

Reliability Analysis of Excavation Induced Basal Heave

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Received: 18 November 2016 / Accepted: 26 May 2017 / Published online: 1 June 2017
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Abstract The present study deals with the reliability analysis of basal heave caused by excavation considering uncertainty in the soil properties. The case study considered in the present work has been analyzed deterministically by Hsieh et al. (Can Geotech J 45:788–799, 2008). Taiwan building code is adopted in the method for analyzing the basal heave failure. The random variables (undrained shear strength and total unit weight of clay) are assumed to be normally distributed and uncorrelated. A series of parametric studies have been conducted to calculate the reliability index on the basis of the matrix formulation for the second moment method by Hasofer and Lind (J Eng Mech ASCE 100(1):111–121, 1974) considering different coefficient of variation of undrained shear strength and total unit weight of clay layers. It has been found that for a particular value of coefficient of variation of total unit weight, the reliability index with respect to occurrence of basal heave failure decreases with increase in the coefficient of variation of undrained shear strength. Moreover, the reliability index also decreases when the coefficient of variation of total unit weight increases. It has also been found that the probability of basal heave failure is lower with respect to factor of safety equals to 1.2, as compared to factor of safety equals to 1.0. Sensitivity analysis shows that the

undrained shear strength of the bottommost layer and total unit weight of the second layer are the most significant random variables affecting the reliability index. Guidelines are provided for reliability based design where, for ‘target’ reliability index of 2.5 and 3.0, the factor of safety can be chosen such that all the related uncertainties are taken into account, especially with regard to undrained shear strength of the bottommost layer and total unit weight of the second layer. Design guidelines have been provided for this purpose.

Keywords Reliability index · Probability of failure · Basal heave · Undrained shear strength · Total unit weight

List of symbols

B	Width of excavation
D_e	Maximum excavation depth (final stage)
D_b	Embedment depth (final stage)
E	Young’s modulus of soil
F	Failure region
G	Shear modulus of soil
K	Bulk modulus of soil
K_o	Coefficient of lateral earth pressure at rest
K_n	Interface normal stiffness between wall and soil
K_s	Interface shear stiffness between wall and soil
M	Vector of mean value of random variables
M_r	Resisting moment
M_d	Disturbing moment

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N	Average SPT value of soil
n	Number of random variables
P_f	Probability of failure
S_i	Undrained shear strength of i th soil layer
t	Thickness of the wall
X	Vector of random variables
γ	Total unit weight of soil
β	Reliability index
λ_i	Sensitivity factor of the i th random variable
$\Phi(\cdot)$	Standard normal cumulative distribution function

1 Introduction

Uncertainty is an integral part of geotechnical engineering problems because the soil is basically random in nature. One of the important geotechnical problems is that of basal heave stability in case of deep strutted excavation for underground construction in clayey soil of very soft to soft consistency. Collapse of the entire bracing system may occur if there is excessive movement of soil caused by basal heave. Conventionally, the stability of excavation against basal heave failure is investigated using deterministic approach where, the factor of safety is calculated as the ratio between resistance and load, which are taken as constant for a particular site condition and geometry of the structure (Terzaghi 1943; Bjerrum and Eide 1956). If the factor of safety is greater than one, then it is considered as stable, otherwise it is unsafe against basal heave failure.

However, it has been found that failure may occur even if the factor of safety is greater than 1.0 or the minimum value as specified in the design code because of the uncertainties involved in the determination of the soil parameters. This uncertainty is considered logically in probabilistic analysis, where the design variables are considered random in nature. In recent years, probabilistic analysis of excavation induced ground deflection, wall movements and damage potential to the adjacent buildings have been conducted by Kung et al. (2007) and Schuster et al. (2009).

For a long time, reliability based design approach has been used for retaining wall (Hoeg and Muruka 1974; Meyerhoff 1982; Smith 1985). Reliability index has been computed by Duncan (2000) for cantilever

retaining wall using Taylor's series for uncorrelated random variables. Sujit et al. (2011) calculated probability of failure against sliding mode of failure for cantilever retaining wall by first order reliability method (FORM), second order reliability method (SORM) and Monte Carlo simulation (MCS) method.

