

# Subgrade Stability Evaluation in Permafrost Regions Based on Unascertained Measurement Model

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**Abstract** With regard to the complex and changeable engineering properties of subgrade in the permafrost regions, a stability-evaluating and order-arranging model of subgrade in permafrost regions is established based on the unascertained measurement theory. Subgrades of Qinghai–Tibet Railway in permafrost regions is taken as examples in view of the unique geological environment conditions. Twelve factors, which influence the subgrade stability, are selected as the evaluation indexes and analyzed qualitatively and quantitatively. The grading standard and stability evaluation index system are constructed. Entropy theory is used to calculate the index weights of evaluation factors, and the stability grade of five typical sections in permafrost regions is judged by credible degree recognition criterion. Meanwhile, the stability of permafrost subgrade is sorted according to evaluation sorting values. It is shown that the results offered by unascertained measurement model assessment are consistent with those by the fuzzy evaluation method. Furthermore, the proposed method can take

into account the unascertained information in stability evaluation, which provides an effective method for the subgrade stability evaluation in permafrost regions.

**Keywords** Permafrost subgrade · Stability evaluation · Unascertained measurement · Entropy · Evaluation ordering

## 1 Introduction

The Qinghai–Tibet railway, which runs through the permafrost regions, has complex and changeable engineering properties. The subgrade construction causes great destruction to the permafrost. In addition, due to human activities, global climate warming rainfall and evaporation, permafrost engineering properties, etc., the permafrost environment along the railway has been degraded to varying degrees, which interferes with the safety of subgrade engineering. Therefore, it is necessary to establish a safety evaluation system and stability analysis method of subgrade engineering in the permafrost regions.

In recent years, many scholars construct modern mathematical evaluation methods to analyze the stability of structures in permafrost regions, such as the analytic hierarchy process (AHP) and fuzzy synthetic evaluation method (Yan 2010; Meng et al. 2015; Ma et al. 2009; Lolaev et al. 2010). These

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research results have enriched the analysis and evaluation of subgrade stability in permafrost regions. The stability evaluation in permafrost regions is an incompatibility problem in essence, which is the result of combined effects of internal and external factors, so it possesses fuzziness, complexity and uncertainty. But the factors considered in the above methods are relatively fixed and have some shortcomings to evaluate the subgrade stability in the permafrost regions. For example, fuzzy evaluation method emphasizes the extreme value, which is subjective and may lead to information loss (Shi et al. 2010). The accuracy of AHP is not satisfactory (Qin and Qin 2011). Therefore, it is necessary to provide a new way to deal with the uncertain information.

Wang (1990) proposed the unascertained information theory, which is different from the fuzzy information (Zadeh 1965), random information (Matheron 1975) and grey information (Deng 1982). On this basis, Liu et al. (2001, 2002) established the unascertained measurement evaluation model, and obtained satisfactory results by using the model to evaluate the reliability of an electricity supply system. Since then, the theory and evaluation model have been widely applied in many diverse fields. Wen et al. (2008) used unascertained measurement theory (UMT) to create an evaluation method to determine the condition of cement concrete pavement. Gong et al. (2008) established a risk-evaluating and order-arranging model of mining underground goaf based on the unascertained measurement theory, which solved the uncertainty problem of many factors, including rock mass structure, geological structure, the influence of groundwater, etc., in the goaf risk evaluation. Liu et al. (2010) constructed an optimization model of underground mining method selection based on the unascertained measurement theory, which can calculate the relative superiority degrees of mining methods to easily determine the optimal method. Yaou and Cui (2012) applied unascertained theory and blind number theory in describing the uncertainty of the mechanical system failure process to help develop appropriate design validation and verification program for reliability assessment. Zhou and Li (2012) established an unascertained measurement (UM) model of blastability of rock mass predictions on the basis of unascertained measurement theory. The model can provide an effective and credible method for estimation the blastability of rock mass. An unascertained

measurement model of water and mud inrush risk evaluation is established founded on unascertained measurement theory, and the results show that the evaluation results agree well with practical construction situation (Li et al. 2017). However, the analysis and research on the subgrade stability evaluation in permafrost region are not widely discussed. In this paper, the unascertained theory is applied to the stability analysis of permafrost subgrade for the first time and the evaluation results are compared with the existing results.

For all this, five typical sections along the Qinghai–Tibet railway are taken as the research background in this paper and the unascertained measurement theory with the membership function is used to evaluate the subgrade stability in the permafrost regions. Twelve influencing factors (ice content, depth of ice-rich content permafrost, permafrost thickness, mean annual ground temperature, annual variation depth, uplift height of artificial permafrost table, geomorphic type, vegetation coverage, annual precipitation, mean annual temperature, subgrade height, engineering measure) are selected as the unascertained evaluation index set and their influences on the subgrade stability of the permafrost regions are analyzed. Then, single index measurement function of the stability evaluation index of permafrost subgrade is established, and the index weight of each evaluation index is calculated by entropy theory. Meanwhile, the stability grade of the permafrost subgrade is judged by credible degree recognition criterion and subgrade stability in the study areas are ordered by evaluation ordering values.

## 2 Unascertained Measurement Theory

Since the concept of the unascertained information is proposed, much work has been done by many scholars, which make the unascertained information a systematic theory and method at present (Wang 1990; Liu et al. 2001). The theory is used to describe the incomplete information and refers to the decision-making-demanded information. The parameters most commonly found in the unascertained measurement theory are shown below.

