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ORIGINAL PAPER

Evaluation of the Instability Risk of the Dam Slopes Simulated with Monte Carlo method (Case Study: Alborz Dam)

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Abstract Embankment dams are one of the most important geotechnical structures that their failures can lead to disastrous damages. One of the main causes of dam failure is its slope instability. Slope Stability analysis has traditionally been performed using the deterministic approaches. These approaches show the safety of slope only with factor of safety that this factor cannot take into account the uncertainty in soil parameters. Hence, to investigate the impact of uncertainties in soil parameters on slope stability, probabilistic analysis by Monte Carlo Simulation (MCS) method was used in this research. MCS method is a computational algorithm that uses random sampling to compute the results. This method studies the probability of slope failure using the distribution function of soil parameters. Stability analysis of upstream and downstream slopes of Alborz dam in all different design modes was done in both static and quasi-static condition. Probability of failure and reliability index were investigated for critical failure surfaces. Based on the reliability index obtained in different conditions, it can be said that the downstream and upstream slope of the Alborz dam is stable. The results show that although the factor of safety for upstream slope in the state of earthquake loading was enough, but the results derived from probabilistic analysis indicate that the factor of safety is not adequate. Also the upstream slope of the Alborz dam is unstable under high and uncontrolled explosions conditions in steady seepage from different levels under quasi-static terms.

Keywords Embankment dam · Probabilistic analysis · Uncertainty · Slope stability · Monte Carlo simulation

1 Introduction

Dams are economically and socially essential structures, and their role in the country's economic and construction development is undeniable. Hence, determining the expected performance evaluation of dams is critical for decision-makers. Therefore, concerning the importance of dams, accurately identify and manage the risks that are subjected to these structures is necessary (Bowles et al. 2011). In recent years, the analysis of geotechnical structures based on risk assessment has been seriously considered by researchers in earth dams. Risk assessment can be beneficial in complex decisions that involve several factors (Vick and Bromwell 1989).

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This viewpoint is due to the uncertain (random) nature of the geotechnical parameters. The origins of errors and uncertainties in geotechnical engineering include physical uncertainties (determination of effective including the value and forces of nature), human errors and uncertainties in modeling (numerical methods) (Jiang et al 2014)

In 1964, Arthur Casagrand introduced the risk definition in geotechnical engineering applications and emphasized that uncertainties are an integral part of any project that should be quantified appropriately (Whitman 1984). Nowadays, it has been proven that just reliability is an ineffective tool for quantifying the uncertainty degree and variability of soil properties. As a result, experts were looking for gadgets to directly and indirectly merge these uncertainties into slope stability calculations, which led to developing the possible analysis of the slope stability in the 1970s (Duncan 1996; Alonso 1976).

According to the researches, several parameters like the overtopping, piping effects and slope instability are the main reason of dam failure (Foster et al. 2000). The stability of upstream and downstream slopes is one of the most critical issues in dam safety management. These slopes should be analyzed for the various possible conditions that may lead to their instability. Therefore, plenty of studies have been performed to investigate these slopes stability. Although the majority of these studies have focused on the deterministic analysis of slope stability, however the probabilistic methods have also used for slope stability analysis.

Conventional methods of slope stability analysis based on the confidence coefficient obtained from deterministic methods cannot explicitly express the inputs parameters uncertainty. For this reason, the uncertainties in geotechnical parameters of materials and construction are ignored in dam failure potential (Alonso 1976). Ideally, a confidence factor higher than one means the slope stability, though, due to the mentioned uncertainties and for safe design, higher reliability coefficients are used. Probabilistic approaches are potent instruments for quantifying uncertainties and for use in slope design analysis. In order to characterize the random features of the variables and probabilistic slope analysis based on the accidental properties, there are two geotechnical data sections in these methods. These investigations can consider the uncertainties associated with the input parameters to obtain the confidence coefficient, reliability, and failure probability.

Probabilistic analysis usually uses a probability of failure or a reliability index in a safety level of the project, although a deterministic analysis utilizes a safety factor. In these methods, the larger reliability index in safer and the smaller ones, weaker performance (Wolff 1996; Mostyn et al. 1993).

There are several probabilistic methods for slope stability analysis; the most popular of these are the First-Order Reliability Method (FORM), First-Order Second Moment (FOSM), Point Estimate Method (PEM), Random Finite Element Method (RFEM) and Monte Carlo Simulation (MCS) (Peterson 1999).

Monte Carlo is one of the methods that can explain the impact of risk and uncertainties mathematically and statistically on problems (Molak 1997). This method was first introduced in 1949 by Newman and Ulam, who proposed the concept initially for random sampling of mathematical problems (U.S. Army Corps of Engineers 2006). The Monte Carlo simulation includes the following four steps (Krahn 2004):

1. Determine an analytical method for slope stability analysis.
2. Specify the input parameters as a probability distribution function.
3. Accidental sampling of input parameters according to their distribution function and performing analysis.
4. Failure probability calculation based on the number of confidence coefficients less than one.

Due to its simplicity and no need for comprehensive statistical and mathematical information, this method is one of the most common methods of probability analysis of slope stability.

The Monte Carlo method effectively models the functional response of the confidence coefficient versus the input variables, which are randomly selected and replaced in the operation function (Molak 1997). In this method, for each random input parameter, given the probability density function form and considering its variation range, the random numbers are generated and the operation function value is calculated according to them. This procedure continues until the density function form of the confidence coefficient is approximated, and the failure probability and the reliability index are calculated. Although the probability density function of each of the random input variables may have any shape, however, the normal, lognormal, triangular, beta and uniform distributions are more prevalent (Zio 2013). The slope reliability can also be obtained by examining the multiple slide surfaces and random responses in this method. The slope reliability is defined by the reliability index and compared with the confidence factor to determine the degree of confidence that can be gained by the confidence factor (Wang et al. 2010).

