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An Improved Coal Permeability Model with Variable Cleat Width and Klinkenberg Coefficient

Gang Wang · Zhiyong Xiao (iD) **· Junhong Yu · Lu Zhang · Linlin Sun**

Abstract The evolution of permeability in coal seams plays an important role in the exploitation of coalbed methane, and the Klinkenberg effect is an important factor affecting the permeability. In most studies, the Klinkenberg coefficient is considered a constant although it is closely related to the cleat width which changes dynamically due to the competing processes of change in effective stress and adsorption induced deformation. In this work, based on the double strain spring model and the effective stress principle, a new stress–strain relationship is proposed for cleat space. In addition, the influence of the adsorption induced swelling effect on coal deformation is considered. By considering both stress-induced strain and adsorption-induced strain, the cleat width model and Klinkenberg coefficient model are established. Finally, we develop an improved permeability model with variable cleat width and Klinkenberg coefficient based on cubic law. This improved permeability model is compared with experimental data and several classical models under conditions of constant effective stress and constant confining stress. Furthermore, the relationship between Klinkenberg coefficient and pore pressure under different conditions is analyzed. Our results indicate that the improved apparent permeability model matches the experimental data well and has a wider range of applications.

Keywords Cleat width · Klinkenberg effect · Double strain spring · Apparent permeability

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List of symbols

σ_{eff}	Effective stress (MPa)
σ_{c}	Confining stress (MPa)
$K_{\text{f,e}}$	"Hard" part modulus of cleat space (MPa)
$K_{\text{f,n}}$	"Soft" part modulus of cleat space (MPa)
K_{bulk}	Bulk modulus of coal (MPa)
$\varepsilon_{\text{c,e}}$	Engineering strain for the cleat width
$\varepsilon_{\text{c,n}}$	Natural strain for the cleat width
ε_{ms}	Matrix block strain caused by adsorption
ε_L	Langmuir strain or maximum adsorption strain
b_{e}	Stressed cleat width for the "hard" part
$b_{0,\text{e}}$	Unstressed cleat width for the "hard" part
b_{n}	Stressed cleat width for the "soft" part
$b_{0,\text{n}}$	Unstressed cleat width for the "soft" part
b_{eff}	Influence of "hard" and "soft" parts on cleat width under the action of effective stress
b_{ms}	Cleat deformation caused by the adsorption strain of matrix block
b_0	Initial averaged cleat width

a_0	Initial width of the matrix block
Δa_s	Width change of matrix block
k_{ab}	Absolute permeability of coal (mD)
k_{ap}	Apparent permeability of coal (mD)
B	Klinkenberg coefficient
C_f	Cleat compressibility (MPa^{-1})
f	Contribution of adsorption to matrix deformation
S_f	Sensitivity of sorption induced strain to the cleat width
ϕ_0	Fracture porosity of the coal sample
P	Pore pressure (MPa)
P_L	Langmuir pressure (MPa)
μ	Dynamic viscosity of gas (cp)
ω	Throat width or cleat width
M	Molecular weight (g/mol)
α	Biot coefficient

Subscript

0	Initial state
CH$_4$	Methane
na	No-adsorption gas

1 Introduction

Coal is often considered as a dual porosity medium containing cleats and pores (Wang et al. 2012; Liu et al. 2011; Wang et al. 2015). Coalbed methane (CBM) exists in cleats and pores in the form of free or dissolved state, and is adsorbed in the organic matter in coal (Cui et al. 2009). In the process of coal mining, CBM has long been considered as a disaster material causing gas outbursts, gas blasting, etc. (Sobczyk 2014; Kim et al. 2014). However, with the development of mining technology and the improvement of environmental protection, coalbed methane, as a clean gas, is being actively exploited by countries around the world (Ji et al. 2015). Many challenges exist for CBM development, and one of the major difficulties is the low permeability and poor connectivity of coal seams. In order to improve the recovery of CBM and reduce the dependence on conventional energy, it is more and more important to study the permeability of coal seam during production.

