Geotech Geol Eng (2018) 36:2057–2076
https://doi.org/10.1007/s10706-018-0445-7

ORIGINAL PAPER

Numerical ANFIS-Based Formulation for Prediction of the Ultimate Axial Load Bearing Capacity of Piles Through CPT Data

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Received: 23 May 2017 / Accepted: 2 January 2018 / Published online: 16 January 2018

Abstract This study explores the potential of adaptive neuro-fuzzy inference systems (ANFIS) for prediction of the ultimate axial load bearing capacity of piles (P_u) using cone penetration test (CPT) data. In this regard, a reliable previously published database composed of 108 datasets was selected to develop ANFIS models. The collected database contains information regarding pile geometry, material, installation, full-scale static pile load test and CPT results for each sample. Reviewing the literature, several common and uncommon variables have been considered for direct or indirect estimation of P_u based on static pile load test, cone penetration test data or other in situ or laboratory testing methods. In present study, the pile shaft and tip area, the average cone tip resistance along the embedded length of the pile, the average cone tip resistance over influence zone and the average sleeve friction along the embedded length of the pile which are obtained from CPT data are considered as independent input variables where the output variable is P_u for the ANFIS model development. Besides, a notable criticism about ANFIS as a prediction tool is that it does not provide practical prediction equations. To tackle this issue, the obtained optimal ANFIS model is represented as a tractable equation which can be used via spread sheet software or hand calculations to provide precise predictions of P_u with the calculated correlation coefficient of 0.96 between predicted and experimental values for all of the data in this study. Considering several criteria, it is represented that the proposed model is able to estimate the output with a high degree of accuracy as compared to those results obtained by some direct CPT-based methods in the literature. Furthermore, in order to assess the capability of the proposed model from geotechnical engineering viewpoints, sensitivity and parametric analyses are done.

Keywords Estimation · Pile axial load bearing capacity · Cone penetration test · Adaptive neuro fuzzy inference systems · Tractable formulation

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1 Introduction

Deep foundations are mainly used in situations that the underlying soil layer is not enough capable of bearing the applied loads where it is not possible to use shallow foundations. The main objective of using piles is to transfer structural loads to a strong layer which is able to support the applied loads. Thus, the safety and stability of pile supported structures depend on the behavior of piles. In order to ensure the stability of the foundation, it is necessary to make an accurate estimation of the pile bearing capacity, particularly for pre-design purposes. This problem has been challenging for many geotechnical engineers and not yet entirely been handled due to many factors and uncertainties affecting the system behavior including complicated behavior of piles in soil, soil disturbance due to pile installation and sampling, and pile load transfer mechanism (Kiefa 1998).

Based on characteristics of load transfer to the underlying layer, the staticaxial load bearing capacity of the pile (Q_u) can be generally considered as the sum of end-bearing capacity of the pile (Q_t), i.e. the bearing capacity of the compact stratum or a stiff layer where applied loads are transferred onto and the shaft friction capacity (Q_s) that loads are carried through friction of the surrounding soil along the shaft and can be represented as the equation as follows (Niazi and Mayne 2013):

$$Q_u = Q_t + Q_s = q_t . A_t + \sum_{i=1}^{n} f_{s(i)} . A_{si} \quad (1)$$

where q_t is the unit end bearing capacity, A_t is the area of the pile at tip, f_s unit shaft resistance of the ith soil layer through which the pile shaft is embedded; A_{si} is the area providing frictional resistance with the adjacent soil in the ith layer against axial displacement

It is worth mentioning that the end-bearing capacity (Q_t) plays the overriding role in granular soils, whereas in cohesive soils the shaft friction capacity (Q_s) dominates. In this regard, considering the type of pile is an important issue for the design and analysis of piles (Tomlinson and Woodward 2014).

The most usually utilized approaches to determine the bearing capacity of piles can be classified into following groups: (1) full scale pile load tests (2) analytical and semi empirical methods (3) correlation with in situ tests. P_u may be obtained using laboratory testing methods; however, it is necessary to conduct several field and laboratory experimentations such as standard penetration test (SPT), unconfined compression test, soil classification, etc. due to the variability and large number of soil properties which are needed as input variables for the static analysis. Hence, laboratory methods would have some certain drawbacks such as being cumbersome, expensive and time consuming. On the one hand, full scale pile load tests cannot be used for other engineering purposes to assess the pile behavior as they are highly expensive. On the other hand, dynamic load tests require special equipment expertise for monitoring, recording and interpreting the obtained data. Among various in situ experimentations, cone penetration test (CPT) may be regarded as the most frequently utilized method for characterization of geo-materials properties at different depth. Besides, CPT is the most commonly used method for soil investigation in Europe as it is rapid, economical and is able to continuously obtain information from soil (Omer et al. 2006). CPT is basically composed of a cylindrical rod with a cone tip which is driven into the soil and measure the tip resistance and sleeve friction due to this intrusion which can be considered as a model pile. Two main parameters obtained from the CPT test, cone resistance and sleeve friction, can be regarded as base and shaft resistance of the pile respectively. In addition, the resistance parameters are used to classify soil strata and to estimate strength and deformation characteristics of soils at different depth. CPT is a simple, quick, and economical test that provides reliable information about undistributed in situ continuous soundings of subsurface soil (Abu-Farsakh and Titi 2004; Eslami 1996; Niazi and Mayne 2013; Omer et al. 2006). Furthermore, it is recently possible to apply this test for a wide range of geotechnical applications adding up different devices to cone penetrometer.

Quite a few approaches are proposed for calculating the axial pile capacity using CPT data. These methods may be mainly classified into two methods (Niazi and Mayne 2013):

(1) *Direct approach* The unit toe bearing capacity of the pile (q_t) is evaluated from the cone tip resistance (q_c), and the unit skin friction of the pile is evaluated from either the sleeve friction (f_s) profile or q_c profile.

(2) *Indirect approach* The CPT data, q_c and fs, are firstly used to evaluate the soil strength parameters such as the undrained shear strength (S_u) and the angle of internal friction ϕ. These parameters are then used to evaluate the qt and the unit skin friction of the pile ϕ using formulas derived based on semi-empirical or analytical methods. In this major quite a few models have been proposed by various researchers.

1.1 Schmertmann (1978) Method

The Schmertmann (1978) method is based on the result of the 108 load tests on pile carried out by Nottingham (1975). The ultimate tip resistance (Q_p) of pile can be calculated as:

$$Q_p = \left(\frac{q_{c1} + q_{c2}}{2}\right) A_{tip} \tag{2}$$

where A_{tip} is pile tip area; q_{c1} and q_{c2} are the minimum of the average cone tip resistances of zones ranging from 0.7 to 4 D below the pile tip, and over a distance 8 D above the pile tip, respectively. The average cone resistance values are obtained from the graphical representation of the failure surface, which is assumed to follow a logarithmic spiral as introduced by Begemann (1963). The method limits the average of q_{c1} and q_{c2} to 15 MPa.

Based on this method, the ultimate shaft resistance (Q_s) of pile is given by:For clay:

$$Q_s = \alpha_c f_s A_s \tag{3}$$

For sand:

$$Q_s = \alpha_s \left[\sum_{d=0}^{8D} \frac{1}{2} f_s A_s + \sum\nolimits_{d=8D}^{L} f_s A_s\right] \tag{4}$$

where D and A_s are pile width or diameter, pile-soil surface area, respectively; a_c and a_s are the ratio of pile shaft resistance to the sleeve friction for sand and clay; a_c varies from 0.2 and 1.2 and is a function of the values of the sleeve friction, while a_s depends on the ratio of the embedment length of pile to the pile width or diameter and varies from 0.4 to 2.4; f_s is the average sleeve friction; product of f_s and α_c must not exceed 120 kPa.

1.2 deRuiter and Bringen (1979) Method

The method proposed by de Ruiter and Beringen (1979) is based on the experience gained in the North Sea. In clay, both unit tip resistance and shaft resistance are determined from undrained shear strength using following equations:

Unit tip resistance in clay:

$$q_p = N_c S_u \tag{5}$$

Unit shaft resistance in clay:

$$f = \alpha' S_u \tag{6}$$

where α' is adhesion factor (0.5 for OC clays and 1 for NC clays); N_c is bearing capacity factor and S_u is the undrained shear strength obtained from the following equation:

$$S_u = \frac{q_{ca}}{N_k} \tag{7}$$

where N_k is the cone factor and varies from 15 to 20.

For unit shaft resistance of a pile in sand, the following equation is used:

$$f = \min\{f_s,\ q_c/300,\ 120\ kPa\} \tag{8}$$

where q_c and f_s are cone resistance and sleeve friction, respectively. Calculation of pile tip resistance in sand with this method is similar to Schmertman (1978) method.

