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TECHNICAL NOTE

Investigation of Bearing Capacity and Failure Pattern in Shell Foundations by FELA Method

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Abstract The development in analysis and design of shell foundation types have led to understand that there are more advantages of shell foundations compared to their conventional plane counterparts. The bearing capacity of foundations depends on the shape of the failure surface and it can be influenced by various parameters, including the footing dimensions, the soil properties and the foundation roughness. In this research, several triangular strip shell foundations were analysed by FELA method and the soil failure surface dimensions under foundations were compared and it was observed that the failure pattern under shell foundations is not dependent on dimensions and the fundamental parameters in determining the failure surface are the friction between foundation and soil, as well as the depth of the foundation, such that by increasing depth of footing, the ratio of bearing capacity of the shell foundations to their plane counterpart will be decreased.

Keywords Shell foundations · Bearing capacity · Numerical modeling · Effect of changing of dimensions · Failure pattern

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List of Symbols

B	The width of foundation
D	The depth of foundation
E	The elastic modulus of the soil
c	The cohesion of soil
q	The ultimate bearing capacity of foundation
B_1	The width of failure wedge
D_1	The depth of failure wedge
γ	The unit weight of soil
v	The poison ratio
φ	The internal friction angle of soil
δ	The friction angle between foundation and soil
α	The peak angle of shell foundation
κ	The bearing capacity ratio

1 Introduction

Evidence shows that that human has figured out the benefits of the use of shell forms since very long time ago and has been using arch and dome forms in buildings and structures. The presence of signs for application of shell forms in nature has inspired to use these structures. Generally, in nature, everything that is important is protected in a crust compartment. The shell of the skull is the protector of brain, and the shell of eggs protects the fetus inside it and the pearl oyster protects the precious pearls. Probably the use of shell foundations also has natural origin. The form of human's feet and arch of bottom of foot is somehow kind of the shell shape and probably pioneers with this vision, established the first shell foundations. The brick arches in foundations has been used practically for a long time and many buildings with such foundations still exist in many part of the world (Fig. 1).

The research on these foundations doesn't have very extended domain. Hanna and Abdel-Rahman (1998) examined the behaviour of the shell foundations in terms of bearing capacity and settlement. Their tests was performed on conical, triangular, and pyramid shells compared to a circular, strip, and square footings. They pointed out that the performance of shell foundations is more effective and also the level of failure zone is deeper than flat foundations. Kurian and Devaki (2005) studied the effect of foundations geometry, soil interface interaction element and soil resistance parameters on bearing capacity and settlement of these foundations under vertical, incline and horizontal loading conditions. Ebrahimi et al. (2013a, b) investigated the effect of edge beam on bearing capacity and stress conditions in soil with numerical modelling of this group of foundations. They also modelled different geometries of footing in terms of bearing capacity, settlement and volume of materials in an effort to determine the optimal form for shell structures. Ramesh and Joy (2015) evaluated the stress and failure conditions in concrete shells by constructing experimental models. Ebrahimi and Khazaei (2015) compared the application of geogrid under cone footings with different dimensions with similar conditions in flat foundations. The results showed that although the use of geogrid in general, increases the bearing capacity and decreases the settlement, this improvement is more obvious in conical footings. Shareena and Rajesh (2017) studied the seismic performance of the hypar and reversed dome foundations using ANSYS software. They showed that, in seismic conditions, shell formations perform better than their flat counterparts.

Although the use of this foundation system has a long history, studies on them are not widely available and should be further investigated and introduced. The effect of the footing roughness, especially on embedded shell foundation did not receive any attention in the literature. The aim of this research is investigation of bearing capacity of shell triangular foundation based on failure pattern and to compare geotechnical behaviour of surface and embedded shell footings. Due to the complexity of stress distribution in subcrustal soil and concrete shells, performing analytical studies is very difficult and sometimes impossible. Therefore, numerical modelling is very useful and may be necessary for the study in these foundations. One of the strongest numerical methods is the limit analysis that is used in this paper.

2 Method of Analysis

Finite element limit analysis (FELA) is a powerful approach for solution of complex stability problems in geotechnical engineering. The technique combines the powerful capabilities of finite element discretization for handling complicated soil stratifications, loadings,

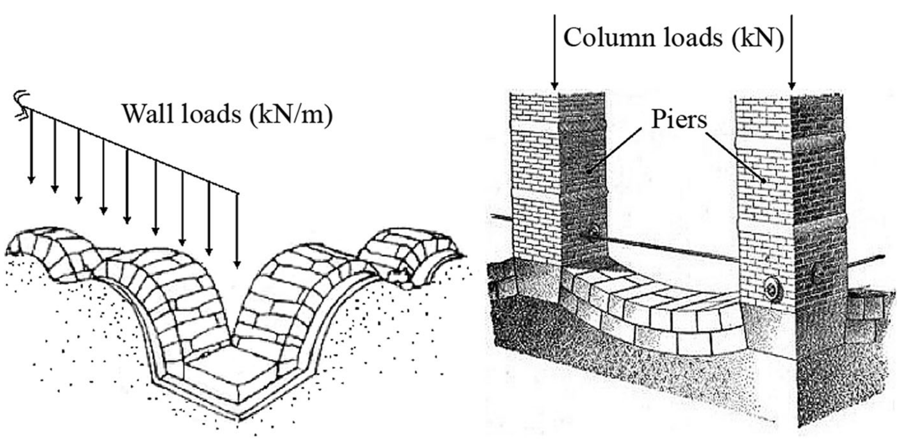

Fig. 1 Brick arch foundations (Kidder 1905)

and boundary conditions using the plastic bound theorem to bracket the exact limit load by upper and lower bound solutions. The underlying bound theorems assume a perfectly plastic material with the associated flow rule (Keawsawasvong and Ukritchon 2017). In OptumG2, both lower and upper bound problems under plane strain condition are formulated as a second-order cone programming (SOCP) [e.g. Makrodimopoulos and Martin (2006, 2007), and full details of their numerical FELA formulation can be found in Krabbenhoft et al. (2015). Using the lower bound and upper bound, the range for a plastic failure of a system is obtained, so the actual failure load of the system is within this range. In other words, two theorems define the range for the failure rate of a system, which can be reduced by calculating the maximum value of lower bound and the minimum value of upper bound (Fig. 2).

