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ORIGINAL PAPER

Study on Stability Prediction of Pillars Based on Bieniawski Pillar Strength Formula: a Case of a Phosphate Mine

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Abstract In order to obtain the stability of pillars in Daqiao Phosphate Mine, the stability of pillars in goaf was studied by theoretical calculation. Based on the results of investigation and the calculation formula of pillar strength, a mathematical formula for calculating pillar stability was established to predict the stability of uninvestigated pillars. The effect of various factors on the stability of the pillar was analyzed by orthogonal test. The research results show that the pillar area, rock column area, and mining depth have the most significant effect on pillar stability. A mathematical formula for predicting pillar stability is constructed by means of dimensional grading method. The critical value of pillar factor is 0.06 in Daqiao Phosphate Mine. When the pillar factor is greater than 0.06, the pillar is stable. When the pillar factor is less than 0.06, the pillar is unstable. The reliability of theoretical calculation method of pillar factor is verified by numerical simulation. A mathematical formula for predicting the stability of pillars is proposed, which can be used as a reference for the stability analysis of goafs in similar mines.

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1 Introduction

With the exploitation of mineral resources, shallow minerals have been facing the problem of depletion. In order to ensure the demand of national production for minerals, it is gradually shifting from open-pit mining to deep mining. For deep mining orebodies, pillars are usually used to support the upper rock mass to ensure the stability of the whole mine stope. If the pillar fails and collapses, it may cause surface subsidence, collapse and other disasters. Therefore, the stability of pillars is related to the safety of production of the mine. It is necessary to study the stability of pillars in stope and determine the stability of pillars.

At present, many scholars studied the stability of stope pillars based on specific mine examples (Wang et al. 2019; Xiao et al. 2015; Qu et al. 2016; Gao et al. 2014; Guo et al. 2015; Xu et al. 2014; Ma et al. 2012; Salamon 1970; Maleki 1992; Heerden 1975). Some scholars have carried out the study of backfill method to improve the stability of goaf (Cao et al. 2018, 2019; Yilmaz et al. 2003, 2013; Yilmaz 2018; Xue et al. 2018; He et al. 2017). Zhang et al. (2017) chose the deep stope of a gold mine as the research background, and used FLAC3D software to establish three kinds of pillar size research programs. It was concluded that the stope was stable when the stope span was about 12 m and the pillar width was about 6 m. Zuo et al. (2016) used the FLAC3D software to establish three-dimensional mechanical analysis model of pillar, goaf and surrounding rock. After analyzing various schemes, it was concluded that the stability of goaf can be ensured by arranging three row pillars in the stope. Zhao and Liu (2017) proposed a pillar state recognition model based on the relationship between Gauss process machine learning algorithm and its main influencing factors. The stability of pillars was identified. Chen et al. (2017) put forward the method of a predicting pillar stability by using contribution rate and unascertained measurement theory. The accuracy of the method was verified by specific engineering examples. Wu et al. (2016) used cusp catastrophe theory, SURPAC, ANSYS and FLAC3D to analyze the pillar stability of a mine. Pillar instability is independent of rock strength. Luo et al. (2015) used point safety factor and orthogonal design experimental method, to analyze the stability of a metal ore pillar. And the quantitative relationship between influencing factors and pillar stability was obtained. Yang et al. (2017) used FLAC3D software to study the pillar stability of a goaf. When the safety pillar thickness was 20 m, the safety production of underground stope can be guaranteed. Through the analysis of the research carried out by the above scholars, it can be seen that pillar stability plays a very important role in mine safety production.

Daqiao Phosphate Mine in Sichuan, China was taken as an example, the range sensitivity analysis of orthogonal test was used to determine the important influencing factors. 3-D laser scanner was used to scan the goaf of Daqiao Phosphate Mine. The point cloud data of the goaf were obtained. The size of the pillar of the goaf was obtained through the point cloud data. The safety factor of each pillar was calculated by Bieniawski pillar strength formula (Bieniawski 1992). Based on the safety factor and influencing factors, a judging formula for predicting the stability of pillars was put forward. The critical factor of unstable pillars was calculated by using the formula. The critical factors of all unstable pillars were counted. Finally, the criteria for determining the stability of pillars in Daqiao Phosphate Mine were put forward. The stability of existing pillars in Daqiao Phosphate Mine can be found in the literature (Wang et al. 2019). The investigation and analysis of the pillar in the goaf and obtained the stability of the pillar are carried out. Based on the theoretical formula, the calculation formula for predicting the stability of ore pillar is put forward. It is verified by numerical simulation. The formula can provide reference for the stability analysis of goaf in similar mines.

2 Description of the Mine Site

Daqiao Phosphate Mine is located in the eastern side of Dadu River from Tianping to Ergutan, Daqiao Township, northern Xiao Liangshan (Fig. 1). The mining area is 2.4119 km^2 and the production scale is 1.5×10^4 t/a. The average thickness of ore body is 0.57 m, and the average grade is 25.41%. The mining depth is from 880 m to 1200 m.