2 Problem Definition and Geotechnical Properties of Alborz Dam

Alborz dam is made of earth rockfill with vertical clay core located on Babrolrud River, 45 km southeast of Babol, a city which is located in the north of Iran. The characteristics of the Alborz dam are summarized in Table 1, and a schematic cross-section of the dam is shown in Fig. 1 (Mazandaran Water Company 1998a, b).

The methods of limit equilibrium analysis based on the two-dimensional study of slope stability are the most common methods of analysis. Although these methods convert 3D problems to two-dimensional with logical simplifications, they are used by many experts due to their ease of use and faster response than 3D analysis. Three-dimensional analysis plays a crucial role when the slope geometry, soil properties, or loading conditions are so complex that a two-dimensional cross-section is not suitable for analysis. Due to the plane strain condition in Alborz dam, two-dimensional analysis can be used with high accuracy for dam cross-section (Geo Slope International 2008).

Table 1 General features of Alborz Dam

Row	Type of dam	Rockfill with clay core
1	Fill volume of body	9.6 MCM
2	Height above foundation	78 m
3	Normal water level	301 m.a.s.l
4	Dam crest elevation	307 m.a.s.l
5	Dam crest length	838 m
6	Maximum water level	305.9 m.a.s.l
7	Minimum water level	258 m.a.s.l
8	Concrete volume	44,000 CM
9	Useful reservoir volume	130 MCM
10	Total reservoir volume	150 MCM

SLOPE/W software version 7.10 of 2007, one of the suites of GeoStudio applications, was used to analyses the slope stability of the dam. The software is designed to be used as a general application for the stability analysis of all types of earth structures. Possibility of using Monte Carlo simulation in slope stability analysis, which is an effective tool in probabilistic analysis of such studies, is one of the prominent features of this software. In the present study, the probability of dam improper performance using this software was calculated.

This software does not take into account the internal friction angle variation with confining stress. Therefore, the sandy and rockfill crust is modeled as zones with regards to the confining stress variation.

Confining stress of 1, 3, 5 and 7 kg/cm^2 are related to each of the distinct regions. The internal friction angle of each area is calculated according to its average confining stress using the following equation:

$$\emptyset = \emptyset_0 - \Delta\emptyset . \log\left({}^{\boldsymbol{\sigma}_3}/_{\boldsymbol{P}_a}\right) \tag{1}$$

$\emptyset_0$: Internal friction angle when the confining stress is equal to the atmospheric pressure. $\Delta\emptyset$: Equation constant that depending on soil type. $\boldsymbol{P}_a$: Atmospheric pressure. $\boldsymbol{\sigma}_3$: Average confining stress of area. $\emptyset$: Internal friction angle of each area.

One of the essential issues in slope stability analysis is the consideration of drained or undrained conditions of materials. In analyses with drained conditions, drained resistances related to effective stresses are used; however, in the analysis of undrained conditions, undrained resistances with total stresses are applied. This condition applies only to the clay core, and other materials are examined in the drained state due to high permeability (Mazandaran Water Company 1998a, b). Dam cross-section modeling was performed using the geotechnical parameters of the dam foundation and body presented in Table 2, 3, 4, 5, and 6, as shown in Fig. 2.

3 Stability Analysis

The stability analysis of upstream and downstream slopes of the Alborz dam was carried out using a bilinear behavioral model for clay core and Mohr-

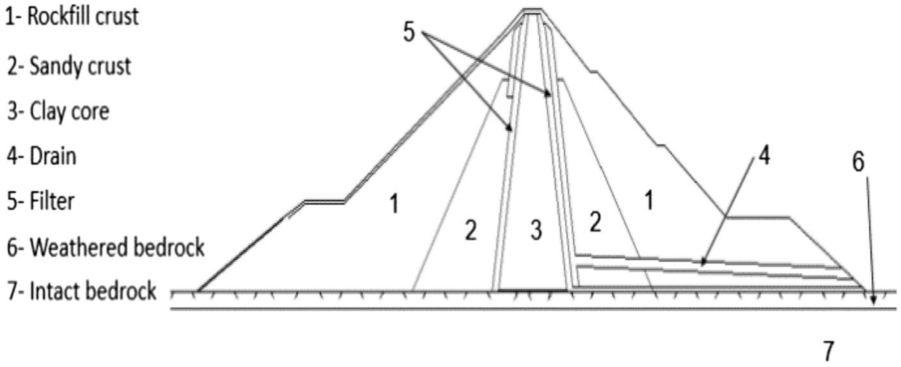

Fig. 1 General section of the Alborz dam

Table 2 Geotechnical parameters of clay core

Row	Parameter		Average	SD
1	Moist unit weight (kN/m^3)		18.5	0.4
2	Saturated unit weight (kN/m^3)		19.5	0.4
3	Internal friction angle (^{o})	UU	4	1
		CU	11	1.5
		CD	18	1.7
4	Cohesion (kPa)	UU	65	8.5
		CU	30	5
		CD	2	1

Table 5 Geotechnical parameters of rockfill

Row	Parameter		Average	SD
1	Moist unit weight (kN/m^3)		20	0.8
2	Saturated unit weight (kN/m^3)		21	0.9
3	Internal friction angle	$\sigma_3 = 1kg/Cm^2$	51	2.1
		$\sigma_3 = 3kg/Cm^2$	45	1.1
		$\sigma_3 = 5kg/Cm^2$	43	0.9
4	Cohesion (kPa)		0	0