1.3 Bustamante and Gianeselli (1982) (LPC) Method

LPC method, also called French method, is based on the experiments conducted by Bustamante and Gianselli (1982) for the French highway department. According to this method, both unit tip and shaft resistance are determined from the cone resistance, neglecting sleeve friction. The unit tip resistance is estimated using following equation:

$$q_p = k_b q_{ca} \tag{9}$$

where q_{ca} is the average of the q_c values over the influence zone which ranges between 1.5 D below and above the pile tip (D is pile diameter); k_b is a factor depending on soil material and pile installation type. The ultimate shaft resistance in this method can be calculated by the following formula:

$$f = \frac{q_c}{k_s} \tag{10}$$

where K_s is a factor that depends on pile material and installation method, and q_c is the average cone resistance value over the pile embedment length.

Furthermore, codes and guidelines, e.g. Eurocode-7 (1997) and ERTC3 (1999), suggest different methods for direct or indirect estimation of pile load bearing capacity based on CPT data (Niazi and Mayne 2013; Omer et al. 2006). Although great efforts have been made to develop appropriate models using analytical methods and assumptions, they often made their conclusions by optimizing the analytical results with their obtained experimental data through empirical methods. In other words, such models are designated through simplifying assumptions and more important they are initiated from few observations and controlling few models through simple statistical regression analyses to find the appropriate model. Besides, the suggested values by guidelines are often too conservative and general. This is highly related to the complex parameters affecting the system behavior which cannot thoroughly be considered for producing constitutive models considering information obtained from CPT results.

Recently, Shahin (2010) have utilized artificial neural networks (ANNs), Alkroosh and Nikraz (2011)and Alkroosh and Nikraz (2012) demonstrated the capability of gene expression programming (GEP) and Kordjazi et al. (2014) used support vector machines (SVM) for prediction of ultimate axial load-carrying capacity of piles through experimental CPT data. It is worth mentioning that the models obtained by those artificial intelligence based approaches represented better performance in comparison with the traditional analytical formulas. Such soft computing techniques may be considered as good alternatives to traditional methods for tackling real world problems as they can automatically learn from observed data to construct a prediction model. Besides, these techniques have become more attractive because of their capability of information processing such as non-linearity, high parallelism, robustness, fault and failure tolerance and their ability to generalize models. Also, they have been successfully applied to many civil engineering prediction problems (Alavi and Sadrossadat 2016; Fattahi and Babanouri 2017; Khandelwal and Armaghani 2016; Sadrossadat et al. 2013, 2016a, b, 2017; Tajeri et al. 2015; Xue et al. 2017; Ziaee et al. 2015; Žlender et al. 2012).

This paper explores the capability of adaptive neuro-fuzzy inference system paradigm to find an optimal model for indirect estimation of the ultimate load bearing capacity of piles through a reliable collection of CPT results. The major criticism associated with ANFIS in comparison to some other soft computing is the fact that it produces black-box models as such techniques usually do not provide practical prediction equations. To cope with these issues, the calculations required for input processing output and model development by ANFIS are explicitly explained and the derived model is represented in present study. In order to verify the robustness of the obtained model several validation and verification study phases are conducted.

2 Adaptive Neuro-Fuzzy Inference Systems

ANFIS combines the advantages of fuzzy inference systems (FIS) with the learning ability of ANN and presents all their benefits in a single framework. The selection of the FIS is the main concern in designating the ANFIS model (Jang et al. 1997). Several FIS systems have been already developed in the literature based on fuzzy reasoning and the employed fuzzy if then rules e.g. (Mamdani 1977; Takagi and Sugeno 1985; Tsukamoto 1979). There are two types of commonly utilized fuzzy inference systems: Mamdani and Takagi–Sugeno (TS) or Sugeno. In Mamdani model both input and output variables are fuzzy (Mamdani 1977), whereas, in TS or the sugeno inference system the output is expressed as a linear function of the input variables which takes a numerical value (Takagi and Sugeno 1985).The main difference between them is the fact that while the Mamdani model uses the human expertise and linguistic knowledge to design the membership functions and if–then rules, TS model uses optimization and adaptive techniques to establish the system modeling and also uses less number of rules. Furthermore, when a numerical or crisp output is required, then, the data-driven rule generation with TS model is selected. Also the output membership function in TS is simpler designed as either linear or constant (Sadrossadat et al. 2016b; Takagi and Sugeno 1985). This inference system is more commonly used in ANFIS for modeling problems (Sugeno and Kang 1988). Considering two input variables (x, y) and one output (f), the two if–then rules in first-order TS type can be represented as follows:

Rule 1: if $x = A_1$ and $y = B_1$, then $f_1 = p_1x + q_1y + r_1$

Rule 2: if $x = A_2$ and $y = B_2$, then $f_2 = p_2x + q_2y + r_2$

where p_i, q_i, and r_i are the consequent parameters obtained from the training, A and B labels of fuzzy set defined suitable membership function.

ANFIS optimizes the model parameters using the ANN architecture. In ANFIS, the input variables are propagated forward in a network similar to the MLP architecture layer by layer. Best consequent parameters are determined by the least-squares method (LSM), while the premise parameters are assumed to be fixed for the current cycle through the training set. Next, the error values propagate backward to adjust the premise parameters, using back propagation gradient descent method (Sadrossadat et al. 2016b; Žlender et al. 2012). In this algorithm, the weighted values are changed to minimize the following error function (E):

$$E = \frac{1}{2}\sum_{n}\sum_{k}\left(t_k^n - h_k^n\right)^2 \tag{11}$$

where t_k^n and h_k^n are, respectively, the calculated output and the actual output value, n is the number of samples and k is the number of output neurons.

As illustrated in Fig. 1, there are five essential layers in which the mathematical computations in ANFIS are performed. The process in each layer may be described as follows (Sadrossadat et al. 2016b):

Layer one Each node in this layer modifies the values of the crisp input variables by using membership functions (fuzzification step). Every node i in this layer is an adaptive node. Parameters in this layer determine the final shape of the membership function and are called *premise parameters*. The output of the ith node of the first layer may be MFs such as linear, triangular, trapezoidal, Gaussian, generalized bell or several other functions. Here, MFs are described by Gaussian functions as selected for the q_{ult} modeling.

$$O_{1,i} = \mu_{A_i}(x) = e^{\frac{-(x-c_i)^2}{2\sigma_i^2}} \quad for \quad i = 1,\ 2 \tag{12}$$

$$O_{1,i} = \mu_{B_i}(x) = e^{\frac{-(x-c_i)^2}{2\sigma_i^2}} \quad for \quad i = 3,\ 4 \tag{13}$$

In equations above, x is the input to node i, and A_i is the linguistic label associated with this node function. So, the $O_{1,i}(x)$ is essentially the membership grade for x and y which is assumed to be a Gaussian function. c_i and σ_i are respectively the center and width of the ith fuzzy set A_i or B_i. These parameters are adjusted

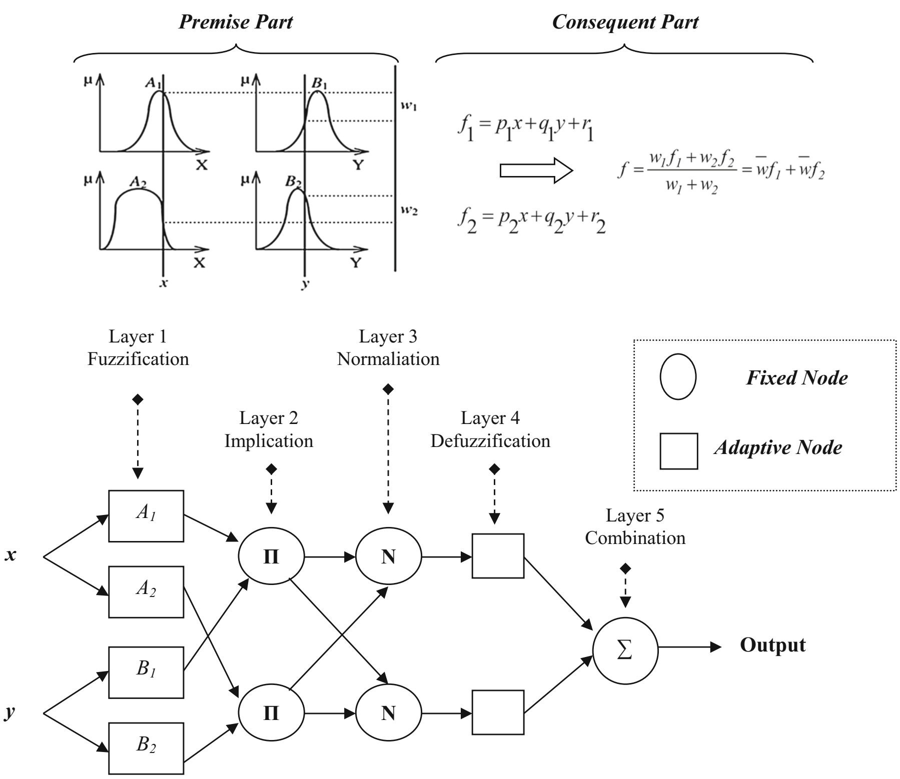


Fig. 1 A typical first-order TS model reasoning and the basic ANFIS architecture

during model optimization and are referred to as *premise parameters*.