Phosphorus deposits occur in the phosphorous dolomite of the upper part of Dengying Formation of the Upper Sinian. The specific stratum information is shown in Fig. 2. The occurrence of ore bodies is identical to that of strata, and the outcrop length is about 13,000 m intermittently. According to the different origins of phosphorus minerals, the ore body is divided into two parts, north and south, with F3 fault as the boundary. The number of the south ore body is I-1. The length of the ore body is 4250 m, and thickness is from 0.42 to 1.02 m. The inclination of the ore body is from 111° to 194°, and dip angle is from 5° to 10°. The bottom of the ore bed is gray-white thin to medium-thick layered phosphorous dolomite with chert strips. The roof is gray-white thin to medium-thick layered phosphorous dolomite. The ore bed is dark grey and grey black phosphorite. The thickness of surface and deep ore bed is not obvious, and the ore bed is stable. The surrounding rock of the ore bed is mainly phosphorous dolomite with clear boundaries with the orebody.

After nearly 40 years of mining, a large area of goaf has been formed in Daqiao Phosphate Mine. The area of new and old goaf is 728,557 m^2, accounting for 30.21% of the mining area, which is the main factor affecting the safety of Daqiao Phosphate Mine. In the early period of irregular mining, the remaining goafs are not treated or handled easily. With the development of mining, the distribution of goaf is more and more complex, the stability of goaf is difficultly

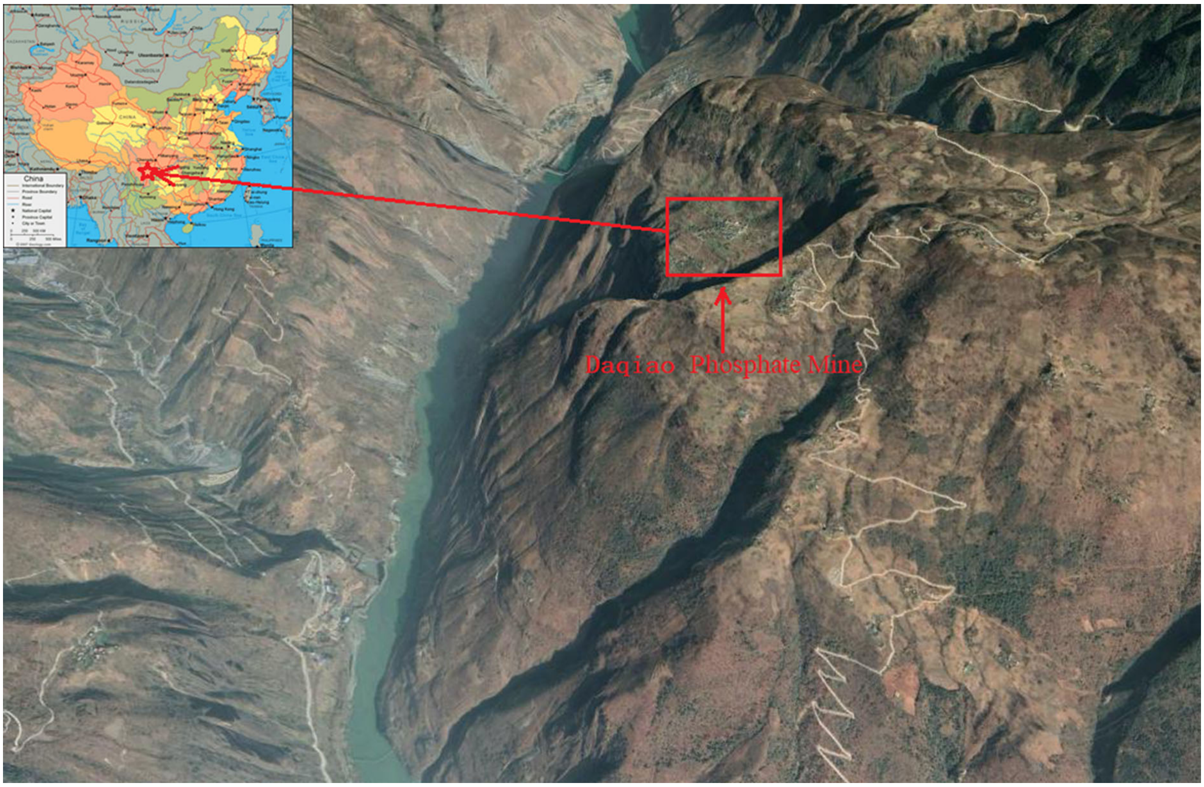


Fig. 1 Daqiao Phosphate Mine, Sichuan, China

obtained. Therefore, it is urgent to carry out the analysis of pillar stability.

3 Materials and Methods

3.1 Theoretical Calculation

According to the design of pillar in Daqiao Phosphate Mine, the strength design formula 1 recommended by Bieniawski is used as the calculation formula of pillar strength (Bieniawski 1992).

$$S_p = \sigma_c\left[0.64 + 0.36\left(W_p/h\right)\right]^\alpha \quad (1)$$

where α is constant, and its value is determined according to the width-height ratio of pillars; S_p is compressive strength of pillars, MPa; Wp is pillar width, m; h is pillar height, m; σ_c is rock compressive strength, MPa.