Table 3 Geotechnical parameters of filter and drain

Row	Parameter	Filter		Drain	
		Average	SD	Average	SD
1	Moist unit weight (kN/m^3)	19.5	0.53	19.5	0.7
2	Saturated unit weight (kN/m^3)	20.5	0.55	20.5	0.6
3	Internal friction angle	37	2	38	2.1
4	Cohesion (kPa)	1	0.5	0	0

Table 4 Geotechnical parameters of sand and gravel

Row	Parameter		Average	SD
1	Moist unit weight (kN/m^3)		20	0.7
2	Saturated unit weight (kN/m^3)		21	0.8
3	Internal friction angle	$\sigma_3 = 3$ kg /Cm^2	42	2.1
		$\sigma_3 = 5kg/Cm^2$	41.5	1.9
		$\sigma_3 = 7kg/Cm^2$	40	1.8
4	Cohesion (kPa)		0	0

Coulomb model for other materials. As it can be seen the parameters are in good agreement with the normal distribution, the normal distribution of these factors is

Table 6 Geotechnical parameters of bedrock and weathered bedrock

Row	Parameter	Intact bedrock		Weathered bedrock
		Average	SD	Average
1	Moist unit weight (kN/m^3)	22.8	1.1	21
2	Saturated unit weight (kN/m^3)	23	1	22
3	Internal friction angle	30	1.3	24
4	Cohesion (kPa)	116	13	60

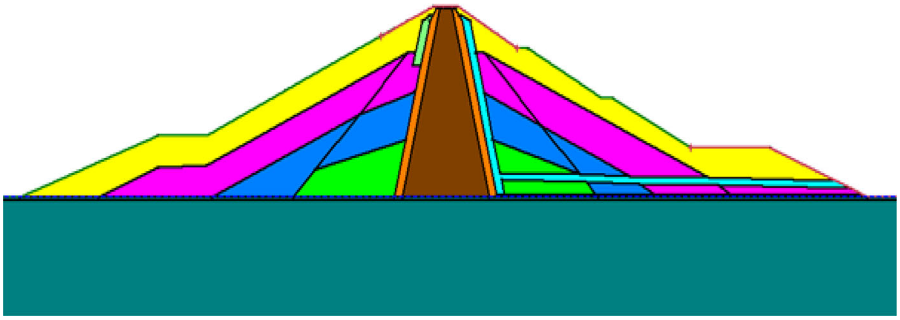

Fig. 2 General Section of the Alborz dam in SLOPE/W software

assigned for dam probabilistic analysis, where the graphs of some parameters are illustrated in Fig. 3a–e.

In order to determine the number of iterations in Monte Carlo method, the Eq. 2 is used. As can be seen, the repetition rate depends on the desirable design confidence level, the normal standard deviation and the number of input parameters (Krahn 2004). The number of Monte Carlo iteration in most engineering projects is very high. But experience has shown that if several thousands of Monte Carlo repetitions are performed, the result of the analysis will not be too sensitive to more iterations and will change very little (Geo Slope International 2008).

$$N = \left(\frac{d^2}{4(1-\varepsilon)^2}\right)^{m} \tag{2}$$

In Eq. 2, N is the number of computational steps of Monte Carlo simulation, d and ε are the normal deviation and the desired confidence level, respectively, and m is the number of random input variables. In this study, a normal standard deviation of 1.28 and the desired confidence level of 80% was selected based on the software suggestion (Abramson 2002). Owing to the large numbers of input parameters in this study, 300,000 Monte Carlo repeats are considered.

It should be noted that the most critical slip surface obtained from the deterministic and probabilistic analysis is not necessarily equal. In the modeling performed in SLOPE/W software, 9200 probable slip surfaces were investigated and 3 of the most critical surfaces of deterministic analysis were studied as the slide surfaces of the probability analysis. In order to make comparisons between methods, limit equilibrium methods, including Bishop, Morgenstern price and Spencer was used to perform the analysis. In this analysis, the safety factor of one (FOS = 1) is considered as the critical confidence factor, which means that in this study, only slope instability is the dam failure reason.

In this research, the reliability index method that is an analytical method was used to obtain the probability of improper performance. In order to calculate improper performance, the stability analysis of upstream and downstream slopes of the dam has been performed in the following cases:

1. End of Construction and before first filling in static and quasi-static conditions
2. Steady seepage from the normal water level in static and quasi-static conditions
3. Seepage from the maximum water level in static conditions
4. Steady seepage from mid-level in static and quasi-static conditions
5. The rapid drawdown in static conditions

