by letting it be too much large. A rational condition to preserve the continuum principle is the Knudsen Number which should be below 0.1. Here, we assume that a measure of Knudsen Number, which is the relative size of the maximum grain to the domain size, is well below 0.05 and guarantees the REV to be a representative of the material point. We mean by the average velocity, the weighted velocity passing through each element, or, the velocity at which, the fluid enters or departs from the boundaries. It is worth mentioning that such averaging requires the velocity vector field to be first parallelly transported (or shifted) to a fixed point and then integrated (Sokolnikoff 1951; Truesdell and Toupin 1960) and averaged over the domain. By using a Cartesian coordinate system, no such shifting is required. In addition, for generating the Representative Elementary Volume (REV) of the soil matrices, the relationship between 2 and 3D measures of the porosity should be considered.

By assuming that the pore spaces are uniformly distributed in the volume, this relationship can be approximately written as $n_{(2D)} = n_{(3D)}^{2/3}$. One may note that this equation is exact if the directional porosity is uniformly distributed along all directions, i.e., in case of an ideally isotropic soil, or *assumed* to be so. Since the volumetric or bulk porosity, n (or $n_{(3D)}$) is always less than unity, this means that the 2D (area) porosity is always greater than the 3D (volumetric or bulk) porosity. For instance, by assuming $n = n_{(3D)} = 0.5$, the two-dimensional porosity will be around 0.64.

A schematic pattern of soil particles arrangement is shown in Fig. 1, in which, a cross section of a three dimensional matrix is developed. Of course, the cross-sectional area seldom passes through the largest dimension (or the largest plane) of a particle; despite, for the sake of convenience and for a better consistency with the GSD curve, we formally assume that the shapes of particles in a soil matrix are similar in both two and three dimensions. However, the two-dimensional porosity, i.e. the actual void spaces among particles, is tried to be maintained as it should be in the plane. In addition, the effect of fine content has been neglected by formally assuming that the soil matrix composes of a clean and nearly uniformly graded particles. Another assumption in developing the soil matrix is that the particles are nearly spherical. This assumption is necessary to help finding a relationship between the number of particles in an arrangement with at a given volumetric porosity. Fortunately, simple mathematical calculation reveals that this assumption is not too much restrictive as long as particles are not needle like or platy in shape. This assumption is applied to initialize the soil matrix and the shapes were slightly changed in order to model the angular grains. Under these assumptions, various assemblages of particles with round, angular and a mix of the two, have been generated and synthetic soil matrices were developed to analyze the passage of the seepage flow.

In the next sections, we first present a series of verifications of the numerical finite element solution of the fluid flow for some available experimental data and then, we will study the role of particles shape on the hydraulic conductivity. Two sets of verifications have been presented: for round particles and for angular particles. In all cases, we formally assume that the soil matrix is relatively isotropic and hence, the hydraulic conductivity can be expressed by only a scalar quantity, or, an isotropic tensor which are both equivalent.

4 Verification with Available Experimental Data for Round Grains

To calculate the hydraulic conductivity by using the solution of the Stokesian incompressible fluid flow as discussed before. A series of experimental data reported in the literature with known GSD curves and relatively accurate measurements, have been recompiled. The GSD curves have been used to generate soil matrices of different forms but the same porosity. For each specimen, a number of different realizations have been made and minimum, maximum and average of the hydraulic conductivity among all realizations are reported. Figure 2, shows the GSD curves of the soil samples used for verifications from two different references, i.e. Jaafar and Likos (2013) and Kedir (2018) with sample points marks on them. The sample points are those for which, the synthetic soils (in numerical models) have been generated in the REV. These samples are mainly clean, uniformly graded sand, classified as Poorly Graded Sand, SP, according to the Unified Soil Classification System

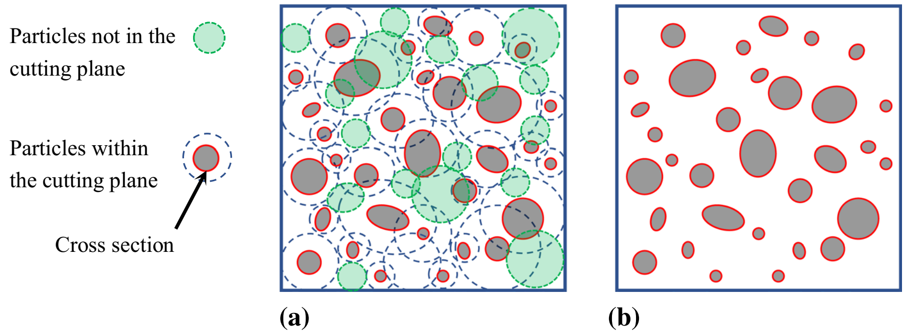


Fig. 1 A schematics assemblage of particles: **a** a three dimensional view and **b** a cross-section passing through an arbitrary plane