The final result, with proper precision, is an average of the upper- and lower bound solution.

$$\text{Result} \approx \frac{UB + LB}{2} \tag{1}$$

2.1 Lower Bound Theorem

Lower bound theorem or static theorem states that the failure load obtained from any elastic–plastic stress field provides a lower estimate of the actual failure rate. Statically Admissible Stress Fields satisfies

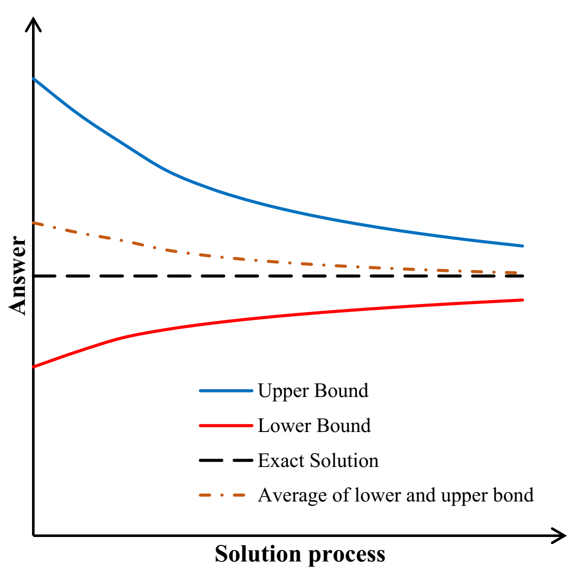

Fig. 2 Bounds of exact solution

equilibrium and the boundary condition and if the stress situation at any point does not exceed the yield criterion, it is admissible in plastic condition.

The general form of equilibrium equation in plane strain condition is expressed as:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + b_i = 0 \tag{2}$$

where σ_{ij} and b_i are the stress tensor components and volumetric force, respectively. This equation should be satisfied at all points of the problem domain.

At the lower bound of limit analysis, boundary conditions have already been identified as boundary stress, so, with the assumption that normal or shear stresses at boundary condition are zero, the boundary condition can be satisfied, also the state of stress at any point in the domain of the problem shouldn't exceed the yield criterion. Under conditions of plane strain, the Mohr–Coulomb yield criterion may be expressed as:

$$F = (\sigma_{11} - \sigma_{22})^2 + (2\sigma_{12})^2 - (2c\cos\phi - (\sigma_{11} + \sigma_{22})\sin\phi)^2 \tag{3}$$

where c and φ are cohesion and friction angle of soil, respectively. The necessity condition for the admissibility of the plastic stress field is the following:

$$F \leq 0 \tag{4}$$

This equation must satisfy at all points in the problem domain. In order to preserve the bounding property of the solution, it is necessary for the linearized yield criterion to circumscribe the parent yield criterion in stress space. By letting $X = \sigma_{11} - \sigma_{22}$, $Y = 2\sigma_{12}$ and $R = 2c\cos\varphi - (\sigma_{11} - \sigma_{22})\sin\varphi$, the Mohr–Coulomb criterion may be expressed as $X^2 + Y^2 = R^2$, which is the equation of a circle (Sloan 1989), in fact, this equation is the Locus of points that located on the perimeter and inside a circle on the X–Y plane that $X = \sigma_{11} - \sigma_{22}$ and $Y = 2\sigma_{12}$.

2.2 Upper Bound Theorem

Upper bound theorem or Kinematic theorem which is based on a perfect rigid-plastic model for soil, calculate the upper bound of the failure load by equalization the internal energy dissipated by a kinematic velocity field with the work done by the external forces. The objective of an upper-bound calculation is to find a velocity distribution u that satisfies compatibility, the flow rule, and the velocity boundary conditions, and which minimizes the internal power dissipation less the rate of work done by prescribed external forces (Lyamin et al. 2007):

$$W_1 = \int_V \sigma\, \dot{\varepsilon} dV - \int_S T_{PRS}^T\, udS - \int_V X_{PRS}^T\, udV \quad (5)$$

where T and X are, respectively, the prescribed surface tractions and body forces.