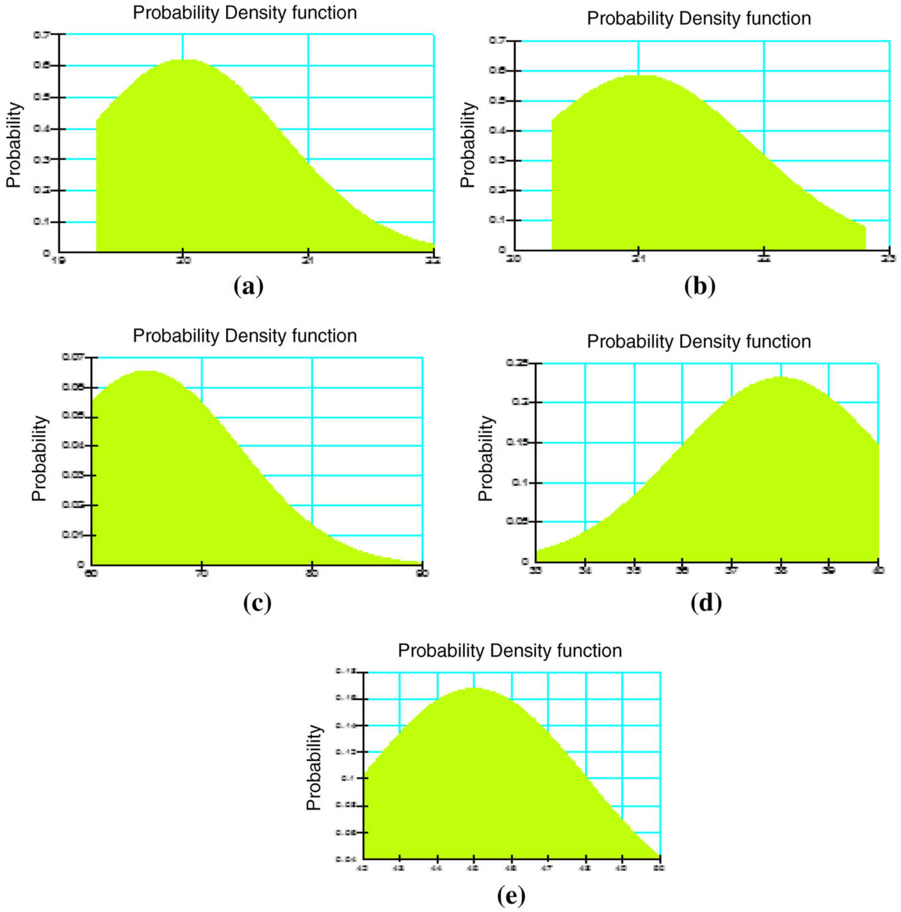

Fig. 3 Normal distribution of Geotechnical Characteristics: **a** crust saturated unit weight, **b** crust moist unit weight, **c** clay core cohesion, **d** internal friction angle of drain, **e** internal friction angle of rockfill. (Confining stress = 1 kg/cm^2)

The forces considered in the stability analysis include:

1. Soil forces including thrust and resistive forces.
2. Water load that accumulates in the dam reservoir.
3. Earthquake force.

One of the most important dam loads is the water load that accumulates in the dam reservoir. The probability of different water levels can be deduced from the information obtained at the time of dam utilization. The annual loading probability can be obtained from the graph of water level (x-axis) versus the annual probability (y-axis) as shown in Fig. 4. To obtain a probability, the occurrence prospect of a range with the center of the desired level must be calculated. The range boundaries are considered with an interval of ±0.5 m away from the desired level.

In this study, earthquake load analysis is done in quasi-static, and design acceleration is calculated according to the magnitude of the earthquake. The earthquake force in the stability analysis is applied using the earthquake coefficient. In this research, the earthquake coefficient is calculated from the proposed Pike method. Unlike conventional quasi-static methods, the factor of the earthquake is not only obtained from the maximum acceleration but also based on the maximum acceleration and the earthquake magnitude. According to the regional seismic report, the maximum acceleration is 0.6 and the magnitude of the earthquake is 7. The earthquake coefficient used in the software is equal to 0.15 obtained by the Pike method. With regards to the Alborz dam reports, the return period is 5000 years, the earthquake loading probability is calculated based on the maximum predictable value for the maximum horizontal

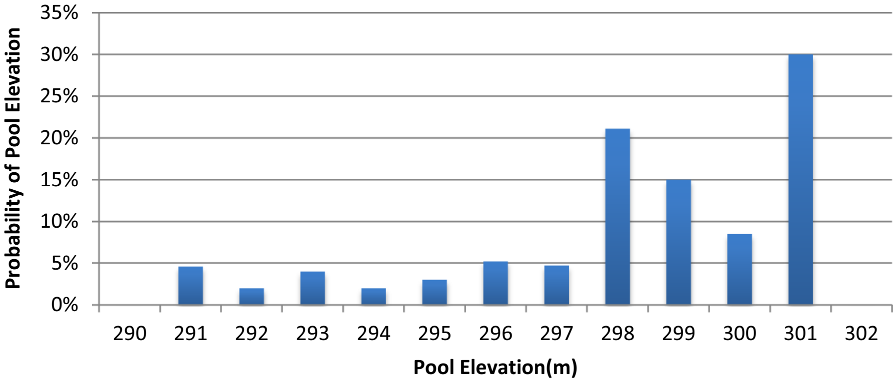

Fig. 4 Water level annual loading probability

acceleration of the area (Mazandaran Water Company 1998a, b).

The modeling is performed and investigated in the above-mentioned modes. It should be noted that at the end of construction and before first filling at the end of the construction process, the maximum active force arising from weight is applied to the slope, while there is insufficient time to drain the pore-water pressure due to the weight of the upper layers. For the stated reasons, clay core material characteristics were introduced into the software under unconsolidated-undrained (UU) conditions, and the Mohr-Coulomb behavioral model was assigned. Due to the high permeability of other materials, the drained condition is considered, and the water level is set equal to the ground level.

Also, in steady seepage condition from the normal level, only the downstream slope of the dam is analyzed because it is affected by the water seepage force. In this case, the water pressure is conservatively considered as hydrostatic pressure. The height of the water level in the upstream is 71 meters, which is the standard level for the dam reservoir.

A bilinear behavioral model is used for the clay core, and in the static and quasi-static analysis, the average envelope curve for the clay core is considered.

Like the steady seepage from the normal level, only the downstream slope is examined in the seepage from the maximum water level. But in this case, the quasi-static condition is not analyzed, because the probability of an earthquake occurrence and seepage from the maximum water level could be neglected the maximum water level in the reservoir is about 76 m. The behavioral model of the clay core is a bilinear with an average envelope curve.

In the steady seepage condition from the mid-level, only the upstream slope of the dam is examined. In order to find the critical mid-level using trial and error, different water levels are analyzed between the standard and minimum levels, and the desired level is obtained with the least confidence factor and the highest failure probability. The mid-level is set to 58 m.

