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TECHNICAL NOTE

Simplified Analytical Method for Calculating the Uplift Capacity of a Single Pile in Nonhomogeneous Soil with a Nonlinear Failure Criterion

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Abstract This paper focuses on the failure of the soil around a pile, and proposes a simplified analytical method for calculating the uplift capacity of a single pile. In this method, a curved failure mechanism is built by taking into consideration the non-homogeneity of the soil. Then the uplift capacity formula for a single pile is deduced based on a nonlinear failure criterion and limit equilibrium method. Further the proposed method is validated by comparison against the numerical simulation results. The effect of the non-homogeneity of the soil and nonlinear strength coefficients on the uplift capacity of the pile is investigated. Results reveal that the uplift capacity of a single pile increases when the non-homogeneity constant of the soil increases, whereas it decreases when the nonlinear strength coefficient increases; increasing the pile length is, to a certain extent, an effective measure to improve the uplift capacity. These conclusions may serve as good references for the future design of uplift piles.

Keywords Uplift capacity · Pile · Non-homogeneous soil · Nonlinear failure criterion · Analytical solution

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1 Introduction

As a key part of uplift structures, the uplift pile has been widely used in construction, electricity transformation, and offshore projects. It is extremely challenging to determine the upper limit of uplift capacity in the design of uplift piles. Therefore, the determination of uplift capacity has attracted increasing attention. The pile failure usually occurs in the following positions: (a) pile itself, (b) pile-soil interface, or (c) the soil. The determination of uplift capacity is supposed to be varied based on different failure types. Current studies mainly focus on the disruption of the pile-soil interface (Levacher and Sieffert 1984; Randolph et al. 1994; Dash and Pise 2003; Huang et al. 2007; Yang and Zou 2008; Zhang et al. 2015; Cheng et al. 2016; Xu et al. 2017; Soundara and Robinson 2017), whereas there are few studies of the soil failure category. Shanker et al. (2007) and Deshmukh et al. (2010) proposed the design of the soil failure surface as a truncated cone. Taking various parameters (including the length, diameter, soil friction angle, unit weight, etc.) into consideration, Shanker et al. (2007) and Deshmukh et al. (2010) proposed semi-empirical calculation formulas for the uplift capacity of a pile in sandy soil respectively. However, the failure patterns in these studies did not include the nonlinear failure characteristics of soil, and thus were not applicable in practical sense. Previous studies of uplift piles (Chattopadhyay and Pise 1986; He 2001; Su et al. 2014) revealed that the failure surface of the soil was curved.

In addition, the natural soil may exhibit remarkably nonhomogeneous characteristic along the depth due to the effect of consolidation pressure, stress history, etc. This characteristic can affect the soil shear strength significantly. Chen (1974), Farzaneh and Askari (2003) and Nian et al. (2008) investigated the stability of nonhomogeneous slope. In this paper, the non-homogeneity of the soil is taken into consideration in the uplift capacity analysis of a single pile. An analytical solution to predict the uplift capacity of a single pile is proposed based on the limit equilibrium method, and the effect of soil non-homogeneity on the uplift capacity of the pile is investigated. This study can provide a good reference for the design of uplift piles in the future.

2 Nonlinear Failure Criterion and Non-homogeneity of the Soil

2.1 Nonlinear Failure Criterion

By virtue of its simplicity and effectiveness in describing soil strength, the Mohr–Coulomb failure criterion has been widely applied in the field of geotechnical engineering. In this criterion, the relationship between normal stress (σ_n) and shear stress (τ_n) are linear in cases of soil failure. However, previous studies (Santarelli 1987; Agar et al. 1987; Yang 2002; Yang and Yao 2017; Zhao et al. 2010) revealed that the strength envelope for soils in the Mohr's plane is a convex curve, namely the normal stress (σ_n) and shear stress (τ_n) are in nonlinear relation. In this paper, a nonlinear power-law failure criterion is adopted, which can be expressed as follows (Yang 2002; Yang and Yao 2017; Zhao et al. 2010):

$$\tau_n = c_0(1 + \sigma_n/\sigma_t)^{\frac{1}{m}}, (m \geq 1) \quad (1)$$

where c_0 and σ_t are the cohesion and tensile strength of the soil, respectively, and both can be obtained from experimental results. As shown in Fig. 1, in the Mohr plane σ–τ, σ_t and c_0 are the intercepts of the strength envelope curve with the σ-axis and τ-axis. m is a dimensionless nonlinear strength coefficient (related to the properties of the soil) that describes the curvature of the envelope and $m \geq 1$.