Layer two The antecedent parts of rules are computed in this layer. This layer consists of the nodes labelled $\prod$, which multiplies the incoming signals and sends the product out.

$$O_{2,\,i} = w_i = \mu_{A_i}(x)\,.\,\mu_B(y) \quad for \quad i = 1,\,2 \tag{14}$$

Layer three Each node in this layer is a fixed node labeled N. The ith node calculates the ratio of the ith ratio of the firing strengths of the rules as follows:

$$O_{3,\,i} = \bar{w}_i = \frac{w_i}{w + w_2} \quad for \quad i = 1,\,2 \tag{15}$$

Layer four The fourth layer is the second adaptive layer of ANFIS architecture, i.e., defuzzification layer. The nodes in this layer are adaptive with linear node functions. Parameters in this layer are called *consequent parameters* which are parameters of output membership functions. These parameters are adjusted during the training of the model considering the utilized datasets.

$$O_{4,\,i} = \bar{w}_i f_i = \bar{w}_i(p_i x + q_i y + r_i) \tag{16}$$

The parameters in this layer ($p_i,\ q_i,\ r_i$) are to be determined and are called consequent parameters.

Layer five The node in this layer is a single fixed node and computes the final model output as the combination of all incoming signals from every fired rule. It is a weighted average combination which is indicated as follows:

$$Overall\ output = O_{5,\,i} = \sum\nolimits_i \bar{w}_i f_i = \frac{\sum_i w_i f_i}{\sum_i f_i} \tag{17}$$

3 Experimental Database

A comprehensive database including the results of the 108 extensive load tests on axially loaded piles has been drawn from the earlier studies by Eslami (1996) and Pooya Nejad (2009). The database consists of information about load test results and pile geometry along with the CPT measurements along the pile embedment length and pile tip. The CPT data consist of cone tip resistance (q_{c1}, q_{c2}, q_{c3}) and sleeve friction values (f_{s1}, f_{s2}, f_{s3}) along the pile embedded length as well as the average cone tip resistance ($\bar{q}_{c-t}$) around the pile failure zone. The pile embedded length is divided into three equal segments with the same thickness, and the average values of q_c and f_s are calculated to consider the variability of the soil properties. Piles are either driven or bored, made up of different materials (concrete, steel, composite), with various shapes (square, round, octagonal, triangle, pipe and H section), and different tip conditions (open and closed). The soil types include sands, clays and silts and mixture of them, in single and multiple layers (Kordjazi et al. 2014).The maintained load test was conducted on most of the cases in the study, in which the pile is loaded in several increments equal to 15% of the design load, each maintained for 5 min. The loading is then continued until reaching 300% of the design load (Fellenius 1975).

The results of the compression load test were plotted in the form of curves describing foundation displacement as a function of applied load. The failure is defined as the point in which excessive displacements take place under a relatively small increase in loading, typically associated with an abrupt change in the load–displacement curve characteristics. In the database, the 80% criterion interpretation method was used in the cases that the failure point was not easily defined (Hansen 1963). It is worth mentioning that various part of the database have been used by various researchers (Alkroosh and Nikraz 2011, 2012; Kordjazi et al. 2014; Shahin 2010). The utilized data and descriptive statistics of variables for developing the ANFIS model are given in Tables 1 and 2, respectively.

4 Numerical Simulation of Bearing Capacity

After reviewing the literature and the structure of the existing models in this field, the main influential parameters which are considered in the database include: type of pile static load test (maintained or constant rate of penetration), pile material (steel, concrete and composite), pile installation method (driven or bored), pile tip condition (close or open), embedment length of pile (L_{emb}), perimeter of the pile (O), cross sectional area of the pile tip (A_t), the average pile tip resistance along the pile embedded length (q_{c1}, q_{c2} and q_{c3}), average sleeve friction along the embedded length of the pile (f_{s1}, f_{s2} and f_{s3}), average cone tip resistance about pile tip failure zone ($\bar{q}_{c-t}$). Among these variables, the average CPT measurements are

Table 1 The utilized data for developing the ANFIS model

References	Type of test	Type of pile	Installation method	End of pile	At (cm^2)	Af (m^2)	qca (MPa)	fsa (kN)	qct (MPa)	Pu (kN)	ANFIS
Abu-Farsakh et al. (1999)	ML	C	DR	CL	5806.4	112.86	1.613	9.39	11.11	5435	6596.38
Albiero et al. (1995)	ML	C	BO	CL	960	10.47	1.497	80.49	3.47	645	786.7
	ML	C	BO	CL	1260	11.99	1.497	80.49	3.29	725	790.56
Altaee et al. (1992)	ML	C	DR	CL	810	12.69	3.86	105.84	3.58	1000	771.83
	ML*	C	DR	CL	810	17.31	5.2	126.32	7.9	1600	1223.36
Avasarala et al. (1994)	ML	C	DR	CL	960	22.72	5.65	71.21	5.43	1260	1344.37
	ML*	C	DR	CL	2500	22.27	14.307	82.51	4.85	2070	1684.98
	ML*	C	DR	CL	960	17.82	7.157	158.06	6.29	1350	1185.56
Bakewell Bridge (unpublished)	ML	C	BO	CL	2827.4	13.74	13.357	121.52	10	518	2024.59
Ballouz et al. (1991)	ML	C	BO	CL	7854	31.8	8.497	105.08	10	4029	4295.64
	ML	C	BO	CL	6575.6	29.1	3.227	68.76	6	3000	2646.23
Briaud et al. (1988)	ML	C	DR	CL	2030	15.49	3.92	181.59	6.16	1330	936.68
	ML	C	DR	CL	1230	9.21	2.753	115.38	2.02	800	515.42
	ML	S	DR	OP	80	8.69	4.953	124.82	15.2	590	960.87
	ML*	S	DR	OP	80	9.92	6.06	164.1	11	1070	1097.89
	ML*	C	DR	CL	1600	13.6	3.69	97.86	12.8	1240	1318.15
	ML*	C	DR	CL	1600	34.01	2.32	67.36	6.35	1330	1743.03
	ML	S	DR	OP	100	9.8	3.98	165.04	6.5	470	756.8
	ML	C	DR	CL	20	18.77	3.063	77.14	4.5	1250	1002.55
	ML	C	DR	CL	30	21.7	4.508	151.04	9.19	1170	1065.12
	ML	C	DR	CL	1600	12.19	4.913	48.24	4.1	600	815.71
	ML	C	DR	CL	1230	18.95	3.173	97.53	3.3	1070	880.18
	ML	C	DR	CL	2030	11.8	5.96	179.1	8.4	1160	1106.8
	ML*	C	BO	CL	960	9.91	6.507	198.21	8.4	1170	919.51
	ML*	C	DR	CL	1600	16.84	2.81	104.38	9.64	1170	1157.38
	ML*	C	BO	CL	960	9.46	3.513	122.07	5.9	720	820.34
	ML*	C	DR	CL	1600	18.3	1.5	170.82	14.2	870	1303.72
	ML*	C	DR	CL	1230	22.68	7.003	145.13	6.05	1070	1361.07
	ML*	C	DR	CL	1600	18.46	5.517	129.14	10	1140	1479.01
	ML	C	DR	CL	1600	18.14	7.357	168.36	7.64	1020	1368.62
	ML*	C	DR	CL	1230	35.43	8.06	147.35	10.25	1560	2188.14
	ML*	C	DR	CL	1600	31.1	1.67	53.35	1.18	1780	1630.87
	ML*	S	DR	OP	100	11.02	8.2	349.64	15.8	2100	1973.32
	ML	C	BO	CL	960	11.91	3.96	93.05	9.2	1390	1175.32
	ML	C	DR	CL	1600	11.99	2.293	70.69	4.2	640	872.52
	ML*	C	DR	CL	2030	19.5	2.367	89.75	11.8	1500	1416.84
	ML	S	DR	OP	80	15.08	2.323	78.38	7.63	1210	1091.54
	ML	C	DR	CL	1600	14.25	5.573	70.27	8.95	1140	1394.58
	ML*	C	BO	CL	960	13.92	3.587	97.75	8.15	1100	1113.51
	ML	C	DR	CL	2030	27.33	5.82	121.63	7.89	1420	1925.69
	ML*	C	DR	CL	1230	9.5	3.763	168.92	6.74	1470	704.08
	ML	S	DR	OP	100	15.19	5.733	125.84	8.95	520	1290.01