Another case in slope stability analysis would be when a rapid drawdown in the water level for the upstream slope occurs. When the water level descends, the water level in the crust also falls but does not decrease rapidly in the core, which causes the reverse flow. This reverse flow affects the upstream slope. The bilinear behavioral model is considered for the clay core using a minimum envelope curve. The critical level is obtained using trial and error like the seepage condition from the mid-level that is about 60 m in this case, which is shown in Fig. 5 as a sample for modeling.

4 Modeling Validation

For simulation validation, the James Hydroelectric Project in northern Quebec, Canada, with approximately 50 km of embankments on soft and sensitive clay was used (Christian et al. 1994). concerning the extensive research on this project, the results of these studies have been used in many papers. The uncertainties and spatial variability of the soil properties of this project were presented by Ladd 1983. Christine et al. used this data to perform a probability analysis of the slope stability of this project (El-Ramly 2001). Therefore, reviewing this project and comparing the

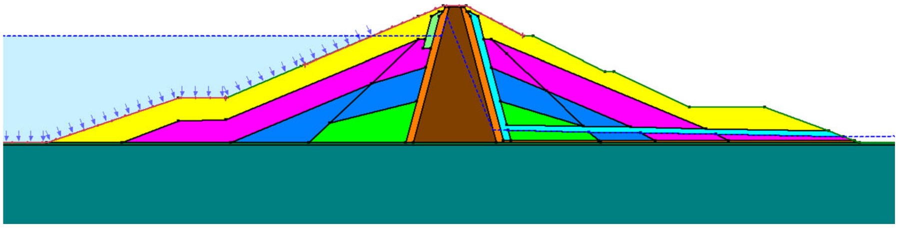

Fig. 5 Section of the Alborz dam in a condition of rapid drawdown (critical level = 60 m)

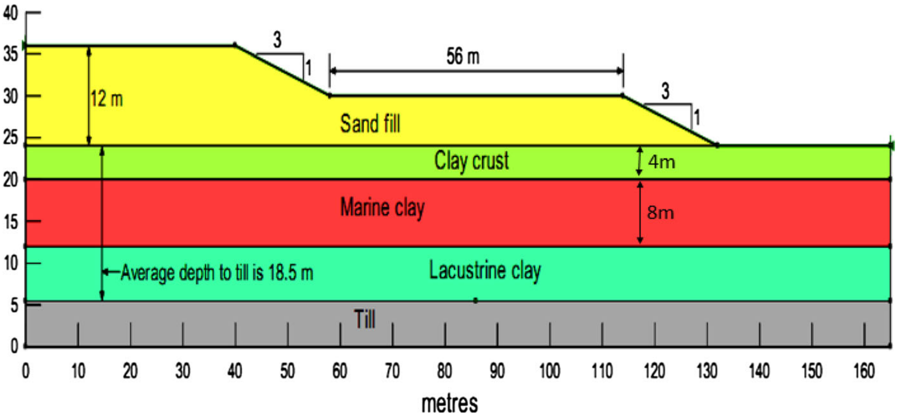

Fig. 6 General cut of James Project

Table 7 Average values and standard deviation of specific variables

Row	Variable parameter	Average	SD
1	Unit weight of sand (kN/m^3)	20	1
2	Sand friction angle (degree)	30	1
3	Clay layer thickness (m)	4	0.48
4	Marine clay cohesion (kPa)	34.5	8.14
5	Lacustrine clay cohesion (kPa)	31.2	8.65
6	Depth of Till layer (m)	18.5	1

results with the results of the mentioned works can be a good criterion for verification of the calculations.

The cutting view of the project design is shown in Fig. 6. The embankment height with a slope of 1: 3 is considered with a 56 m in length platform in the middle.

By investigating the project data, it was concluded that in the slope stability calculations, variable parameters such as unit weight of sand, clay layer crust thickness, marine clay undrained resistance, lacustrine clay undrained resistance and position of the till layer relative to surface should be considered (Ladd 1983; Christian et al. 1994). Table 7 shows the average and standard deviation of each of these variables (Table 8).

The normal distribution is assumed for all variables. Unfortunately, SLOPE/W software has some limitations in considering all of the mentioned variables. For example, the layer thickness parameter cannot be directly entered into the program, but these uncertainties must be applied indirectly. In studies by experts, a circular slide surface that has continued to the hard glacial layer is considered as a critical failure mode (Christian et al. 1994; El-Ramly 2001). This slip surface can be seen in Fig. 7.

The deterministic confidence factor calculated based on the Bishop's method by SLOPE/W software with the slip surface shown in Fig. 7 is 1.458, which is very close to the results of previous studies. Christian et al. (1994) calculated this value of about 1.453 and El-Ramly et al. (2002) around 1.46.

The analysis carried out by Christian et al. (1994) was of FOSM type with a confidence index of 2.66 (They did not present the results of their research as the probability of failure). The results obtained from the analysis performed by the SLOPE/W software and those of other mentioned experts indicate a good agreement.

Table 8 Comparison between different analyses

Row	–	SLOPE/W Analysis	EI-Ramly Analysis	Christian Analysis
1	Probability of failure (%)	0.48	0.47	–
2	Reliability index	2.54	2.32	2.66

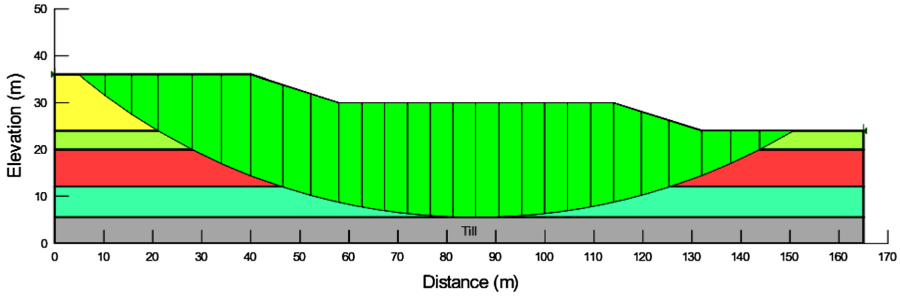

Fig. 7 Critical slip surface position

5 Analysis of Results

The slope stability analysis for the three critical slip surfaces was performed for all the conditions, as mentioned earlier, the results of which are presented for each case in Table 9, Table 10, Table 11, Table 12, Table 13, Table 14, Table 15, Table 16.