2.2 Non-homogeneity of the Soil

The non-homogeneity of the soil can affect the soil strength significantly. If the soil strength is described by the cohesion (c) and internal friction angle (φ), the cohesion is usually assumed to be non-homogeneous (Chen 1974; Farzaneh and Askari 2003; Nian et al. 2008). As shown in Fig. 2, it is assumed that the cohesion increases linearly with the depth. Specifically, the pile length, the cohesion at the top surface, and the cohesion at the pile bottom are denoted L, c'_0, and $c'_0 + \eta c'_0$, respectively, then the cohesion c_0 at any depth can be expressed as:

$$c_0 = c'_0(1 + \eta d/L) \quad (2)$$

where η ($\eta \geqslant 0$) is a parameter describing the non-homogeneity of the soil.

3 The Shape of the Soil Failure Surface and Determination of the Uplift Capacity of a Single Pile

Previous studies (Chattopadhyay and Pise 1986; He 2001; Ilamparuthi and Muthukrishnaiah 1999; Su et al. 2014) revealed that the failure surface of the soil is

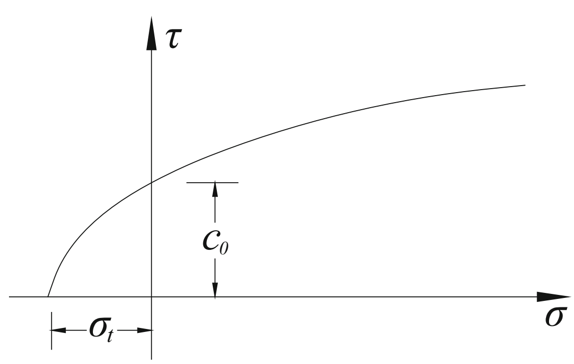

Fig. 1 Nonlinear failure criterion in the Mohr's plane σ–τ

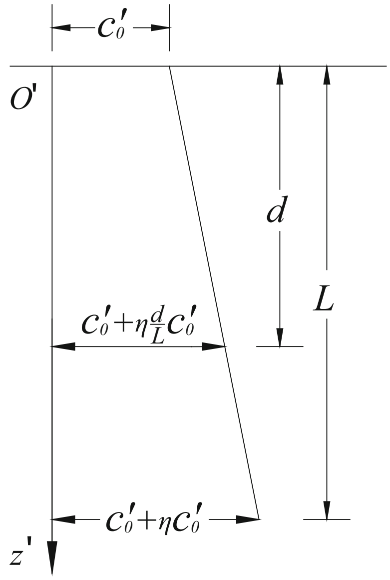

Fig. 2 Non-homogeneity model of the soil

curved (symmetric bugle) when a single pile is uplifted, and the angle between the failure surface and the horizontal surface at the top of the pile is $\pi/4-\varphi/2$. Based on the assumption that the bottom of the failure surface is tangential to the pile surface, He (2001) proposed an equation for the failure surface of a single pile:

$$x=r+\frac{L}{(n+1)\tan(\pi/4-\varphi/2)}\left(\frac{z}{L}\right)^{n+1} \tag{3}$$

where r is the pile radius, L is the pile length, and n is a constant related to the properties of soil ($n\geqslant 0$). When $n\to\infty$, the equation is $x=r$, which means failure occurs along the pile-soil interface; when $n=0$, the equation is $x=r+z/\tan(\pi/4-\varphi/2)$, which means failure occurs along a straight line, and it can be categorized as a cone failure; when n is any other value, the failure is a curved surface.

We assume that the soil failure obeys the nonlinear power-law failure criterion, and the cohesion of the soil increases linearly along the pile depth. A mechanical model for predicting the uplift capacity of a single pile can be constructed accordingly, as shown in Fig. 3.

