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ORIGINAL PAPER

Research on Rock Strength Prediction Based on Least Squares Support Vector Machine

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Abstract In order to estimate the strength parameters of rock, as the direct method by conducting rock mechanical tests is time-consuming and expensive, an indirect method based on soft computing technique is proposed. Least squares support vector machine (LS-SVM) is utilized to develop rock uniaxial compressive strength (UCS) and shear strength (SS) prediction models by considering indirect parameters such as rock density, point load strength, P-wave velocity and slake durability index. The results show that according to the rock physical and mechanical parameters of four rock types, empirical relationships based on statistical regression method are rock type specific, only linear relations existed between point load strength and rock strengths are acceptable with high determination coefficients for whole rock types. The LS-SVM models built for rock UCS and SS prediction have greater determination coefficients than the regression models. The prediction values based on LS-SVM prediction models for rock UCS and SS are both extremely close to the measured values, which indicates the applicability of LS-SVM is supported for estimation of strength parameters of rock.

Keywords Uniaxial compressive strength (UCS) · Shear strength (SS) · Prediction model · Soft computing technique · Least squares support vector machine (LS-SVM)

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1 Introduction

Strength parameters of rock are the most basic and important mechanical parameters for engineering geologists, geotechnical engineers and mining engineers. These parameters have great importance in rock engineering such as tunnel and dam design, rock blasting and drilling, mechanical rock excavation and slope stability (Ceryan et al. 2013a). There are two methods for assessing the strength properties of rocks. First is the direct method by conducting laboratory tests on crafted specimens; the other, known as the indirect method, uses the previously derived empirical equations or models from literatures (Baykasoglu et al. 2008; Madhubabu et al. 2016). The direct method by performing rock mechanical tests in the laboratory should follow the testing procedures standardized by the International Society for Rock Mechanics (ISRM) (Ulusay and Hudson 2007). Although the method is relatively simple, it is time-consuming and expensive;

also, it requires well prepared and high-quality core specimens (Heidari et al. 2012), and this cannot always be extracted from weak, highly fractured, weathered and thinly bedded rocks (Ceryan et al. 2013a). For these reasons, indirect methods from empirical equations with parameters such as sonic velocity (Sharma and Singh 2008; Kahraman 2001; Moradian and Behnia 2009), Schmidt rebound number (Sachpazis 1990; Yagiz 2009), point load strength (Basu and Aydin 2006; Mishara and Basu 2012; Singh et al. 2012) have been proposed for predicting rock strengths (Fener et al. 2005). These indirect parameters are all easy to obtain because of low requirements for rock cores and simple testing equipment. And these tests can be performed at the engineering field. Compared to the traditional rock mechanical tests, the indirect prediction method can make the acquisition of rock strength parameters much easier, faster and more economical.

At present, there are two commonly used mathematical methods for performing prediction problems (Singh et al. 2016), which are statistics and soft computing technique. Using the statistical method, statistical regression models between rock strength indices and indirect parameters (e.g. point load strength, P-wave velocity, Schmidt rebound number) could be built with linear (Sachpazis 1990; Basu and Aydin 2006; Sharma and Singh 2008), power (Kahraman 2001; Yagiz 2009) and exponential (Moradian and Behnia 2009) functions. Except that simple prediction models with single one indirect parameter have been proposed (Fattahi 2016), some studies have dealt with models relating all determined indices simultaneously with rock strength parameters (i.e. multiple regression analysis) (Mishara and Basu 2013). As the complexity of rock types, these empirical equations have limitations, notably being site and type specific (Sarkar et al. 2010). The statistical regression method will not be applicable when many rock types exist in a rock data base for prediction, hence new technologies should be proposed for the estimation of rock strength parameters.

With the development of intelligent computing, methods such as neural networks (Dehghan et al. 2010; Singh and Verma 2012; Torabi et al. 2014; Mert 2014; Mohamad et al. 2015), genetic algorithm (Beiki et al. 2013), fuzzy inference (Yilmaz and Yuksek 2009; Rezaei et al. 2012), and support vector machine (Ceryan et al. 2013b) have been put forward and are widely used in rock mechanics and rock engineering with great superiority. Least squares support vector machine (LS-SVM) is a pattern recognition and regression analysis structure based on statistics theory and structure risk minimum criterion. It integrates square error variable into traditional support vector machine that substitutes least square linear system for quadratic programming to resolve function estimating problems, which shows good generalization performance and calculation speed in dealing with multivariate nonlinear problems by small samples (Tan et al. 2014; Xu et al. 2015).