For the rapid drawdown in the water level by investigating the deterministic confidence coefficient at different water levels in Table 13, the level of 290 m has the lowest factor of safety. The analysis results for the critical slip surfaces of the downstream slope of the dam at the seepage from the mean level of 290 m with the lowest factor of safety are presented in Table 14.

Also, by investigating deterministic factor of safety under the static condition at different water levels at steady seepage from the mid-level, it was concluded that the level of 288 m had the lowest factor of safety. Table 16 illustrates the analysis results of the different water levels in a steady seepage from the mid-level.

The critical slip surfaces and the probability density function corresponding to the critical slip surfaces for the upstream slope of the dam in steady seepage from mid-level are illustrated in Figs. 8, 9, 10, and 11.

Table 9 Analysis results of downstream slope at the end of construction and before first filling

Row	Stability analysis method	Critical slip surfaces	Average of confidence coefficient	Reliability index	SD	Probability of slope failure	Confidence coefficient obtained from deterministic analysis
1	Bishop	1	1.54	4.96	0.11	0	1.51
		2	1.47	9.48	0.049	0	1.45
		3	1.51	11.8	0.044	0	1.5
2	Morgenstern-price	1	1.54	4.94	0.11	0	1.51
		2	1.46	9.44	0.049	0	1.44
		3	1.57	11	0.052	0	1.55
3	Spencer	1	1.54	4.94	0.11	0	1.51
		2	1.50	10.1	0.05	0	1.47
		3	1.55	11.2	0.05	0	1.54

Table 10 Analysis results upstream slope at the end of construction and before first filling

Row	Stability analysis method	Critical slip surfaces	Average of confidence coefficient	rEliability index	Standard deviation	Probability of slope failure	Confidence coefficient obtained from deterministic analysis
1	Bishop	1	1.6	5.3	0.11	0	1.57
		2	1.6	10.12	0.06	0	1.57
		3	1.57	11.5	0.05	0	1.55
2	Morgenstern-price	1	1.6	5.3	0.11	0	1.57
		2	1.61	9.8	0.063	0	1.59
		3	1.61	11.5	0.053	0	1.59
3	Spencer	1	1.61	5.36	0.11	0	1.58
		2	1.62	9.95	0.063	0	1.6
		3	1.63	11.7	0.053	0	1.6

Table 11 Analysis results of downstream slope at the steady seepage from the normal water level

Row	Stability analysis method	critical slip surfaces	Average of confidence coefficient	Reliability index	SD	Probability of slope failure	Confidence coefficient obtained from deterministic analysis
1	Bishop	1	1.51	5.24	0.1	0	1.48
		2	1.44	8.9	0.05	0	1.43
		3	1.47	20.1	0.023	0	1.46
2	Morgenstern-price	1	1.51	5.22	0.1	0	1.48
		2	1.43	8.16	0.53	0	1.42
		3	1.48	16.5	0.029	0	1.48
3	Spencer	1	1.51	5.22	0.098	0	1.49
		2	1.47	9.35	0.05	0	1.45
		3	1.49	20.43	0.024	0	1.49

Table 12 Analysis results of downstream slope at the steady seepage from the maximum water level

Row	Stability analysis method	Critical slip surfaces	Average of confidence coefficient	Reliability index	SD	Probability of slope failure	Confidence coefficient obtained from deterministic analysis
1	Bishop	1	2.11	12.55	0.089	0	2.08
		2	2.04	14.72	0.07	0	2.02
		3	2.07	16.93	0.063	0	2.05
2	Morgenstern-price	1	2.09	13.2	0.083	0	2.06
		2	2	14.24	0.071	0	1.99
		3	2.10	16.52	0.067	0	2.08
3	Spencer	1	2.12	12.33	0.091	0	2.09
		2	2.04	14.27	0.073	0	2.02
		3	2.10	16.68	0.066	0	2.08

Table 13 Analysis results in different water levels at the rapid drawdown

Row	Water level (m)	Deterministic factor of safety
1	301	1.92
2	295	1.9
3	**290**	**1.87**
4	288	1.88
5	285	1.89
6	280	1.91
7	275	1.95

Based on the analysis results, it can be said that the upstream slope of the Alborz dam is completely stable in all the conditions mentioned above and with regard to the minimum reliability index for the downstream slope in different loading modes at the end of construction and before the first filling is equal to 4.94, it seems that the dam's downstream slope is cautiously designed and the dam slope can be considered with more inclination. Since the Probability of failure is zero; therefore, the annual economic risk is zero for the upstream slope of the Alborz Dam, and only the accurate dam maintenance operation is required.

The results of the analysis show that in the most critical case, the Probability of failure is zero. In other words, all the Monte Carlo repetitions have a confidence coefficient higher than unit value. The reliability index at the most critical is also high. When the reliability value is more than 3.5, the slope of the dam has excellent performance and is stable. Also, when the reliability index is less than 1.5, the slope has a poor performance (U.S. Army Corps of Engineers 2006).