The main migration channels of gas in coal seams are cleats and pores, which are different from conventional reservoirs. Coal deformation is affected by the combination of mechanical and chemical effects (Wang et al. 2018b; Cao et al. 2016). For deformation caused by mechanical loading, Biot (1941) proposed that pore pressure could not completely offset the influence of confining stress, and developed the effective stress principle by introducing Biot coefficient. For deformation caused by adsorption, Langmuir (1918) proposed an isotherm equation to describe the adsorbed gas content as a function of pressure and this equation was later extended to link the effect of adsorption on swelling for coal and shale. Importantly, these two processes compete with each other in controlling the net deformation and permeability. During CBM production as the pore pressure in the coal decreases, on one hand, the effective stress increases and the pore volume is compressed, and as a result the permeability decreases. On the other hand, the gas desorption from the matrix to the cleat causes the coal matrix to shrink and thus widens the cleat, which increases the permeability of the system. The net permeability of coal depends on the competitive action of these two processes. However, a large number of experimental studies have shown that the influence of adsorption strain is overestimated when calculating permeability in coal seam (Connell et al. 2010; Liu and Rutqvist 2009). Therefore, the influence factor f is introduced to evaluate the contribution of adsorption to deformation, and the value of f is 0–1(Zhou et al. 2016; Guo et al. 2014). In addition, when the pore pressure is low, the gas migration in the coal seam is not completely the same (Anez et al. 2014; Ashrafi Moghadam and Chalaturnyk 2014; Li et al. 2018; Cui et al. 2018; Si et al. 2018; Wang et al. 2019; He et al. 2017). When the average free path of gas molecules is the same order of magnitude as the cleat width, the molecular velocity at the pore wall is not zero, causing the permeability measured in the laboratory greater than the absolute permeability of coal sample, this phenomenon is called the Klinkenberg effect (Klinkenberg 1941). In order to better understand the flow state of gas in cleats, Knudsen (1909) proposed the Knudsen number to determine the flow regime, which was defined as the average free path of molecules divided by the pore size. When $0.001 < Kn < 0.1$, the flow in the cleat is completely in the state of slip flow (Anez et al. 2014). In this paper, the effects of effective stress, adsorption induced swelling, and Klinkenberg effect on permeability are studied.

In order to study the evolution of permeability in coal seams, a large number of permeability models have been proposed in the literature. Seidle and Huitt (1995) put forward a model to explain the relationship between permeability and effective stress, ignoring the elastic property of coal. Palmer and Mansoori (1998) developed a model describing permeability changes caused by adsorption strain. Shi and Durucan (2004) proposed a permeability model based on the uniaxial strain condition, which can match most experimental data. Based on the different stiffness of cleats and matrix, Wang et al. (2012) established a permeability model by assuming that the system is transformed from globally unconstrained to locally constrained. Liu et al. (2009) proposed a dual-strain spring model, suggesting that different Hooke's law should be applied to calculate the deformation of coal seams when the fracture and matrix are subjected to effective stress. Liu and Rutqvist (2009) then considered the influence of adsorption strain on the basis of the dual-strain spring model, and considered that there was an internal interaction between matrix and fracture. Chen et al. (2012) considered that part of the adsorption strain had an effect on cleat width, and introduced the concept of sensitivity of adsorption strain to cleat width to describe the effect of adsorption strain on permeability.

Due to the influences of effective stress and adsorbed strain, the cleat width is constantly changing, which causes the Klinkenberg effect to change as well. The Klinkenberg coefficient, an important parameter controlling the Klinkenberg effect, is closely related to the cleat width and can be calculated from the image of the reciprocal of apparent permeability and mean pore pressure (Harpalani and Chen 1997). When considering the Klinkenberg effect, only a few studies have taken the variation of cleat width into account. Wang et al. (2014) only considered the effect of adsorption-induced strain when the variable Klinkenberg coefficient model was established. Zhou et al. (2016) developed a cleat width model based on effective stress principle,.However, his model lacked further validation.

The effects of variable Klinkenberg effect and variable cleat width on permeability are studied in this paper. Based on previous works, this paper argues that different Hooke's law should be applied to cleat space when they are subjected to same effective stress, and that the cleat width is affected by both matrix deformation and fracture deformation under the action of effective stress. On this basis, the adsorption and its induced swelling effect is considered. Different from the previous permeability models, we believe that the adsorption and swelling only partially affects the coal deformation. Therefore, the sensitivity of expansion and deformation on cleat width is introduced to describe this change. In addition, the Klinkenberg effect cannot be ignored because the pore pressure used in the laboratory and the actual pore pressure in the field are often low. Considering the fact that the Klinkenberg coefficient varies with different gas migration, an improved Klinkenberg coefficient model is established. Then an improved apparent permeability model is derived based on the cubic law and its fidelity is subsequently verified under the conditions of constant effective stress and constant confining stress.

2 Development of Fracture Aperture Model Under the Influence of Effective Stress and Gas Adsorption

2.1 Evaluation of Effective Stress Effect

Coal can be considered as a dual porosity medium containing micropores, mesopores and macropores. Gas is stored in and flows through such pore systems. Due to the unique deformation characteristics of the cleat, its width in the coal seam is easily changed, which has a significant impact on the flow behavior and permeability in the coal. Compared with the fracture system, the permeability of coal matrix can be as much as 8 orders of magnitude smaller than that of the fracture system (Wang et al. 2018a; Liu and Rutqvist 2009). Therefore, the permeability of coal matrix is sometimes neglected in numerical models. Liu et al. (2009) argued that different Hooke's law should be applied to rock regions with significant differences in stress–strain behavior. We expanded the concept by dividing cleat space into "hard" and "soft" parts, these two parts follow different Hooke's law when subject to the same stress. This theory is similar to previous works (Chen et al. 2012; Berryman 2006; Wang et al. 2018c).