Table 1 Properties of soil in numerical modelling

B (m)	γ (kN/m^3)	c (Pa)	φ (°)	E (MPa)	ν	δ (°)
0.25 to 32	18.4	0	37	182	0.3	0 to rough

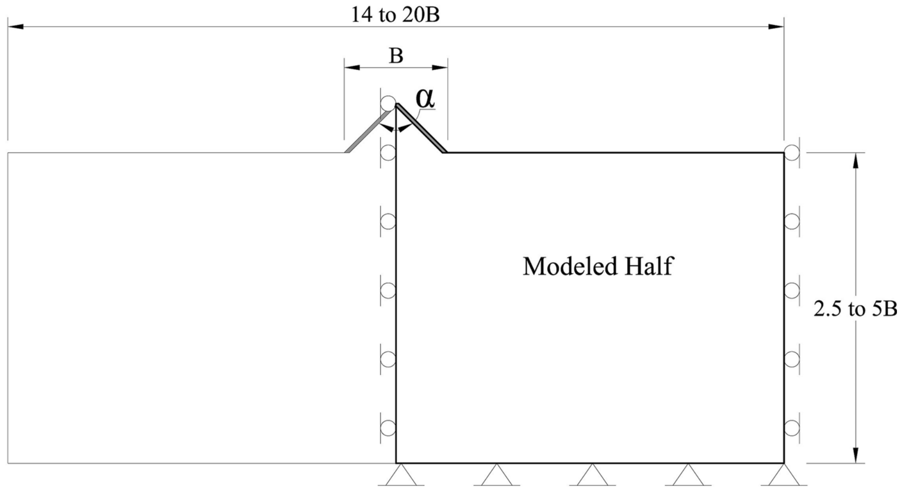

Fig. 3 Idealization of the plane strain strip foundation problem

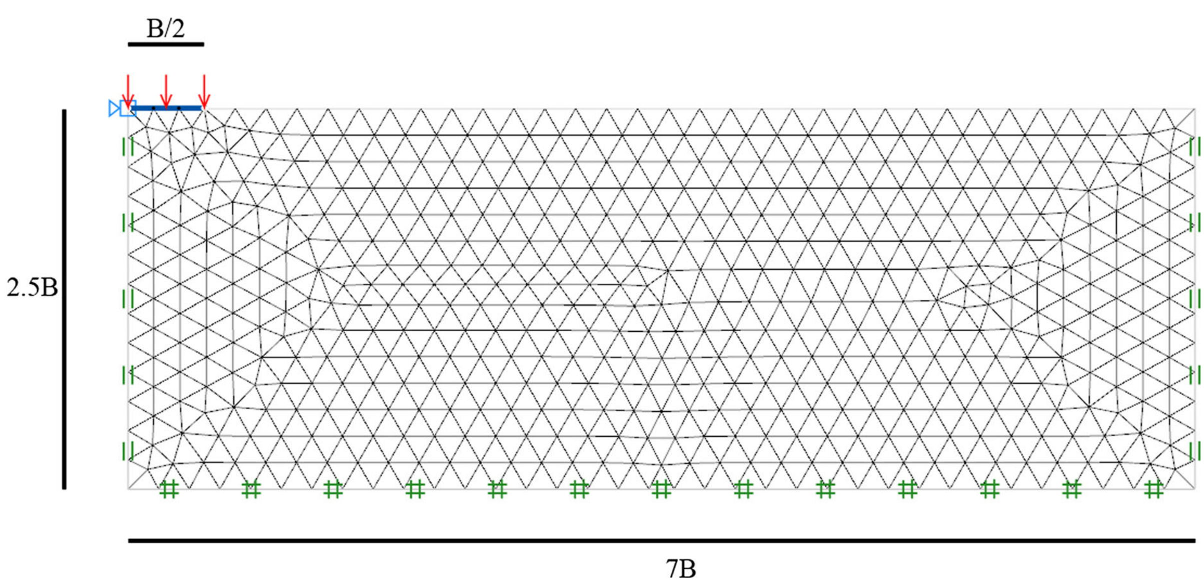

Fig. 4 mesh network and boundary conditions of the model for shallow foundation

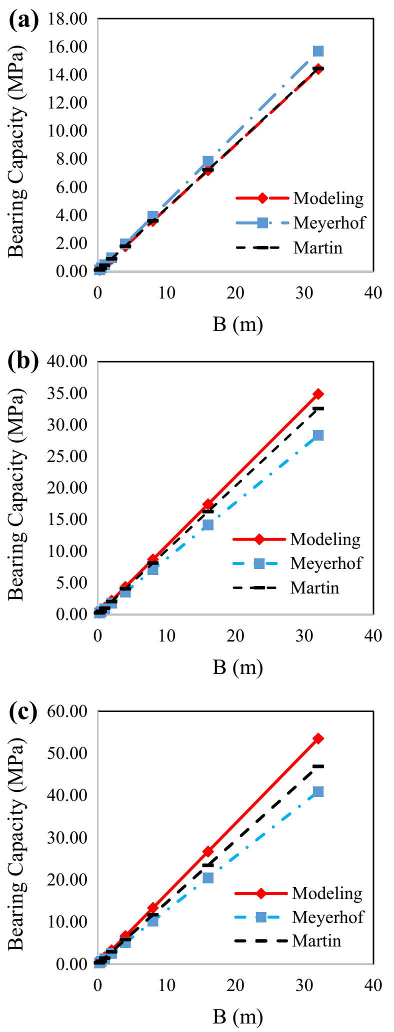

Fig. 5 Comparison of bearing capacity of shallow foundations for **a** D/B = 0.0, **b** D/B = 0.5, and **c** D/B = 1.0

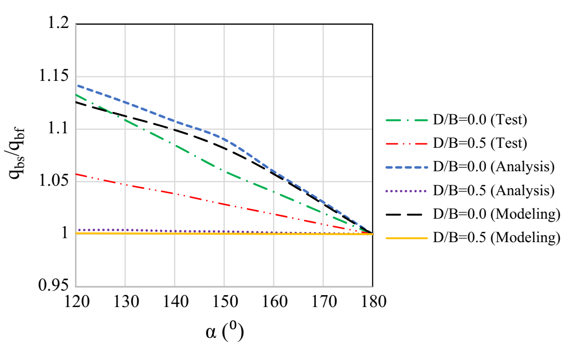

Fig. 6 Validation of the results of shell foundations modelling by FELA method

An upper bound estimate on the true collapse load can be obtained by equating W_1 to the rate of work done by all other external loads, given by:

$$W_2 = \int_S T^T udS + \int_V X^T udV \tag{6}$$

For a cohesionless soil, there is no energy dissipation. In a bearing capacity problem, this means that the bearing capacity comes entirely from the self-weight of the soil. Additionally, minimization of W_1 implies maximization of W_2, which is due entirely to the tractions applied on the soil mass by the foundation.