Table 15 Analysis results in different water levels at the steady seepage from mid-level

Row	Water level (m)	Deterministic factor of safety
1	301	1.92
2	295	1.91
3	290	1.89
4	289	1.89
5	288	1.88
6	287	1.89
7	285	1.91
8	280	1.92
9	275	1.94

Since existing uncertainties include both human and machine errors, it is assumed that as a result of high and uncontrolled explosions as well as the excessive density at the site, the gravel in the dam crust is finer and the mean internal coefficient of friction is reduced to 42 degrees. Here the conditions defined above are called critical conditions and the stability analysis of the upstream slope of the dam in the seepage state from different levels was also performed in critical conditions.

Figure 4 shows the probability of water level, and the probability of slope failure at different levels has presented in Table 17.

According to the analysis results, the upstream slope of the Alborz dam is unstable at seepage from

Table 14 Analysis results of upstream slope at the rapid drawdown

Row	Stability analysis method	Critical slip surfaces	Average of confidence coefficient	Reliability index	SD	Probability of slope failure	Confidence coefficient obtained from deterministic analysis
1	Bishop	1	2.13	11.8	0.096	0	2.10
		2	1.96	14.02	0.069	0	1.95
		3	2.14	18.8	0.061	0	2.12
2	Morgenstern-price	1	2.14	11.5	0.099	0	2.12
		2	1.89	13.7	0.065	0	1.87
		3	2.15	18.7	0.062	0	2.12
3	Spencer	1	2.15	11.26	0.103	0	2.12
		2	1.92	14.06	0.065	0	1.91
		3	2.11	18.8	0.059	0	2.09

Table 16 Analysis results of upstream slope at the steady seepage from mid-level

Row	Stability analysis method	Critical slip surfaces	Average of confidence coefficient	Reliability index	SD	Probability of slope failure	Confidence coefficient obtained from deterministic analysis
1	Bishop	1	1.27	4.25	0.064	0	1.25
		2	1.19	4.75	0.041	0	1.19
		3	1.23	5.74	0.041	0	1.22
2	Morgenstern-price	1	1.29	4.32	0.069	0	1.27
		2	1.18	4.18	0.044	0	1.17
		3	1.27	6.17	0.044	0	1.26
3	Spencer	1	1.30	4.36	0.068	0	1.28
		2	1.21	4.55	0.046	0	1.2
		3	1.28	6.7	0.043	0	1.27

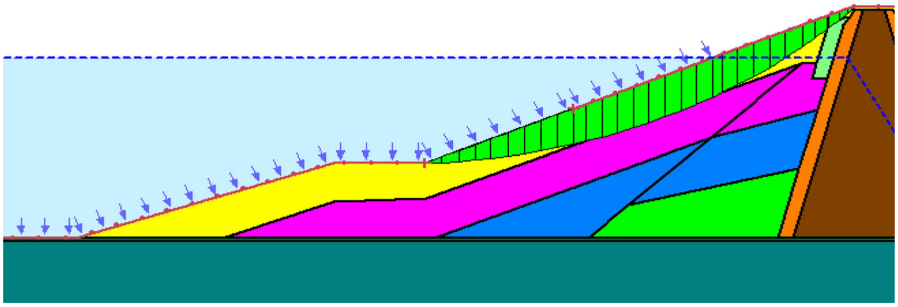

Fig. 8 The first critical slip surface for the upstream slope of the dam in steady seepage from mid-level (288 m)

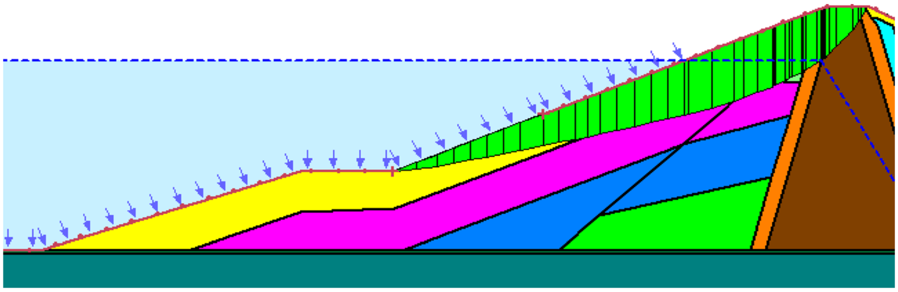

Fig. 9 The second critical slip surface for the upstream slope of the dam in steady seepage from mid-level (288 m)

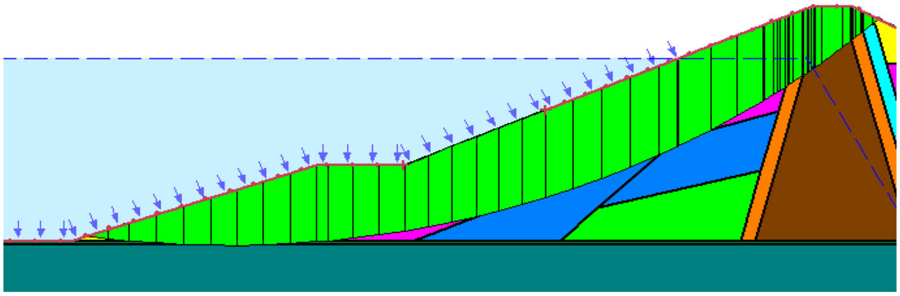

Fig. 10 The third critical slip surface for the upstream slope of the dam in steady seepage from mid-level (288 m)