Natural strain (volume change divided by rock volume under current stress state) is used to represent "soft" part deformation, and engineering strain (volume change divided by rock volume under stress-free state) is used to represent "hard" part deformation, as

shown in Fig. 1. These two parts follow different Hooke's law. We assume that the cleats are embedded into the rock sample to undergo the effect of effective stress σ_{eff}. For the "hard" part, Hooke's law can be expressed as

$$d\sigma_{\text{eff}} = K_{\text{f,e}} d\varepsilon_{\text{c,e}} \tag{1}$$

where $K_{\text{f,e}}$ is the "hard" part modulus of cleat space, $\varepsilon_{\text{c,e}}$ is the engineering strain for the cleat width, which can be defined as

$$d\varepsilon_{\text{c,e}} = -\frac{db_{\text{e}}}{b_{0,\text{e}}} \tag{2}$$

where b_{e} is the stressed cleat width for the "hard" part, and $b_{0,\text{e}}$ is the unstressed cleat width for the "hard" part. This is done by applying a boundary condition: $b_{\text{e}} = b_{0,\text{e}}$ when $\sigma_{\text{eff}} = 0$. By simplifying and integrating Eqs. (1) and (2), we obtain

$$b_{\text{e}} = b_{0,\text{e}}\left(1 - \frac{\sigma_{\text{eff}}}{K_{\text{f,e}}}\right) \tag{3}$$

For the "soft" part, Hooke's law can be expressed as

$$d\sigma_{\text{eff}} = K_{\text{f,n}} d\varepsilon_{\text{c,n}} \tag{4}$$

where $K_{\text{f,n}}$ is the "soft" part modulus of cleat space, $\varepsilon_{\text{c,n}}$ is the natural strain for the cleat width, which can be defined as

$$d\varepsilon_{\text{c,n}} = -\frac{db_{\text{n}}}{b_{\text{n}}} \tag{5}$$

where b_{n} is the stressed cleat width for the "soft" part. This is done by applying a boundary condition: $b_{\text{n}} = b_{0,\text{n}}$ when $\sigma_{\text{eff}} = 0$. By simplifying and integrating Eqs. (4) and (5), we obtain

$$b_{\text{n}} = b_{0,\text{n}} \exp\left(-\frac{\sigma_{\text{eff}}}{K_{\text{f,n}}}\right) \tag{6}$$

where $b_{0,\text{n}}$ is the unstressed cleat width for the "soft" part. The deformation of cleat width in non-absorbent media are composed of two components: "hard" part and "soft" part, the volumetrically averaged cleat width (b) is given by

$$b_0 = b_{0,\text{e}} + b_{0,\text{n}} \tag{7}$$

Through the above analysis, it can be obtained that the cleat width b_{eff} under effective stress σ_{eff} is

$$b_{\text{eff}} = b_{\text{e}} + b_{\text{n}} \tag{8}$$

Combining Eqs. (3), (6) and (8), the influence of "hard" part and "soft" part on cleat width under the action of effective stress can be obtained as

$$b_{\text{eff}} = b_{0,\text{e}}\left(1 - \frac{\sigma_{\text{eff}}}{K_{\text{f,e}}}\right) + b_{0,\text{n}} \exp\left(-\frac{\sigma_{\text{eff}}}{K_{\text{f,n}}}\right) \tag{9}$$

Since the modulus of "hard" part $K_{\text{f,e}}$ is much larger than the modulus of "soft" part $K_{\text{f,n}}$ and the "soft" part deformation is much larger than the "hard" part deformation under high effective stress, $1 - \sigma_{\text{eff}}/K_{\text{f,e}} \approx 1$ is obtained. The above equation can be simplified to:

$$b_{\text{eff}} = b_{0,\text{e}} + b_{0,\text{n}} \exp(-C_{\text{f}} \cdot \sigma_{\text{eff}}) \tag{10}$$
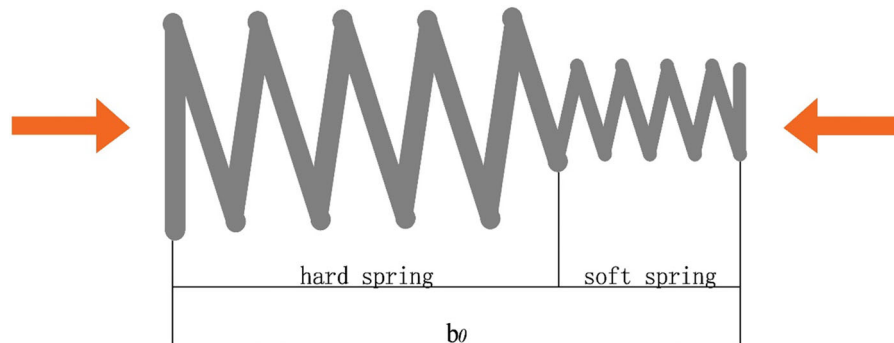
Fig. 1 Dividing cleat space into "hard" and "soft" parts, natural strain is used to represent "soft" part deformation; engineering strain is used to represent "hard" part deformation

where $C_f = 1/K_{f,n}$ is defined as the cleat compressibility in coal.