According to the research results (Bieniawski 1978), when the width-height ratio of a pillar is more than 5, $\alpha = 1.4$. When the width-height ratio is less than 5, $\alpha = 1.0$. According to the investigation of pillars in the goaf of Daqiao Phosphate Mine, the width-height ratio of pillars is generally less than 5, so $\alpha = 1.0$ is chosen.

The W_p can be calculated by following formula 2.

$$W_p = 4A_p/L_p \quad (2)$$

where L_p is the pillar perimeter, m; A_p is pillar area, m^2.

In the Fig. 3, the calculation diagram of the average stress of irregular columns is given (Wang et al. 2019). σ_p is expressed as the ratio of the weight of a rock pillar on a single pillar to the area of the rock pillar (formula 3).

$$\sigma_p = \gamma * H * \frac{A_p}{A_r} \quad (3)$$

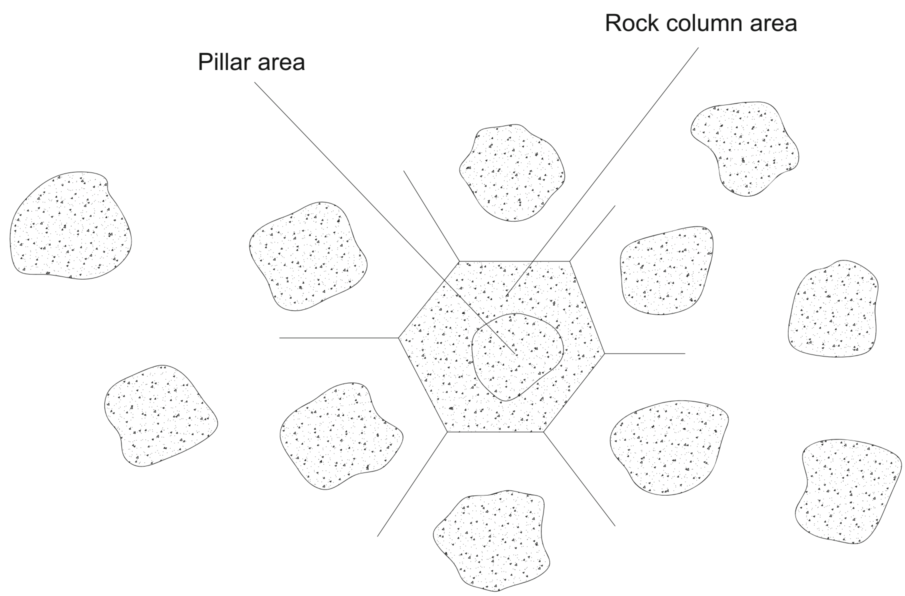
where γ is rock weight, kN/m^3; H is mining depth, m; A_p is pillar area, m^2; A_r is rock column area, m^2.

The mining condition of Daqiao Phosphate Mine shows that the pillar shape of goaf is irregular. When evaluating the stability of pillars, the safety factor of

Stratigraphic age	Strata thickness	Histogram	Lithology description
Quaternary (Q)	10-30m		Quaternary includes Holocene alluvial proluvial soil, residual Deluvial soil, etc.It is mainly pebble soil, distributed in the riverbed, terrace and gully bottom of Dadu River, with a thickness of 10-30m.
Permian (P_1)	50m		It is distributed in the mountain top area in the east of the mining area, which is dark gray clay rock with carbon chips and silt shale. The plant fossils are pseudointegrated with xiafulongmaxi formation, with a thickness of more than 50m.
Silurian (S_1)	40m		It is distributed in the high and middle mountain area, with gray black shale mingled with gray thin-layer bioclastic limestone, producing graptolite, brachiopod and other fossils, with a thickness of about 40m.
Ordovician $(O_3+O_2+O_1)$	385m		It is distributed in the high and middle mountain area. The lower part is gray and light gray medium-sized fine-grained feldspathic quartz sandstone with gray silty clay rock and brachiopod fossils. The upper part is light gray and gray medium layered dolomite with silty clay rock, with a thickness of about 385m.
Cambrian $(Є_3+Є_2+Є_1)$	580m		It is mainly composed of sandstone, argillaceous limestone and dolomite. At the bottom is grayish black argillaceous siltstone with banded grayish white marl, which is in integrated contact with the underlying Sinian Dengying Formation. In the middle and lower part, there are phosphorous bioclastic rocks, thin phosphorite layer, low grade, trilobite, Ostracoda, soft tongue snail and other fossils. The thickness is about 580m.
Sinian (Z)	365-598m		It is distributed in the narrow valley and steep cliff on both sides of Dadu River and consists of a set of carbonate rocks, which are gray, light gray, medium to thick layer grape like dolomite and silty sand like dolomite, with developed dissolution structure. The upper part is a set of gray, light gray thick bedded phosphorous microcrystalline dolomite with tunnel band and thin layer dolomite, which is the production horizon of phosphorite. The thickness is about 365-598m.

Fig. 2 Stratigraphic histogram of the Daqiao Phosphate Mine

Fig. 3 Pillar area and rock column area



pillars is calculated according to the following formula (4). The formula (4) is obtained by the formula (1), formula (2), and formula (2).

$$K = \frac{\sigma_c * A_p\{0.64 + 0.36[4A_p/(L_p * h)]\}}{\gamma * H * A_r} \quad (4)$$

where K is safety factor, γ is rock weight, kN/m^3; H is mining depth, m; A_p is pillar area, m^2; A_r is rock column area, m^2; L_p is the pillar perimeter, m; h is pillar height, m; σ_c is compressive strength, MPa.