A circular wedge of thickness Δz at height z above the tip of the pile is taken for analysis. Based on Eq. (2), the soil cohesion at height z at the failure surface can be written as:

$$c_0=c_0'\left[1+\eta\left(L-z-\frac{\Delta z}{2}\right)\Big/L\right] \tag{4}$$

Meanwhile, assuming that k_0 is the lateral pressure coefficient in the soil mass, the normal stress at the failure surface can be represented by:

$$\sigma^*=\sigma_z\left(\cos^2\theta+k_0\sin^2\theta\right) \tag{5}$$

Furtherly, in order to simplify the calculation, we assume that the lateral pressure coefficient along the pile length is defined as a constant, and the unit weight of the pile is equal to that of the soil mass (both are γ). The vertical stress at height z corresponding to the soil failure surface can be obtained by $\sigma_z=\gamma(L-z-\Delta z/2)$.

Based on the nonlinear power-law failure criterion and Eqs. (4)–(5), the shear stress at the soil failure surface can be obtained by:

$$\begin{aligned}\tau^*&=c_0(1+\sigma^*/\sigma_t)^{\frac{1}{m}}\\&=c_0'\left[1+\eta\left(L-z-\frac{\Delta z}{2}\right)/L\right]\\&\quad\cdot\left\{1+\gamma\left(L-z-\frac{\Delta z}{2}\right)\frac{\left(\cos^2\theta+k_0\sin^2\theta\right)}{\sigma_t}\right\}^{\frac{1}{m}}\end{aligned} \tag{6}$$

According to the equilibrium equation of vertical forces of the circular wedge at height z, the following equation can be obtained:

$$\begin{aligned}\Delta P=&\pi q\Delta x(2x+\Delta x)+\pi\Delta q(x+\Delta x)^2+\pi\gamma\Delta z\left(x+\frac{\Delta x}{2}\right)^2\\&+2\pi\Delta z\left(x+\frac{\Delta x}{2}\right)(\tau^*-\sigma^*\cot\theta).\end{aligned} \tag{7}$$

Substituting Eqs. (5)–(6) into Eq. (7) yields:

$$\begin{aligned}\frac{\Delta P}{\Delta z}=&\pi q\frac{\Delta x}{\Delta z}(2x+\Delta x)+\pi\frac{\Delta q}{\Delta z}(x+\Delta x)^2+\pi\gamma\left(x+\frac{\Delta x}{2}\right)^2\\&+2\pi\left(x+\frac{\Delta x}{2}\right)\times\left\{c_0'\left[1+\eta\left(L-z-\frac{\Delta z}{2}\right)/L\right]\right.\\&\times\left[1+\gamma\left(L-z-\frac{\Delta z}{2}\right)\cdot\frac{\left(\cos^2\theta+k_0\sin^2\theta\right)}{\sigma_t}\right]^{\frac{1}{m}}\\&\left.-\gamma\left(L-z-\frac{\Delta z}{2}\right)\left(\cos^2\theta+k_0\sin^2\theta\right)\cot\theta\right\}\end{aligned} \tag{8}$$

In the limit, Eq. (8) can be written as:

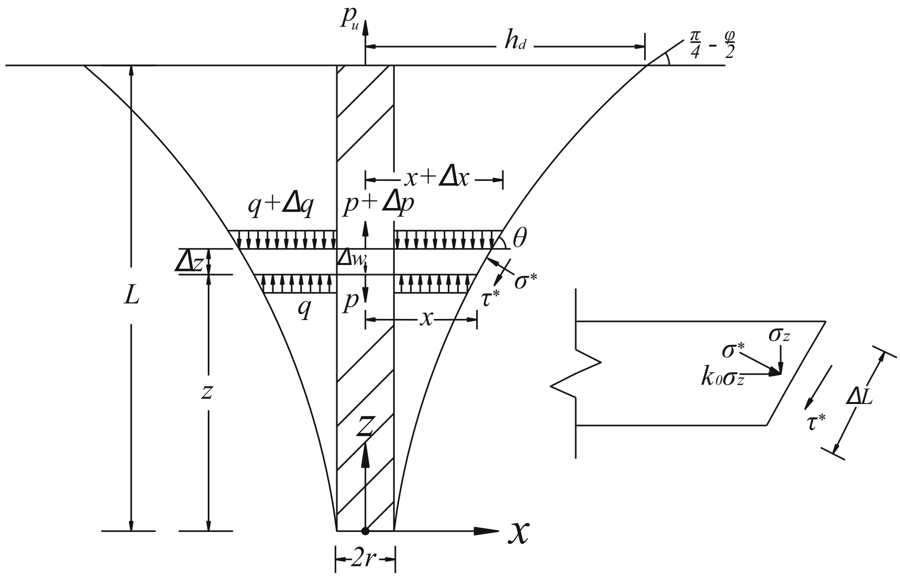

Fig. 3 The calculation model of the uplift capacity for a single pile

$$\begin{aligned}\frac{\mathrm{d}P}{\mathrm{d}z} = {} & 2x\pi q\frac{\mathrm{d}x}{\mathrm{d}z} + \pi x^2\frac{\mathrm{d}q}{\mathrm{d}z} + \pi\gamma x^2 + 2\pi x \\ & \times \left\{c_0'[1+\eta(L-z)/L]\right. \\ & \times \left[1+\gamma(L-z)\left(\cos^2\theta + k_0\sin^2\theta\right)/\sigma_t\right]^{\frac{1}{m}} \\ & \left. - \gamma(L-z)\left(\cos^2\theta + k_0\sin^2\theta\right)\cot\theta\right\}\end{aligned} \tag{9}$$

where

$$\begin{aligned} & q = \gamma(L-z) \\ & \cot\theta = \frac{\mathrm{d}x}{\mathrm{d}z} = \frac{1}{\tan(\pi/4-\varphi/2)}\left(\frac{z}{L}\right)^n \\ & \sin\theta = \frac{1}{\sqrt{1+\cot^2\theta}}; \quad \cos\theta = \frac{\cot\theta}{\sqrt{1+\cot^2\theta}}\end{aligned} \tag{10}$$

Based on Eq. (9), the uplift capacity of the pile can be obtained by:

$$\begin{aligned}P_{\mathrm{u}} = \int_0^L & \left\{\frac{2x\pi\gamma(L-z)}{\tan(\pi/4-\varphi/2)}\left(\frac{z}{L}\right)^n + 2\pi x\right. \\ & \times \langle c_0'[1+\eta(L-z)/L][1+\gamma(L-z) \\ & \left(\cos^2\theta + k_0\sin^2\theta\right)/\sigma_t]^{\frac{1}{m}} \\ & \left. - \gamma(L-z)\left(\cos^2\theta + k_0\sin^2\theta\right)\cot\theta\rangle\right\}\mathrm{d}z\end{aligned} \tag{11}$$

The extremal conditions of Eq. (11) are:

$$\begin{aligned} & \mathrm{d}P_{\mathrm{u}}/\mathrm{d}n = 0 \\ & \mathrm{d}^2P_{\mathrm{u}}/\mathrm{d}n^2 < 0\end{aligned} \tag{12}$$

The value of n and the uplift capacity of the pile can be obtained by Eq. (12) with numerical methods. In this paper, a computer program using MATLAB software is developed to determine the uplift capacity and the critical failure mechanism for a single pile.

4 Results and Discussions

4.1 Comparison with the Numerical Calculation Results

It is worth noticing that the non-homogeneity and nonlinear failure characteristics of the soil have not been incorporated into the uplift capacity analysis for a single pile in any reported studies. Referring to Eq. (1), the nonlinear power-law failure criterion can be converted to the well-known Mohr–Coulomb criterion by setting the coefficient $m = 1$. The corresponding cohesion c and internal friction angle φ in Mohr–Coulomb criterion should meet the following condition:

$$\begin{cases} c = c_0 \\ \tan\varphi = c_0/\sigma_t \end{cases} \tag{13}$$

Furtherly, to verify the effectiveness of the proposed method in this paper, the method of numerical simulation by means of FLAC^{3D} Software 3.0 is employed for analysis when $m = 1$. The built-in Mohr–Coulomb constitutive model is applied to simulate the failure characteristics of the soil. Meanwhile, the non-homogeneity model in Fig. 2 is embedded in the calculation process through writing *FISH* language. The constructed numerical model corresponding to varying pile length is shown in Fig. 4. Wherein, the model is 15 m long, 15 m wide, the pile is installed in the central place, and the model height is twice of the pile length. The top of the model is the free boundary, the bottom boundary is fixed with no horizontal and vertical displacement, and the side boundary is fixed with no horizontal displacement. The soil parameters are as follows: cohesion $c = 40$ kPa, internal friction angle $\varphi = 21°$, unit weight $\gamma = 20$ kN/m^3, tensile strength $\sigma_t = 80$ kPa, parameter $\eta = 0.05$–0.25 for pile length $L = 1$–5 m.