The aim of this study is to build prediction models for estimation of rock strength properties. For this purpose, the relations between indirect parameters (rock density, point load strength, P-wave velocity and slake durability index) and rock strengths (uniaxial compressive and shear strength) were analyzed with rock physical and mechanical test results of four rock types. The prediction models were built based on LS-SVM, and statistical performances were tested.

2 Rock Physical and Mechanical Test Results

The physical and mechanical test results of four rock types, which were limestone, slate, quartzite and quartz mica schist, were used in this research with 10 data sets for each rock type. Rock samples were collected from different locations in Luhri region, Himachal Pradesh and rock parameters were determined according to the standard testing methods of ISRM (1981) (Sarkar et al. 2010). The relations between rock indirect parameters (rock density, P-wave velocity, point load strength and slake durability index) and rock strengths (UCS and SS) were shown in Figs. 1 and 2, respectively. With regard to a specified rock type, statistical regression models can be obtained after performing regression analysis to the data points, and the relations and correlation coefficients (R) were listed in Table 1.

Seen from the Table 1, it shows that the relations between indirect parameters and rock strengths all have a high correlation coefficient, except one which is less than 0.85. Although linear models with high correlation coefficients exist for each rock type to predict its rock UCS and SS based on the indirect parameters, these models are rock type specific and

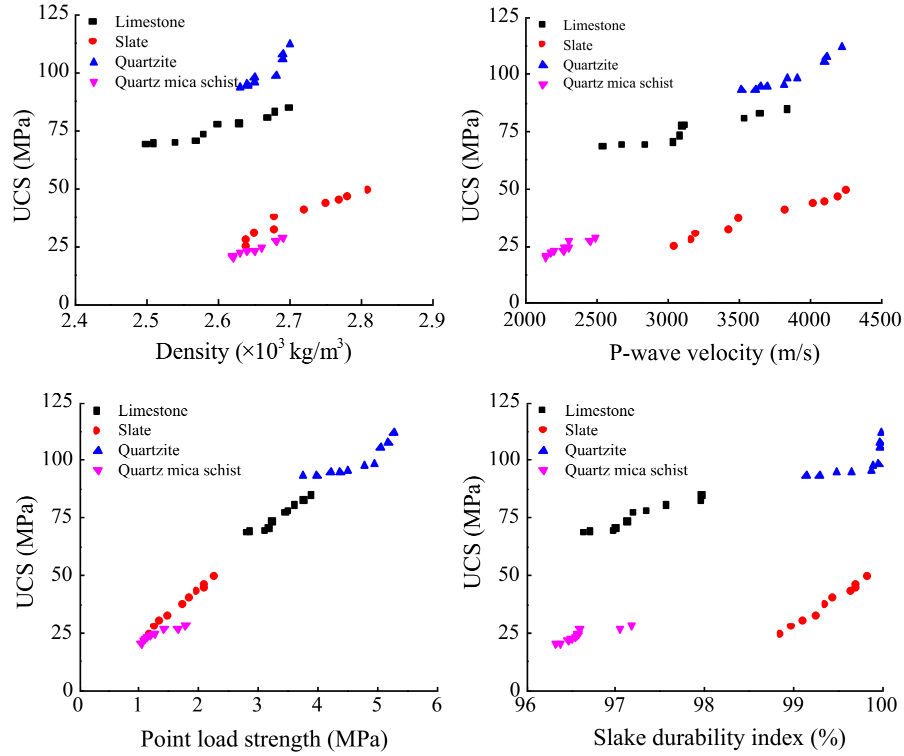

Fig. 1 Relations of rock indirect parameters with rock uniaxial compressive strength