Table 1 continued

References	Type of test	Type of pile	Installation method	End of pile	At (cm^2)	Af (m^2)	qca (MPa)	fsa (kN)	qct (MPa)	Pu (kN)	ANFIS
	ML	S	DR	OP	80	14.82	2.22	78.64	7.63	1240	1083.65
	ML	C	BO	CL	960	7.24	5.62	171.09	14.88	880	1212.65
	ML	C	DR	CL	1230	7.79	5.85	196.6	7.6	1050	797.2
	ML	S	DR	OP	80	15.03	2.323	78.38	7.63	1260	1091.11
	ML	S	DR	OP	100	23.27	6.737	146.65	9.75	1370	1422.8
Brown et al. (2006)	CRP*	C	BO	CL	2827.4	19.08	2.123	75.88	1.7	2205	1516.94
	ML	C	BO	CL	2827.4	10.98	2.123	75.88	6	1800	1011.02
Campnella et al. (1989)	ML	S	DR	CL	620	17.31	2.22	20.28	6.75	630	1096.34
	ML	S	DR	CL	820	14.12	1.63	13.08	0.85	290	660.78
	ML*	S	DR	CL	820	32.05	5.393	27.15	2.3	1100	1022.61
	ML*	S	DR	OP	6580	194.65	5.25	37.09	4.2	7500	7416.21
Hill (1987)	ML	C	DR	CL	3080	52.76	9.083	125.08	21.8	5785	5542.89
Fellenius et al. (2004)	ML	CO	DR	CL	1294.6	58.1	2.653	33	2	1915	2313.7
Fellenius et al. (2007)	ML	C	DR	CL	1225	8.75	4.573	308.41	5.74	1500	1638.77
Finno (1989)	ML	S	DR	CL	1280	22	6.91	63.02	1.15	1010	727.79
	ML	S	DR	CL	1990	21.69	6.91	63.02	1.15	1020	1034.93
Florida Dept. of Trans. (2003)	CRP	C	DR	OP	7458.7	111.47	2.45	23.97	11.14	10,910	9854.63
Gambini (1985)	ML	S	DR	CL	860	10.5	5.863	88.37	21.4	625	1342.03
Harris and Mayne (1994)	ML	C	BO	CL	4536.5	41.09	5.457	121.4	20	2782	3405.32
Haustoefer et al. (1988)	ML	C	DR	CL	1260	14.66	5.067	83.99	9.7	1300	1386.65
	ML	C	DR	CL	2030	26.02	10.703	122.5	12.62	4250	2713.89
Horvitz et al. (1981)	ML	C	BO	CL	960	17.59	5.347	63.33	5.83	900	1222.12
Laier (1994)	ML	S	DR	OP	140	45.2	11.403	134.14	19.5	2130	2504.47
Matsumoto et al. (1995)	ML	S	DR	OP	410	20.83	24.707	81.91	27.11	4700	4299.11
	ML	S	DR	OP	410	20.83	24.707	81.91	27.11	3690	4299.11
Mayne (1993)	ML	C	BO	CL	4420	40.13	5.383	118.33	5.72	4500	3467.08
McCabe and Lehane (2006)	ML	C	DR	CL	625	6.07	0.833	10	0.25	60	555.72
Neveles (1994)	ML	C	DR	CL	2920	35.37	9.663	50.62	11.55	3600	3290.86
	ML*	S	DR	CL	3420	38.14	9.663	49.22	11.26	3650	3519.46
Nottingham (1975)	ML	S	DR	CL	590	13.2	6.5	76.55	6.3	675	1253.69
	ML	C	DR	CL	940	16.92	5.753	70.56	4.37	810	1074.6
	ML	C	DR	CL	2020	27.15	6.29	130.52	5.74	1755	1812.3
	ML*	C	DR	CL	2030	16.67	12.803	323.05	11.77	1845	2133.64
	ML*	S	DR	CL	590	19.54	7.883	97.37	22.3	1620	1911.08
	ML*	C	DR	CL	1230	22.46	5.77	70.21	8.87	1485	1638.61
	ML*	C	DR	CL	2030	14.58	3.437	52.86	9.5	1140	1284.09

Table 1 continued

References	Type of test	Type of pile	Installation method	End of pile	At (cm^2)	Af (m^2)	qca (MPa)	fsa (kN)	qct (MPa)	Pu (kN)	ANFIS
Omer et al. (2006)	ML	C	DR	CL	1320	11.08	6.36	182.2	18	2475	1944.09
	ML	C	DR	CL	1320	11.08	6.643	195.16	20	2257	2656.78
	ML	C	DR	CL	1320	11.08	6.643	195.16	20	2670	2656.78
	ML	C	DR	CL	1320	11.08	6.36	182.2	18	2796	1944.09
O'neil (1986)	ML	S	DR	CL	590	11.29	2.467	42.33	4.5	780	904.51
	ML	S	DR	CL	590	11.29	2.94	64.43	2.8	800	750.18
O'neil (1988)	ML	S	DR	CL	590	5.45	5.527	39.05	5.8	490	674.17
Paik and Salgado (2003)	ML	S	DR	OP	325.72	14.43	11.457	69.8	20	1140	650.88
	ML	S	DR	CL	995.38	7.81	11.457	69.8	6	1620	1249.29
Peixoto et al. (2000)	ML	C	DR	CL	324	9.47	1.97	135.58	2.69	260	571.9
Reese et al. (1988)	ML	C	BO	CL	5030	61.23	9.913	267.36	13.08	5850	6217.2
	ML	C	BO	CL	5030	61.23	9.637	266.39	18.25	7830	7484.64
Tucker and Briaud (1988)	ML	S	DR	OP	140	23.88	10.177	86.23	19.71	2900	1180.94
	ML	S	DR	CL	960	16.03	17.25	75.17	20.8	1300	1402.42
	ML	S	DR	CL	1260	18.62	17.25	75.57	20.5	1800	1849.23
Tumay and Fakhroo (1981)	ML	S	DR	CL	960	31.8	1.37	20.38	7.72	1710	1528.13
	ML	C	DR	CL	6360	108.29	2.93	34.47	2.35	3960	4250.15
	ML	C	DR	CL	960	10.58	8.53	136.97	1.25	900	364.34
	ML*	C	DR	CL	1075	46.31	1.353	19.81	4.68	2160	1940.96
	ML*	S	DR	CL	1260	47.72	1.66	21.78	4.68	2800	2173.48
	ML*	C	DR	CL	2030	66.51	1.663	21.2	3.76	2950	3395.14
	ML*	C	DR	CL	5630	60.13	1.56	38.34	2.36	2610	2848.99
Urkkada Ltd (1995)	ML*	S	DR	CL	710	29.91	3.17	26.84	1.58	1690	970.35
	ML*	S	DR	CL	710	27.08	3.057	29.29	1.43	1240	889.79
US Dept. of Transportation (2006)	ML	C	DR	CL	3721	41.99	6.673	74	5.6	3100	4011.99
	ML	CO	DR	CL	3038.46	33.63	6.673	74	5.6	2551	3169.4
	ML	CO	DR	CL	2752.5	32	6.673	74	1.7	2500	2913.29
Viergever (1982)	ML	C	DR	CL	630	9.36	3.553	27.85	2.56	700	443.11
Yen et al. (1989)	ML	S	DR	CL	2910	66.22	5.23	53.63	9.85	4330	3944.29
	ML*	S	DR	CL	2910	66.22	6.177	57.82	9.8	4460	4121.02

ML maintained load, *CRP* constant rate of penetration, *C* concrete, *CO* composite, *S* steel, *DR* driven, *BO* bored, *CL* closed, *OP* open

*Test samples

included in the model to account for the soil properties variability. These values have been used as input variables for establishing various models by researchers (Alkroosh and Nikraz 2011, 2012; Kordjazi et al. 2014; Shahin 2010). The selected variables as independent input variables in present study are different from those provided by other AI based approaches. A_t and A_s are considered to be as input variables in the model development in order to account for the pile geometry as well as the fact that these parameters have direct influence on the bearing capacity in terms of physical behavior and from geotechnical engineering viewpoints. It is worth mentioning that the A_s is calculated by multiplying the perimeter and the pile embedded length, and has not been considered in the previously published models in the literature for

Table 2 Descriptive statistics of variables in database used for developing ANFIS model

