



Determination of Friction Capacity of Driven Pile in Clay Using Gaussian Process Regression (GPR), and Minimax Probability Machine Regression (MPMR)

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Abstract Friction capacity (f_s) of driven pile in clay is key parameter for designing pile foundation. This study employs Gaussian Process Regression (GPR), and Minimax Probability Machine Regression (MPMR) for determination of f_s of driven piles in clay. GPR is a Bayesian nonparametric regression model. MPMR is a probabilistic model. Pile length (L), pile diameter (D), effective vertical stress (σ'_v), undrained shear strength (S_u) have been used as input variables of GPR and MPMR. The output of the models is f_s . The developed GPR, MPMR models have been compared with the Artificial Neural Network (ANN). GPR also gives the variance of predicted f_s . The results prove that the developed GPR and MPMR are efficient models for prediction of f_s of driven piles in clay.

Keywords Minimax Probability Machine Regression · Gaussian Process Regression · Variance · Artificial Neural Network · Pile foundation · Clay · Friction capacity

1 Introduction

The determination of friction capacity (f_s) of driven pile in clay is a challenging task for geotechnical engineers. The different methods are available for determination of f_s of driven pile in clay (Chandler 1968; Tomlinson 1971; McClelland 1972; Burland 1973; Meyerhof 1976; Parry and Swain 1977a, b). The available methods are not reliable (Randolph et al. 1979). Randolph et al. (1979) has nicely explained the mechanism of driven pile in clay. Goh (1995) successfully adopted Artificial Neural Network (ANN) for determination of f_s of driven pile in clay. However, ANN has some limitations such as black box approach, arriving at local minima, low generalization capability, absence of probabilistic output, etc. (Park and Rilett 1999; Kecman 2001).

The article uses Gaussian Process Regression (GPR) and Minimax Probability Machine Regression (MPMR) for determination of f_s of driven pile in clay. GPR is a probabilistic non-parametric modeling approach. It estimates distributions over functions from training data. Researchers have successfully used GPR for solving different problems in engineering (Pal and Deswal 2010; Xia and Tang 2011; Ni et al. 2012). MPMR is developed based on Minimax Probability Machine Classification (MPMC) by constructing a dichotomy classifier (Strohmann and Grudic 2002). MPMR has been successfully used in the various field of engineering (Yang et al. 2010; Zhou and Xia 2011; Zhou et al. 2013). This study uses

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the database collected by Goh (1995) (see Table 1). The dataset contains information about pile length (L), pile diameter (D), effective vertical stress (σ'_v), undrained shear strength (S_u) and f_s . The developed GPR and MPMR have been compared with the ANN (Goh 1995). This article is organized as follows. The details of GPR for prediction of f_s of driven piles in clay are described in Sect. 2. Section 3 describes MPMR model. The results and discussion are provided in Sect. 4. In Sect. 5, the major conclusions are drawn.

2 Details of GPR

In GPR, the problem is to determine a function $y = f(x)$ from the following dataset

$$D = \{x_i, y_i\}_{i=1}^N, x \in R^d, y \in R \tag{1}$$

where x is input, y is output, R^d is d -dimensional vector space and R is one dimensional vector space. In this study, L, D, σ'_v , and S_u are used as input variable. f_s is output of the GPR. So, $x = [L, D, \sigma'_v, S_u]$ and $y = [f_s]$.

The joint distribution of y is given by

$$p(y) = N(0, K(x, x) + \sigma^2 I) \tag{2}$$

where $K(x_i, x)$ is kernel function and I is identity matrix.

For a new input x_{N+1} , the distribution of y_{N+1} is Gaussian with mean and variance:

$$\mu = K(x_{N+1}, x)(K(x, x) + \sigma^2 I)^{-1} y \tag{3}$$

$$\sigma = k(x_{N+1}, x) - \sigma^2 I - K(x_{N+1}, x)(K(x, x) + \sigma^2 I)^{-1} K(x, x_{N+1}) \tag{4}$$

The optimum value of hyperparameters of the GPR for a particular data set can be derived by maximizing the log marginal likelihood using common optimization procedures.