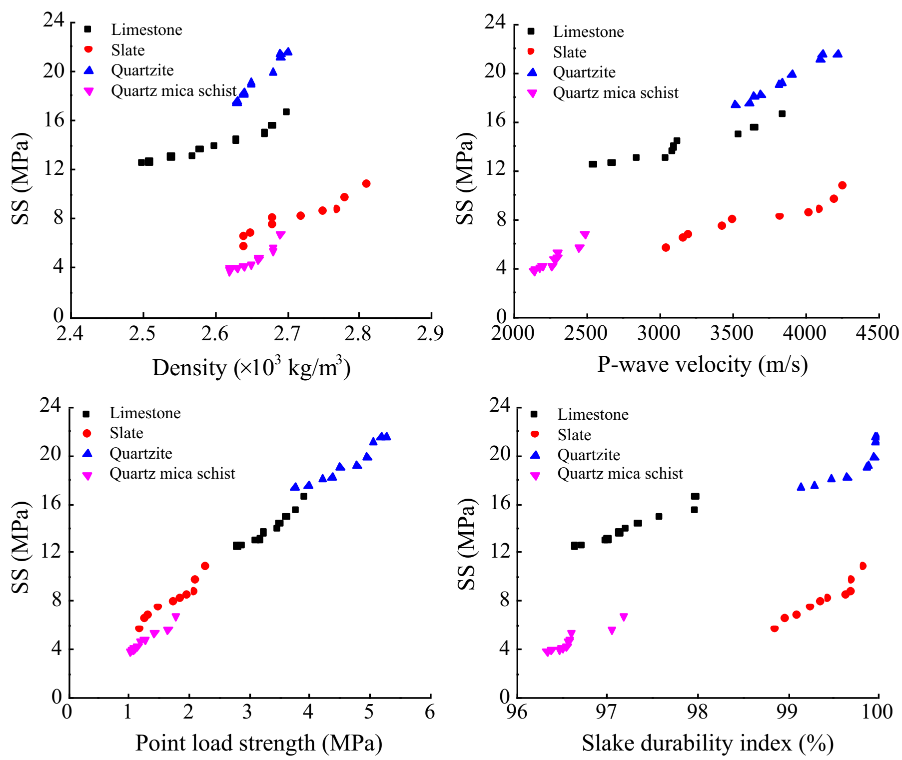

Fig. 2 Relations of rock indirect parameters with rock shear strength

Table 1 Model of rock strength and indirect parameter

Rock type	UCS prediction model		SS prediction model	
	Relation	*R*	Relation	*R*
Limestone	$\sigma_c = 83.01771\rho - 140.52602$	0.97239	$\tau = 18.4101\rho - 33.85443$	0.96704
	$\sigma_c = 0.01357v_p - 32.43269$	0.9427	$\tau = 0.0031v_p + 4.21616$	0.96571
	$\sigma_c = 15.94158I_{s(50)} + 21.70189$	0.97124	$\tau = 3.53332I_{s(50)} + 2.12777$	0.96537
	$\sigma_c = 12.48476I_{d2} - 1139.10134$	0.96203	$\tau = 2.82578I_{d2} - 260.8571$	0.97648
Slate	$\sigma_c = 132.25792\rho - 321.22048$	0.97232	$\tau = 23.31731\rho - 55.21154$	0.95614
	$\sigma_c = 0.0183v_p - 29.8372$	0.98672	$\tau = 0.00315v_p - 3.5499$	0.94658
	$\sigma_c = 21.69128I_{s(50)} - 0.19307$	0.99695	$\tau = 3.7598I_{s(50)} + 1.49798$	0.96386
	$\sigma_c = 25.17049I_{d2} - 2464.25764$	0.99389	$\tau = 4.37172I_{d2} - 426.4848$	0.96285
Quartzite	$\sigma_c = 234.69697\rho - 524.90194$	0.93522	$\tau = 57.77273\rho - 134.3344$	0.9823
	$\sigma_c = 0.0274v_p - 6.09763$	0.95752	$\tau = 0.00665v_p - 6.25701$	0.99143
	$\sigma_c = 11.44086I_{s(50)} + 46.71827$	0.88095	$\tau = 2.92085I_{s(50)} + 5.89341$	0.95966
	$\sigma_c = 15.57315I_{d2} - 1453.547$	0.71684	$\tau = 4.3759I_{d2} - 417.01922$	0.85947
Quartz mica schist	$\sigma_c = 109.11015\rho - 265.40024$	0.99273	$\tau = 34.74355\rho - 87.39663$	0.9336
	$\sigma_c = 0.02207v_p - 26.14492$	0.94067	$\tau = 0.00761v_p - 12.5365$	0.95796
	$\sigma_c = 9.93096I_{s(50)} + 11.45668$	0.93445	$\tau = 3.5272I_{s(50)} + 0.29846$	0.9802
	$\sigma_c = 8.86992I_{d2} - 832.95135$	0.87724	$\tau = 3.2177I_{d2} - 306.11955$	0.93871

with little applicability when the rock type of a different investigation site is unknown and the strengths need to be predicted. A model which considers rock types as many as possible may have a probability to predict an unknown rock type, that is to say, a prediction model considering these four rock types simultaneously will be a probable prediction model in this research. While the scatter plots in Figs. 1 and 2 show that there is no such a statistical prediction model that could express the relations between the indirect parameters (rock density, P-wave velocity and slake durability index) and rock strengths for these four rock types, that is to say, just given one parameter, the rock strength parameters cannot be predicted. Only the point load strength of these rocks may have a linear relation with rock UCS and SS for the whole data points, which can be expressed as followed:

$$\sigma_c = 21.9483I_{s(50)} - 1.13531 \quad R^2 = 0.98174 \tag{1}$$

$$\tau = 4.13715I_{s(50)} + 0.19087 \quad R^2 = 0.98623 \tag{2}$$

3 Prediction Model of Rock Strength Using Least Squares Support Vector Machine

3.1 Foundation of LS-SVM Technique

LS-SVM is now widely used and can achieve acceptable results. The two problem types for which LS-SVM is used are regression and classification problems. Prediction belongs to the regression problem (Xu et al. 2015).

For regression problems, suppose **T** is a training set and n is a sample number. In this case:

$$\mathbf{T} = \{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\} \tag{3}$$

where $\boldsymbol{x}_i \in R^n$ is the input vector and $\boldsymbol{y}_i \in R$ is the output variable that corresponds to $\boldsymbol{x}_i$.

The optimization problem of LS-SVM can be described as follows:

$$\begin{cases} \min\limits_{w,b,e} J(\boldsymbol{w}, \mathbf{b}, \mathbf{e}) = \dfrac{1}{2}\mathbf{w}^T\mathbf{w} + \dfrac{\mu}{2}\sum\limits_{i=1}^{n}\mathbf{e}_i^2 \\ s.t. \quad \mathbf{y}_i = \mathbf{w}^T\varphi(\mathbf{x}_i) + \mathbf{b} + \mathbf{e}_i \quad i = 1, 2, \ldots, n \end{cases} \tag{4}$$

where $\boldsymbol{w}$ is the weight vector, μ is the regularization parameter (also called the penalty parameter), $\boldsymbol{e}_i$ is the error variance, $\varphi(\cdot)$ denotes nonlinear mapping from the input space to high-dimensional feature space, and $\boldsymbol{b}$ is a partial vector.

The function Lagrange of the optimization problem (4) is as follows:

$$L(\boldsymbol{w},\boldsymbol{b},\boldsymbol{e},\boldsymbol{\alpha}) = J(\boldsymbol{w},\boldsymbol{b},\boldsymbol{e}) - \sum_{i=1}^{n} \boldsymbol{\alpha}\{\boldsymbol{w}\varphi(\boldsymbol{x}) + \boldsymbol{b} + \boldsymbol{e} - \boldsymbol{y}\} \tag{5}$$

where $\boldsymbol{\alpha}_i$ is the Lagrange multiplier and sample ($\boldsymbol{\alpha}_i \neq 0$) is the support vector.

The following equations are obtained based on the Karush–Kuhn–Tucker (KKT) condition, which is a necessary and sufficient condition for the optimal solution of the object function in a nonlinear optimization problem:

$$\frac{\partial L}{\partial \boldsymbol{w}} = 0 \quad \frac{\partial L}{\partial \boldsymbol{b}} = 0 \quad \frac{\partial L}{\partial \boldsymbol{e}_i} = 0 \quad \frac{\partial L}{\partial \boldsymbol{\alpha}_i} = 0 \Rightarrow \begin{cases} \boldsymbol{w} = \sum_{i=1}^{n} \boldsymbol{\alpha}_i \varphi(x_i) \\ \sum_{i=1}^{n} \boldsymbol{\alpha}_i = 0 \\ \boldsymbol{\alpha}_i = \mu e_i \quad i = 1,2,\ldots,n \\ \boldsymbol{w}^T\varphi(x_i) + \boldsymbol{b} + \boldsymbol{e}_i - \boldsymbol{y}_i = 0 \quad i = 1,2,\ldots,n \end{cases} \tag{6}$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{b}$ are obtained by solving the right part of Eq. (6). The output value $\boldsymbol{y}(\boldsymbol{x})$ of the new input vector $\boldsymbol{x}$ can be calculated through the following formula:

$$\boldsymbol{y}(\boldsymbol{x}) = \sum_{i=1}^{n} \boldsymbol{\alpha}_i K(\boldsymbol{x},\boldsymbol{x}_i) + \boldsymbol{b} \tag{7}$$

where $K(\boldsymbol{x},\boldsymbol{x}_i) = \varphi(\boldsymbol{x})^{\mathrm{T}}\varphi(\boldsymbol{x}_i)$ is called the kernel function. The radial basis function (RBF) is one of the most popular kernel functions for SVM. The RBF can be described in the following way:

$$K(\mathbf{x},\mathbf{x}_i) = e^{-\frac{\|\mathbf{x}_i - \mathbf{x}\|^2}{2\sigma^2}} \tag{8}$$

where σ^2 is the squared bandwidth, which is optimized through an external optimization technique during the training process (Cao et al. 2008; Xu et al. 2015).