3 Properties of Model

In this research, the Mohr–Coulomb failure criterion was used to model soil behaviour. This criterion, in addition to requiring less parameters, is suitable for modelling the surface footings and displays the soil behaviour under these foundations. Griffths (1982) showed how the finite element methods, using the Mohr–Coulomb criterion, determine the exact quantities of bearing capacities factor, Nc, Nq and Nγ, in theory. Numerical modelling results are close to the lower bound of experimental values, and also the amount of Nγ depends on the roughness of the foundation. In order to study in the various conditions of roughness, modelling was performed for two different friction angle of soil and footing interface element. The properties of soil that used in this research are presented in Table 1.

For achievement the ideal geometry with consideration of soil-foundation interaction, the study domain involving the foundation and part of the soil below and around the foundation that is effective in bearing capacity and deformation (settlement) was determined large enough (between 7 and 10 times the footing's width from the side, and 2.5–5 times the footing's width from the bottom of the below the foundation). The boundaries of the domain are far from the place of foundation and the depth and width of the sand layer are chosen such that the boundary effect on foundation behaviour is minimized, therefore the effects of stress and deformation in these points are low and negligible. The plane strain model analysed is shown in Fig. 3.

Table 2 Bearing capacity of foundations (kPa) for δ = 0

Peak angle	Width (m)							
	0.25	0.5	1	2	4	8	16	32
(D/B = 0.00)								
α = 90°	104.48	210.13	419.58	838.31	1675.87	3350.51	6704.30	13,407.77
α = 120°	97.40	195.50	391.00	781.69	1560.64	3026.40	6257.74	12,538.10
α = 150°	81.10	163.69	327.07	653.07	1306.33	2612.24	5227.33	10,448.68
α = 180°	57.20	114.23	228.71	457.28	913.81	1830.27	3660.68	7328.63
(D/B = 0.50)								
α = 90°	270.78	543.39	1088.45	2172.21	4364.17	8704.37	17,443.05	34,864.81
α = 120°	271.93	544.63	1088.45	2175.61	4351.69	8707.75	17,379.12	34,862.54
α = 150°	271.14	541.44	1086.23	2172.26	4336.58	8655.27	17,378.28	34,823.31
α = 180°	208.28	416.47	833.27	1668.52	3330.80	6659.36	13,334.70	26,667.30
(D/B = 1.00)								
α = 90°	418.08	836.04	1673.17	3351.08	6698.21	13,366.21	26,746.90	53,503.52
α = 120°	418.40	837.09	1672.19	3338.43	6668.65	13,370.11	26,714.98	53,512.12
α = 150°	417.96	836.06	1668.51	3343.13	6669.91	13,327.74	26,652.99	53,370.48
α = 180°	363.63	727.39	1453.52	2907.12	5809.91	11,633.88	23,273.44	46,622.06

Table 3 Bearing capacity of foundations (kPa) for δ = rough

Peak angle	Width (m)							
	0.25	0.5	1	2	4	8	16	32
(D/B = 0.00)								
α = 90°	109.94	220.64	440.82	881.43	1763.20	3544.81	7126.36	14,080.97
α = 120°	110.24	221.67	442.82	885.27	1770.90	3555.53	7089.25	14,252.15
α = 150°	111.56	222.63	444.86	889.40	1777.64	3552.15	7121.16	14,231.53
α = 180°	111.19	223.62	447.06	894.56	1785.24	3595.35	7200.55	14,407.98
(D/B = 0.50)								
α = 90°	272.27	545.57	1084.38	2178.08	4371.72	8728.58	17,453.70	34,899.08
α = 120°	272.12	544.63	1088.41	2175.82	4350.08	8705.43	17,424.12	34,881.84
α = 150°	272.32	543.91	1089.12	2179.03	4351.52	8693.70	17,396.34	34,835.54
α = 180°	272.83	545.47	1089.62	2181.89	4353.72	8698.22	17,436.83	34,870.17
(D/B = 1.00)								
α = 90°	419.47	838.50	1676.64	3350.61	6711.45	13,392.57	26,782.24	53,588.47
α = 120°	416.48	833.76	1674.61	3339.77	6670.25	13,356.40	26,739.83	53,447.93
α = 150°	418.59	835.64	1672.92	3345.72	6660.77	13,322.51	26,695.80	53,438.64
α = 180°	418.11	836.27	1670.03	3332.60	6668.58	13,357.89	26,722.56	53,519.88

The base of the sand layer is fixed in both the horizontal and vertical directions. The right and left vertical boundaries are fixed in the horizontal direction but free in the vertical direction.