A risk based approach to failure was considered by Goh et al. (2008) and using basic structural reliability concepts, it has been shown that the assumption of a linear limit state surface can be used to provide reasonable estimates of the reliability index or probability of failure. Moreover, design charts have also been provided to estimate the probability of basal heave failure in clay. Some of the other studies involving probabilistic analysis of basal heave of excavation include that of Tang and Kung (2011), Luo et al. (2011), Wu et al. (2010, 2012). The probabilistic analysis of case study on basal heave problem (Hsieh et al. 2008) is incorporated in the method proposed by Taiwan building code and performed employing Monte Carlo simulation method (Tang and Kung 2012). In the present study the probability of basal heave failure has been estimated on the basis of the second moment method proposed by Hasofer and Lind (1974) using an Excel spreadsheet developed by Low and Tang (1997).

2 Basal Heave Analysis

In the present study, the method for basal heave analysis proposed by Taiwan building code (Tang and Kung 2011) is adopted, where a circular failure surface centered at the lowest strut level is assumed which is similar to that proposed by Terzaghi (1943). The schematic diagram of basal heave problem with all geometrical parameters and soil properties are shown in Fig. 1. The excavation case study and the soil properties (undrained shear strength and total unit weight) are taken from Hsieh et al. (2008). The embedded retaining wall is supported by three levels of strut at 1.4, 3.7 and 6.0 m, respectively below the ground level. The final excavation depth (D_e) and the overall depth of the wall (D) are 9.3 and 15.4 m, respectively. Thus, the embedment depth of the wall (D_b) and D_b/D_e are given by 6.1 and 0.66, respectively. From the variation of undrained shear strength profile with depth, the mean values of undrained shear strengths of layer-I, II, III and IV i.e. S_1 , S_2 , S_3 and

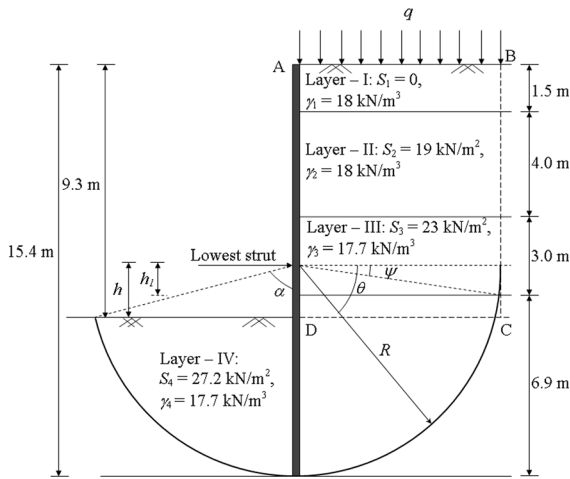


Fig. 1 Schematic diagram of slip circle method for basal heave stability analysis and soil profile

S_4 are taken as 0, 19.0, 23.0 and 27.2 kN/m^2 , respectively. The mean values of total unit weight of the layers i.e. $\gamma_1, \gamma_2, \gamma_3$ and γ_4 are 18.0, 18.0, 17.7 and 18.0 kN/m^3 , respectively. The factor of safety (F_S) against basal heave is defined as the ratio of resisting moment (M_r) to disturbing moment (M_d) and is given by,

$$F_S = \frac{M_r}{M_d} = \frac{R \int_0^{\frac{\pi}{2} + \alpha} S(Rd\theta)}{(W + q) \frac{R}{2}} \geq 1.2 \tag{1}$$

where S is the undrained shear strength of clay along the circular failure surface; W is the total weight of the soil (weight of the soil block ABCD in Fig. 1); R is the radius of the failure arc; q is the surcharge above the ground surface on the retained side outside the excavation area and is taken as 20 kN/m^2 ; θ is the angle between the failure surface and the horizontal direction; α is angle of failure arc in excavation zone and is calculated as $\alpha = \cos^{-1}(h/R)$, where h is the distance between the lowest strut level and final excavation level. As the slip surface passes through soil layers (III and IV), the factor of safety from Eq. (1) takes the following form,

$$F_S = \frac{2R[S_3\psi + S_4(\frac{\pi}{2} + \alpha - \psi)]}{W + q} \geq 1.2 \tag{2}$$

where ψ is the angle made by the failure arc (within the third soil layer) with the lowest strut level and is given by $\psi = \tan^{-1}(h_1/R)$, h_1 is the distance between lowest

strut level and top of layer-IV. For the geometry of the problem considered, h, h_1 and R are 3.3, 2.5 and 9.4 m, respectively. So, α and ψ are given by 1.21 and 0.27 rad, respectively.