Set  $R_1, R_2, \dots, R_n$  as  $n$  evaluation objects of the pending evaluation, and the evaluation object set is  $R = \{R_1, R_2, \dots, R_n\}$ . Each evaluation object  $R_i$  ( $i = 1, 2, \dots, n$ ) has  $m$  evaluation indexes

$U_1, U_2, \dots, U_m$  and  $U = \{U_1, U_2, \dots, U_m\}$ , which is the evaluation index set. Set  $C_k (k = 1, 2, \dots, p)$  as the  $k$ -th evaluation grade, and the  $k$ -th stability grade is superior to the  $(k + 1)$ -th one, i.e.  $C_k > C_{k+1}$ . If  $\{C_1, C_2, \dots, C_p\}$  satisfies  $C_1 > C_2 > \dots > C_p$ ,  $\{C_1, C_2, \dots, C_p\}$  is called ordinal classification of the evaluation index set  $U$ .

2.1 Unascertained Measurement of Single Index

Let  $\psi_{ik} = \varphi(x_i \in C_k)$  express the degree where  $x_i$  belongs to  $k$ -th evaluation grade of  $C_k$ , and  $\varphi$  satisfies the flowing conditions:

$$0 \leq \varphi(x_i \in C_k) \leq 1, (k = 1, 2, \dots, p) \tag{1}$$

$$\varphi\left(x_i \in \bigcup_{l=1}^p C_l\right) = 1, (i = 1, 2, \dots, m) \tag{2}$$

$$\varphi\left(x_i \in \bigcup_{l=1}^k C_l\right) = \sum_{l=1}^k \psi(x_i \in C_p), (k = 1, 2, \dots, p) \tag{3}$$

where Eqs. (1)–(3) are respectively called “non-negative boundedness”, “normalization” and “additivity” of  $\varphi$  in the evaluation set.

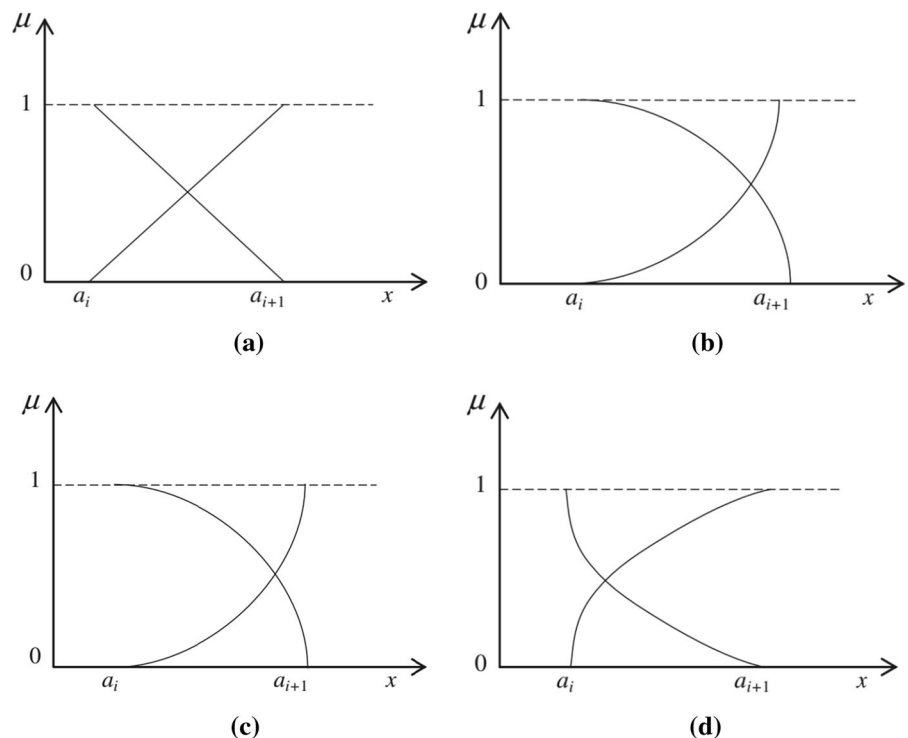
Thus  $\varphi$  is called unascertained measurement and the matrix  $(\psi_{ik})_{m \times p}$  can be written as

$$\psi_{ikm \times p} = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \cdots & \varphi_{1p} \\ \varphi_{21} & \varphi_{22} & \cdots & \varphi_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_{m1} & \varphi_{m2} & \cdots & \varphi_{mp} \end{bmatrix}$$

which is called the measurement evaluation matrix of single index.

When “uncertainty” is described by the unascertained set, the key is to construct a rational function of the single index measurement. Normally, there are four common kinds for the construction type of single index function, including straight-line type, para-curve type, sine curve type and exponential curve type, which are shown in Fig. 1. Among all the types listed above, the straight-line type is the most widely used and simplest unascertained measurement function. Therefore, the function of the straight-line type is adopted to calculate the parameters in the present

**Fig. 1** The graph of the unascertained measurement function by different construction type. **a** The straight-line type, **b** the para-curve type, **c** The sine curve type, **d** the exponential curve type



study. The expression for the corresponding index functions in the interval  $[a_i, a_{i+1}]$  is given in Eq. (4).

$$\begin{cases} \mu_i(x) = \begin{cases} \frac{-x}{a_{i+1} - a_i} + \frac{a_{i+1}}{a_{i+1} - a_i} & a_i \leq x < a_{i+1} \\ 0 & x > a_{i+1} \end{cases} \\ \mu_{i+1}(x) = \begin{cases} 0 & x < a_i \\ \frac{x}{a_{i+1} - a_i} - \frac{a_i}{a_{i+1} - a_i} & a_i \leq x < a_{i+1} \end{cases} \end{cases} \quad (4)$$

## 2.2 Determination of Evaluation Index Weight

Delphi method and other subjective weighting methods are adopted to ascertain the evaluation weight, which makes the results exist deviation during evaluation due to the pure subjective judgement and one-sidedness. Therefore, entropy is adopted to ascertain the index weight, which can avoid human disturbance and use the effective information of data (Zhou and Li 2012; Jaynes 1957; Shamilov 2007).