4 Verification

In order to verification of results of software in this modelling, the analysis of shallow foundation with

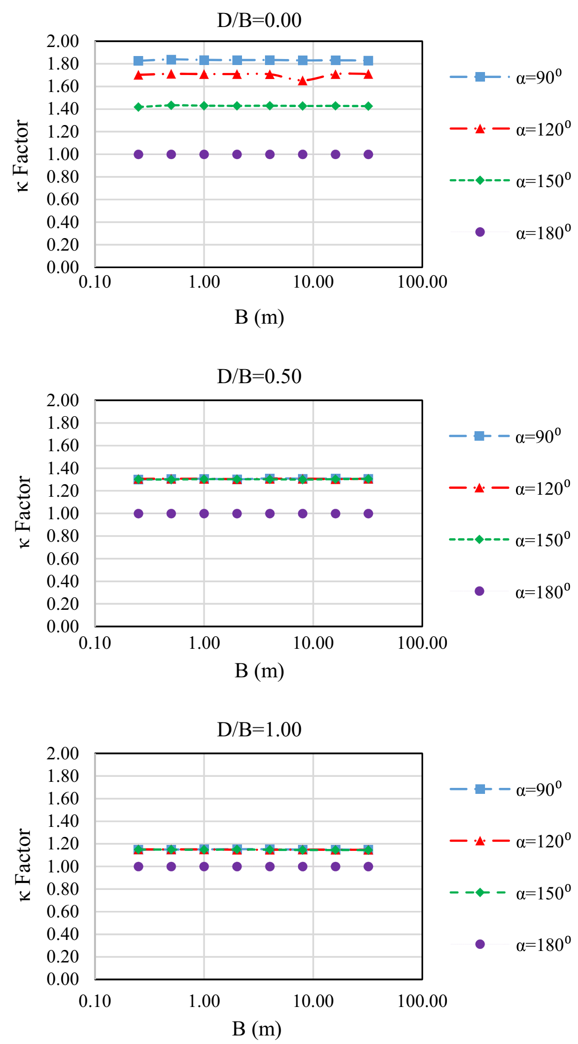

Fig. 7 κ Parameter for δ = 0

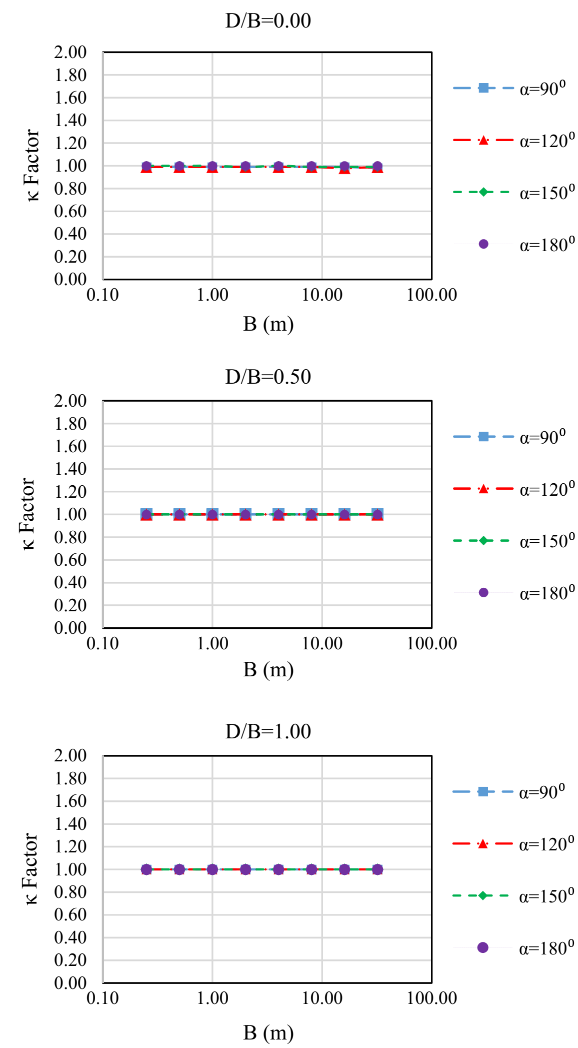

Fig. 8 κ Parameter for δ = rough

rough base was compared with bearing capacity result of Martin's (2005a, b) and Meyerhof's (1951) researches. The shape of mesh network and boundary conditions of the model for shallow footing with 1.0 m width is shown in Fig. 4.

The comparison results is shown in Fig. 5.

Also, in order to validate the software for shell foundations modelling, the results of the software were compared with the studies by Yamamoto et al. (2009) and presented in Fig. 6. The Yamamoto et al. have determined the ratio of bearing capacity of shell foundations (q_{bs}) to their plane counterparts (q_{bf}) with similar width by analytical and experimental methods.

The results of numerical modelling have a suitable accuracy especially for shallow foundations.

5 Results and Comparisons

5.1 Investigation of Footing's Dimensions on Bearing Capacity

The foundations with various widths (B = 0.25 m to 32 m) and three types of depth ratios (D/B = 0.0, 0.5, and 1.0) were modelled by FELA method. The result for ultimate bearing capacity are shown in Tables 2 and 3.