The total unit weight of the soil are taken from Hsieh et al. (2008), while the undrained shear strength of each layer are estimated as average value at the layer top and bottom as obtained from its variation with depth measured from UU test (Tang and Kung 2012).

3 Reliability Analysis of Basal Heave

Reliability index (β) is calculated on the basis of matrix formulation for the second moment, Hasofer–Lind reliability index defined as

$$\beta = \min_{X \in F} \sqrt{(X - M)^T C^{-1} (X - M)} \tag{3}$$

where X = vector of random variables; M = vector of mean value of random variables; C = covariance matrix of random variables; and F = failure region. The reliability index (β) is defined as the shortest distance from the transformed failure surface to the origin of the reduced variate space. In the present study, the procedure explained by Low and Tang (1997) is adopted, in which the Eq. (3) is used to obtain the value of β with the help of “SOLVER”, an optimization tool available in Microsoft’s Excel.

There are two different scenarios regarding the failure. One is the theoretical definition of failure, where there is no specific value of factor of safety (F_S) to indicate the failure and in the other scenario, failure is defined with respect to specific value of factor of safety (design failure). In other words, the system will fail, if F_S is less than that particular value. The performance function can be defined accordingly. For example if the specific value of F_S is taken as 1.0, then the performance function is given by $F_S - 1 = 0$. In the present study, design failures are considered for $F_S = 1.0$ and $F_S = 1.2$ (Taiwan building code). Hence, the performance function $g(\cdot)$ are given by Eq. (4) and Eq. (5) for, $F_S = 1.2$ and $F_S = 1.0$ criterion, respectively.

$$g(R, q, a, y, W, S) = M_r - 1.2M_d \tag{4}$$

$$g(R, q, a, y, W, S) = M_r - 1.0M_d \tag{5}$$

The minimum reliability index (β) is calculated using the optimization tool available in Excel. After performing optimization, the probability of failure (P_f) can be calculated from Eq. (6):

$$P_f = 1 - \Phi(\beta) = \Phi(-\beta) \quad (6)$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function.

It can be found from Eq. (2), that factor of safety (F_S) against basal heave is a function of six parameters, R , q , α , ψ , W and S . In practice, the parameters (R , q , α and ψ) can be exactly determined on the basis of the design of excavation. In the present study, W and S are considered as uncertain parameters. From the variation of undrained shear strength of soil (UU test results) with depth below ground surface (Hsieh et al. 2008) it is confirmed that shear strength must be considered as a random variable. However, from only six data points it is difficult to estimate COV of the shear strength. Hence, from other UU test results, it is found that COV of undrained shear strength of Taipei clay varies from 0.2 to 0.3 (Tang and Kung 2012). It has been found from previous work (Duncan 2000), that COV of total unit weight of soil varies from 3 to 7%, which is not large as compared to COV of undrained shear strength, which varies from 13 to 40%. However, to investigate the effect of uncertainty in the unit weight on the probability of failure it is also considered as random variable in the present study and thus the weight W of the soil block also becomes uncertain. In the present excavation problem, the weight of the soil block ABCD (in Fig. 1) depends on the total unit weight of four soil layers (γ_1 , γ_2 , γ_3 and γ_4). So, all the total unit weights are considered as random variables. However, the circular failure surface passes through only third (layer-III) and fourth (layer-IV) soil layers. So, undrained shear strength of the third and fourth soil layers (S_3 and S_4) are considered as random variables. It is assumed that all the random variables are normally distributed which may be correlated or, uncorrelated.

4 Comparison of Different Methods for Calculating Probability of Failure

For the validation study, the total unit weight of all the layers are considered as deterministic variables, while the undrained shear strength of layer-III and IV are