According to the codes (China Metallurgical Construction Association 2014), the safety factor of the pillar is $K < 1.0$, the pillar is unstable. The $1.0 \leq K < 1.5$, the pillar is basically stable. The $K \geq 1.5$, the pillar is stable.

3.2 Orthogonal Range Analysis

According to the analysis of factors affecting the stability of the pillar of Daqiao Phosphate Mine, the following six factors are considered in this analysis:

1. Mining depth (H);
2. Rock column area (A_r);
3. Pillar area (A_p);
4. Pillar height (h);
5. Pillar perimeter (L_p);
6. Rock bulk density (γ).

According to the occurrence conditions, mining technology and allowable mining depth of Daqiao Phosphate Mine, the levels of six influencing factors are controlled within an appropriate range (Table 1). Five levels are selected for each factor in the test.

There are 6 factors in Table 1, 5 horizontal values for each factor. The L_{25} (5^6) table is used. The test plan and analysis results are shown in Table 2. The pillar safety factor is calculated by formula 3.

According to the range R of Table 2, the range of the pillar area is 18.62. The range of the rock column area is 15.15. The range of the mining depth is 11.29. The effect on the pillar stability of Daqiao Phosphate Mine are as follow: pillar area A_p > rock column area A_r > mining depth H > pillar perimeter L_p > pillar height h > rock bulk density γ. The pillar area, rock column area, and mining depth have the most significant influence on pillar stability. Under certain conditions of pillar mining depth and rock mass strength, the stability of goaf can be guaranteed by controlling pillar width and height.

3.3 Fitting Analysis of Pillar Safety Factor

The variations of the pillar area, rock column area, mining depth, pillar perimeter, and pillar height have the most significant effect on pillar stability. Only the above three important factors, such as the pillar area, rock column area, and mining depth, are fitted and analyzed. The fitting results of other factors are similar to the three factors.

In order to analyze the quantitative relation between the safety factor of pillar and the three main influencing factors and avoid the cross-influence among variables, the factors to be studied are taken as variables, and the other two main influencing factors are set as fixed values (Chen et al. 2018). Finally, the quantitative relation between pillar safety factor and single influencing factor is obtained. The relation between pillar safety factor and main influencing factors and fitting analysis are shown in Figs. 4, 5 and 6, respectively.

Table 1 Schemes of test factors for calculating pillar stability

Level	Six factors					
	Mining depth H (m)	Pillar height h (m)	Rock column area A_r (m^2)	Pillar area A_p (m^2)	Pillar perimeter L_p (m)	Rock bulk density γ (kN/m^3)
1	100	1	20	10	10	25
2	200	1.5	40	20	15	26
3	300	2	60	30	20	27
4	400	2.5	80	40	25	28
5	500	3	100	50	30	29

Table 2 Stability calculation scheme and result analysis table for pillars

Test	Six factors and safety factor						
	Mining depth H (m)	Pillar height h (m)	Rock column area A_r (m^2)	Pillar area A_p (m^2)	Pillar perimeter L_p (m)	Rock bulk density γ (kN/m^3)	Safety factor K
1	100	1	20	10	10	25	24.96
2	100	1.5	40	20	15	26	22.15
3	100	2	60	30	20	27	19.11
4	100	2.5	80	40	25	28	16.73
5	100	3	100	50	30	29	14.90
6	200	1	40	30	25	29	18.37
7	200	1.5	60	40	30	25	15.36
8	200	2	80	50	10	26	30.58
9	200	2.5	100	10	15	27	1.14
10	200	3	20	20	20	28	12.00
11	300	1	60	50	15	28	32.38
12	300	1.5	80	10	20	29	0.97
13	300	2	100	20	25	25	1.95
14	300	2.5	20	30	30	26	14.03
15	300	3	40	40	10	27	18.96
16	400	1	80	20	30	27	2.22
17	400	1.5	100	30	10	28	5.66
18	400	2	20	40	15	29	26.48
19	400	2.5	40	50	20	25	15.60
20	400	3	60	10	25	26	0.80
21	500	1	100	40	20	26	6.50
22	500	1.5	20	50	25	27	28.44
23	500	2	40	10	30	28	0.94
24	500	2.5	60	20	10	29	2.47
25	500	3	80	30	15	25	2.88
K_1	19.57	16.89	21.18	5.76	16.53	12.15	/
K_2	15.49	14.52	15.21	8.16	17.01	14.81	/
K_3	13.66	15.81	14.02	12.01	10.84	13.98	/
K_4	10.15	9.99	10.68	16.81	13.26	13.54	/
K_5	8.25	9.91	6.03	24.38	9.49	12.64	/
Range R	11.29	6.98	15.15	18.62	7.52	2.66	/

From Fig. 4, it can be seen that the pillar safety factor increases with the increase of pillar area. Through function fitting analysis, the relation between pillar safety factor and mining depth is negative exponential function. The fitting correlation coefficient is 0.9997, which shows that the relation between pillar safety factor and pillar area follows the increasing law of the following exponential function (formula 5).

$$K = a_1 * e^{b_1 * Ap} \tag{5}$$

where K is safety factor, Ap is pillar area, m^2; a_1, b_1 is constant, and mainly depend on rock column area, mining depth and other factors.