κ parameter was defined as:

$$\kappa = \frac{q_{u(shell\ foundation)}}{q_{u(strip\ foundation)}} \tag{7}$$

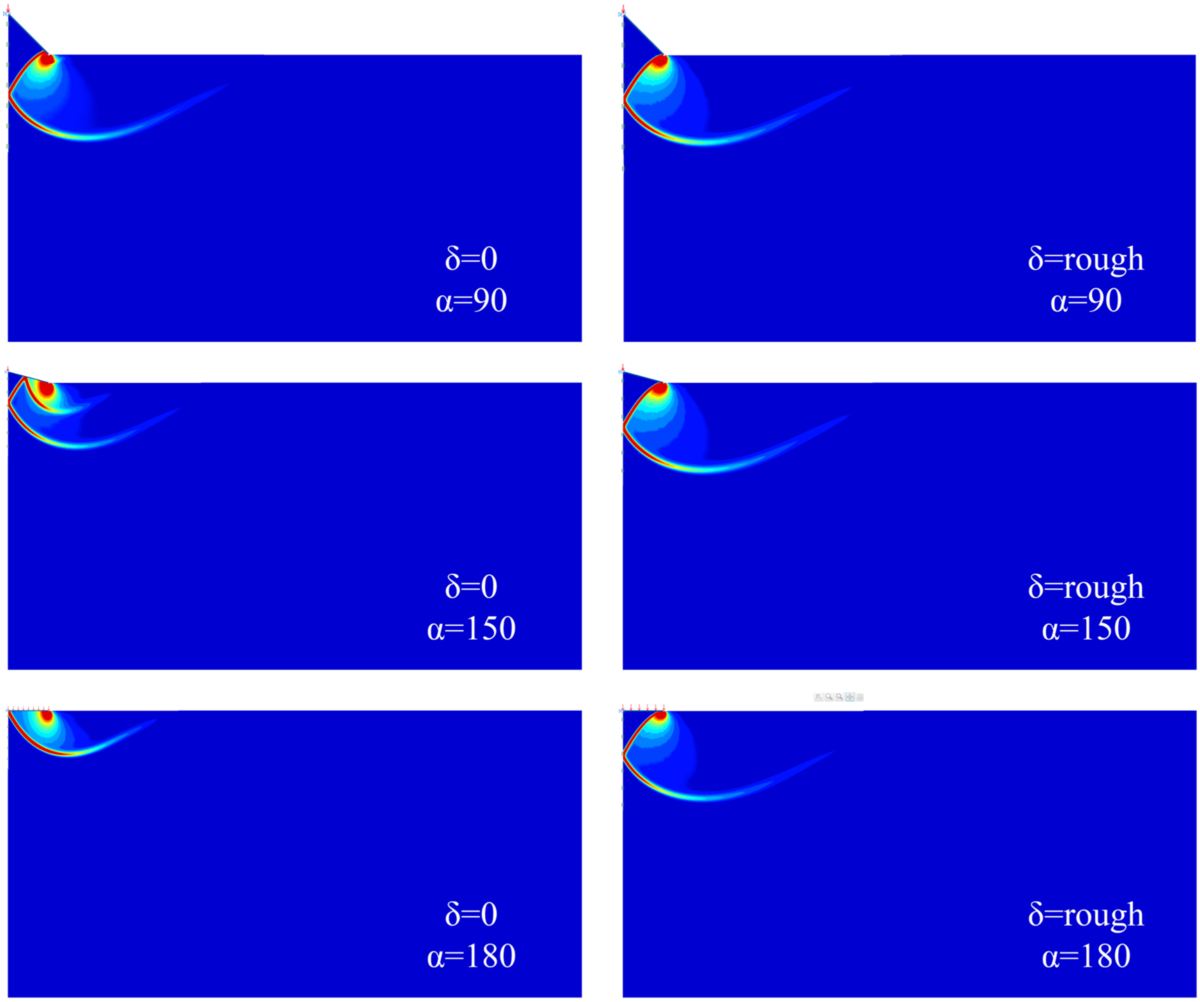

Fig. 9 Shear failure in soil for D/B = 0.0

Values of κ parameter can be shown such as Figs. 7 and 8.

In the buried foundations, as shown, by decreasing the peak angle to less than 150°, there will be no change in the bearing capacity ratio, and thus the reduction of the peak angle to less than this does not affect the bearing capacity.

5.2 Study in Pattern Failure on Shell Foundations

Failure surface in soil at ultimate load for shell foundations with peak angles of 90°, 150° and 180° for the three different depths of footing, in the case of the minimum and the maximum of friction angle between the soil and foundation, are shown in Figs. 9, 10, and 11.

It should be noted that the failure pattern for all foundation dimensions is similar to Figs. 9, 10 and 11, and as it is seen the plastic strains at the soil under foundations are not dependent on the footing dimensions and depend on the shape, roughness and depth of the foundations. In general, the failure surface is appeared in the form of a logarithmic spiral in accordance with the Terzaghi's theory, but it must be taken into account that the hypotheses of the Terzaghi's theory are based on the principle that the friction between soil and foundation is maximum and does not include all real conditions. The failure pattern in different conditions of the friction angle between soil and foundation is very different for strip and shell