After the initial equilibrium of ground stress in the numerical model, a vertical load velocity is applied on the top surface of the pile to simulate the uplift failure process. Figure 5 presents the calculating results of the pile uplift capacity based on the numerical method and the theoretical method in this paper.

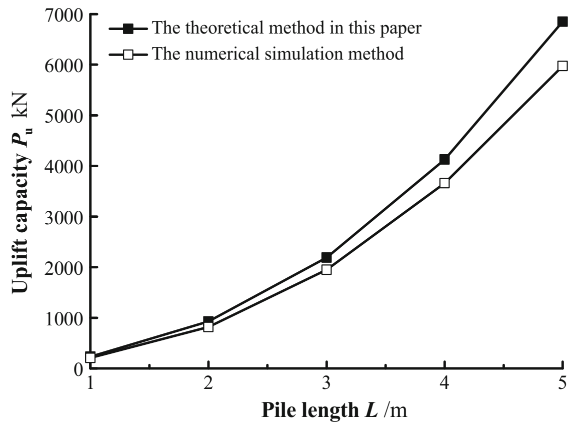

Fig. 5 The uplift capacity of the pile under varying methods

It can be found from Fig. 5 that, the uplift capacity of the pile calculated by the proposed theoretical method is close to those calculated by the numerical method. The results based on the latter method are slightly smaller, with the maximum difference of 14.3%. Therefore, the agreement between the two methods shows that the proposed method in this paper is valid.

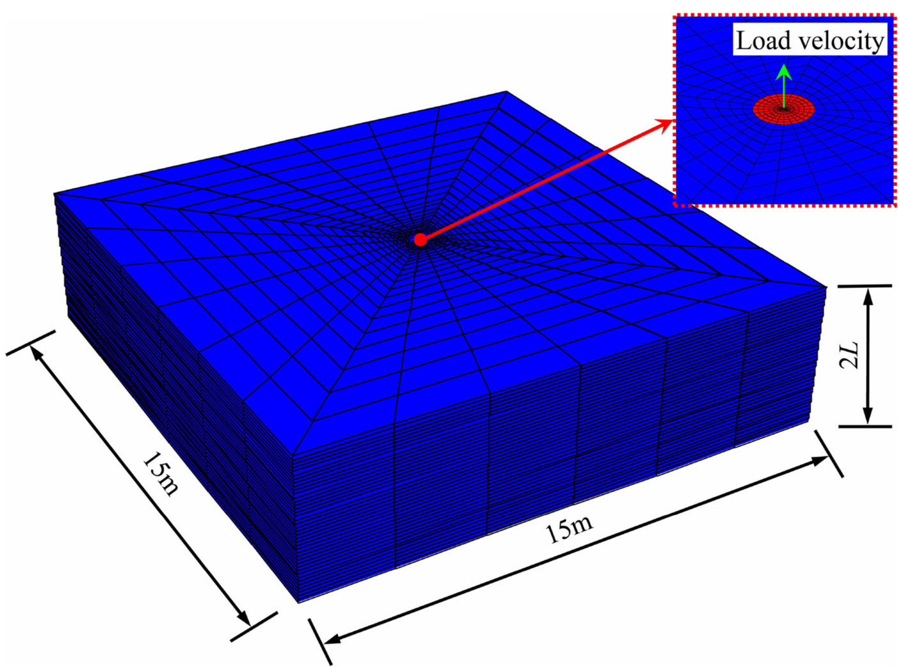

Fig. 4 Numerical calculation model

4.2 Comparison Analysis Under Varying Soil Parameters

To investigate the effect of soil non-homogeneity and nonlinear strength coefficients on the uplift capacity of the pile, the following parameter values are adopted for analysis: $\gamma = 20$ kN/m^3, $r = 125$ mm, $c'_0 = 40$ kPa, $\varphi = 21°$, $\sigma_t = 80$ kPa, $k_0 = 0.8$, and the pile length is 1, 2, 3, and 4 m, respectively.