2.2 Evaluation of Strain Effect Induced by Adsorption

The transport of gas in coal seams is obviously different from that of other rocks, because during the transport, there are processes of gas adsorption or desorption and coal seam expansion or shrinkage. The shrinkage and swelling of coal matrix may increase or decrease the width of cleats, which will affect the porosity and permeability of fractures in the reservoir. A large number of studies have shown that there is a close relationship between coal volume expansion and gas adsorption capacity (Wang et al. 2011, 2018a; Li and Feng 2015). The relationship between them can be expressed by the Langmuir type equation

$$\varepsilon_{ms} = \frac{\varepsilon_L P}{P_L + P} - \frac{\varepsilon_L P_0}{P_L + P_0} \qquad (11)$$

where ε_{ms} represents the strain caused by adsorption on the matrix block, ε_L represents the maximum strain caused by adsorption, P_L represents the Langmuir pressure, and specifically represents the pore pressure when the measured strain equals $0.5\varepsilon_{ms}$, P represents the pore pressure, and P_0 represents the initial pore pressure. The width change of matrix block Δa_s can be written as

$$\Delta a_s = a_0 d\varepsilon_{ms} \qquad (12)$$

where a_0 is the initial width of the matrix block.

Under the condition of constant volume, the expansion and contraction of matrix are directly regulated by the decrease and increase of cleat width. In the literature, it is usually assumed that the swelling strain is completely regulated by the change cleat width (Wang et al. 2012; Karacan 2007). However, this assumption has been considered to overestimate the impact of swelling strain. Therefore, a factor f was introduced to estimate the contribution of adsorption-induced strain to cleat width, as shown in Fig. 2. The matrix width is considered to be unit 1, so the contact part is equivalent to f, and the value ranges from 0 to 1. Therefore, the variation of cleat width caused by the adsorption strain of matrix block can be expressed as

$$b_{ms} = -f\Delta a_s \qquad (13)$$

By using the cubic model, combining Eqs. (11), (12) and (13), and Eq. (13) can then be rewritten as

$$b_{ms} = -f\frac{3b_0 \cdot \varepsilon_L}{\phi_0}\left(\frac{P}{P_L + P} - \frac{P_0}{P_L + P_0}\right) \qquad (14)$$

defining the evolution of cleat width as a function of sorption contributing factor, initial cleat width, initial porosity, the material coefficients of the Langmuir strain and pressure, and the change in gas pressure. This is the component response required to determine the influence of shrinkage deformations on permeability evolution during CBM production.

2.3 Development of Cleat Width Model

As shown above, we have derived the variation of cleat width under effective stress and sorption-induced strain respectively. However, as we know, in the process of CBM exploitation, due to gas migration and pore pressure changes, the strain caused by desorption and effective stress will affect the change of cleat width simultaneously. Combining Eqs. (10) and (14), the cleat width under the effects of effective stress and desorption can be expressed as

$$b = b_{eff} + b_{ms} = b_{0,e} + b_{0,n}\exp(-C_f \cdot \sigma_{eff})$$
$$- f\frac{3b_0 \cdot \varepsilon_L}{\phi_0}\left(\frac{P}{P_L + P} - \frac{P_0}{P_L + P_0}\right) \qquad (15)$$

Based on the double-strain spring model, the "hard" part deforms less under the action of stress, then $b_{0,e} \ll b_0$, and $b_{0,e} \approx 0$, therefore Eq. (15) can be simplified as

$$b = b_0\left[\exp(-C_f \cdot \sigma_{eff}) - 3S_f\varepsilon_L\left(\frac{P}{P_L + P} - \frac{P_0}{P_L + P_0}\right)\right] \qquad (16)$$

where we define $S_f = f/\phi_0$ to describe the sensitivity of sorption induced strain to the cleat width. Under the same boundary conditions, the larger the S_f, the greater contribution of sorption-induced strain to the cleat width. In other words, the change of cleat width is more sensitive to sorption induced strain when the contributing factor is large. In the following, we will consider S_f as a variable and adjust it to match the experimental data.