The dependence of unascertained measurement model on each evaluation index is different. Suppose that the evaluation index weight  $w_j$  ( $0 \leq w_j \leq 1$ ,  $\sum_{j=1}^m w_j = 1$ ) is the relative important extent of evaluation index  $U_j$  compared with other indexes, which reflects the importance of single index in the whole evaluation index set.

Then  $w_j$  is called the weight of evaluation index  $U_j$ , which can be calculated by the entropy:

$$v_j = 1 + \frac{1}{\lg p} \sum_{i=1}^p (\varphi_j)_i \lg(\varphi_j)_i \quad (5)$$

$$w_j = v_j / \sum_{i=1}^n v_i \quad (6)$$

wherein  $\varphi_j$  represent the elements of  $j$ -th row in the measurement evaluation matrix of single index.  $v_j$  is the measure of the evaluation index  $U_j$  whether or not to benefit the recognition of the unascertained measurement, which is called the peak value of the evaluation index and satisfies  $0 \leq v_j \leq 1$ . When  $(\varphi_j)_i = 1/p$ ,  $v_j = 0$  and the unascertained measurement is conducive to identification, which means that the effect of  $U_j$  is small. For given the measurement evaluation matrix of single index, Eqs. (5) and (6) yield the evaluation index weight  $w_j$ .

## 2.3 Multi-index Unascertained Measurement

Parameter  $\Psi_k$  denoted the degree of the assessment object belonging to the  $k$ -th evaluation grade and is defined as

$$\Psi_k = \sum_{i=1}^m w_j \psi_{ik}, \quad (i = 1, 2, \dots, n; k = 1, 2, \dots, p) \quad (7)$$

where  $\Psi_k$  satisfies  $0 \leq \Psi_k \leq 1$  and  $\sum_{k=1}^p \Psi_k = \sum_{k=1}^p 1^p \sum_{i=1}^m w_j \psi_{ik} = \sum_{i=1}^m (\sum_{k=1}^p \psi_{ik}) w_j = 1$ .  $\Psi_k$  is called multi-index measurement evaluation vector of the evaluation object  $R_i$ .

After obtaining the index weight, single index matrix is derived and the value of unascertained measurement can be figured out by the Eq. (7).

## 2.4 Evaluation Criterion and Results Ordering

It is necessary to adopt the credible degree recognition criterion to analyze the evaluation object.  $\lambda$  is set as credible degree, whose range of values is  $0.5 \leq \lambda \leq 1$ . If the evaluation space satisfies  $C_1 > C_2 > \dots > C_p$ , and

$$k_0 = \min \left\{ k : \sum_{l=1}^k \Psi_l \geq \lambda, k = 1, 2, \dots, p \right\} \quad (8)$$

it can be judged that evaluation object  $R_i$  belongs to the  $k_0$ -th evaluation grade.

In addition, it is necessary to order the stability degree of the evaluation object after determining which evaluation grade of the evaluation object belongs to. Since  $C_1 > C_2 > \dots > C_p$ , let the score value of  $C_l$  be  $n_l$ , and thus  $q_{R_i}$  is given by

$$q_{R_i} = \sum_{l=1}^p n_l \Psi_{il} \quad (9)$$

where  $n_l > n_{l+1}$ , and  $q_{R_i}$  is the ordering value of evaluation object.  $q = \{q_{R_1}, q_{R_2}, \dots, q_{R_n}\}$  is ordering modulus, and  $R_i$  can be sorted in stability according to the size of  $q_{R_i}$ .

## 3 Evaluation Index System of Subgrade Stability

There are numerous factors affecting the stability of permafrost regions, and the correlations among them are complex. Zhang et al. (2011) introduced