In carrying out the formulation, the data has been divided into two sub-sets; such as:

- (a) A training dataset: This is required to construct the model. In this study, 45 out of the 65 cases of pile load test are considered for training dataset.

Table 1 Dataset used in this study

L (m)	D (cm)	σ'_v (kPa)	S_u (kPa)	F_s (kPa)
14.1	15	96	26	27
13	15	102	15	26
11.7	20	54	23	14
17.5	14.3	87	23	26
15.9	15	49	17	12
8.1	13.5	37	13	11
7.7	16.5	32	15	9
10	13.5	33	10	12
12	15.5	39	12	10
10.2	22	19	15	8
24.2	15	146	19	29
17.1	15	109	57	24
12.7	23.2	38	19	17
10	17	82	36	28
14.3	26	89	22	22
22.5	47	60	45	23
5.5	30.5	44	30	38
19.2	61	142	31	30.7
15.2	35.6	448	104	109.2
12.2	35.6	718	162	162
43.9	30.5	162	38	30
96	61	354	80	44
73.8	61	273	67	47.6
22.6	76.7	651	170	192.1
30.5	32.5	153	45	29.3
45.7	32.5	148	52	21.8
13.7	32.5	112	45	42.3
5.5	16.9	51.6	129.5	76.7
29	33	105	39	39.8
12.2	16.8	33	16	9.9
14	35.1	59	30	23.4
39.6	27.4	297	165	80.9
30.5	61	91	52	30.7
25.9	32.5	99	61	34.2
13.1	27.4	80	110	53.9
20.1	61	105	208	91.5
9.1	45	54	144	73.4
16.8	61	87	100	55
13.7	32.5	112	137	64.4
18.3	76.2	115	335	154.1
4.6	16.9	43	1205	84.6
33.6	32.5	121.4	35.4	30.4
33.6	32.5	108	48.8	27.1
20.3	32.5	158.2	112.8	53
30.5	51	102.8	24.4	23.5

Table 1 continued

L (m)	D (cm)	σ'_v (kPa)	S_u (kPa)	F_s (kPa)
8	13.5	27	9	9
9.4	29.3	52	29	18
14.6	16	67	29	16
11.6	17.5	57	27	18
9.6	19.2	42	15	13
21.6	45.7	147	31	28.8
36.9	30.6	149	28.2	30.5
66.4	32.5	223	60	31.2
11.6	11.4	44	21	13.4
22.9	32.5	91	52	27
13.8	19	27	21	13
25.3	27.4	244	185	88.8
14.9	52.8	66	53	27.6
18.3	32.5	51	33	32
48.2	61	152	64	37.8
32	27.4	141	115	59.8
13.4	27	81	22	20
24.2	15	147	19	30
15.5	17.5	80	72	35
12.8	32.5	110	96	54.7

- (b) A testing dataset: This is required to estimate the model performance. In this study, the remaining 20 data is considered as testing dataset.

The data is normalized between 0 and 1. The normalization has been done by using the following equation.

$$d_{normalized} = \frac{(d - d_{min})}{(d_{max} - d_{min})} \tag{5}$$

where d = any data (input or output), d_{min} = minimum value of the entire dataset, d_{max} = maximum value of the entire dataset, and $d_{normalized}$ = normalized value of the data.

Radial basis function ($K(x_i, x) = \exp\left\{-\frac{(x_i - x)(x_i - x)^T}{2s^2}\right\}$, s is width of radial basis function) has been used as a kernel function. The program of GPR has been developed by using MATLAB.

3 Details of MPMR

This section will describe briefly MPMR for prediction of f_s of driven piles in clay. MPMR has the following form.

$$y = \sum_{i=1}^N \beta_i K(x_i, x) + b \tag{6}$$

where $K(x_i, x)$ is kernel function, x is input, y is output, N is number of data, β_i and b are output of the MPMR algorithm.

In this study, $x = [L, D, \sigma'_v, S_u]$ and $y = [f_s]$.