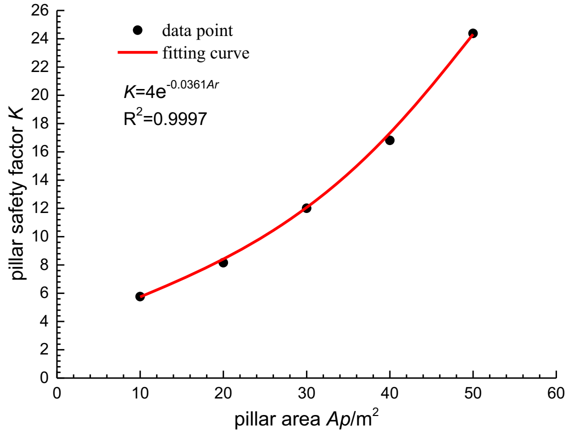


Fig. 4 Fitting analysis diagram of pillar area

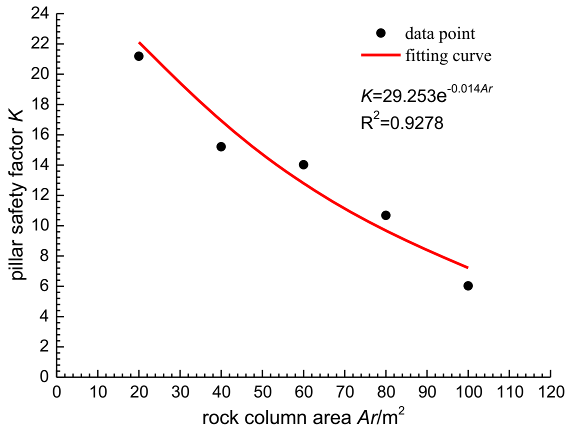


Fig. 5 Fitting analysis diagram of rock column area

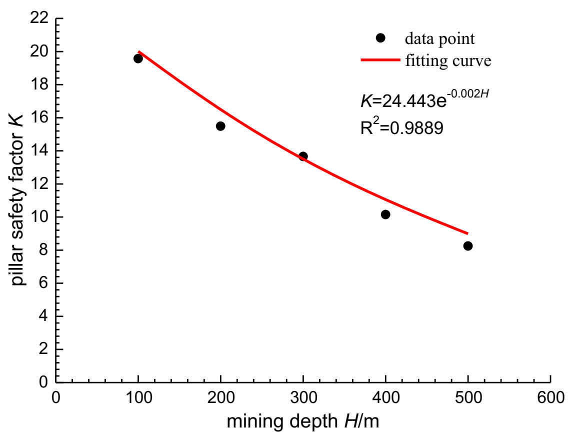


Fig. 6 Fitting analysis diagram of mining depth

As shown in Fig. 5, the pillar safety factor decreases with the increase of rock column area. Through function fitting analysis, the relation between pillar safety factor and rock column area is an exponential function. The final fitting correlation coefficient is 0.9278, which shows that the relation between pillar safety factor and rock column follows the descending law of the following exponential function (formula 6).

$$K = a_2 * e^{-b_2 * Ar} \tag{6}$$

where K is safety factor; Ar is rock column area, m^2; a_2, b_2 is constant, and mainly depend on pillar area, mining depth and other factors.

From Fig. 6, it can be seen that the pillar safety factor decreases with the increase of mining depth. Through function fitting analysis, the relation between pillar safety factor and mining depth is negative exponential function. The final fitting correlation coefficient is 0.9889, which shows that the relation between pillar safety factor and mining depth follows the descending law of the following exponential function (formula 7).

$$K = a_3 * e^{-b_3 * H} \tag{7}$$

where K is safety factor; H is mining depth, m; a_3, b_3 is constant, and mainly depend on pillar area, rock column area and other factors.

4 Experimental Results

4.1 Prediction of Pillar Stability

The goaf areas in Daqiao Phosphate Mine are large. The shape and size of pillars are different each other. The distribution is complex. More than 10 typical goafs are selected in this field investigation of goaf. The stability, area, height, circumference and ground pressure activities of pillars in goaf are investigated in detail. According to the knowledge of statistics, 436 pillars in 10 goafs investigated were used as sample data for data analysis.

Due to the large area of the goaf, it is difficult to fully investigate the pillar in each goaf. Because of the complexity of the buried depth, span and size of pillars, it is difficult to distinguish uninvestigated pillars stability. Further research is needed to distinguish uninvestigated pillars stability. According to the orthogonal range analysis, the effect on the pillar stability of Daqiao Phosphate Mine are as follow: mining depth $H >$ rock column area $A_r >$ pillar area $A_p >$ pillar perimeter $L_p >$ pillar height $h >$ rock bulk density γ. The formula 4 obtained by Bieniawski pillar strength formula shows that the stability of pillars is related to rock column area, pillar perimeter, pillar area, pillar height, orebody strength, mining depth and rock bulk density.