considered as random variables. The properties of the soil layers are taken from Tang and Kung (2012) and shown in Table 1. As the surcharge load is not mentioned in Tang and Kung (2012), a value of 20 kN/m² has been taken. In different studies the surcharge load is taken as 10 kN/m² (Wu et al. 2012; Luo et al. 2012). However, in the present study, it has been considered that abnormal vehicle loads such as industrial loads are present near the excavation area. Hence, the surcharge loading has been selected on the basis of HB type of highway traffic loading with 45 units (Puller 2003) which is 20 kN/m². The coefficient of variation of undrained shear strength of the third and fourth soil layers (CV_γ) are considered as equal and varied through 0.1, 0.2, 0.3, 0.4 and 0.5. The variation of probability of failure (P_f) with CV_γ for two criterion of factor of safety i.e. $F_S = 1.2$ and $F_S = 1.0$ as obtained by Rosenblueth's point estimate method (Christian and Baecher 1999), advanced second moment method using Microsoft Excel's SOLVER tool (Low and Tang 1997) and Monte Carlo simulation (Tang and Kung 2012) are shown in Fig. 2. Numerous reliability based analysis has been done using "SOLVER" tool available in Microsoft Excel (Low and Tang 1997; Low et al. 2001; Low and Tang 2001). It can be found from Fig. 2, that, probability of failure obtained by advanced second moment method using Microsoft Excel's SOLVER tool is marginally higher than that obtained by Tang and Kung (2012) and the maximum variation is around 3 and 11% for $F_S = 1.2$ and 1.0 criterion, respectively. Moreover, it can be found that very high potential of basal heave failure is more consistent with the results obtained from the present study, where, probability of failure as obtained in the present analysis are 0.91 and 0.82 as compared to 0.82 and 0.73 (Tang and Kung 2012), for $F_S = 1.0$ criterion, against CV_γ of 0.20 and 0.30, respectively. For the other criterion ($F_S = 1.2$), the probability of failure as found from the present analysis are around 1.0 and 0.97 as compared to 1.0 and 0.95 (Tang and Kung 2012) for COV of 0.20 and 0.30, respectively. Since the probability of failure obtained in the present analysis are always higher than that obtained by Tang and Kung (2012), it can be inferred that the results obtained from the present study reflects the actual failure condition at site more closely than that obtained by Tang and Kung (2012). Thus the advanced second moment method gives more accurate results because it is more closely related as compared to that

Table 1 Parameters for basal heave problem considered for validation study

Parameter	Notation	Statistics of parameter	
		Mean	Coefficient of variation
Total unit weight of layer-I	γ_1	18.0 kN/m ³	–
Total unit weight of layer-II	γ_2	18.0 kN/m ³	–
Total unit weight of layer-III	γ_3	17.7 kN/m ³	–
Total unit weight of layer-IV	γ_4	18.0 kN/m ³	–
Undrained shear strength of layer-III	S_3	23.0 kN/m ²	0.1–0.5
Undrained shear strength of layer-IV	S_4	27.2 kN/m ²	0.1–0.5
Surcharge	q	20 kN/m ²	–

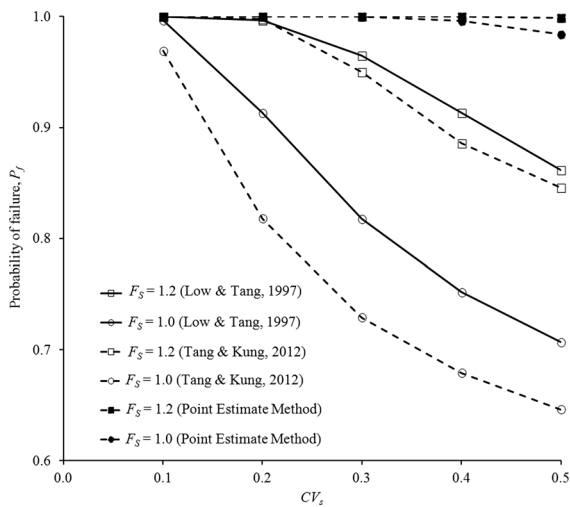


Fig. 2 A comparison of variation of probability of failure with COV of undrained shear strength determined by Excel spreadsheet (Low and Tang 1997) and Monte Carlo Simulation (Tang and Kung 2012)

obtained by Monte Carlo simulation (Tang and Kung 2012) to the actual basal heave failure recorded in the excavation case at site. Moreover, it can be observed from Fig. 2, that among all the methodologies adapted for calculating minimum reliability index (or, probability of failure) point estimate method generates much higher value of probability of failure as compared to the other two ones and nature of variation of P_f with CV_s is also much different as compared to the other methods. Apart from this, it has been already found from previous studies (Rosenblueth 1981; Christian and Baecher 1999) that when the number of random variables becomes large, the computation of reliability index becomes more tedious because number of weighted factors become 2^n , where, n is the number of random variables. Various parametric have

been done to investigate the influence of each random variable on the reliability index. The total number of random variables considered is six (total unit weight of four soil layers and undrained shear strength of the two bottom layers). Therefore, point estimate method will take long time in computation and thus all the analyses in the present paper has been performed using advanced second moment method (with the help of Excel’s SOLVER tool) for computing the probability of failure against basal heave.