One dataset is obtained by shifting all of the regression data + ϵ along the output variable axis. The other dataset is obtained by shifting all of the regression data - ϵ along the output variable axis. A regression surface is the classification boundary between these two classes. More details about MPMR are given by Strohmann and Grudic (2002). MPMR uses the same training dataset, testing dataset, and normalization technique as used by GPR. Radial basis function has been used as kernel function. MATLAB has been used to develop the MPMR

4 Results and Discussion

The performance of GPR model depends on the value of Gaussian Noise (ϵ) and s . The design values of ϵ and s have been determined by trial and error approach. The developed GPR gives best performance at $\epsilon = 0.01$ and $s = 0.3$. Figures 1 and 2 shows the performance of training and testing dataset

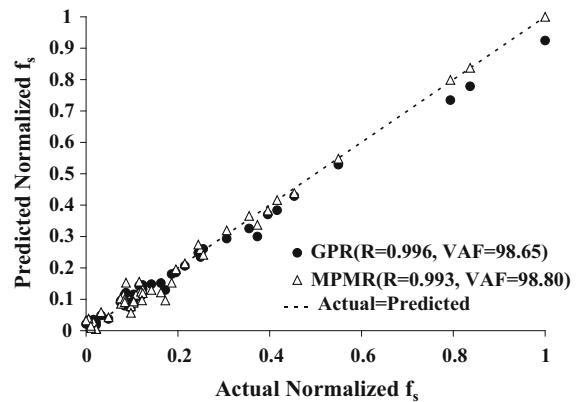


Fig. 1 Performance of training dataset

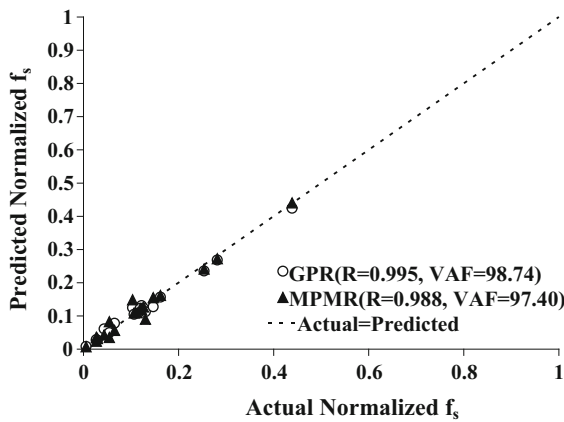


Fig. 2 Performance of teasing dataset

respectively. The performance of GPR has been assessed in terms of Coefficient of Correlation (R) and variance account for (VAF). The value of R is calculated from the following equation.

$$R = \frac{\sum_{i=1}^n (f_{sai} - \bar{f}_{sa})(f_{spi} - \bar{f}_{sp})}{\sqrt{\sum_{i=1}^n (f_{sai} - \bar{f}_{sa})^2} \sqrt{\sum_{i=1}^n (f_{spi} - \bar{f}_{sp})^2}} \quad (7)$$

where f_{sai} and f_{spi} are the actual and predicted f_s values, respectively, \bar{f}_{sa} and \bar{f}_{sp} are mean of actual and predicted f_s values corresponding to n patterns. For a good model, the value of R should be close to one.

The following equation has been used to determine VAF

$$VAF = \left[1 - \frac{\text{var}(f_{sa} - f_{sp})}{\text{var}(f_{sa})} \right] \times 100 \quad (8)$$

For a good model, the value of VAF should be close to 100 (Erzin and Cetin 2013). It is observed from Figs. 1 and 2 that the value of R and VAF is close to 1 and 100 respectively. So, the developed GPR predicts f_s reasonable well. The variance is also obtained from the developed GPR. Figures 3 and 4 illustrates the variance of training and testing dataset respectively. These figures can be used to determine uncertainty. It can be also used to determine probability of failure.

The design values of σ and ϵ have been determined by trial and error approach for developing the MPMR model. The developed MPMR gives best performance at $s = 0.8$ and $\epsilon = 0.007$. Figures 1 and 2 shows the performance of training and testing dataset respectively. The value of R and VAF is close to one and 100 respectively for training as well as testing datasets.