Fig. 2 The adsorption strain of the contact part affects the matrix deformation, and the other part affects the cleat width
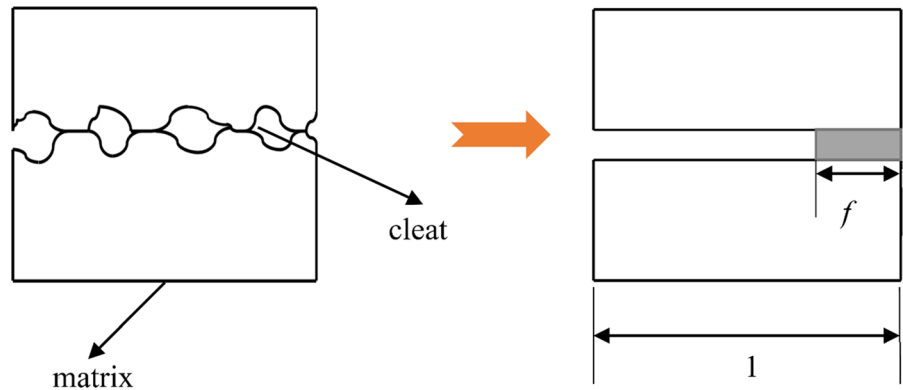
3 Improved Variable Klinkenberg Coefficient and Apparent Permeability Model

3.1 Improved Variable Klinkenberg Coefficient Model

In the previous section, we obtained the cleat width model closely related to the Klinkenberg coefficient. However, the flow in the cleat is not necessarily Darcy flow. When gas flows in the coal seam, the gas molecules collide with each other. The velocity of the gas molecules near the fracture wall is not zero, causing the apparent permeability greater than the absolute permeability. Klinkenberg proposed an equation to describe the linear relationship between apparent permeability and absolute permeability as

$$k_{ap} = k_{ab}\left(1 + \frac{B}{P}\right) \tag{17}$$

where k_{ab} is the absolute permeability of coal, k_{ap} is the actual measured apparent permeability, and B is the Klinkenberg coefficient, which can be obtained from the following equation

$$B = \frac{16c\mu}{\omega}\sqrt{\frac{2RT}{\pi M}} \tag{18}$$

where c is a constant of 0.9, μ is the dynamic viscosity of gas, ω is the throat width, R and T are gas constant and absolute temperature, respectively. M is molecular weight. The Klinkenberg coefficient B is usually obtained by the linear relationship between the apparent permeability K_{ap} measured in the laboratory and the reciprocal of the average pore pressure P. However, when permeability is measured by sorptive gas, the Klinkenberg coefficient cannot be obtained

directly by this method, because the slippage effect and the swelling/shrinkage caused by adsorption/desorption affect the value of B. Therefore, we should first separate the slippage effect and adsorption effect with inert gas (Wang et al. 2015), and then use it to get the Klinkenberg coefficient of adsorptive gas CH_4 from

$$B_{CH_4} = \frac{b_{na}}{b_{CH_4}}\frac{\mu_{CH_4}}{\mu_{na}}\sqrt{\frac{M_{na}}{M_{CH_4}}} \times B_{na} \tag{19}$$

where b is the cleat width; μ is the dynamic viscosity of gas; M is the gas molar mass; B is the gas Klinkenberg coefficient. The subscripts na and CH_4 represent non-adsorbed gas and methane, respectively.

When the non-adsorbent gas passes through the coal seam, its cleat width is only affected by the effective stress. While when CH_4 passes through the coal seam, the cleat width is affected by both the effective stress and the strain caused by adsorption. Based on Eq. (16), the cleat width for non-adsorbent gas and CH_4 can be written as

$$b_{na} = b_0 \exp(-C_f \cdot \sigma_{eff}) \tag{20}$$

$$b_{CH_4} = b_0\left[\exp(-C_f \cdot \sigma_{eff}) - 3S_f\varepsilon_L\left(\frac{P}{P_L + P} - \frac{P_0}{P_L + P_0}\right)\right] \tag{21}$$

By substituting Eqs. (20) and (21) into Eq. (19), we can obtain a new Klinkenberg coefficient model under the coupled effects of adsorption and effective stress as

$$B_{CH_4} = \frac{\exp(-C_f \cdot \sigma_{eff})}{\left[\exp(-C_f \cdot \sigma_{eff}) - 3S_f\varepsilon_L\left(\frac{P}{P_L + P} - \frac{P_0}{P_L + P_0}\right)\right]}\frac{\mu_{CH_4}}{\mu_{na}}\sqrt{\frac{M_{na}}{M_{CH_4}}} \times B_{na} \tag{22}$$

3.2 Improved of Apparent Permeability Model

According to the cubic law, the ratio of the absolute permeability under stress to the initial absolute permeability is

$$\frac{k_{\mathrm{ab}}}{k_{\mathrm{ab},0}} = \left(\frac{b}{b_0}\right)^3 \tag{23}$$