5 Effect of Different Parameters on Probability of Failure (P_f)

5.1 Effect of COV of Undrained Shear Strength of Soil on Probability of Failure (P_f)

For all the studies other than validation, both total unit weight and undrained shear strength of the soil layers are considered as random variables. A number of analysis is done by varying COV of undrained shear strength (CV_s) of third and fourth soil layers through 0.1, 0.2, 0.3, 0.4 and 0.5, while keeping COV of the unit weights (CV_γ) as constant i.e. 0.05, based on coefficient of variation given in Luo et al. (2012) considering two different failure criterion ($F_s = 1.0$ and 1.2) and the results are shown in Fig. 3. The mean values of total unit weight and undrained shear strength are as per Table 1. It can be found from Fig. 3, that probability of failure (P_f) decreases with increase of COV of undrained shear strength and for any value of CV_s , P_f calculated on the basis of $F_s = 1.2$ criterion is higher than that $F_s = 1.0$ (maximum variation is around 18%). Moreover, it is also found from Fig. 3, that the variation between

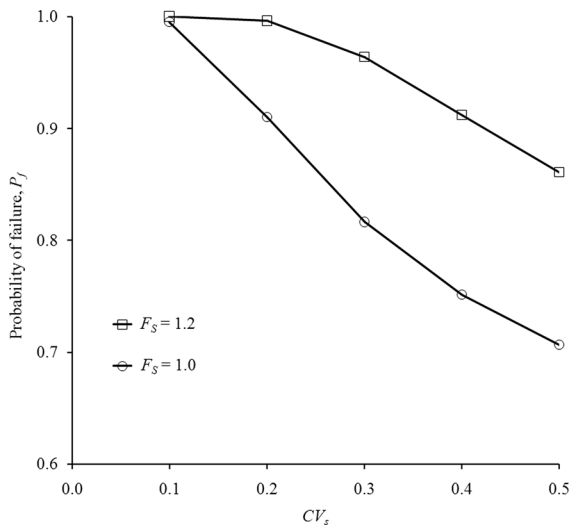


Fig. 3 Variation of probability of failure with COV of undrained shear strength

probability of failure obtained for two different criterion of F_S increases with increase in CV_S .

5.2 Effect of COV of Total Unit Weight of Soil on Probability of Failure (P_f)

As the COV of undrained shear strength of Taipei clay lies between 0.2 and 0.3 (Tang and Kung 2012), probability of failure is calculated by varying COV of total unit weight (CV_γ) through 0.05, 0.10, 0.15, 0.20, 0.25 and 0.30 keeping coefficient of variation of undrained shear strength (CV_S) of third and fourth soil layers as 0.20 and 0.30, respectively, considering two different failure criterion i.e. $F_S = 1.0$ and 1.2. The variation of probability of failure (P_f) with CV_γ is shown in Fig. 4. It can be found from Fig. 4, that probability of failure decreases with increase of COV of total unit weight and the maximum decrement is around 9% when CV_γ varies from 0.05 to 0.3, $F_S = 1.0$ and $CV_S = 0.2$. Moreover, it can be found from Fig. 4, that, for the range of CV_γ considered in the present study, the probability of failure calculated considering $CV_S = 0.2$ are higher than that calculated using $CV_S = 0.3$ by (3–4)% and (7–10)% for $F_S = 1.2$ and 1.0, respectively. It can also be depicted from the figure that the difference in P_f values remains almost constant for $F_S = 1.2$, while, it gradually reduces for $F_S = 1.0$, when CV_γ varies from 0.05 to 0.3. It can be concluded that for a particular factor of safety

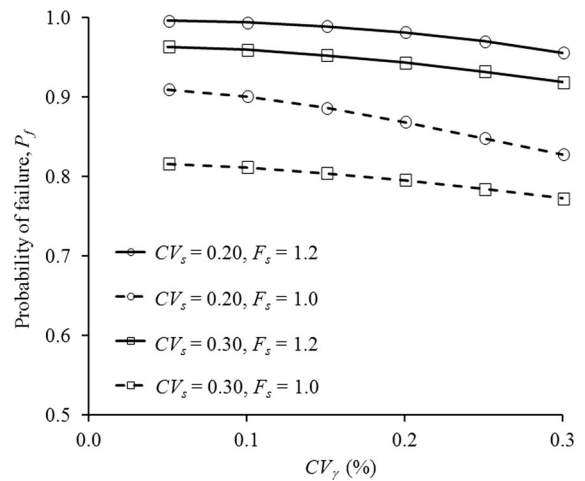


Fig. 4 Variation of probability of failure with COV of total unit weight of soil when COV of undrained shear strength are 0.20 and 0.30

criterion, lower coefficient of variation produces higher probability of failure of against basal heave (Table 2).