1. The effect of soil non-homogeneity on the uplift capacity of the pile

Furtherly, we define the slope of the straight line in Fig. 2 as a non-homogeneity constant. The magnitude of this constant represents the degree of soil non-homogeneity, and it can be expressed as: $\lambda = \eta c'_0/L$ ($\lambda \geqslant 0$). Figure 6 presents the pile uplift capacity under varying non-homogeneity constants when $m = 1.8$. It can be found from Fig. 6 that, with the increase of the non-homogeneity constant, the shear strength of the soil increases along the depth, and the uplift capacity of the pile increases accordingly. Additionally, such effect of the non-homogeneity constant is more obvious with the increase of the pile length.

2. The effect of the nonlinear strength coefficient on the uplift capacity of the pile

Figure 7 presents the pile uplift capacity under varying nonlinear strength coefficients when $\lambda = 2$ kPa/m. It is found from Fig. 7 that, with the

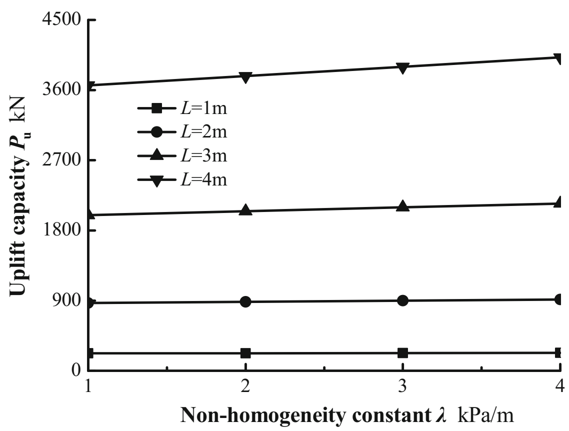

Fig. 6 The uplift capacity of the pile under varying non-homogeneity constants

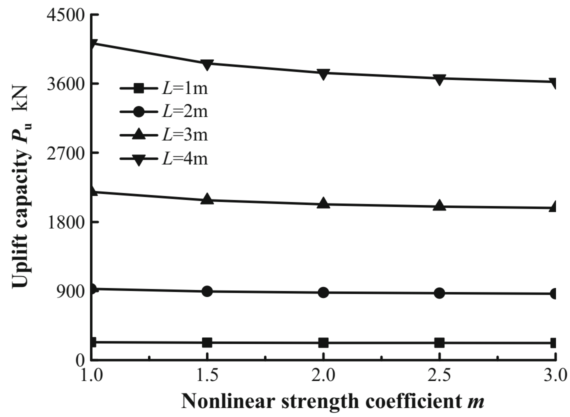

Fig. 7 The uplift capacity of the pile under varying nonlinear strength coefficients

increase of the nonlinear strength coefficient, the shear strength of the soil decreases, and the uplift capacity of the pile decreases accordingly. Specifically, this trend is more obvious when the nonlinear coefficient is smaller or the pile length is relatively bigger.

5 Conclusions

1. This paper focuses on the failure of the soil around a pile, and proposes a simplified analytical method for calculating the uplift capacity of a single pile. In this method, a curved failure mechanism is built by taking into consideration the non-homogeneity of the soil. Then the uplift capacity formula for a single pile is deduced based on a nonlinear failure criterion and limit equilibrium method. Further the proposed method is validated by comparison against the numerical simulation results.
2. When the cohesion at soil-pile interface exceeds the soil strength or the pile length is relatively short, the uplift capacity of the pile can be calculated based on the assumption of surrounding soil failure, which can provide good references for the design and testing of uplift piles in the future.
3. The uplift capacity of a single pile increases as the soil non-homogeneity constant increases and the nonlinear strength coefficient decreases. Increasing pile length (within a certain range) is an effective way to improve the uplift capacity. Additionally, the effect of the non-homogeneity constant and the nonlinear strength coefficient on

the pile uplift capacity become more obvious with increasing pile length.

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Compliance with Ethical Standards

Conflict of interest The authors declare that they have no conflict of interest.

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