Therefore, the absolute permeability evolution can be rewritten as

$$\frac{k_{ab,P}}{k_{ab,i}} = \frac{b_p^3}{b_i^3} = \left\{ \frac{\left[\exp(-C_{\mathrm{f}} \cdot \sigma_{\mathrm{eff},p}) - 3S_{\mathrm{f}}\varepsilon_L\left(\frac{P}{P_L+P} - \frac{P_0}{P_L+P_0}\right)\right]}{\left[\exp(-C_{\mathrm{f}} \cdot \sigma_{\mathrm{eff},i}) - 3S_{\mathrm{f}}\varepsilon_L\left(\frac{P_i}{P_L+P_i} - \frac{P_0}{P_L+P_0}\right)\right]} \right\}^3 \tag{24}$$

$k_{\mathrm{ab},P}$ and $k_{\mathrm{ab},i}$ are absolute permeability at P MPa and i MPa, respectively. b_P and b_i are cleat width at P MPa and i MPa, respectively. $\sigma_{\mathrm{eff},P}$ and $\sigma_{\mathrm{eff},i}$ are effective stresses at P MPa and i MPa, respectively. Equation (22) and (24) are then substituted into Eq. (17) to obtain the ratio of apparent permeability models at P MPa and i MPa.

$$\frac{k_{\mathrm{ap,p}}}{k_{\mathrm{ap,i}}} = \frac{k_{\mathrm{ab,p}}}{k_{\mathrm{ab,i}}} \frac{1 + B_P/P}{1 + B_i/P_i}$$

$$= \left\{ \frac{\left[\exp(-C_{\mathrm{f}} \cdot \sigma_{\mathrm{eff,p}}) - 3S_{\mathrm{f}}\varepsilon_L\left(\frac{P}{P_L+P} - \frac{P_0}{P_L+P_0}\right)\right]}{\left[\exp(-C_{\mathrm{f}} \cdot \sigma_{\mathrm{eff,i}}) - 3S_{\mathrm{f}}\varepsilon_L\left(\frac{P_i}{P_L+P_i} - \frac{P_0}{P_L+P_0}\right)\right]} \right\}^3$$

$$\times \frac{1 + \frac{\exp(-C_{\mathrm{f}}\cdot\sigma_{\mathrm{eff,p}})}{\left[\exp(-C_{\mathrm{f}}\cdot\sigma_{\mathrm{eff,p}}) - 3S_{\mathrm{f}}\varepsilon_L\left(\frac{P}{P_L+P}-\frac{P_0}{P_L+P_0}\right)\right]} \frac{\mu_{\mathrm{CH_4}}}{\mu_{\mathrm{na}}}\sqrt{\frac{M_{\mathrm{na}}}{M_{\mathrm{CH_4}}}} \times B_{\mathrm{na}} \Big/ P}{1 + \frac{\exp(-C_{\mathrm{f}}\cdot\sigma_{\mathrm{eff,i}})}{\left[\exp(-C_{\mathrm{f}}\cdot\sigma_{\mathrm{eff,i}}) - 3S_{\mathrm{f}}\varepsilon_L\left(\frac{P_i}{P_L+P_i}-\frac{P_0}{P_L+P_0}\right)\right]} \frac{\mu_{\mathrm{CH_4}}}{\mu_{\mathrm{na}}}\sqrt{\frac{M_{\mathrm{na}}}{M_{\mathrm{CH_4}}}} \times B_{\mathrm{na}} \Big/ P_i} \tag{25}$$

This allows the apparent permeability to be evaluated as a function of stress, coal properties, depletion, and Klinkenberg effect.

4 Application and Evaluation of the Model

4.1 Evaluation of the Model Under the Condition of Constant Effective Stress

Fracture compressibility can be calculated from $C_{\mathrm{f}} = 1/K_{\mathrm{f,n}} = \alpha/\phi K_{\mathrm{bulk}}$. Using parameters listed in Table 1, we obtain $C_{\mathrm{f}} = 0.0328$ MPa^{-1}, The parameter S_{f} was determined by fitting the model to the permeability data using a least-squares analysis. Through matching the experiment data, The parameter S_{f} is equal to 23.The improved apparent permeability model was then used to verify the coal seam of Sun Juan Basin, and the experimental data were derived from the literature (Harpalani and Chen 1997). During the experiment, the effective stress was maintained at 5.4 MPa, and the experimental temperature was maintained at 44 °C. When the stress reached stability, the pore pressure decreased from 6.2 MPa to 0.62 MPa. σ_0 and P_0 are assumed to be zero. Parameters are substituted into Eq. (25) and compared with experimental data, as shown in Fig. 3.

Our model divided cleat space into "hard" and "soft" parts, these two parts follow different Hooke's law when subject to the same stress, and takes only part of adsorption induced strain having an impact on the cleat width, while considering variable Klinkenberg effect simultaneously. The model shows a good match with the experiment data and the Pearson correlation coefficient value reached 0.973. When not considering slippage effect ($B = 0$), the model becomes Eq. (24), and the results show that the Klinkenberg effect has a significant impact on permeability.