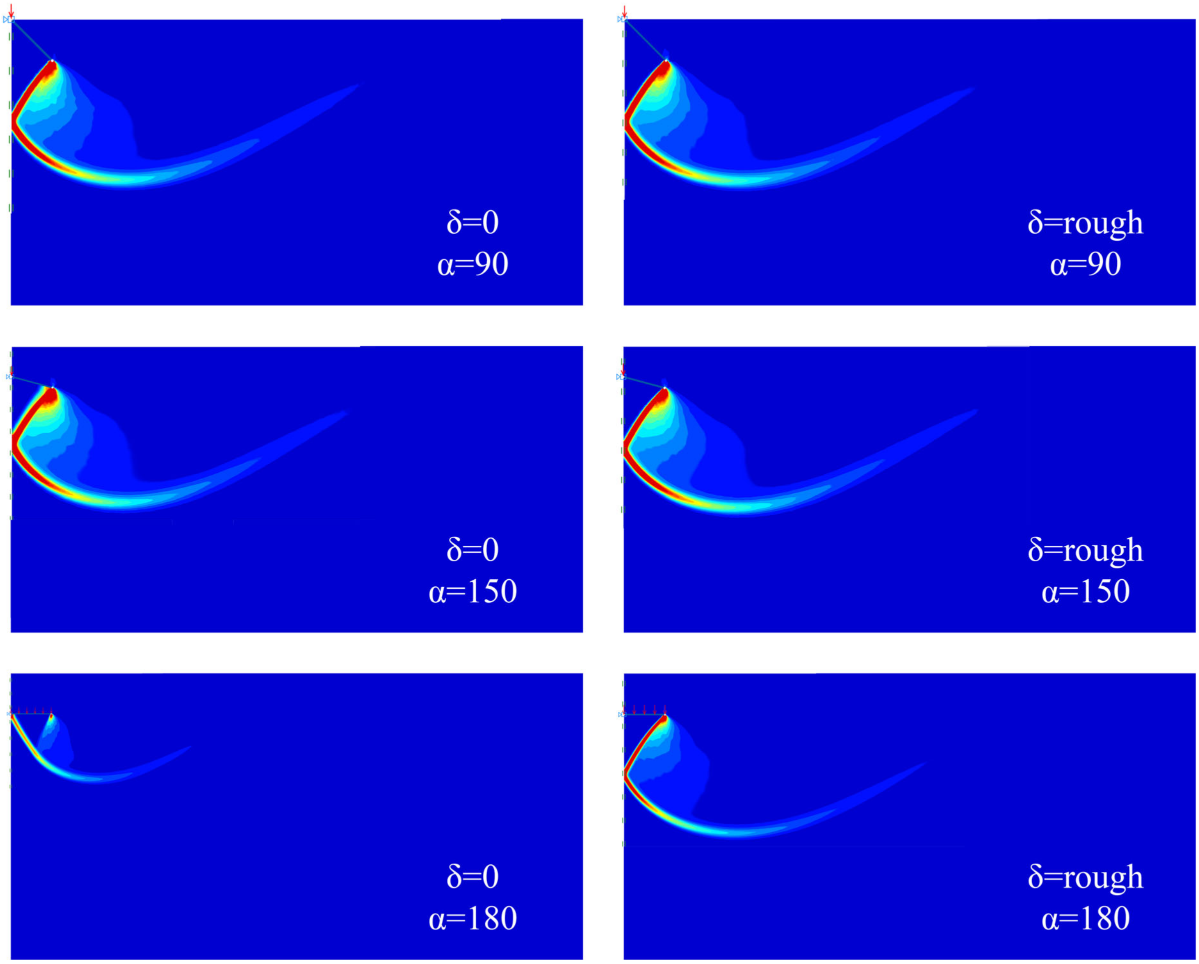

Fig. 10 Shear failure in soil for D/B = 0.5

foundations when peak angle is more than 150°, which will cause a difference in the load bearing capacity. Therefore, the capacity of using shell foundation, especially in embedment conditions, is similar to the increasing roughness, and in this case, it can be used to calculate the load bearing capacity conventional methods based on the logarithmic spiral failure pattern, such as Terzaghi's and Meyerhof's relations. The ratio of depth (D_1) and width (B_1) of the failure surface to the width of the foundation are presented in Table 4. This ratio is calculated for the different sizes of the footings. As shown in the Table 4, shape of the failure surface is not dependent on the foundation width.

For surface foundations, by reducing the peak angle, the failure pattern is changed gradually, and tends to the form of failure surface in soil under the rough foundations, but it should be noted that this condition in the case of poor soil ($\varphi < 20°$) due to the instability of the soil slope, reduces bearing capacity, so it is not recommended to use surface shell foundations for cohesionless soils with a small internal friction angle, but for stronger soils, decreasing the peak angle to 90° will increase the load bearing capacity and reducing peak angle more than this, does not effect on the bearing capacity.

For embedded foundations, by decreasing peak angle, the confining soil under footing leads to increase foundations bearing capacity, therefore, for buried smooth foundation, it is better to use shell foundation systems, however, in this case, the excessive reduction of the peak angle does not help to increase the load bearing capacity and is recommended not to select the peak angle less than 150°,