6 Sensitivity Analysis

The influence of each random variable i.e. the total unit weight of all the four soil layers ($\gamma_1, \gamma_2, \gamma_3$ and γ_4) and undrained shear strength of third and fourth layers (S_{u3} and S_{u4}) on the probability of failure can be expressed by sensitivity factor (λ_i) measured in terms of the direction cosines of the position vector of the design point in the transformed U -space. Sensitivity factors indicate the relative importance of the uncertainty in random variables on the computed reliability. As per sign convention, the loading and resistance variables are associated with negative and positive factors, respectively. Sensitivity analysis for a particular random variable is conducted by varying that variable through the specified range of values, while keeping all other variables fixed at a particular value. Each of the total unit weight ($\gamma_1, \gamma_2, \gamma_3$ and γ_4) is varied through 16.0, 17.0 and 18.0 kN/m³. The undrained shear strength S_3 and S_4 are varied through 20.0, 25.0 and 30.0 and 35.0, 40.0 and 45.0 kN/m², respectively. To determine the sensitivity factor for γ_1 , it is varied through 16.0, 17.0 and 18.0 kN/m³, while $\gamma_2, \gamma_3, \gamma_4, S_3$ and S_4 are kept as 17.0, 17.0, 17.0, 25.0 and 40.0 kN/m², respectively. Similarly, to determine the

Table 2 Parameters for basal heave problem considered in sensitivity analysis

Parameter	Notation	Statistics of parameter	
		Mean	Coefficient of variation
Total unit weight of layer-I	γ_1	18.0 kN/m ³	0.05 ^a
Total unit weight of layer-II	γ_2	18.0 kN/m ³	0.05 ^a
Total unit weight of layer-III	γ_3	17.7 kN/m ³	0.05 ^a
Total unit weight of layer-IV	γ_4	18.0 kN/m ³	0.05 ^a
Undrained shear strength of layer-III	S_3	23.0 kN/m ²	0.1–0.5
Undrained shear strength of layer-IV	S_4	27.2 kN/m ²	0.1–0.5
Surcharge	q	20 kN/m ²	–

^a Based on coefficient of variation value given in Luo et al. (2012)

sensitivity factor for S_3 , it is varied through the above values while, keeping $\gamma_1, \gamma_2, \gamma_3$ and γ_4 as 17.0 kN/m³ and S_4 as 40.0 kN/m². The analysis for all the combinations of the values are conducted for $D_b/D_e = 0.66$ as obtained from the excavation case study (Hsieh et al. 2008) and surcharge = 20.0 kN/m². The sensitivity factors of the random variables are shown in Table 3. It can be found from Table 3, that the most sensitive random variable (load) causing basal heave failure is the total unit weight of layer-II, because it is the thickest one and thus contributes maximum to the total weight of the soil block above the final excavation surface and thus affect the driving moment most significantly as compared to the unit weight of the other three layers. However, the most sensitive random variable resisting basal heave failure is the undrained shear strength of the bottommost soil layer. This is obvious because, the major portion of the failure circular arc (Fig. 1) passes through the bottommost soil layer and thus influences the maximum resisting moment to the largest extent as compared to the undrained shear strength of the other soil layer (layer-III).

As the reliability index is most sensitive to the total unit weight of the second layer and undrained shear strength of the fourth layer, so reliability index is calculated against factor of safety for mean values of γ_2 (16.0, 17.0 and 18.0 kN/m³) and S_4 (35.0, 40.0 and 45.0 kN/m²). The variation of reliability index (β) with

factor of safety (F_S) for different total unit weight of second layer (γ_2) and undrained shear strength of fourth layer (S_4) are shown in Figs. 5 and 6, respectively. The values of the other parameters are shown in the respective figures. It can be found from Fig. 5, that for a given factor of safety, the reliability index increases with increase in γ_2 . However, it can be found from Fig. 6, that the reliability index decreases with increase in undrained shear strength of layer-IV for a particular factor of safety. The β – F_S variation in terms of mean values of γ_2 and S_4 are very much relevant under practical situations. For example, if, the factor