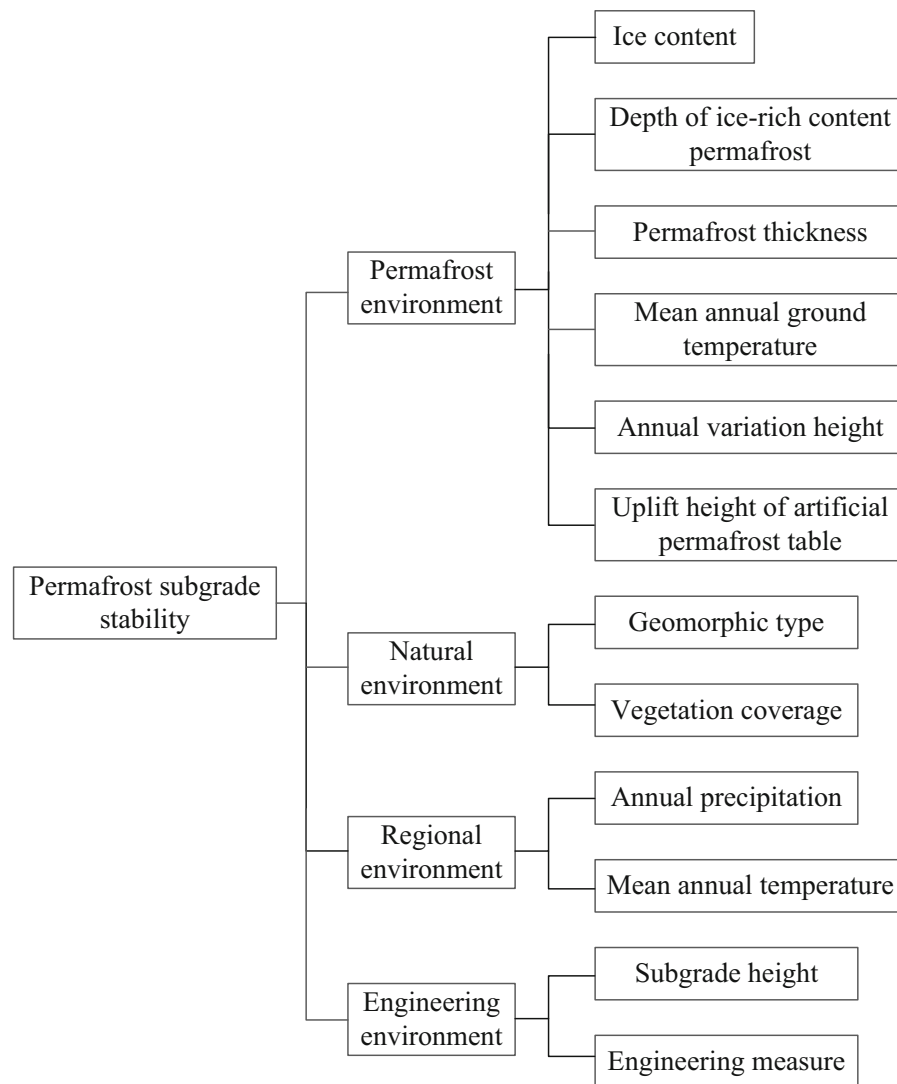
environmental and geological properties of frozen soil and permafrost engineering of Qinghai–Tibet railway in detail. Yan (2010) selected four main aspects affecting the subgrade stability based on the engineering experience to evaluate the stability of typical sections along Qinghai–Tibet railway. The factor level includes 12 factors such as ice content, ground temperature, mean annual temperature, engineering measure, etc. Meng et al. (2015) evaluated the safety evaluation of permafrost regions based on fuzzy theory by selecting permafrost type, permafrost conditions, landform, climate, subgrade structure and engineering defect as the criterion level. In addition, many scholars analyze the geological conditions, environment and other factors, a variety of engineering strategies are utilized to mitigate the effect of the influencing factors to the permafrost foundation. Lolaev et al. (2010) pointed out the strategies that cool the roadbed by implementing air convection embankments, ventilation ducts, thermosiphons, or combinations of these are effective in reducing embankment temperatures and stabilizing the roadbed. Wu et al. (2004, 2014) analyzed the permafrost change under the climate change and engineering action based on the monitoring data, the measurements (cooling roadbed and active protecting permafrost) could control the permafrost thermal state. For Qinghai–Tibet railway, the long-term monitoring system on permafrost regions has been established to ensure the safe operation of the Qinghai–Tibet railway. Its main goal is to monitor permafrost changes, including artificial permafrost table, permafrost temperature, subgrade settlement and the variation of seasonal freezing depth under the subgrade in seasonally frozen soil areas and thawed areas of permafrost regions. Meanwhile, a weather system consisting of weather stations has been established in the whole monitoring network. The weather system could monitor temperature, air pressure, precipitation, evaporation, etc. (Wu et al. 2008; Yu 2008; An et al. 2010).

On the basis of relevant studies about permafrost stability evaluation indexes and treatment measures, a comprehensive evaluation index system is established, which takes four aspects of permafrost environment, natural environment, regional environment, engineering environments as criterion level (Yan 2010; Meng et al. 2015; Ma et al. 2009; Zhang et al. 2011; Wu et al. 2004; Regehr et al. 2013). Twelve factors influencing

the subgrade stability are selected as the evaluation indexes, which indicated by  $U_1, U_2, U_3, \dots, U_{12}$  respectively. The evaluation index system is shown in Fig. 2. The stability evaluation indexes of the permafrost regions are divided into four grades, and its evaluation index set is  $\{C_1, C_2, C_3, C_4\} = \{\text{grade I, grade II, grade III, grade IV}\}$ . Grade I, grade II, grade III and grade IV represent stable engineering geology, relatively stable engineering geology, generally stable engineering geology, and bad engineering geology, respectively. Stable engineering geology means that subgrade engineering is in a stable state, which can guarantee the safe and reliable operation of the railway. Relatively stable engineering geology means that subgrade engineering is in a basically stable state, which basically guarantees the safe and reliable operation of the railway, and the measurements do not need to be taken. Generally stable engineering geology means that there are hidden problems in the stability of subgrade engineering, and measurements need to be taken to ensure the safe and reliable operation of the railway. Bad engineering geology means that there are safety problems in the stability of subgrade engineering, which seriously affect the traffic. The trains need to be restricted, and subgrade treatment measures should be adopted. The values of evaluation indexes come from the references (Yan, 2010; Meng et al. 2015; Ma et al. 2009). Quantitative indexes are evaluated by measured values and qualitative indexes are converted into semi-quantitative index by assignment method whose corresponding assignment is that  $\{\text{grade I, grade II, grade III, grade IV}\} = \{4, 3, 2, 1\}$ , then the classification criteria and assignments are shown in Table 1.

#### 4 Engineering Application

Qinghai–Tibet railway extends about 550 km across the lower latitudes of the Qinghai–Tibet Plateau, and the topography is high, whose average elevation is above 4000 m. The annual mean temperature is  $-7.0$  to  $-3.0$  °C in permafrost regions, which provides the necessary ambient temperature for the development of permafrost. Approximately 275 km of warm permafrost with mean annual ground temperature higher than  $-1.0$  °C, and 231 km of ice-rich permafrost with volumetric ice content 20%. Approximately 134 km of the railway is constructed on the warm



**Fig. 2** Evaluation index system of permafrost subgrade stability

and ice-rich region that is extremely sensitive to engineering activities and climate change (Zhang et al. 2011; Wu et al. 2004, 2014). Different geological structures and topography, hydrogeological conditions, lithology, surface traits have led to the variation of permafrost in regional climatic conditions, which make differences in the stability of permafrost subgrade. Five typical sections along the Qinghai–Tibet railway (DK1, DK2, DK3, DK4, DK5 are respectively located in Kumar River High Plains, Ulan Moron Basin, Buqu River Valley, Wenquan Fault Basin and Tanggula mountain) are chosen for comprehensive evaluation calculation of subgrade

stability. Permafrost distribution along the Qinghai–Tibet Railway is shown in Fig. 3. The statistical values of twelve influencing indexes of five typical sections are listed in Table 2.