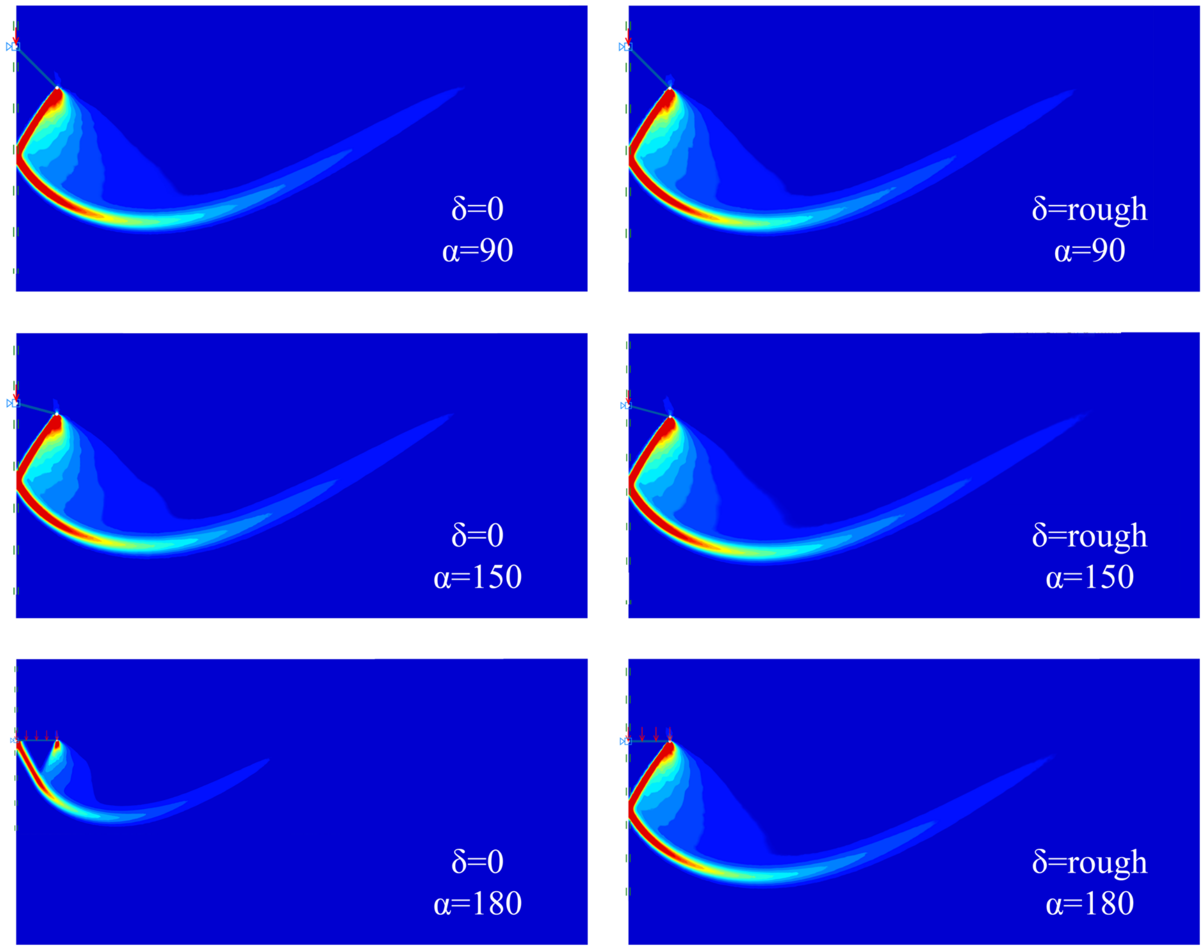

Fig. 11 Shear failure in soil for D/B = 1.0

Table 4 The ratio of depth and width of the failure surface to the width of the foundation

	δ = 0°			δ = rough		
	α = 90°	α = 150°	α = 180°	α = 90°	α = 150°	α = 180°
D/B = 0.0						
B_1/B	6.3	5.2	3.9	6.5	6.5	6.4
D_1/B	2.1	1.6	1.1	2.2	2.2	2.2
D/B = 0.5						
B_1/B	9.3	9.2	5.3	9.5	9.4	8.8
D_1/B	3.2	3.1	1.7	3.2	3.2	2.9
D/B = 1.0						
B_1/B	10.9	10.6	6.8	10.9	10.9	10.7
D_1/B	3.5	3.4	2.1	3.6	3.6	3.5

although the selected value can be vary with the depth of the foundation.

6 Conclusion

The use of modern foundation systems has always been the subject of attention by researchers and engineers of geotechnical science. Systems that can increase the bearing capacity and reduce settlement, in addition to being cost-effective. Perhaps it would be possible to identify shell foundations as the inescapable future of these modern systems, which, due to their particular form, would significantly reduce the use of materials and making them more economical than conventional foundations, and also due to the specific failure pattern under foundation, their bearing capacity will be greater than their plane counterparts.

The study in failure pattern of shell foundations has been neglected in literature, so it has been thought, in all conditions, bearing capacity of shell foundations is more than plane foundations, but study in failure surface shows bearing capacity ratio depends on various parameters such as foundation roughness, soil strength, depth of foundation and etc. and in some cases, the bearing capacity of shell foundations can be less than flat foundations. The results of this study are generally:

1. The failure pattern in the foundations is independent of the dimensions of the footing, and depends on shape, roughness, and depth of the foundation, and with increasing roughness of the foundation, the failure surface tends to the supposed wedge in the load bearing capacity theories.
2. Increasing the roughness of the foundation will increase the bearing capacity, and the use of shell structures will create conditions similar to the increasing of roughness in the foundations where the friction between soil and foundation is minimum.
3. For surface smooth foundation, by reduction of the peak angle, the bearing capacity is increased and tends to the bearing capacity of rough foundations. Although the use of these systems in poor soils ($\varphi < 20°$) is not advisable due to the instability of the soil slope, but for stronger soils, the decline of the peak angle from 180° to 90°, increases the bearing capacity but if the peak angle decreases more than 90° the bearing capacity will be constant.
4. Use of embedded shell foundations in the condition where the friction angle between the soil and footing is the minimum will increase the bearing capacity. In this case, depending on the depth of the foundation, by decreasing the peak angle between 10° and 20° ($\alpha = 170°$ to 160°), the bearing capacity is increased and then fixed. So the excessive reduction in the peak angle does not help to increase the load bearing capacity.
5. In general, the use of shell foundations for the conditions where the friction angle of the interface element between the soil and the foundation is minimal, increases the bearing capacity, so the use of this system of foundations, under these conditions, and in particular for surface footings are recommended on strong soil.

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