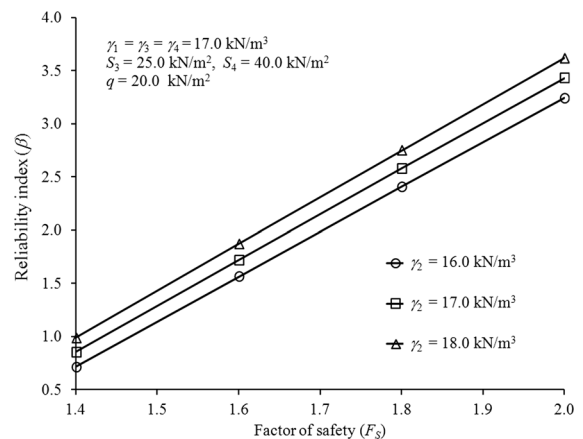


Fig. 5 Variation of reliability index with factor of safety for normalized total unit weight of second clay layer

Table 3 Sensitivity factors for basal heave failure

Random variable		γ_1	γ_2	γ_3	γ_4	S_3	S_4
Sensitivity factor (λ_i)	$D_b/D_e = 0.66$	-0.0370	-0.0990	-0.0740	-0.0200	0.0131	0.1034

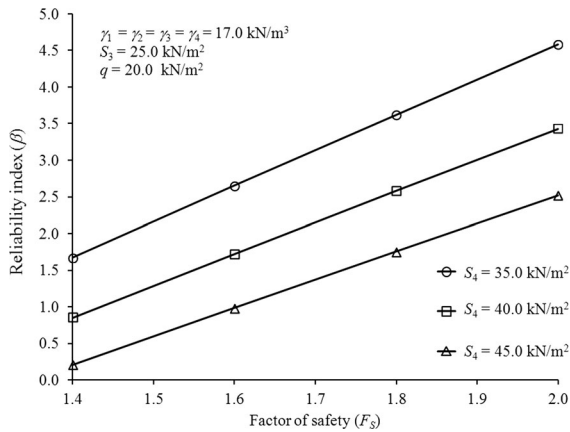


Fig. 6 Variation of reliability index with factor of safety for undrained shear strength of fourth clay layer

of safety in deterministic approach is given by 1.6, the reliability index can vary from 1.57 ($\gamma_2 = 16.0$ kN/m³) to 1.87 ($\gamma_2 = 18.0$ kN/m³). Moreover, when S_4 varies from 35.0 to 45.0 kN/m², the reliability index varies from 2.65 to 0.99, for a factor of safety of 1.6. So, a particular factor of safety does not ensure a unique reliability index when any one random variable takes different values.

7 Reliability Based Design for Basal Heave Failure

In deterministic method, the structure is designed to achieve a target factor of safety against a particular mode of failure. The factor of safety is independent of the variations which is inherent in the design variables. It has been observed that although the factor of safety is greater than the desired value, the structure failed. This indicates that there is requirement for incorporation of the uncertainties associated with various design variables. So, reliability based design becomes relevant, where, the uncertainties associated with the design variables are taken into account by treating them as random variables. Reliability based approach is significant for assuring any desired level of safety. In the design methodology based on reliability analysis of basal heave failure, the embedded strutted retaining wall is to be designed for a ‘target’ reliability index (β_{target}) considering all the possible uncertainties prevailing at the particular site. In conventional design methods, the designer adopts deterministic approach and uses concept of constant factor of safety as

mentioned in the relevant design codes. In the present study, design tables have been developed for basal heave failure for two typical target reliability indices (2.5 and 3.0), so that the designer can select appropriate value of factor of safety corresponding to the target reliability index which is related to the safety of the structure.