According to the field investigation, there is no significant difference in pillar height and orebody compressive strength. The overlying strata is dolomite, so there is no difference in rock bulk density. Because the pillar height, orebody strength and rock bulk density can be obtained from the investigated pillars, the parameters with little change of pillars are ignored. A mathematical formula for predicting pillar stability is constructed by means of dimensional grading method. The formula 8 is related to mining depth, rock column area, pillar perimeter, and pillar area. The pillar stability is defined as pillar factor δ.

$$\delta \propto \frac{A_p^2}{L_p * H * A_r} \tag{8}$$

where δ is pillar factor; A_p is pillar area, m^2; L_p is pillar perimeter, m; H is mining depth, m; A_r is rock column area, m^2.

The pillar factor δ is directly proportional to the square of pillar area, and inversely proportional to pillar perimeter, mining depth and rock column area. Among the effect of factors on the pillar stability, the size of pillar factor, the pillar perimeter, pillar area and mining depth are easy to obtain specific data in actual situation. However, the rock column area is difficult to obtain.

The rock column area of each pillar is basically the same, when the pillar layout in the stope is relatively standard. In order to make it easy for engineer to judge the stability of pillars, The influence factor of rock column area is deleted. The pillar information of 10 goafs obtained by scanning is brought into the formula 9, and the pillar factors are obtained. Through the research and analysis of 436 pillar factor data, the critical value of directly evaluating the safety and stability of pillars is obtained. Finally, it is found that pillars are generally in an unstable state when the pillar factor ≤ 0.06. The statistical results are shown in the Fig. 7. The red dots in the figure indicate unstable pillars, and the pillar factors ≤ 0.06. The black dots represent stable and basically stable pillars, and the pillar factors > 0.06.

For field workers, it is easy to judge the stability of a pillar in goaf. The data of pillar perimeter, pillar area and mining depth only need to be obtained. The stability of pillars can be judged by substituting them into the following formula 9. The formula can predict the stability of the pillars in the goaf which has not been investigated.

$$\delta = \frac{A_p^2}{L_p * H} \tag{9}$$

where δ is pillar factor; A_p is pillar area, m^2; L_p is pillar perimeter, m; H is mining depth, m.

Finally, the criteria for determining pillar stability are as follows: pillar stability occurs when pillar factor > 0.06; pillar instability occurs when pillar factor

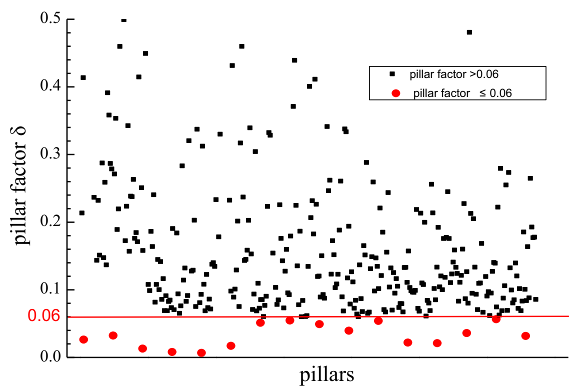


Fig. 7 Statistical analysis of pillar factors

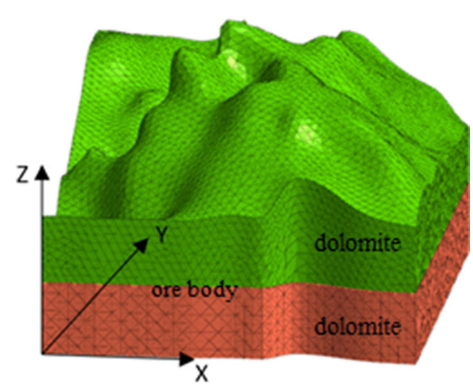


Fig. 8 Three-dimensional model

Table 3 Mechanical parameters of rock mass

Rock	Bulk density γ (kN/m^3)	Elastic modulus E (GPa)	Tensile strength σ_t (MPa)	Cohesive force C (MPa)	Internal friction angle φ (°)	Poisson ratio μ
Dolomite	27.4	14.964	6.5	1.8	38	0.24
Orebody	27.8	9.887	5.3	1.6	37	0.25

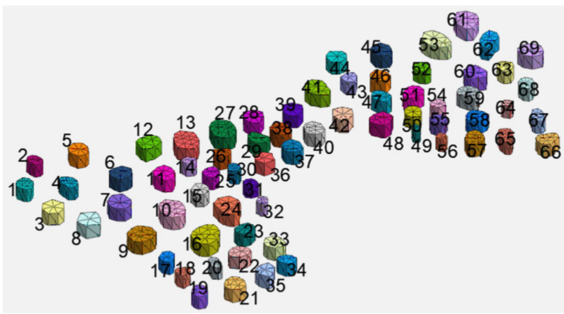


Fig. 9 9-2 Goaf pillars

$\leq$ 0.06. The formula is derived for Daqiao Phosphate Mine and has some limitations. It is only a reference for other mines to carry out similar research.