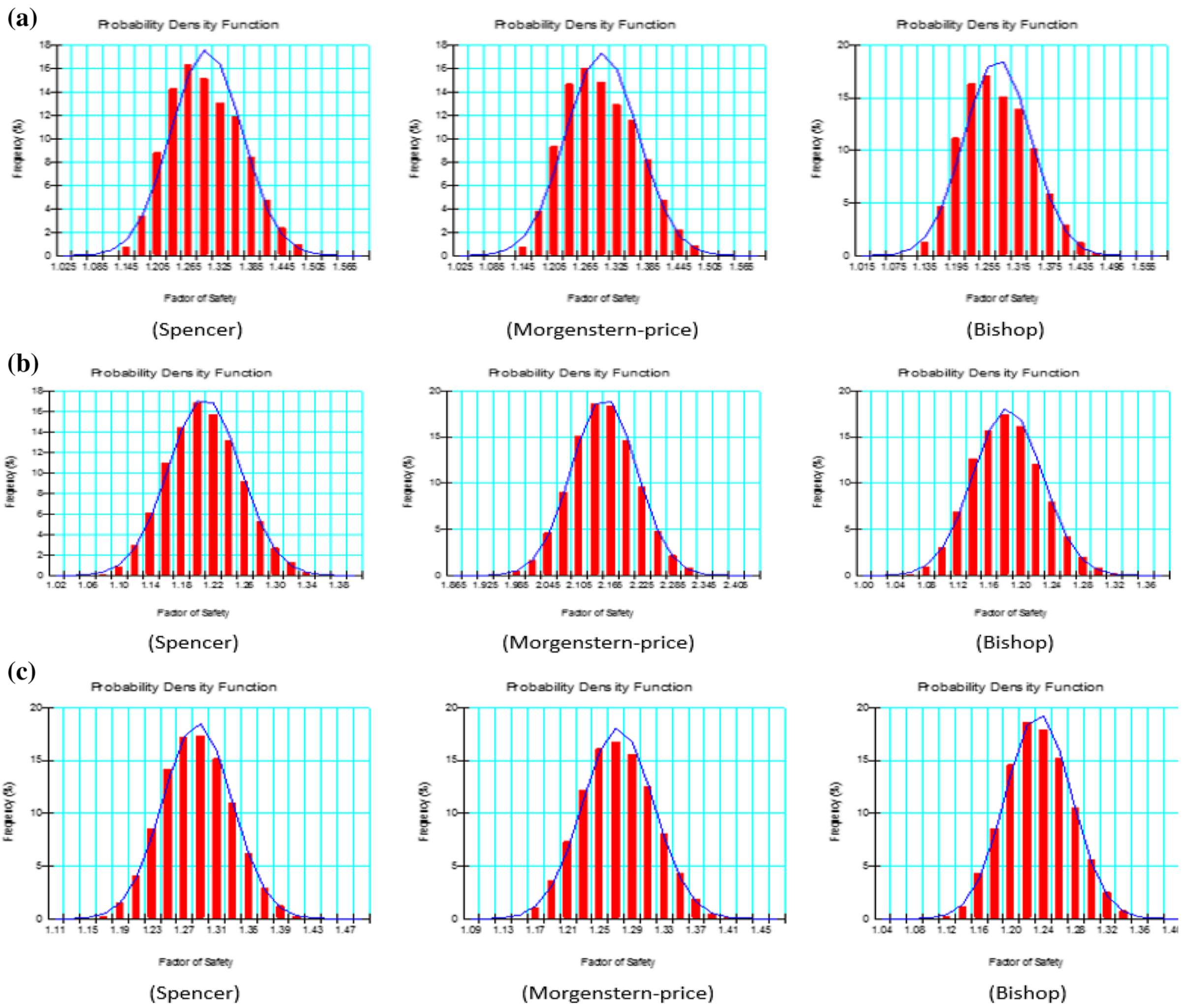

Fig. 11 Probability density function corresponding to the critical slip surfaces for the upstream slope of the dam in steady seepage from mid-level (288 m). **a** The first critical slip surface, **b** The second critical slip surface, **c** The third critical slip surface

different levels under critical conditions. Therefore, it can be said that the dam is harmful in this case, and it is necessary to take preventive actions.

6 Conclusions

Probabilistic analysis of the stability of the Alborz earth dam slope in different conditions was investigated using Monte Carlo method and the following results were obtained:

1. Based on the reliability index obtained in different conditions, it can be surely said that the downstream and upstream slope of the Alborz dam is stable. Although, it can be said with high confidence that the downstream slope of the Alborz dam is uneconomical.
2. The upstream slope of the Alborz dam is unstable under high and uncontrolled explosions conditions in steady seepage from different levels under quasi-static terms. Although the probability of critical conditions is very unlikely to occur, it can be said that the upstream slope has an excellent performance.
3. The deterministic factor of safety in the seepage from the mid-level under critical conditions is more than one (1.12), which is acceptable according to the design regulations in the country. But

Table 17 Probability of failure of the dam slope at different levels

Row	Water level (m)	Probability of water level	Probability of failure (%)
1	288	0	0.97
2	289	0	1.34
3	290	0	1.74
4	291	4.6	2.13
5	292	2	2.5
6	293	4	2.8
7	294	2	3.1
8	295	3	3.6
9	296	5.2	4.15
10	297	4.7	4.7
11	298	21.1	5.2
12	299	15	5.7
13	300	8.5	6.01
14	301	30	6.32
15	302	0	6.7

the reliability index value of 1.2 indicates that confidence in the obtained reliability factor is low, and there is a possibility of poor performance.

4. The reliability index of different critical levels represents that all methods like Bishop, Morgenstern price and Spencer should be used for the study, because the minimum reliability index in various analyses is always obtained from one of the methods.
5. As it could be seen in the dam analysis under critical conditions, it should be noted that human and machine errors do not directly enter the software, though it can have a significant impact on the dam's malfunction. The results of this study, like other researches, show that for valuable structures, the decisive estimation is not reliable enough, and the risk-based approaches provide better analysis without considering the uncertainties.

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