3.2 Establishment of Prediction Model and Results Analysis

Based on the above fundamental theory of LS-SVM, an input matrix including rock density, point load strength, P-wave velocity and slake durability index and two output matrixes including rock UCS and SS were separately built in the MATLAB environment. The radial basis function (RBF) was used as the kernel functions for LS-SVM. 80% (32 datasets) of the total samples were used for training and the other 20% (8 datasets) samples were used for testing. The training and testing data used in LS-SVM were listed in Tables 2 and 3. The raw data of rock UCS show that the values range from 20.32 to 112.25 MPa, which means that the prediction models built with LS-SVM are suitable for rocks with these similar strengths. The prediction results based on LS-SVM were shown in Figs. 3 and 4, and the determination coefficients (R^2) for training and testing results were calculated, respectively. The results show that with regard to rock UCS prediction, the R^2 values for training and testing results are 0.9997 and 0.9995, respectively. And with regard to rock SS prediction, the R^2 values for training and testing results are 0.9990 and 0.9964, respectively. It can be concluded that the accuracy of rock strength prediction models based on LS-SVM is extremely high and the models could be accepted for prediction.

3.3 Performance Validation of LS-SVM Prediction Models

In order to check the validation of prediction models based LS-SVM, the relations of predicted values versus measured values were plotted in Figs. 5 and 6. The error in the predicted value is represented by the distance that each data point plots from the 1:1 diagonal line (Kahraman et al. 2016). It can be seen that the predicted values for both models are almost lying on the diagonal line.

To verify the performance of the models, four statistical criteria viz. squared correlation coefficient (R^2), variance account for (VAF), root mean squared error ($RMSE$), and mean absolute percentage error ($MAPE$) were chosen to be the measure of accuracy. Let y_i be the actual value and $\hat{y}_i$ be the predicted value of the ith observation, $\bar{y}$ be the mean value of the

Table 2 Training data used in LS-SVM

Parameter	Data							
P-wave velocity (m/s)	3845.35	3656.2	3124.58	3106.42	3089.78	2845.98	2679.14	2549.63
Point load strength (MPa)	3.91	3.78	3.51	3.47	3.25	3.12	2.87	2.81
Density (gm/cm^3)	2.7	2.68	2.63	2.6	2.58	2.54	2.51	2.5
Slake durability index (%)	97.98	97.97	97.35	97.21	97.14	96.98	96.72	96.65
UCS (MPa)	84.5	82.46	77.68	77.2	72.98	69.15	68.84	68.43
Shear strength (MPa)	16.56	15.48	14.35	13.89	13.56	12.96	12.54	12.49
P-wave velocity (m/s)	4260.12	4199.09	4026.97	3832.05	3502.15	3202.15	3172.76	3050.38
Point load strength (MPa)	2.28	2.11	1.98	1.86	1.74	1.34	1.27	1.18
Density (gm/cm^3)	2.81	2.78	2.75	2.72	2.68	2.65	2.64	2.64
Slake durability index (%)	99.84	99.71	99.65	99.45	99.36	99.11	98.98	98.86
UCS (MPa)	49.25	46.13	43.18	40.35	37.19	30.05	27.54	24.46
Shear strength (MPa)	10.78	9.66	8.53	8.17	7.98	6.78	6.5	5.64
P-wave velocity (m/s)	4225.14	4119.56	3910.22	3841.06	3820.25	3650.12	3617.35	3521.13
Point load strength (MPa)	5.27	5.18	4.95	4.78	4.5	4.21	3.98	3.75
Density (gm/cm^3)	2.7	2.69	2.68	2.65	2.65	2.64	2.63	2.63
Slake durability index (%)	99.98	99.97	99.95	99.89	99.87	99.48	99.29	99.14
UCS (MPa)	112.25	108.01	98.43	98.02	95.51	94.54	93.3	93.21
Shear strength (MPa)	21.57	21.46	19.86	19.12	18.99	18.06	17.55	17.42
P-wave velocity (m/s)	2489.25	2450.64	2300.23	2278.45	2265.13	2178.6	2145.56	2142.39
Point load strength (MPa)	1.78	1.65	1.26	1.19	1.14	1.08	1.05	1.03
Density (gm/cm^3)	2.69	2.68	2.66	2.66	2.65	2.63	2.62	2.62
Slake durability index (%)	97.18	97.05	96.58	96.57	96.55	96.47	96.38	96.33
UCS (MPa)	28.45	27.1	24.78	24.36	23.18	22.1	20.58	20.32
Shear strength (MPa)	6.78	5.72	4.89	4.75	4.28	4.06	3.95	3.8