The interaction between the Qinghai–Tibet railway subgrade and permafrost is a complex thermal–mechanical process, and the stability of the subgrade is an external manifestation of this complex process. The amount of subgrade deformation induced by frost heave is small based on the monitoring data, and the main problem is subgrade settlement deformation (Yu 2008). Therefore, stable subgrade refers to subgrade settlement stability in this paper, which includes

**Table 1** Classification criteria of quantitative and qualitative indexes. (Reproduced with permission from Yan (2010), Meng et al. (2015) and Ma et al. (2009))

Parameter	Evaluation indexes	Stability grades			
		Grade I	Grade II	Grade III	Grade IV
$U_1$	Ice content (IC)/%	< 10	10–20	20–50	$\geq 50$
$U_2$	Depth of ice-rich content permafrost (DICP)/m	$\geq 10$	5–10	3–5	2–3
$U_3$	Permafrost thickness (PT)/m	$\geq 20$	15–20	10–15	< 10
$U_4$	Mean annual ground temperature (MAGT)/°C	< – 2.0	– 2.0 to – 1.0	– 1.0 to – 0.5	$\geq - 0.5$
$U_5$	Annual variation depth (AVD)/m	5–8	8–10	10–15	$\geq 15$
$U_6$	Uplift height of artificial permafrost table (UHAPT)/m	$\geq 1.0$	0.5–1.0	1–0.5	< 0
$U_7$	Vegetation coverage (VC)/%	$\geq 80$	50–80	10–50	< 10
$U_8$	Geomorphic type	Ridge	Hillside, plain	Valley	Basin
$U_9$	Annual precipitation (AP)/mm	< 50	50–150	150–300	$\geq 300$
$U_{10}$	Mean annual temperature (MAT)/°C	< – 3.0	– 3.0 to – 2.0	– 2.0 to – 1.0	$\geq - 1.0$
$U_{11}$	Subgrade height (SH)/m	< 1.5	1.5–3.0	3.0–6.0	$\geq 6.0$
$U_{12}$	Engineering measure	Thermal pipe, air-cooled riprap subgrade, rubble slope	Rocky air-cooled embankments, rubble slope	Heat insulation materials	Common soil subgrade

stable settlement and stable sedimentation rate. According to the regulations of ‘Code for design of railway earth structure (TB 1000–2016)’ in China, the settlement of subgrade should meet the following requirements: Qinghai–Tibet railway’s post-construction settlement should not exceed 200 mm, and its settlement rate should not exceed 50 mm/a.

4.1 Construction of the Single Index Measurement Function

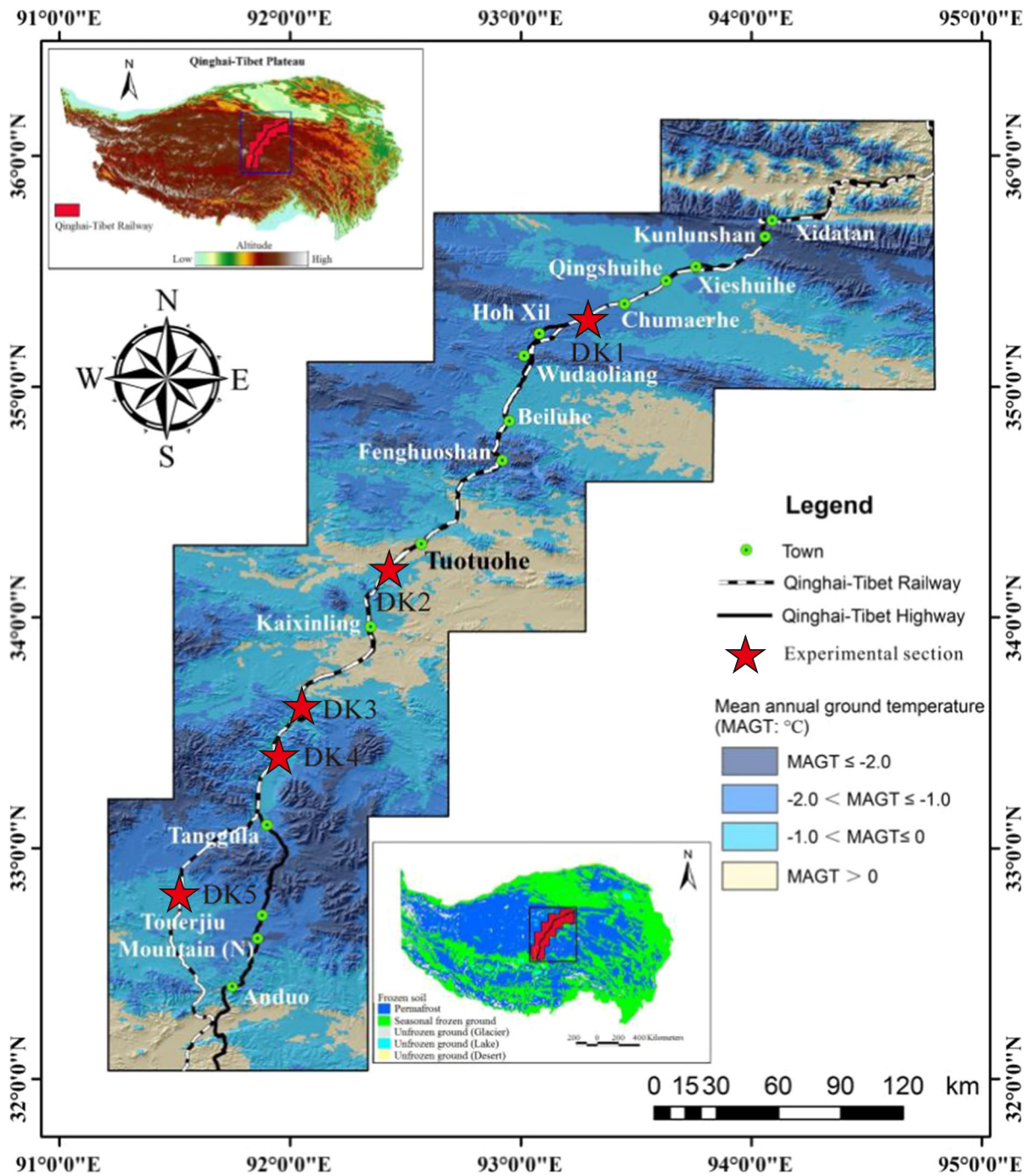
The single index measurement functions are constructed to get the unascertained measurement value of the evaluation index of subgrade stability, on the basis of the straight-line type function and classification criteria of quantitative indexes and qualitative indexes (Table 1). According to the scores in Table 2 and taking DK2 section as an example, the single index measurement function of ice content (IC), depth of ice-rich content permafrost (DICP), permafrost thickness (PT), mean annual ground temperature (MAGT), annual variation depth (AVD), uplift height of