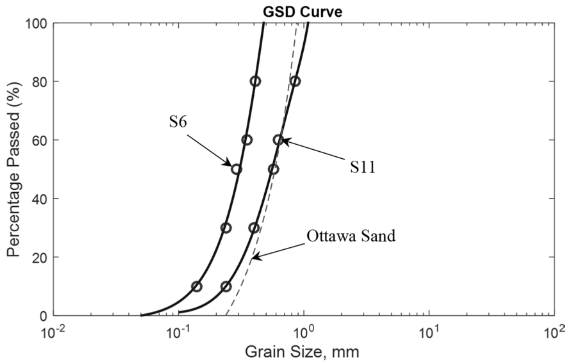


Fig. 2 GSD curves for experimental samples used for verifications (Data from by Jaafar and Likos 2013-S6 and Kedir 2018-S11)

(USCS). The GSD curves are fairly similar to the famous Ottawa Sand (20–30) which is also depicted on the same plot for comparison. Table 2, shows the soil properties of each sample, measured hydraulic conductivity and the predicted hydraulic conductivity by the finite element method. In addition, Figs. 3 and 4 show a sample of REVs, simulations and numerical solutions for the flow velocity and pressure fields. Results indicated that simulations can predict the hydraulic conductivity of the soil matrix with a reasonable accuracy.

5 Verification with Available Experimental Data for Angular Grains

In this section, attempt is made to show the effect of particles shape on the hydraulic conductivity and also to verify the numerical procedure against some experimental data considering the shape of the particles. Verification for the shape of particles is somehow difficult as there is no or very few such experimental data is available. One exception is a research made conducted by Cabalar and Akbulut (2016) where both round and angular particles (as well as a combination of them) have been used. A one-dimensional fluid flow has been modeled in a series of assemblages of particles for a fixed GSD curve. Angular particles are generated by assuming that these particles are bulky in shape and possess four to six edges. Two factors were changed in all these REVs: (1) the particles shape and (2) the arrangement of particles.

Table 2 Verifications with some available experimental data for sands with round particles

Sand type	Volumetric porosity, n	Soil characteristics				Predicted hydraulic conductivity (FEM simulations) k(m/s)			Measured hydraulic conductivity (experimental) k(m/s)		
		C_u	C_c	D_{10} (mm)	D_{50} (mm)	k^{min}	k^{max}	k^{ave}	k^{min}	k^{max}	k^{ave}
Jaafar and Likos (2013)—S6	0.35	2.4	0.84	0.14	0.29	1.4×10^{-4}	2.1×10^{-4}	1.7×10^{-4}	NA	NA	1.9×10^{-4}
Kedir (2018)—S11	0.42	2.21	0.91	0.24	0.47	2.4×10^{-4}	3.0×10^{-4}	2.9×10^{-4}	NA	NA	2.7×10^{-4}

NA Not Available (not reported)

Fig. 3 Simulations for some various arrangements of particles with a fixed GSD curve and bulk porosity. Sample S6: **a** FE mesh, **b** velocity field and **c** pressure field in kPa (Data from Jaafar and Likos 2013)