It has been found from the sensitivity analysis (Table 3), that the reliability index is influenced significantly by two random variables which are total unit weight of the second soil layer (γ_2) and undrained shear strength of the fourth layer (S_4). So, factor of safety are calculated against target reliability index of 2.5 and 3.0, for various combinations of γ_2 and S_4 and provided in Table 4. The final depth of excavation is considered as 10 m, while the embedment depths are varied through 8 and 10 m, thus, D_b/D_e is varied through 0.8 and 1.0, respectively. The coefficient of variation of the total unit weight of all the layers is taken as 5%, while that of S_3 and S_4 are varied through 20 and 30%, respectively. The surcharge and the mean value of the undrained shear strength of the third layer are taken as 20 and 25 kN/m², respectively. It can be found from Table 4 that, for $D_b/D_e = 0.8$ and the prevailing site condition ($S_3 = 25$ kN/m² with a COV of 20%, $S_4 = 40$ kN/m² with a COV of 20% and $\gamma_2 = 17$ kN/m³ with a COV of 5%), the recommended factors of safety to be considered in the design shall be 1.73 and 1.84, for target reliability index of 2.5 and 3.0, respectively. It can also be observed from Table 4, that for a particular value of D_b/D_e , S_4 and β_{target} , the factor of safety decreases with increase of γ_2 . This is due to the fact that as total unit weight increases, the disturbing moment also increases and thus the factor of safety decreases accordingly for a desired level of reliability ($\beta_{\text{target}} = 2.5$ or, 3.0). It can also be observed from Table 4, that for a particular value of D_b/D_e , γ_2 and β_{target} , the factor of safety increases with increase of S_4 . This can be attributed to the fact that when all other parameters remain constant, increase in undrained shear strength results in increase of resisting moment, which results in increase in factor of safety for a target reliability index.

8 Summary and Conclusions

In the present study, basal heave failure is analyzed probabilistically using Hasofer–Lind reliability index

Table 4 Recommended factors of safety for sliding failure for target reliability index ($\beta_{\text{target}} = 2.5$ and 3.0

γ_2 (kN/m ³)		Reliability index [COV of S_3 and $S_4 = 20\%$] $\gamma_1 = \gamma_3 = \gamma_4 = 17.0$ kN/m ³ , $S_3 = 20$ kN/m ² , $q = 20$ kN/m ²					
		$\beta_{\text{target}} = 2.5$			$\beta_{\text{target}} = 3.0$		
		S_4 (kN/m ²)			S_4 (kN/m ²)		
		35	40	45	35	40	45
0.8	16	1.55	1.76	1.97	1.65	1.88	2.11
	17	1.52	1.73	1.93	1.62	1.84	2.06
	18	1.49	1.69	1.89	1.58	1.80	2.02
1.0	16	1.59	1.80	2.02	1.69	1.92	2.16
	17	1.55	1.77	1.98	1.66	1.88	2.11
	18	1.52	1.73	1.94	1.62	1.85	2.07
		Reliability index [COV of S_3 and $S_4 = 30\%$] $S_3 = 20$ kN/m ²					
		$\beta_{\text{target}} = 2.5$			$\beta_{\text{target}} = 3.0$		
		S_4 (kN/m ²)			S_4 (kN/m ²)		
		32	36	40	32	36	40
0.8	16	1.80	2.05	2.29	1.95	2.22	2.48
	17	1.76	2.00	2.24	1.91	2.17	2.43
	18	1.72	1.96	2.20	1.87	2.13	2.38
1.0	16	1.84	2.09	2.35	1.99	2.27	2.55
	17	1.80	2.05	2.30	1.95	2.22	2.50
	18	1.76	2.01	2.25	1.91	2.18	2.44

by applying Microsoft Excel’s ‘SOLVER’ optimization tool for embedded strutted retaining wall constructed by excavation and installation of support members (struts) in cohesive soil profile. The results obtained in the probabilistic analysis are validated against the field data of an excavation case study (Tang and Kung 2012). The effect of coefficient of variation of the undrained shear strength (CV_S) and the total unit weight of the soil layers (CV_γ) are investigated using ‘SOLVER’ tool. It is found that the probability of failure decreases with increase in CV_S , however, it remains almost constant with increase in CV_γ . Sensitivity analysis results showed that the reliability index is affected primarily by the total unit weight of the second layer and the undrained shear strength of the fourth layer. It is found that for a particular factor of safety, the reliability index decreases and increases with increase of total unit weight and decrease of undrained shear strength, respectively. Thus, in the conventional deterministic approach, which assumes a

constant factor of safety against basal heave failure can actually result in substantial variation in the risk involved in failure, depending on the variability of the design variables. The F_S - β plots can be effectively used in the design of the underground strutted retaining wall against basal heave failure corresponding to target reliability. The design table prepared under the particular site condition and the geometry of the excavation provides some guideline regarding reliability based design which is more rational as compared to the deterministic design methods. Similar type of tables can also be prepared for other prevailing site conditions and the geometry of the proposed excavation. In the present study, the most effective method of probabilistic analysis for the particular problem of basal heave has been determined and also the design tables are provided (for different combination of the design parameters), which will be adequate in obtaining a overall picture of the factor of safety to be achieved for a target reliability index.

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