Parameter	A_t (cm^2)	A_f (m^2)	$q_{cave\text{-}shaft}$ (MPa)	$f_{save\text{-}shaft}$ (kN)	$q_{cave\text{-}tip}$ (MPa)	P_{um} (kN)
Mean	1736.02	26.46	5.84	101.89	8.82	1965.86
Median	1230	17.98	5.388	81.91	7.63	1340
Mode	960	11.08	6.673	74	20	1140
Standard deviation	1674.09	26.35	4.23	66.29	6.19	1702.20
Kurtosis	3.28	16.80	6.49	2.46	0.44	8.06
Skewness	1.86	3.53	2.13	1.35	1.02	2.47
Range	7834	189.2	23.874	340.25	26.86	10,850
Minimum	20	5.45	0.833	9.39	0.25	60
Maximum	7854	194.65	24.707	349.64	27.11	10,910
Count	108	108	108	108	108	108

indirect estimation of P_u of piles through interpreting information obtained by CPT methods which can be taken into account as a highly significant input variable. Thus, the proposed formulation of P_u is considered as a function in terms of following parameters:

$$P_u = f\,(A_t,\ A_s,\ \bar{q}_{c-s},\ \bar{q}_{c-t},\ \bar{f}_s) \tag{18}$$

where, A_t is the pile tip cross sectional area, A_s is the shaft area, $\bar{q}_{c-s}$ is the average cone tip resistance along the embedded length of the pile,$\bar{q}_{c-t}$ is the average cone tip resistance over influence zone, and $\bar{f}_s$ is the average sleeve friction along the embedded length of the pile. It should be noted that the failure zone is defined in accordance with Eslami (1996), in which it extends to 4 D below and above the pile tip when soil tip is located in homogenous soil; 4 D below and 8 D above the pile tip in the case that the pile tip is located in a nonhomogeneous strong layer underneath a weak layer; and 4 D below and 2 D above the pile tip when the pile tip is situated in a weak layer with a strong layer above in which D is the equivalent diameter of pile cross section.

5 Data Preprocessing for Model Development

Artificial intelligence based computing techniques as well as statistical regression approaches generally use datasets for developing models. Thus, some issues regarding the data preprocessing must be taken into account for providing more accurate models considering the limited range of data (Sadrossadat et al. 2016a; Ziaee et al. 2015). In fact, the correlation between independent input variables and the output, different scales of data, distributions of variables considering their range in the employed database highly affect the prediction accuracy obtained by modeling techniques. In this regard, normalization techniques may be used to adjust the scale of the data. Normalization of data may be regarded as adjusting data values provided on different scales to a notionally common scale. It increases the speed of training in machine learning algorithms and is especially efficient where the range of raw data vary widely. More, it is recommended to normalize or standardize the inputs in order to reduce the chances of getting stuck in local optima or unchanged outputs (Xue et al. 2017; Ziaee et al. 2015). Feature scaling is a method can be used to standardize the range of variables or features of data. Feature standardization makes the values of each feature in the data have mean close to 0 the unit-variance. In addition, this technique can be used to restrict the range of values in the dataset between any arbitrary values a and b. the general form of the formula used for feature scaling to normalize the raw data of variables to a range of [a, b]:

$$X_n = a + (a - b)\frac{X_{\min} - X}{X_{\max} - X_{\min}} \tag{19}$$

where $X_{\max}$ and $X_{\min}$ are the maximum and minimum values of the variable and X_n is the normalized value. In the present study, $a = 0.05$ and $b = 0.95$.

A major problem in generalization of the obtained models is the overfitting. It is the case in which the error on the datasets obtained by the model is driven to a very small value, but when new datasets are presented to the model, the error becomes very large. A commonly used approach to avoid overfitting is to test the model on another group of data which are not used in the training process. To avoid overfiting, It is recommended that the available database should be classified into three sets: (1) training, (2) validation, and (3) test subsets (Alavi and Sadrossadat 2016; Sadrossadat et al. 2016a, b; Shahin et al. 2004; Ziaee et al. 2015). The training set is utilized to fit the models, the validation set is used to estimate prediction error for model selection and the test set which is a group of unseen datasets is used for the evaluation of the generalization error of the selected model. In present study, 70% of the data sets are taken for the training and validation processes. The remaining 30% of the data sets are used for the testing of the obtained ANFIS models.

6 ANFIS Model Development

In order to develop prediction models through ANFIS algorithm, a code was written in MATLAB 2011b environment based on genfis3 which is an advanced fuzzy inference technique used in MATLAB. There are many difficulties in developing fuzzy models due to the large number of degrees of freedom which needs expert knowledge. During the input processing output modeling, quite a few parameters are required to be found e.g. the number and the type of MFs, rules, selection of the logical operations and etc. These values may be achieved using the process of trial and error or using optimization algorithm based approaches.

Considering the complexity and complicated behavior of P_u the determination of MFs would be difficult. In addition, a TS model is needed to fuzzify crisp or numerical values. On the other hand, simplification of fuzzy models is important to make the rule simple and interpretable. This can be achieved by optimizing the number of fuzzy sets for each input variable and or reducing the number of rules (Sadrossadat et al. 2016b). Besides, these values directly affect the complexity of the obtained ANFIS models which is a significant aim of this paper. Herein, the fuzzy c-means clustering (FCM), is chosen due to its efficiency and simplicity (Sadrossadat et al. 2016b). In present study, the number of clusters is considered to be 4 in order to generate simpler formulation ANFIS-based model which is obtained after several run. It is worth mentioning that the number of clusters, MFs and rules are considered to be equal in genfis3. Besides, input MFs are considered to be Gaussian functions as follows, where linear functions were used as output MFs as was mentioned in Sect. 2. In inference method, AND is *prod*, OR is *probor*, implication is *prod* and aggregation is *sum*. The detailed definitions of these represented expressions are described in Matlab functions.

The structure of ANFIS model for predicting the P_u of piles is represented in Fig. 2.

The general form of fuzzy rule extracted from ANFIS model can be represented as follows where i varies between 1 to 4:

If A_t *is* $in1cluster(i)$ *and* A_s *is* $in2cluster(i)$ *and* $\bar{q}_{c-s}$ *is* $in3cluster(i)$ *and* $\bar{q}_{c-t}$ *is* $in4cluster(i)$ *and* $\bar{f}_s$ *is* $in5cluster(i)$ *then* P_u *is* $out1cluster(i)$.

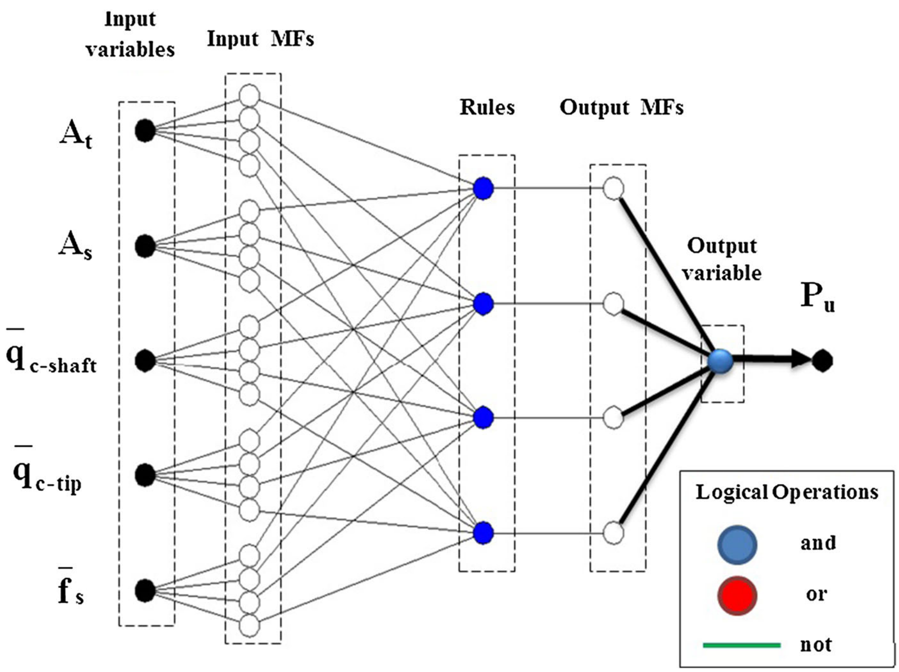
In expression above, A_t is $in1cluster1$ indicates that A_t is considered as the first input variable which is selected from the first cluster. As was mentioned, in present study *and* is used in constructing rules in inference method which means MFs should be multiplied.