4.2 Validation of Numerical Simulation

In order to further analyze the stability of the goaf in Daqiao Phosphate Mine, MIDAS-GTS/NX finite element software was used to establish the three-dimensional model of the goaf pillar. The size of the three-dimensional model was consistent with the prototype. The model size is 3000 m in X direction, 5000 m in Y

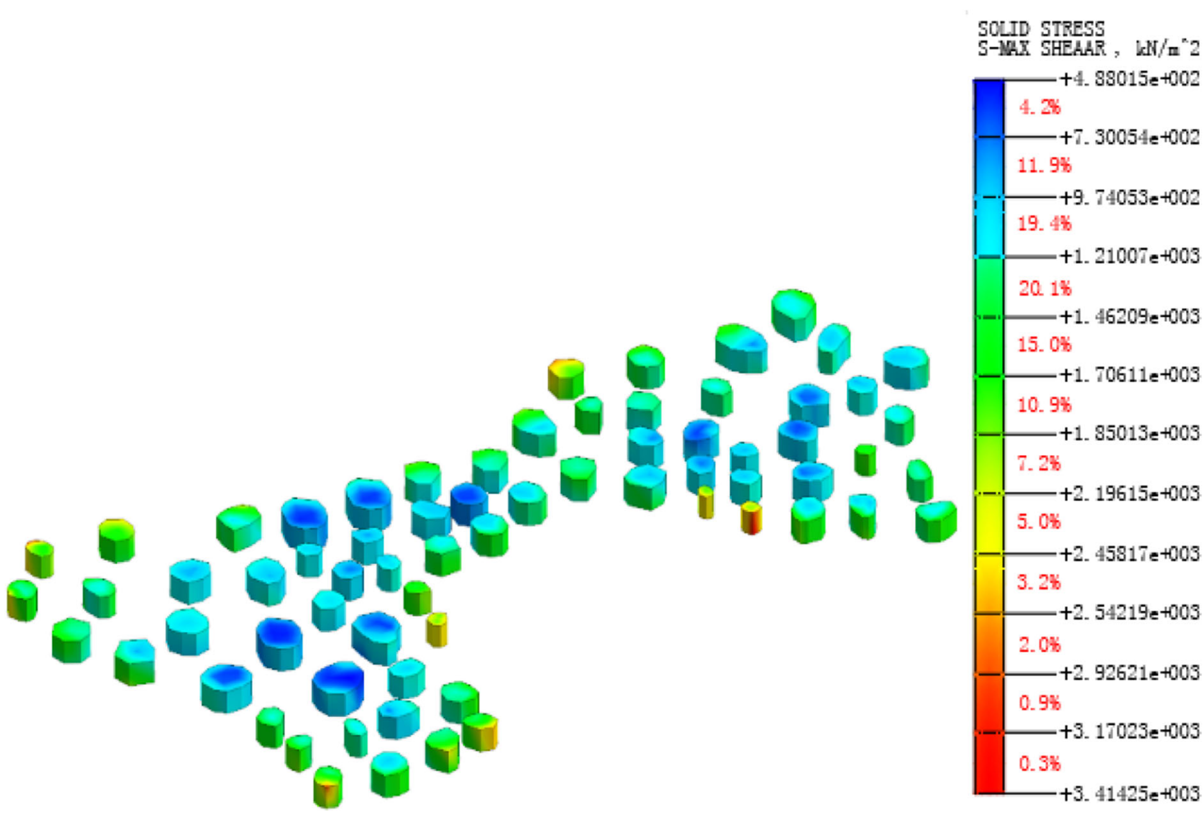


Fig. 10 Shear force of 9-2 goaf pillars

direction, and 1500 m in Z direction. The pillar and rock mass are simulated by solid element, and the Mohr–Coulomb yield criterion is adopted (Jiang et al. 2018). The burial depth of each goaf is established according to the actual situation. The three-dimensional model is shown in Fig. 8. In the three-dimensional model, the upper and lower parts are dolomite. The ore body is in the middle part. The mechanical parameters of rock are obtained through laboratory tests. The mechanical parameters of rock mass are obtained by empirical formulas and codes. (Table 3).

In order to verify the accuracy of the formula 9 for predicting pillar stability, 69 pillar models of mine 9-2 goaf (Fig. 9) are established. Nonlinear mechanical analysis of three-dimensional model was carried out. The shear force (Fig. 10) and axial force (Fig. 11) of pillar are obtained. The results calculated by formula 9 and numerical simulation are mutually validated to determine the reliability of the results calculated by formula 9.

The pillar factors of 69 pillars in 9-2 goaf are calculated by formula 9. The pillars with pillar factors $\leq$ 0.06 are given as shown in Table 4.