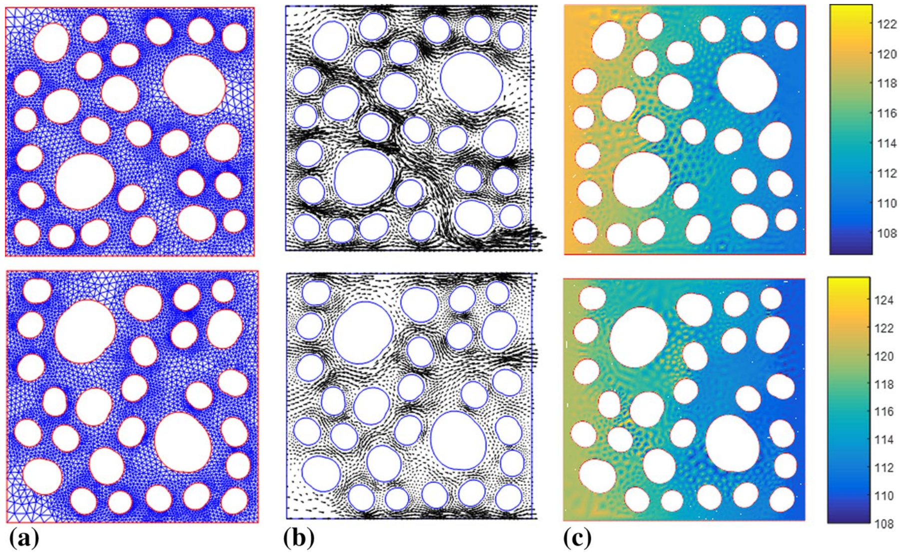
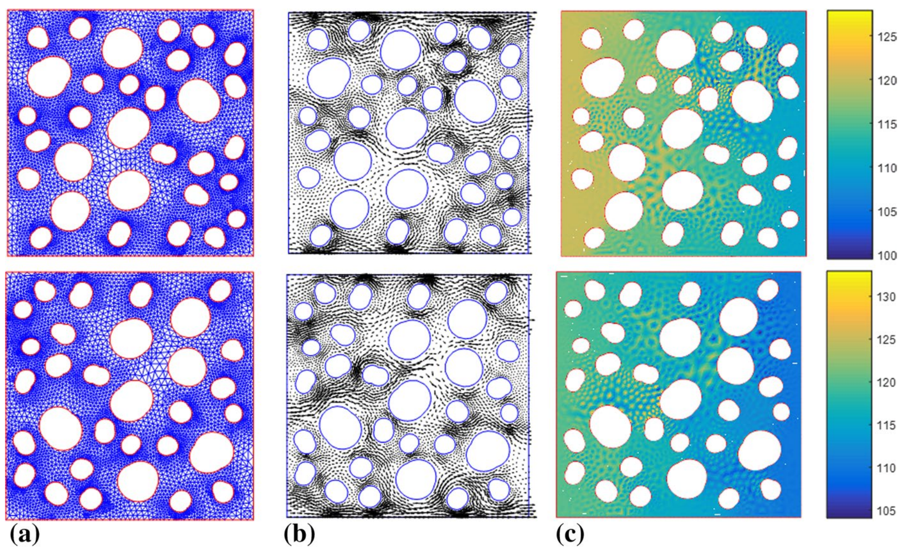


Fig. 4 Simulations for some various arrangements of particles with a fixed GSD curve and bulk porosity. Sample S11: **a** FE mesh, **b** velocity field and **c** pressure field in kPa (Data from Kedir 2018)



While the first variable results in a different soil matrix to study the effect of the particle shape, the second one represents different realization for a particular soil matrix. Therefore, by changing the arrangement of particles, a range for the hydraulic conductivity of certain GSD curve with a specific shape of particles will be obtained. It should be noted that the porosity for all these samples has been kept nearly constant. The soil sample used for verification is classified under SP, i.e. poorly graded sand, according to the Unified Soil Classification System with $C_u = 3.47$. The GSD curve of this soil sample along with the Ottawa Sand are depicted in Fig. 5 with sampling points marked on it.

Table 3, contains a summary of the results of the analyses conducted on this soil for angular particles. The finite element mesh, the pressure field and the velocity field for a series of analyzed samples with various percentages of round particles are also illustrated in Fig. 6. The results for the rest of synthetic and/or real samples are also presented afterwards, with a discussion on the effect of particles shape.

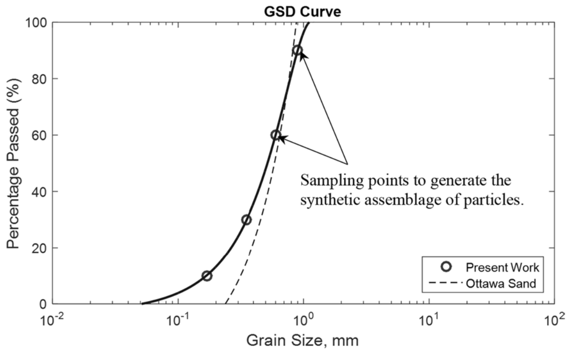


Fig. 5 GSD curve for selected experimental tests (Data from by Cabalar and Akbulut 2016) and the GSD curve for the Ottawa Sand