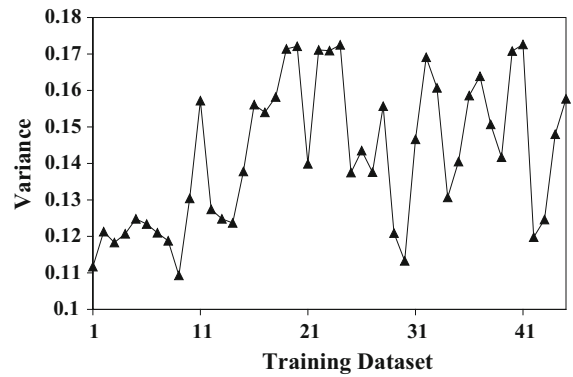


Fig. 3 Variance of training dataset

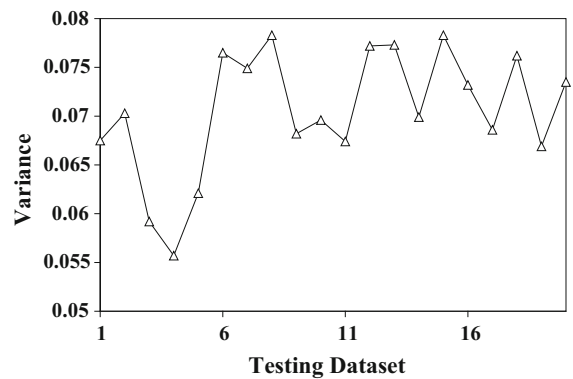


Fig. 4 Variance of testing dataset

Therefore, the developed MPMR proves his capability for prediction of f_c of driven piles in clay. The developed GPR and MPMR have been compared with the ANN model developed by Goh (1995). The comparison has been carried out in terms of Root Mean Square Error (RMSE) and Mean Absolute Error (MAE). The values of RMSE and MAE have been determined by using the following relation.

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (f_{sai} - f_{spi})^2}{n}} \quad (9)$$

$$MAE = \frac{\sum_{i=1}^n |f_{sai} - f_{spi}|}{n} \quad (10)$$

where f_{sa} and f_{sp} are actual and predicted f_s values respectively and n is the number of data. Figure 5 shows the bar chart of RMSE and MAE values of the ANN, GPR and MPMR models. It is observed from Fig. 5 that the developed GPR and MPMR outperform the ANN model. The performance of GPR and MPMR

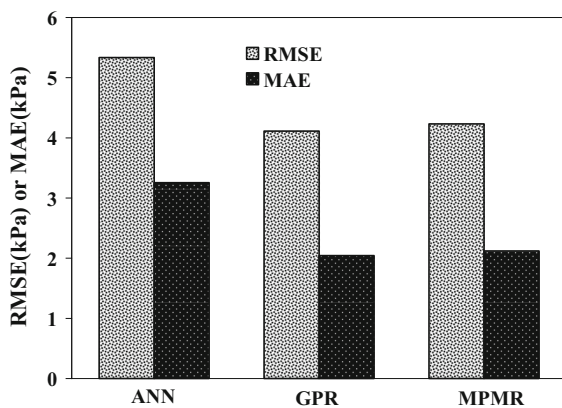


Fig. 5 Bar chart of RMSE and MAE values of the GPR, ANN and MPMR models

is almost same. The developed GPR produces variance of the predicted f_s . However, the ANN and MPMR models do not give variance of the predicted f_s . GPR assumes data distribution for developing the model. However, ANN and MPMR do not assume any data distribution.

5 Conclusion

This article describes GPR and MPMR for prediction of f_s of driven piles in clay. 65 datasets have used to develop the GPR and MPMR models. The data division and normalization technique have been described in the manuscript. The developed GPR and MPMR give reasonable performance. The predicted variance is useful for determination risk. The performance of GPR and MPMR is better than the ANN model. This study gives excellent tools based on the developed GPR and MPMR for prediction of f_s of driven piles in clay.

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