4.2 Evaluation of the Model Under the Condition of Constant Confining Stress

When considering the Klinkenberg effect on permeability, Klinkenberg coefficient B is a key parameter. Sampath and Keighin (1982) provided the method to calculate the Klinkenberg coefficient when the confining stress was constant, and the pore pressure of the three groups selected by them was all less than 0.24 MPa. According to Eqs. (18) and (19), when pore pressure is low, Klinkenberg effect is more profound

Table 1 Parameters obtained from the literature

Parameters	Value
ϕ_0 (%)	1.225 (Liu et al. 2012)
P_L (MPa)	4.16 (Mitra et al. 2012)
ε_L	0.01075 (Mitra et al. 2012)
B_{He}	0.94 (Harpalani and Chen 1997)
α	0.925 (Guo et al. 2014)
K_{bulk} (MPa)	2299 (Wang et al. 2014)
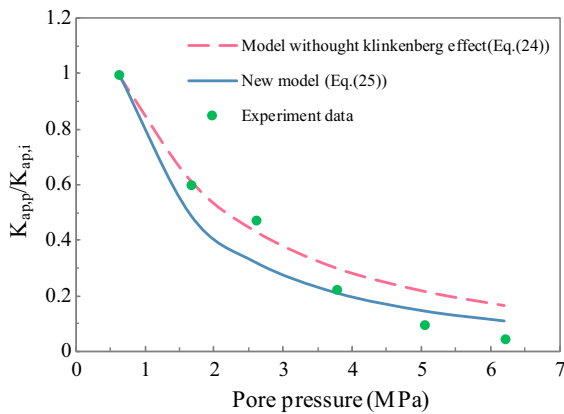
Fig. 3 Experimental permeability data were compared with model results

(Anez et al. 2014; Guo et al. 2014). Therefore, when calculating the Klinkenberg coefficient, we selected the data from Robertson and Christiansen (2005) with low pore pressure from 0.8 MPa to 5.6 MPa when confining stress was constant at 6.895 MPa (1000 psi). The Klinkenberg coefficient B_{N2} of N_2 we calculated is 0.66. When calculating the Klinkenberg coefficient, Harpalani and Chen (1997) ignored the influence of cleat width change and assumed that the cleat width in coal when helium flows is equal to that when methane flows ($b_{He} = b_{CH4}$). However, cleat width is variable due to the influence of gas type and stress effect. In our improved permeability model, the effects of adsorption and effective stress are both taken into account. In addition, we believe that the adsorption and swelling only partially affects the coal deformation. Then the Klinkenberg coefficient of CH_4 can be obtained according to the Eq. (22).

In this section, the data of Anderson 01 coal sample in Robertson and Christiansen (Robertson and Christiansen 2005) were used for verification. During the test, the confining stress remained constant at 6.895 MPa (1000 psi), and the pore pressure changed from 0.8 MPa to 5.6 MPa. Methane data were selected for the comparison. In addition, because Biot coefficient α was not measured, and the sorption contributing factor f cannot be directly measured neither, hence in the process of model validation, we took the fracture compressibility C_f and the sensitivity factor S_f as variables. The sensitivity factor S_f and fracture compressibility factor C_f were calculated as 7.33 and 0.065 MPa^{-1} by matching the curve, respectively. In the process of verification, relevant parameters of S-D

model, C-B model and P-M model are directly obtained from Guo et al. (2014). The model comparison results are shown in Fig. 4.

Comparing the model without slippage effect (Eq. (24)) with the new model developed in this work, it can be seen that the influence of Klinkenberg effect on permeability is significant and cannot be ignored. Since S-D model, C-B model, and P-M model all consider the influence of total swelling strain on permeability, it is observed from the trend in Fig. 4 that these models can overestimate permeability in the whole process. In addition, since these models are based on the assumption of uniaxial strain and the experimental data were measured under constant confining stress, these models cannot match the experimental data well. According to the new model developed in this study, it could be inferred that only part of the adsorption strain has an effect on coal permeability, and the Klinkenberg coefficient changes with different gas migration, so the model results match well with the experimental data.