6 Effect of the Particles Shape

In this section, the effect of articles shape on the hydraulic conductivity of granular soils is inspected. One may note that there are many factors affecting the hydraulic conductivity. For instance, regardless of the character of the flow itself, the percentage of fines can highly affect the hydraulic conductivity. Only for clean granular soils (with infinitesimal amount of fines) at a relatively loose to medium density and with a fairly uniformly graded bulky particles, the results of this study would be applicable. A series of synthetic matrices of round and angular grains, also combinations of the two, have been generated and the variation of the hydraulic conductivity through these matrices has been investigated against the percentage of round particles. It is found that the hydraulic conductivity exhibits a fairly linearly increasing trend with the percentage of round particles.

To further inspect this effect and also to verify that the numerical results are in agreement with real soil data, some available test data on samples of sand with various particles shape has been collected. Unfortunately, in very few experiments a complete information on the shape of the samples, i.e. by inspecting the number of angular edges and percentage of round particles, were reported. In some cases, images presented in the papers were inspected to find out an overall imagination of the particles shape. In addition, not all particles are in practice, ideally angular or round, but they may be needle like or platy and most particles may have some angular edges whereas others round ones which are classified under sub-round

Table 3 The effect of particles shape on the hydraulic conductivity of granular soils

Sand Type	Volumetric porosity, n	Soil characteristics				Predicted hydraulic conductivity (FEM simulations) k(m/s)			Measured hydraulic conductivity (experimental) k(m/s)		
		C_u	C_c	D_{10} (mm)	D_{50} (mm)	k^{min}	k^{max}	k^{ave}	k^{min}	k^{max}	k^{ave}
Cabalar and Akbulut (2016)—S6	0.63	3.5	1.10	0.19	0.4	1.6×10^{-4}	2.3×10^{-4}	1.7×10^{-4}	NA	NA	2.0×10^{-4}

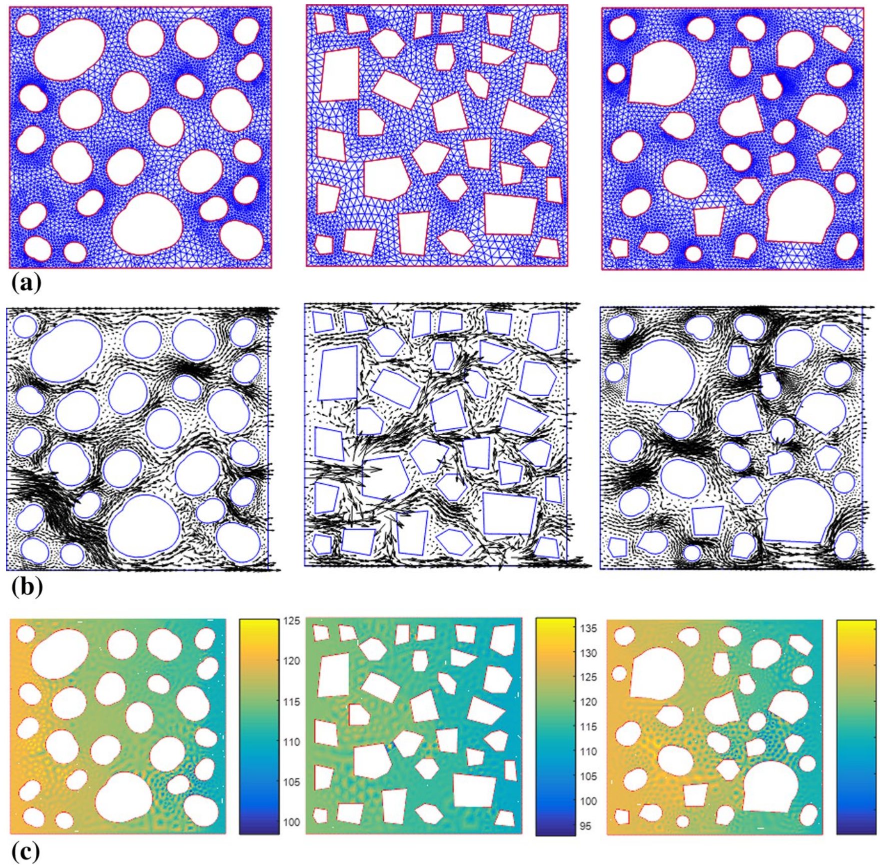


Fig. 6 FE simulations of the fluid flow in a poorly graded sand with a fixed GSD curve and bulk porosity at various percentages of round and angular particles: **a** finite element mesh, **b** velocity field and **c** pressure field in kPa (Data from Cabalar and Akbulut 2016)

and/or sub-angular. Therefore, the results obtained from real experiments are not accurate, although they possess sufficient accuracy considering the complexities involved in the study of the fluid flow in porous media. It is an optimistic view that the results can be considered as being valid under the abovementioned assumptions and for a such a complicated problem.