Consequently, the weighted average method (wave) is utilized as the defuzzification method. It is one of the most frequently used methods in fuzzy applications (Lee 1990; Mishra et al. 2015; Sadrossadat et al. 2016b; Yilmaz and Yuksek 2009). Wave is typically applied to symmetrical output MFs such as those provided in this study, i.e. the Gaussian MF. It is formed by weighing each MF, using its respective maximum membership value which is the center of symmetrical MF. The algebraic expression of wave is given as follows:

$$c^* = \frac{\sum_{i=1}^{n} \mu_{Ai}(c_i).c_i}{\sum_{i=1}^{n} \mu_{Ai}(c_i)} \tag{20}$$

In equation above, c^* is the defuzzified real-valued output where $\mu A_i(x)$ is the ith MF and c_i is the center of the ith fuzzy set A_i, respectively.

Fig. 2 The structure of ANFIS model for predicting the ultimate axial load bearing capacity of the pile



7 Explicit Formulation of the Obtained ANFIS Model

Considering the aforementioned issues for development of the models and the process of input processing output in ANFIS in addition to the ANFIS architecture, the obtained model can be explicitly represented via a complex formula as the following equations. It is noteworthy that normalized variables are used as input and output variables in the training process as explained before. Therefore, the first step is to normalize values input variables and accordingly, in order to obtain a real value of P_u, the output should be denormalized. Herein, the normalization is calculated through following equation:

$$X_n = 0.05 + (0.9)\frac{X_{\max} - X}{X_{\max} - X_{\min}} \tag{21}$$

where $X_{\max}$ and $X_{\min}$ are the maximum and minimum values of the variable and X_n is the normalized value. Considering input variables in this study, w_i can be represented as follows:

$$w_i = e^{\frac{-(A_t^n - c_i)^2}{2\sigma_i^2}} \times e^{\frac{-(A_s^n - c_i)^2}{2\sigma_i^2}} \times e^{\frac{-(q_{c-t}^{-n} - c_i)^2}{2\sigma_i^2}} \times e^{\frac{-(q_{c-s}^{-n} - c_i)^2}{2\sigma_i^2}} \times e^{\frac{-(\bar{f}_s^n - c_i)^2}{2\sigma_i^2}} \tag{22}$$

Table 3 The Gaussian MF properties for corresponding input variables in ANFIS model

MF (i)	Input variable	σ_i	c_i
1	A_t	0.175	0.519
	A_f	0.108	0.27
	q_{ca}	− 0.286	0.112
	f_{sa}	− 2.986	− 0.185
	q_{ctip}	− 0.496	0.173
2	A_t	− 1.827	− 0.32
	A_f	− 0.74	− 1.254
	q_{ca}	6.155	1.619
	f_{sa}	1.172	− 0.197
	q_{ctip}	− 0.364	0.1
3	A_t	0.375	0.13
	A_f	0.385	− 0.067
	q_{ca}	0.9	0.409
	f_{sa}	0.325	0.327
	q_{ctip}	0.264	0.233
4	A_t	4.917	1.208
	A_f	1.812	2.629
	q_{ca}	− 1.236	0.113
	f_{sa}	− 1.336	1.463
	q_{ctip}	− 1.355	1.319

in which n means the normalized value of the variable, c_i and σ_i are the center and width of the ith Gaussian membership function for the corresponding input variable which can be calculated through the obtained values after training process summarized in Table 3. It should be noted that the number of MFs, clusters and rules are obtained to be 4 for the optimal model presented in present study. Therefore, i varies between 1 and 4.

Accordingly, the firing strengths of weights and the output variable which gives normalized values can be calculated using Eqs. (23) and (24):

$$\bar{w}_i = \frac{w_j}{\sum_{j=1}^{4} w_i} \tag{23}$$

$$p_u^n = \sum_{i=1}^{4} \bar{w}_i (a_i . A_t^n + b_i . A_s^n + c_i . \bar{q}_{c-t}^n + d_i . \bar{q}_{c-s}^n + e_i . \bar{f}_s^n + f_i .) \tag{24}$$

where a_i to d_i are the linear function coefficients which are obtained after training process and are summarized in Table 4.

Finally, considering the maximum and minimum values of P_u in the range of datasets used for ANFIS model development in this paper, the de normalization function can be calculated as follows:

$$P_u = 11452 - 12055 \times P_u^n \tag{25}$$

In order to precisely assess this complex formula, an example is provided in this paper. The values of sample number 39 in Table 1 are assumed to be calculated via the obtained ANFIS model. In that sample, A_t, A_s, $\bar{q}_{c-s}$, $\bar{f}_s$, and $\bar{q}_{c-t}$ are 960 cm^2, 13.92 m^2, 3.587 MPa, 97.75 kPa and 8.15 MPa respectively.

Firstly the values should be normalized using Eq. 21. The normalized values of A_t, A_s,$\bar{q}_{c-s}$, $\bar{f}_s$, and $\bar{q}_{c-t}$ are 0.12, 0.0448, 0.1153, 0.2597, 0.2941, respectively. Considering Eq. 22 and the obtained values of c_i and σ_i for each variable and corresponding MF, w_1, w_2, w_3 and w_4 are calculated 0.008, 0.163, 0.866, and 0.177, respectively. Accordingly, $\bar{w}_1$ to $\bar{w}_4$ can be calculated. The P_u factor is obtained equal to 0.097 through Eq. 24 and considering the aforementioned values of coefficients in Table 4. After denormalizing the obtained value, the final value of P_u is calculated equal to 1113.5 kN.

8 Results and Discussions

8.1 Performance Analysis of ANFIS Model

Although There would be several models obtained by ANFIS prediction technique; however, an optimal model should meet some criteria before the selection. In this regard, quite a few procedures and statistical criteria have been suggested by researchers in the literature. In this paper, correlation coefficient (R), root mean square error (RMSE) and mean absolute error (MAE) are employed to assess the accuracy of the ANFIS model. It is suggested that there is a strong correlation between the predicted and observed values if $R > 0.8$ or $R^2 > 0.64$ which means the prediction capability of the model is acceptable (Sadrossadat et al. 2013, 2016a, b; Smith 1986). It is noteworthy that only considering R as a model performance evaluation criterion would not be sufficient to examine the accuracy of a model because it is insensitive to additive and proportional differences between model predictions and observed values. Therefore, RMSE and MAE are considered as additive criteria to obtain a robust model. The lower the RMSE and MAE values, the better the model performance would be. These

Table 4 The obtained coefficients of linear output MFs after training the ANFIS model

MF (i)	Weights					
	a_i	b_i	c_i	d_i	e_i	f_i
1	− 1.298	0.0024	0.940	0.210	− 0.967	1.195
2	− 0.159	4.463	− 4.73	1.986	− 0.631	0.965
3	− 0.316	− 0.232	1.127	− 0.490	0.288	0.098
4	1.343	0.932	0.227	0.628	1.867	− 1.844

parameters are calculated using the following equations:

$$R = \frac{\sum_{i=1}^{n}(o_i - \bar{o}_i)(p_i - \bar{p}_i)}{\sqrt{\sum_{i=1}^{n}(o_i - \bar{o}_i)^2 \sum_{i=1}^{n}(p_i - \bar{p}_i)^2}} \quad (26)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(p_i - o_i)^2}{n}} \quad (27)$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|p_i - o_i| \quad (28)$$

where o_i and p_i are the actual and predicted output values for the ith output, respectively, $\bar{o}_i$ and $\bar{p}_i$ are the average of the actual and predicted outputs, and n is the number of samples.

In order to represent the capability of the obtained ANFIS models, the predicted versus observed values are demonstrated in Fig. 3. It can be figured out that the ANFIS-based model with high R and low RMSE and MAE values is able to predict the actual values with a high degree of accuracy. Besides, close R, RMSE and MAE values on the training and testing data suggests that it has both good predictive abilities and generalization performance in addition to the fact that overfitting is avoided.

Furthermore, in order to ensure about the prediction performance of the ANFIS model, its prediction capability should be assessed and compared with those of other conventional models. In this regard, the same test datasets employed for producing ANFIS model are considered and the comparative analyses results are represented in Fig. 4. It should be noted that test data are unseen in input processing output procedure which means that they have not been employed for producing the ANFIS model. Therefore, they can be considered as new data for the generated ANFIS model and the predictability and generalization performance of the ANFIS model can be check using them. It is worth mentioning that the obtained values made by CPT-based methods are extracted from the previously published research by Eslami (1996) as these methods cannot estimate P_u factor using the available information which can be regarded as a negative point of those approaches as they require further information to estimate the P_u factor. It has also been shown that the results obtained by different methods vary differently for the same case (Briaud 1988; Abu-Farsakh and Titi 2004).

As can be seen, the obtained ANFIS model with higher R value and less RMSE and MAE outperforms other models. This way of observing mismatches or differences between predicted and measured values made by different models can also represent how models can estimate the target value, e.g. Schertmann's model overestimates the P_u factor in many cases.