artificial permafrost table (UHAPT), vegetation coverage (VC), annual precipitation (AP), mean annual temperature (MAT) are shown in Fig. 4.

The values of each evaluation index for DK2 section are respectively substituted into the corresponding unascertained measurement functions in Fig. 4. Thus, the measurement evaluation matrix of single index of the subgrade can be obtained in Eq. (10).

$$(\mu_{ik})_{12 \times 4} = \begin{bmatrix} 0 & 0.25 & 0.75 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0.14 & 0.86 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0.31 & 0.69 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$





**Fig. 3** Permafrost distribution along the Qinghai–Tibet Railway (*Notes* permafrost distribution in the map is modified based on the map by Li and Cheng (1996) and Liu et al. (2017), and data are from the China Cold and Arid Scientific Data Center)



**Table 2** Evaluation index values. (Reproduced with permission from Yan (2010), Meng et al. (2015) and Ma et al. (2009))

Location	$U_1$	$U_2$	$U_3$	$U_4$	$U_5$	$U_6$	$U_7$	$U_8$	$U_9$	$U_{10}$	$U_{11}$	$U_{12}$
DK1	65	8.9	< 10	- 1.22	14	1.82	13	3	402	- 4.5	6.0	3
DK2	30	12.3	≥ 20	- 0.05	6	1.83	35	1	402	- 4.1	5.5	3
DK3	< 10	≥ 10	≥ 20	- 2.71	9	2.40	90	2	322	- 4.1	3.5	3
DK4	30	5.7	< 10	- 0.32	12	- 2.80	25	1	322	- 4.1	2.2	1
DK5	40	6.3	< 10	- 0.08	8	- 0.50	90	4	495	- 5.0	3.0	1

4.2 Calculation of the Multi-index Measurement Evaluation Vector

For DK2 section, the weights of each subgrade stability evaluation index are calculated by Eqs. (4) and (5), which are  $w_1 = 0.068$ ,  $w_2 = 0.088$ ,  $w_3 = 0.088$ ,  $w_4 = 0.088$ ,  $w_5 = 0.088$ ,  $w_6 = 0.073$ ,  $w_7 = 0.088$ ,  $w_8 = 0.088$ ,  $w_9 = 0.088$ ,  $w_{10} = 0.066$ ,  $w_{11} = 0.088$ ,  $w_{12} = 0.088$ , respectively. Then, the multi-index measurement evaluation vector of permafrost subgrade stability of Qinghai–Tibet Railway in the DK2 section can be obtained according to the index evaluation matrix (10) and Eq. (7), which is {0.4611, 0.1609, 0.1138, 0.2642}. The proportional relation of the four grades is shown in Fig. 5. It can be known that  $C_1$  accounts for the largest proportion, which means that stable engineering geology, followed by  $C_4$ ,  $C_2$ ,  $C_3$ . In terms of the proportion of individual indicators, the subgrade stability of DK2 section may be considered poor. However, since the proportion of  $C_1$  and  $C_2$  is more than 50%, the subgrade stability of DK2 section is inclined to be stable.