Another exception is a relatively close numerical study made by Garcia et al. (2009) on the hydraulic conductivity of an assemblage of particles corresponding to clean, uniformly graded Ottawa Sand. They used 6 different particle shapes ranging from needle-like grains to fully spherical ones. They kept the bulk porosity within a close range of 0.34 to 0.38. They reported the hydraulic conductivity, normalized by the D_{max}^2 and the kinematic viscosity. Here we recompiled the dataset of Garcia et al. (2009) for samples A, B and C in their studies and formally assumed that particle shapes of G1, G2 and G5 being angular (or nearly angular) whereas the rest (i.e. G3, G4 and G6) being rounded. In addition, by making use of their suggested equation, the hydraulic conductivities have been scaled for the bulk porosities assumed in this study and then, normalized by the area porosity. After such manipulations, their reported data has been used as an outsource for further comparison. The results of the present work show a good agreement with their simulations.

Figure 7a, represents the results of the numerical analyses on synthetic soils at different amounts of round particles and various area porosities, ranging between 0.5 and 0.7. Once the results are normalized by n^2, all show a fairly similar trend. A linear increase in the hydraulic conductivity with the percentage of round particles can be observed. A more detailed inspection has been made by implementing a series of experimental data on real soils, as well as the dataset of Garcia et al. (2009) scaled for area porosity of 0.53, all normalized by their area porosity. This is shown, along with previous data, on Fig. 7b. These soils have more or less similar GSD curves and are all classified as SP. Again, the linear trend of the increase

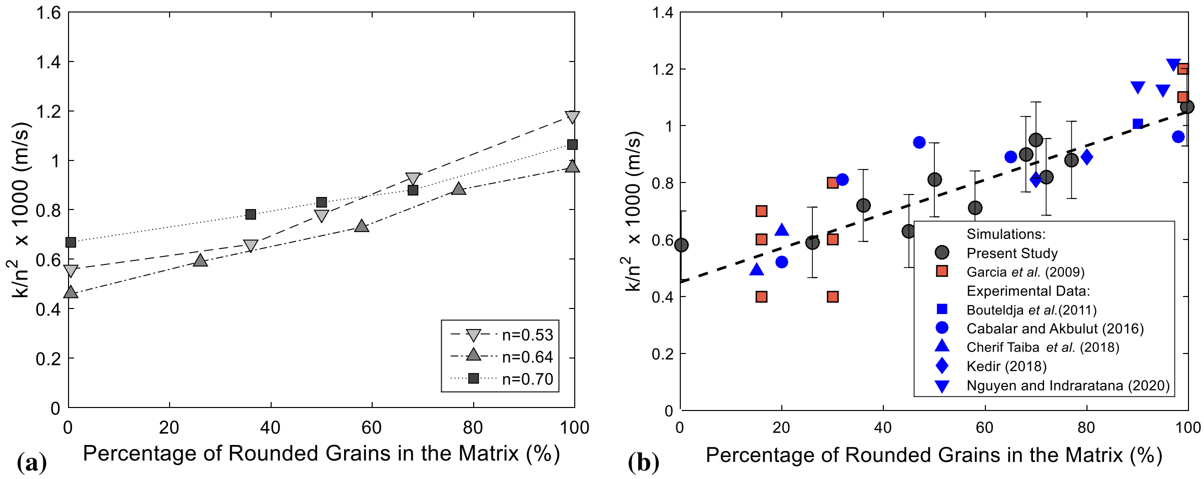


Fig. 7 Effect of the percentage of round particles on the hydraulic conductivity of granular soils: **a** synthetic data (k^{ave}) and **b** synthetic data, available experimental data and dataset of Garcia et al. (2009)

in the hydraulic conductivity with the percentage of round particles is evident. It should be emphasized that this conclusion and comparison are made with care, as the measurement of the hydraulic conductivity, maintaining different samples of the same soil type at a fixed porosity, generation of soil matrices and numerical analyses, contain errors which are inevitable. However, the general conclusion for particular class of clean, uniformly graded granular soils at relatively loose to medium density, is valid at an acceptable level of accuracy.