4.3 Evaluation of Variable Klinkenberg Coefficient Under Different Conditions

According to Eq. (22), the Klinkenberg coefficient is related to effective stress and adsorption, which is similar to the improved apparent permeability model. In this following, the variable Klinkenberg coefficient will be analyzed under the condition of constant effective stress and constant confining stress. We assume both σ_0 and P_0 are 0. The relationship between
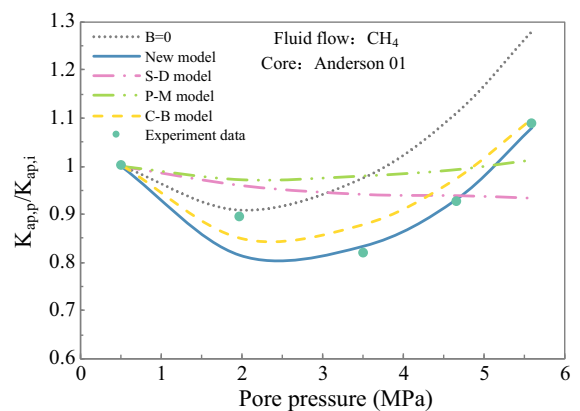
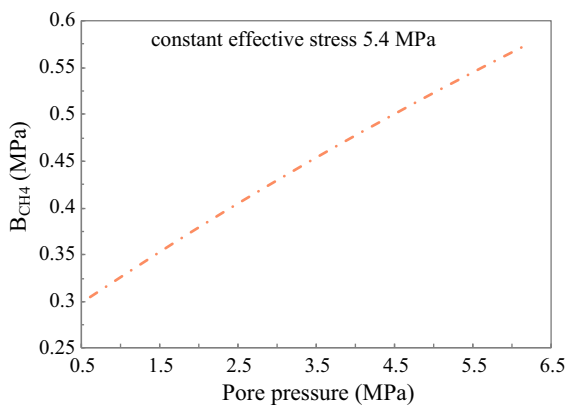**Fig. 4** Comparison of Anderson 01 experimental data with several models

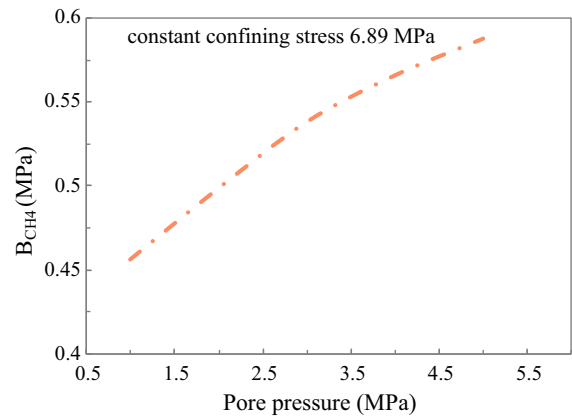the Klinkenberg coefficient and pore pressure is shown in Fig. 5 and 6.

Constant effective stress: The effective stress is 5.4 MPa, σ_c _ P = 5.4 MPa. The sensitivity factor S_f of adsorption induced strain to cleat width is 23. The fracture compressibility C_f is 0.0328 MPa^{-1}. The relationship between the Klinkenberg coefficient and pore pressure is shown in Fig. 5.

Constant confining stress: The confining stress is 6.89 MPa, σ_c = 6.89 MPa. The sensitivity factor S_f of adsorption induced strain to cleat width is 7.33. The fracture compressibility C_f is 0.065 MPa^{-1}. The relationship between the Klinkenberg coefficient and pore pressure is shown in Fig. 6.

As shown in Fig. 5, the Klinkenberg coefficient increases with the increase of pore pressure. This is because with the increase of pore pressure, the gas moves from the fracture to the matrix, and the adsorbed gas expands the matrix, narrowing the cleat width, so the Klinkenberg coefficient increases. When the confining stress is constant, the increasing trend of Klinkenberg coefficient gradually slows down, as shown in Fig. 6. This is because the effect of adsorption induced swelling on cleat width begins to dominate. With the continuous increase of pore pressure, the effect of effective stress on cleat width will gradually dominate, so the Klinkenberg coefficient will increase more and more slowly.

Fig. 6 The relationship between the Klinkenberg coefficient and pore pressure under constant confining stress

5 Conclusions

In this work, the effects of cleat space deformation on cleat width under the variable Klinkenberg effect and effective stress during gas migration in coal are analyzed. Based on the assumption that only part of the adsorption induced strain has an impact on deformation, an improved apparent permeability model is established. The model is verified under two conditions: constant effective stress and constant confining stress. The following conclusions can be drawn from this study.

(1) Based on the double-strain spring model, the influence of "hard" and "soft" parts deformation on cleat width under effective stress is considered, and it is considered that only partial adsorption strain affects coal deformation, then the cleat width model is established.

(2) On the basis of the cleat width model, the variable Klinkenberg effect of different gas migration is considered, and the variable Klinkenberg coefficient model is established. When the effective stress is constant, the Klinkenberg coefficient increases with the increase of pore pressure. When the confining stress is constant, the increase of Klinkenberg coefficient gradually slows down with the increase of pore pressure.

(3) The improved apparent permeability model was verified under the condition of constant effective stress, which matches the experimental data well. Under the condition of constant confining

Fig. 5 The relationship between the Klinkenberg coefficient and pore pressure under constant effective stress

stress, the improved model was compared with several commonly used models, and the results showed that the improved model was capable of matching the laboratory data with constant confining stress, while the other models could not well match the laboratory data well. Therefore, our newly developed model could be applicable to wider applications.

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