4.3 Comprehensive Evaluation

Credible degree recognition is performed using Eq. (8) and the multi-index measurement evaluation vector of permafrost subgrade in DK2 section. Here,  $\lambda$  is set as 0.5,  $k_0 = 0.4611 + 0.1609 = 0.6220 > \lambda = 0.5$ , so the grade of subgrade stability belongs to grade II, which means “relatively stable engineering geology”. Similarly, the stability of permafrost subgrade in other samples along Qinghai–Tibet railway can be evaluated, and the evaluation ordering values of permafrost subgrade stability in the five typical sections can be obtained on the basis of Eq. (10), the evaluation results are shown in Table 3. Meanwhile, the fuzzy evaluation results in the reference (Meng

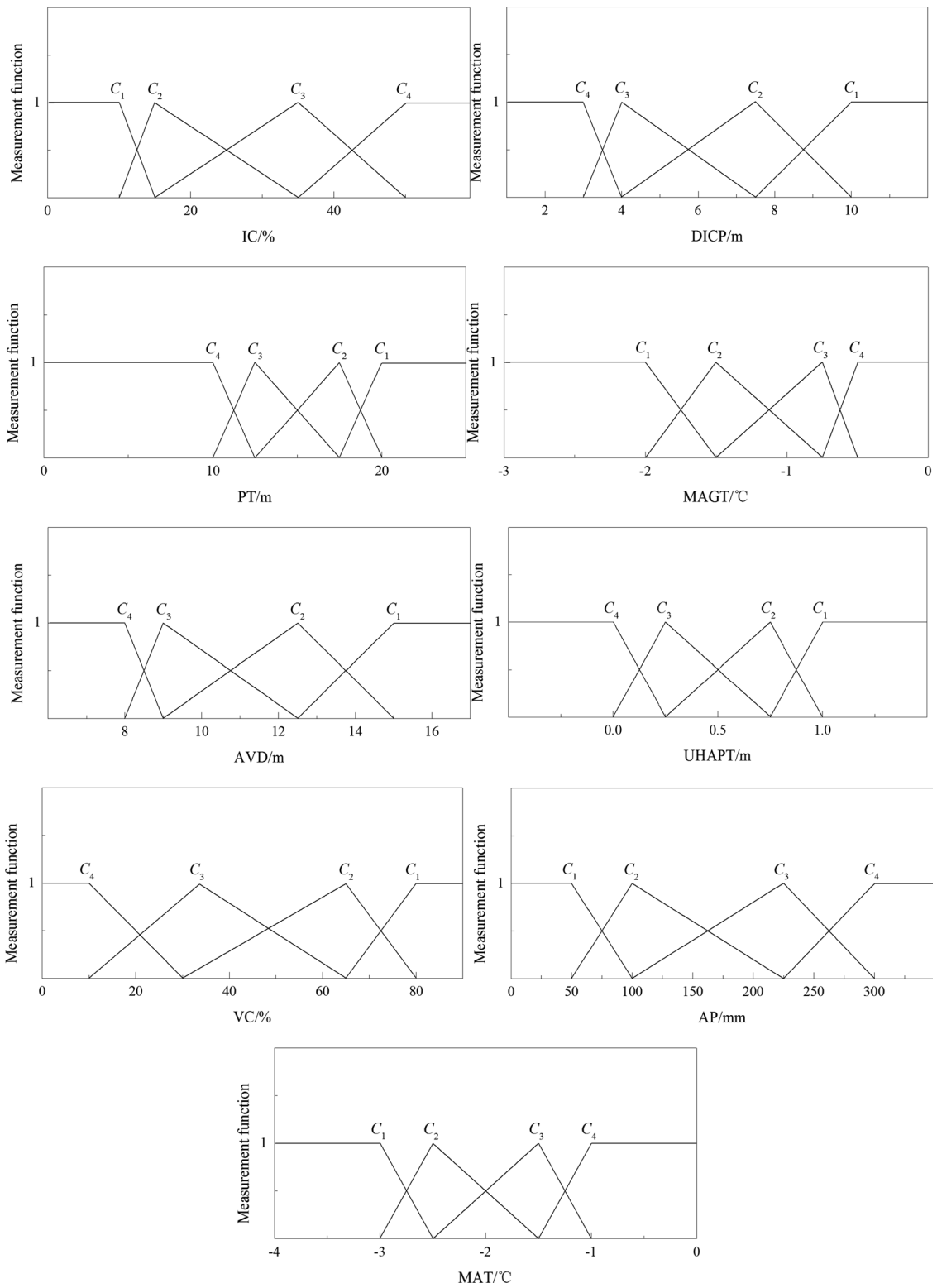
et al. 2015) are also listed in Table 3 to compare and analyze the results of the different evaluation methods. Table 4 shows the annual settlement of typical sections in the permafrost regions of the Qinghai–Tibet railway (Qinghai–Tibet Railway Company 2014).

The results show that the subgrade stability of DK3 section is slightly better than that of DK2 section, the stability grade of DK3 section belongs to grade I. The subgrade stability of the other three sections (DK1, DK4, DK5) are less stable than that of DK2 section, and their stability grade belongs to the same grade, which is grade III.

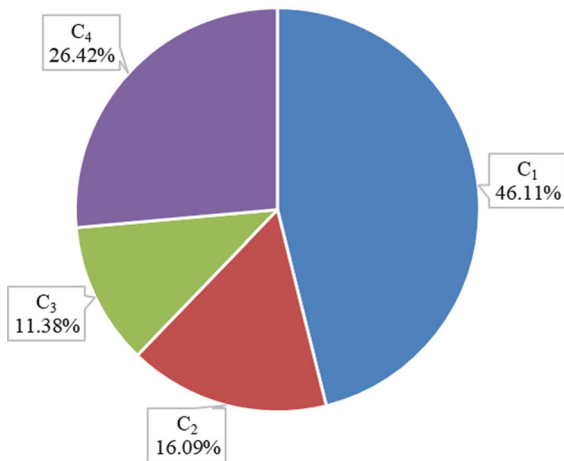
In addition, the permafrost subgrade stability along Qinghai–Tibet railway is sorted according to evaluation ordering values, and the unascertained superiority degrees of the five sections (DK1, DK2, DK3, DK4, DK5) are 2.494, 2.831, 3.312, 2.019, 2.331 respectively. It shown that the subgrade stability of the five samples exhibits the following order from best to worst: DK3, DK2, DK1, DK5, DK4. From the evaluation ordering values, it can be seen that the evaluation results are consistent with analysis results of the fuzzy evaluation method in the reference (Meng et al. 2015).