Moreover, another statistical analysis procedure is employed here for evaluating external capability of the ANFIS model on testing datasets which has been proposed by various researchers (Abu-Farsakh and Titi 2004; Alkroosh and Nikraz 2011, 2012; Kordjazi

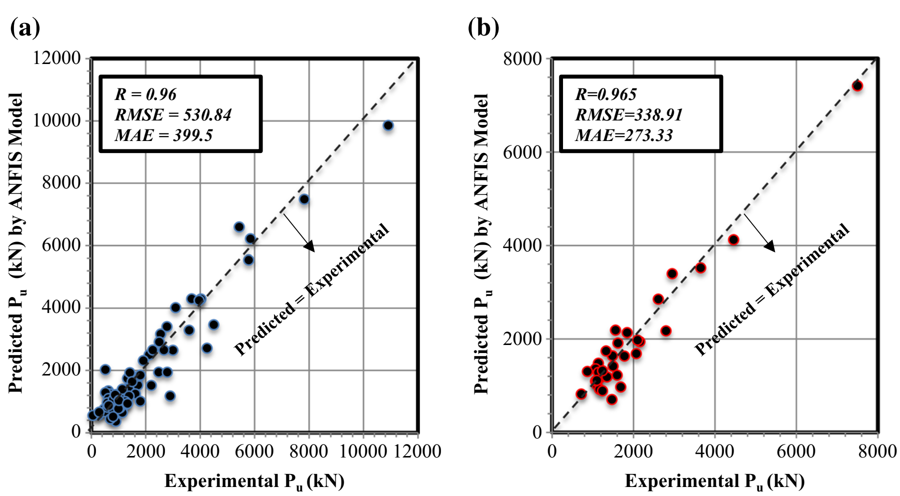


Fig. 3 Predicted versus experimental P_u values using the optimal ANFIS model: **a** training (learning and validation) datasets, **b** testing datasets

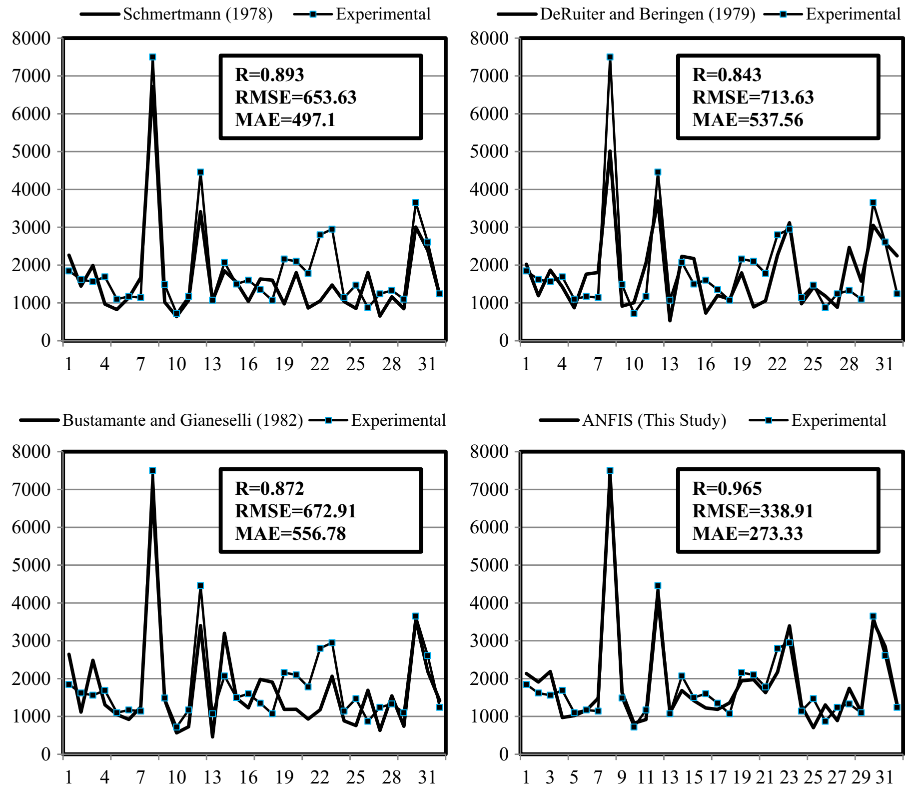


Fig. 4 A comparative plot of experimental and predicted P_u (kN) values using different models for test data

et al. 2014). This procedure can be done based on four criteria:

(1) The equation of the best fit line of estimated (P_u) versus measured pile capacity (P_m) with the corresponding coefficient of determination.

(2) The arithmetic mean (μ) and standard deviation (σ) of P_{up}/P_{um}. It can be suggested that a model is highly capable of predicting the target values when μ (P_{up}/P_{um}) is closer to 1 and σ (P_{up}/P_{um}) is closer to 0.

(3) The 50% cumulative probability ($P_{50\%}$) of P_{up}/P_{um}. The closer the value of $P_{50\%}$ to 1, the better model would be. In order to calculate $P_{50\%}$, P_{up}/P_{um} values estimated by the model should be arranged in an ascending order (1, 2, 3,…, i,…, n). Thereafter, the cumulative probability can be calculated using the following equation:

$$P = \frac{i}{n+1} \tag{29}$$

where i is the order number given for the calculated ratio of P_{up}/P_{um}, n is the number of data which is considered to be the number of testing data here.

(4) The coefficient of efficiency (E) which compares the predicted and observed values of ultimate axial bearing capacity and evaluates how well the model is able to explain the total variance of the data. This parameter can be calculated through the following equations:

$$E = \frac{E_1 - E_2}{E_1} \tag{30}$$

$$E_1 = \sum_{t=1}^{r} \left(P_{um} - \bar{P}_{um}\right)^2 \tag{31}$$

$$E_2 = \sum_{t=1}^{r} \left(P_{up} - P_{um}\right)^2 \tag{32}$$

The overall performance of the ANFIS model and each traditional CPT-based model may be considered as an overall rank index (RI) which is defined as the sum of the ranks from the different criteria (i.e.

Table 5 The results of the statistical analysis for different models for test data

Method	Best fit calculation			Arithmetic calculation of pup/pum			Cumulative probability		Coefficient of efficiency		Overall rank	
Model	P_{fit}/P_m	R	R1	Mean	SD	R2	P_p/P_m at P50	R3	E	R4	RI	Final rank
ANFIS	0.97	0.96	1	1	0.22	1	0.99	2	0.93	1	5	1
Schmertmann (1978)	0.82	0.89	2	0.9	0.35	3	0.89	5	0.74	2	12	2
DeRuiter and Beringen (1979)	0.84	0.84	4	1.03	0.4	2	0.93	3	0.69	4	13	3
Bustamante and Gianeselli (1982)	0.87	0.87	3	0.93	0.4	4	0.79	6	0.73	3	16	4

RI = R1 + R2 + R3 + R4). The lower the RI, the better the estimation performance of the model is. The results of the statistical analysis are summarized in Table 5.

8.2 Parametric Analysis

Quite a few studies were done in order to evaluate the generalization, validation and prediction capability of the proposed ANFIS model through some statistical criteria in addition to the fact that the results were compared with those of obtained by some CPT-based methods proposed by various researchers in the literature. Although the ANFIS model results represented that the ANFIS model is of great prediction capability, the ANFIS model is required to be assessed from engineering viewpoints in terms of physical behavior. For this purpose, the response and behavior of the models may be evaluated to different input variables and should be compared with those experimentally or theoretically provided in the literature. To cope with this issues, a parametric analysis can be performed as recommended by several researchers (Alavi and Sadrossadat 2016; Sadrossadat et al. 2013; Tajeri et al. 2015; Ziaee et al. 2015). The parametric analysis represents the response of the ANFIS-based P_u model to variations of variables. The mentioned method can be done through varying only one independent input variable while the other variables are remained constant at their mean values. Thereby, a set of artificial data is generated for each variable according to their range in the employed database. The obtained values are then presented to the model and the output is calculated. This procedure is repeated using other variable, one by one, until the model response is obtained for all of the predictor variables.

It is expected that P_u must increase with increasing the amount of A_t, A_s, $\bar{q}_{c-s}$, $\bar{f}_s$, $\bar{q}_{c-t}$ as have been represented by researchers in the literature (Abu-Farsakh and Titi 2004; Alkroosh and Nikraz 2011, 2012; Eslami 1996; Kordjazi et al. 2014). As is represented in Fig. 5, the obtained results of the parametric study conform that the ANFIS model can predict the P_u factor regarding the physical behavior and from engineering viewpoints.