From Figs. 10 and 11, it can be seen that the maximum shear force of pillars in 9-2 goaf is 3.41 MPa and the maximum axial force is 6.76 MPa. Large shear and axial forces are produced in pillars 2, 5, 18, 19, 32, 34, 42, 44, 49 and 56. Table 3 shows that

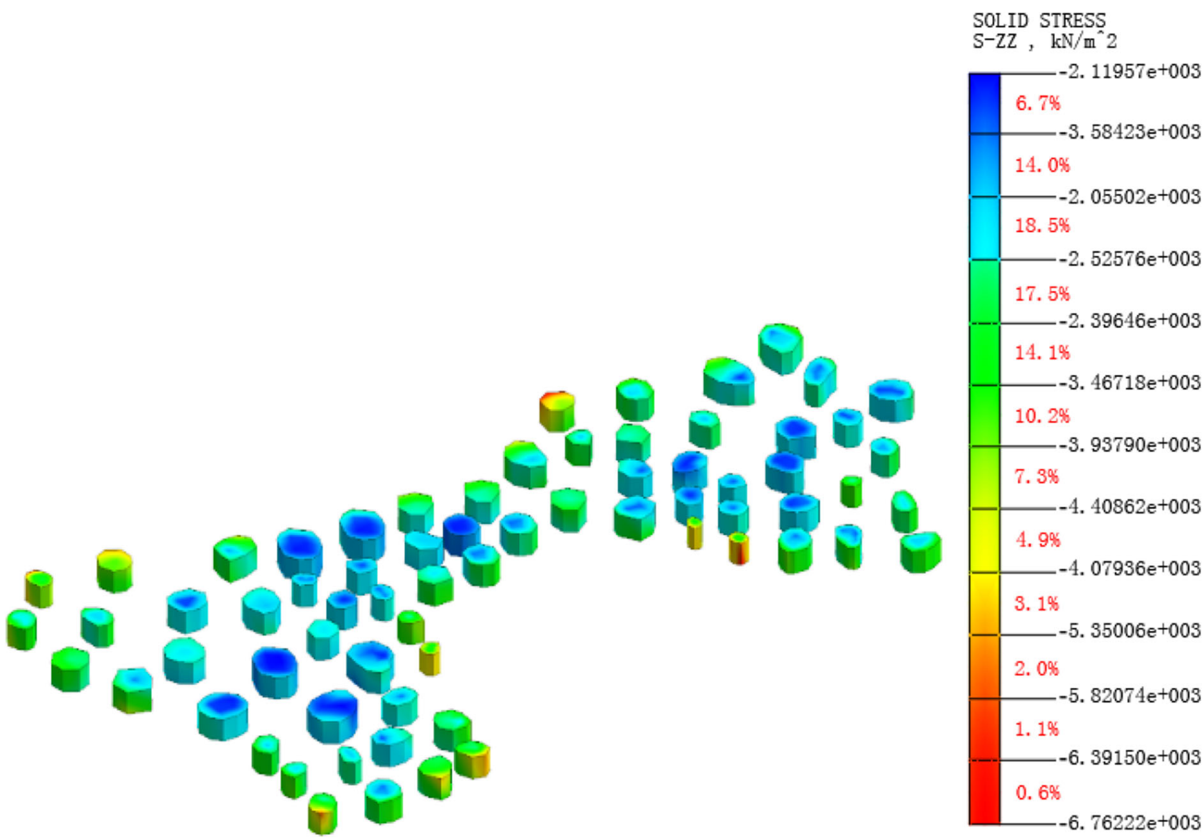


Fig. 11 Axial force of 9-2 goaf pillars

Table 4 Pillars with pillar factor $\leq$ 0.06

Pillar number	2	5	19	32	34	44	49	56
Pillar factor	0.051	0.047	0.021	0.034	0.058	0.043	0.012	0.044

the maximum compressive strength of the orebody is 5.3 MPa. From Fig. 11, it can be concluded that the pressure produced by some pillars is greater than the maximum compressive strength of the orebody. Pillars will be crushed. This phenomenon is in line with the actual situation on site. The results of numerical simulation and field investigation show that 2, 5, 19, 32, 34, 44, 49 and 56 pillars have been destroyed, which is basically consistent with the results calculated by theoretical calculation formula 9. It shows that the stability of pillars in goaf has been obtained by field investigation, theoretical calculation and numerical simulation. The reliability of the proposed formula for predicting pillar stability has been verified.

5 Conclusions

Based on a mine case, the stability of goaf pillars is studied. Based on the results of investigation and the calculation formula of pillar strength, a mathematical formula for calculating pillar stability was established to predict the stability of uninvestigated pillars. The conclusions are as follows:

1. The effect on the pillar stability of Daqiao Phosphate Mine are as follows: pillar area $A_p >$ rock column area $A_r >$ mining depth $H >$ pillar perimeter $L_p >$ pillar height $h >$ rock bulk density γ. The pillar area, rock column area, and mining depth have the most significant influence on pillar stability. The relation between pillar safety factor and pillar area, rock column area and mining depth are an exponential function.
2. A mathematical formula for predicting pillar stability is constructed by means of dimensional grading method. The formula is related to mining depth, pillar perimeter, and pillar area.
3. Through the research and analysis of 436 pillar factor data, the critical value of directly evaluating the safety and stability of pillars is obtained. Finally, it is found that pillars are generally in an unstable state when the pillar factor ≤ 0.06. The pillars are generally in a stable state when factors > 0.06.
4. According to the theoretical calculation formula of pillar stability prediction in goaf, numerical simulation method is used to verify the preparation of the formula prediction. The formula for predicting pillar stability is obtained from Daqiao Phosphate Mine. The formula can provide reference for the stability analysis of goaf in similar mines.

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