The effect of single factor in unascertained measurement model is analyzed. The changes in ice content ( $U_1$ ), depth of ice-rich content permafrost ( $U_2$ ), annual ground temperature ( $U_4$ ), uplift height of artificial permafrost table ( $U_6$ ), annual temperature ( $U_{10}$ ) and subgrade height ( $U_{11}$ ) affect the evaluation results significantly, and can lead to changes in the evaluation grade, while other factors show little influence on the final result. Therefore, the factors ( $U_1, U_2, U_4, U_6, U_{10}, U_{11}$ ) can be considered as the main factors affecting the stability of subgrade.

Based on these six factors mentioned above, a reduced-order model is established, and the evaluation results are the same as those of the 12-factors model.



**Fig. 4** Unascertained measurement function of evaluation indexes



**Fig. 5** Proportional relation map

From another perspective, the model is universal, which can consider the situation with few factors, but also consider the situation with many factors. If the factor data is complete, the results will be more accurate. If the data is incomplete, as long as the main factors are grasped, accurate prediction results can be obtained. There are few artificial factors in the model, which proves that the model is better than other models.

In actual engineering, there is no cracks or slips in the subgrade at DK2 and DK3 sections, then no treatment measure is taken. It can be considered that the stability of DK3 is slightly better than that of DK2 based on the settlement data. In response to the settlement problem in the subgrade engineering of the Qinghai–Tibet Railway in permafrost regions, reinforcement measures have been taken for grade III subgrades. For DK4 section, the annual settlement in 2012 is 27 mm before the reinforcement measures,

and it is not more than 11 mm after the stability reinforcement of subgrade. Similarly, the settlements are respectively 14 mm and 5 mm after the subgrades of DK5 section and DK1 section are reinforced. Based on the settlement data before and after reinforcement measures, the stability order from best to worst is DK1, DK5, DK4. From the evaluation results, it can be seen that the evaluation results are accordant with the actual situation basically.

Through above analysis, both the stability grade results and evaluation ordering values are consistent with analysis results of the fuzzy evaluation method and accordant with the actual situation. It is proved that the unascertained measurement theory is feasible for the stability evaluation of permafrost subgrade. Although the performance of the fuzzy evaluation model and unascertained measurement model are the same in terms of the evaluation results, fuzzy evaluation method emphasizes too much the importance of the extreme value, which is apt to cause information loss, and the determination of index weight has subjective randomness (Shi et al. 2010). In addition, it is hard for the fuzzy evaluation method to distinguish the differences between the adjacent categories (Li 1997; Qin and Qin 2011). However, the entropy theory and credible degree recognition criterion are used in the process of subgrade stability evaluation for unascertained measurement model, which can overcome the shortcomings of fuzzy model and make the evaluation results more objective.

Therefore, it can be seen from the above analysis that the performance of the unascertained measurement model is relatively superior to the fuzzy model. The method is reliability and practicability by combining unascertained measurement and entropy theory in evaluating the stability of the permafrost subgrade,

**Table 3** Evaluation results of subgrade stability in permafrost regions

Location	Comprehensive unascertained measurement				Unascertained measurement evaluation results		Fuzzy evaluation results <sup>a</sup>	
	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	Evaluation ordering values	Stability grade	Evaluation values	Stability grade
DK1	0.3091	0.0626	0.2522	0.3761	2.494	III	0.485	III
DK2	0.4611	0.1609	0.1138	0.2642	2.819	II	0.736	II
DK3	0.5972	0.2034	0.1141	0.0853	3.312	I	0.945	I
DK4	0.1978	0.1083	0.2089	0.4850	2.019	III	0.426	III
DK5	0.3565	0.0853	0.0906	0.4676	2.331	III	0.440	III

<sup>a</sup>The fuzzy evaluation results come from Meng et al. (2015)

**Table 4** Annual settlement of subgrade in permafrost region

Location	Stability grade	Annual settlement/mm			
		2011	2012	2013	2014
DK1	III	16	18	20	5
DK2	II	7	9	6	11
DK3	I	5	3	5	8
DK4	III	32	27	11	4
DK5	III	30	25	27	14

which offers technical support on guiding the safety construction, and it is also a new method for subgrade stability evaluation.

## 5 Conclusions

As to the complexity and uncertainty of the stability evaluation in permafrost region, unascertained measurement theory and entropy theory are introduced in this paper. The characteristics and conditions of the geological environment in the permafrost regions are considered, and twelve influencing factors are selected as the stability evaluation indexes of subgrade. Meanwhile, the evaluation indexes of the subgrade stability grading are constructed. Thus the evaluation model of subgrade stability in permafrost regions is established.

To reduce the influence of subjective judgment and single factor, the evaluation index weights are calculated by entropy theory, which can improve the credibility of evaluation results. Meanwhile, the evaluation results of the permafrost subgrade stability are obtained based on credible degree recognition criterion, and the stability of five typical permafrost sections are sorted according to evaluation ordering values. The order from best to worst is DK3, DK2, DK1, DK5, DK4, and the stability grades are grade I, II, III, III, III.

Unascertained measurement evaluation model can determine the stability grade of permafrost subgrade and sort the stability, which objectively reflects the stability of permafrost subgrade and provides a new way for the stability evaluation in permafrost subgrade. Moreover, it should be noted that the accuracy and credibility of stability evaluation results depend on the adopted evaluation index and the methods of constructing unascertained measurement function.

Therefore, the subgrade stability affected by each influence factor should be analyzed in detail and the more widely applicable unascertained measurement function should be constructed to improve the reliability of stability evaluation model in permafrost subgrade.

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