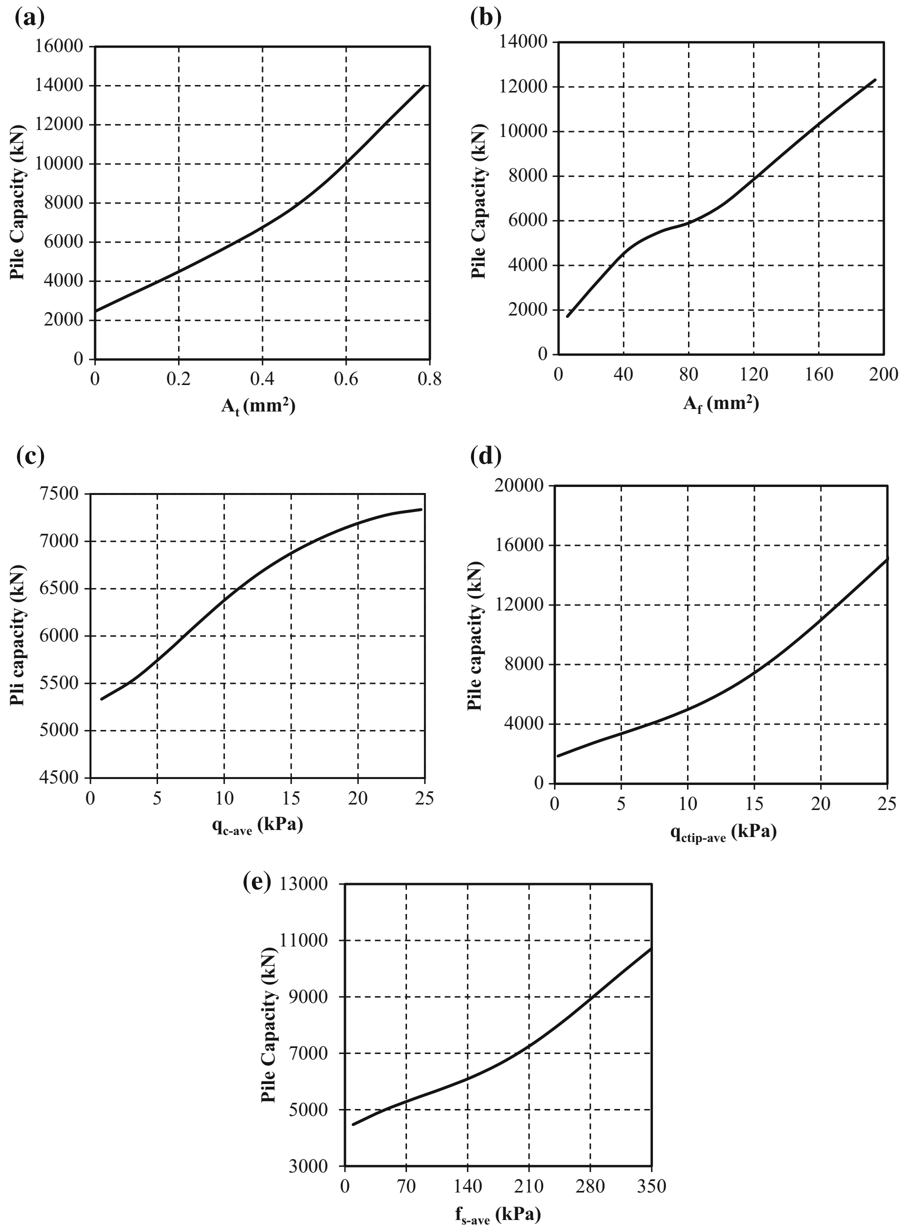
8.3 Sensitivity Analysis

Another significant issue to assess a model is to find out the contribution and importance of each parameter which can be achieved through a sensitivity analysis (SA). The results of SA on model represent how much a model is affected by the variation of each independent input variable. The SA conducted in the present study is based on varying each input from its minimum to maximum value regarding to its range in the utilized database and the model output is calculated while the other inputs are fixed at their means (Kiani et al. 2016). According to the strictly increasing tendency of all variables as represented in Fig. 6, this type of SA is used here. The percent of each obtained output difference for each variable is computed which is often referred to as the sensitivity index (SI). The SI (%) values can be calculated using the following equations:

$$L_i = f(x_{\max}) - f(x_{\min}) \tag{33}$$

$$SI_i = \frac{L_i}{\sum_{j=1}^{n} L_i} \times 100 \tag{34}$$

Fig. 5 A parametric analysis of proposed ANFIS model for indirect estimation of P_u



where $f(x_{max})$ and $f(x_{min})$ are the predicted values of maximum and minimum of the independent input variable over the i^{th} input domain, and n is the number of involved variables.

The process of SA was conducted and the obtained SI (%) results for each variable are demonstrated in Fig. 6. As is illustrated, the obtained ANFIS model is more sensitive to the variation of the considered input variable, i.e. the shaft area of pile (A_s), followed by $\bar{f}_s$, $\bar{q}_{c-t}$, $\bar{q}_{c-s}$ and A_t. The obtained results can be verified by those obtained by (Abu-Farsakh and Titi 2004; Eslami 1996; Kordjazi et al. 2014). All in all, the SI value for each variable used in an equation or a model is unique. Besides, it would be recommended that engineers are required to highly know about the significance and effect of each variable on models and equations they use for design purposes.

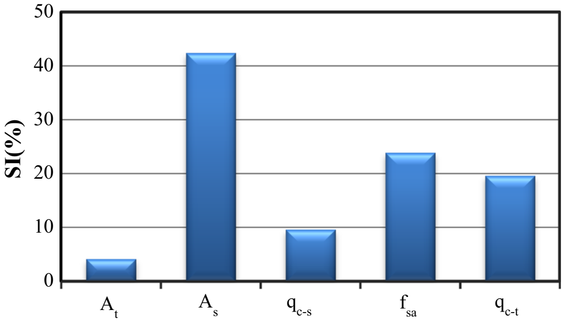


Fig. 6 The significance of each input variable in ANFIS model

9 Summary and Conclusion

This paper aimed at investigating the robustness of ANFIS method for estimating the ultimate axial load bearing capacity of piles using CPT data which is a crucial problem in geotechnical engineering. In this regard, a collection of data was used for the development of the models. The selected database contained information about the pile installation method, pile material, full-scale pile load test and CPT data. However, the model was developed to simultaneously take into account A_t, A_s, $\bar{q}_{c-s}$,$\bar{f}_s$, $\bar{q}_{c-t}$ as input variables obtained from pile load test and CPT results due to the fact that these variables are more meaningful from geotechnical engineering viewpoints. It is well-known that there exist some practical equations to estimate the ultimate axial load bearing capacity of piles based on experimental results obtained by CPT, as was presented in the paper. It should be highlighted that the strength of ANFIS algorithm, as a predictive tool, absolutely lies in its high precision for prediction and approximation purposes; however, the main weakness of ANFIS modeling technique which has scholarly been visited in the existing literature is the fact that it has not been able to generate explicit models or equations which can be used for hand-calculation aims. In other words, ANFIS has been considered as a black-box predictive tool. In this paper, the obtained optimal ANFIS model was converted to an explicit tractable formula which can be used for pile design uses. Additionally, engineers are required to know about the degree of accuracy, the physical behavior of the model, the relative significance of each variable and the validation and verification of the models they use in their computations or calculations. In this regard, several criteria were offered to assess the generalization performance of the model and sensitivity and parametric analyses are conducted and discussed. The parametric analysis results demonstrated that the obtained model output, i.e. P_u, increases with increasing A_t, A_s, $\bar{q}_{c-s}$, $\bar{f}_s$, $\bar{q}_{c-t}$ as it was expected. The relative importance values of input variables obtained by the sensitivity analysis indicate the all input variables influence on the output to a large extent the obtained ANFIS takes into account all input variables. However, the sensitivity analysis results obtained by each model would be unique and the proposed ANFIS model is more sensitive to A_s variation followed by $\bar{f}_s$, $\bar{q}_{c-t}$, $\bar{q}_{c-s}$ and A_t. The obtained results of the parametric and sensitivity studies approve that the ANFIS model can accurately predict the P_u factor regarding the physical behavior and from engineering viewpoints. The results of several comparative performance analyses of existing models confirmed that the proposed ANFIS model is up to standard for indirect estimation of the ultimate load bearing capacity of piles as it notably outperformed traditional models in terms of generalization and predictability. Finally, numerical models highly depend on the data used in their process of model development. The capability of such models is mostly limited to the range of the data, information and soil strata used for their calibration, and also the formulation depends on the available variables, in the database. To rise above this, the model should be retrained and improved to make more accurate predictions for a wider range, more variables or other types of soils.

Compliance with Ethical Standards

Conflict of interest The authors declare that they have no conflict of interest.

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