For the sake of practical applications, a very simple equation can be suggested to express the hydraulic conductivity of such granular soils including the shape factor which is defined by the fraction of rounded particles. By looking at the empirical equations suggested for the hydraulic conductivity, it can be immediately observed that the hydraulic conductivity is often expressed as a function of the porosity and also the mean or effective grain size. For the sands under study, which are of fairly similar mean grain size, the following general equation can be suggested which involves the porosity as well as the fraction of the rounded particles, p_r, representing some measure of the shape factor:

$$k = \alpha\left(1 + \beta p_r\right)n^2_{(2D)} = \alpha\left(1 + \beta p_r\right)n^{4/3}_{(3D)} \quad (7)$$

In this equation, k is the hydraulic conductivity in mm/s, $n_{(2D)}$ is the area porosity (which can be approximately related to the volumetric or bulk porosity by $n_{(2D)} = n^{2/3}_{(3D)}$), p_r is the percentage of round particles (in decimal) in the entire soil matrix and α and β are two semi-empirical constants (with units of k) obtained by numerical simulations as well as experimental data. For the sands under study, these two constants can be approximately assumed as $\alpha \cong 0.4$ and $\beta = 1.5$.

Finally, we made a further study on the accuracy of the suggested equation when compared to the simulated data. Figure 8, illustrates the two set of data, i.e. simulated soil matrices and the hydraulic conductivity approximated by Eq. (7). This is evident that the accuracy of this equation is reasonably high.

7 Conclusions

The present study is specified to an inspection on the effect of particles shape on the hydraulic conductivity. A series of soil matrices at various porosities but with similar GSD curves were generated. The percentage of round particles changes and the fluid flow equation in different soil matrices were studied. Verifications against some available experimental data showed a fairly good accuracy of the numerical models, at least for a particular type of clean and uniformly graded sand with medium density. Further inspection of the generated data as well as a few found in the literature revealed that the hydraulic conductivity shows a fairly

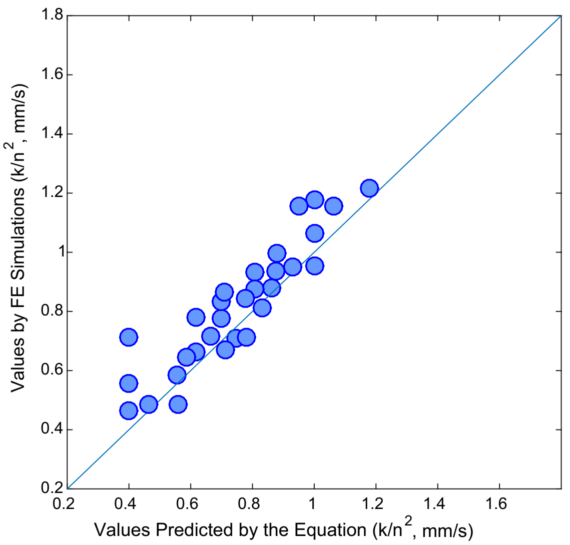


Fig. 8 Predicted (by the equation) versus simulated (by the FE analyses) hydraulic conductivities

linear trend with the increase in the percentage of round particles.

Of course, a general conclusion for the effect of particles shape cannot be drawn regardless of the other factors such as the fine contents, the directional porosity, the mean and/or effective grain size, the type of the flow and other factors related to the properties of the permeant, etc. However, by assuming that the medium is approximately isotropic and the soil matrix comprises only bulky particles which are characterized by angular and/or round particles and also for relatively loose to medium dense sands, the conclusions would be valid. By inspection of synthetic as well as some few available experimental data it was found that the linear trend can be expressed by a quite simple equation involving the porosity as well as the shape factor. This equation with suggested constant would be suitable for granular materials characterized under uniformly graded clean sands. It should be however emphasized that the fluid flow in porous media is itself a very challenging problem and this study is not free of inevitable approximations. In spite of it, for practical problems, it is expected to be a helpful estimate.

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Declarations

Conflict of interest The authors have not disclosed any competing interests.

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