Table 3 Testing data used in LS-SVM

Parameter	Data							
P-wave velocity (m/s)	3548.13	3047.13	4104.56	3435.69	4102.37	3695.79	2302.68	2200.07
Point load strength (MPa)	3.62	3.19	2.1	1.5	5.05	4.37	1.42	1.1
Density (gm/cm^3)	2.67	2.57	2.77	2.68	2.69	2.64	2.68	2.64
Slake durability index (%)	97.58	97.01	99.7	99.25	99.97	99.65	96.6	96.5
UCS (MPa)	80.25	70.05	44.38	32.1	105.69	94.96	27.02	22.8
Shear strength (MPa)	14.91	13.01	8.78	7.43	21.13	18.25	5.38	4.17

observations and N be the number of observations, then R^2, *VAF*, *RMSE* and *MAPE* could be defined below (Fattahi 2016), and the results were shown in Table 4. The performance indices show that the rock strength models based on LS-SVM are excellent because the values are all extremely close to the best condition at which the R^2 is 1, *VAF* is 100, *RMSE* and *MAPE* is 0 (Armaghani et al. 2016a, b). The prediction models built based on LS-SVM are applicable for the prediction of strength parameters of rock.

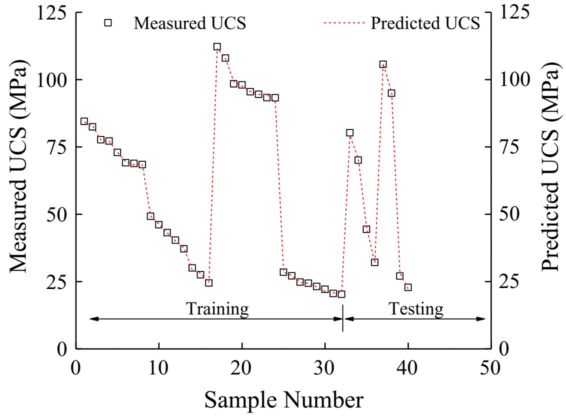

Fig. 3 Comparison of Rock measured and predicted UCS

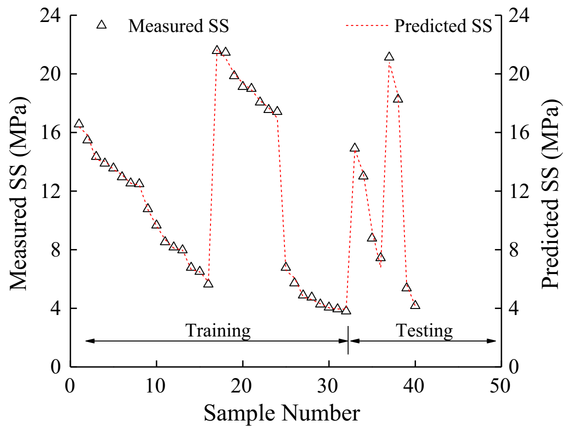

Fig. 4 Comparison of Rock measured and predicted SS

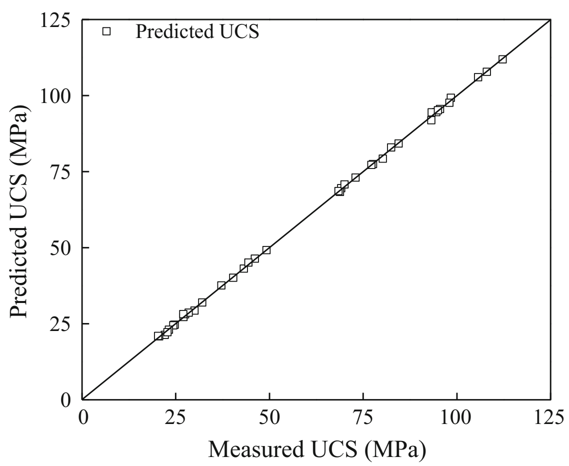

Fig. 5 Relation between predicted and measured UCS values

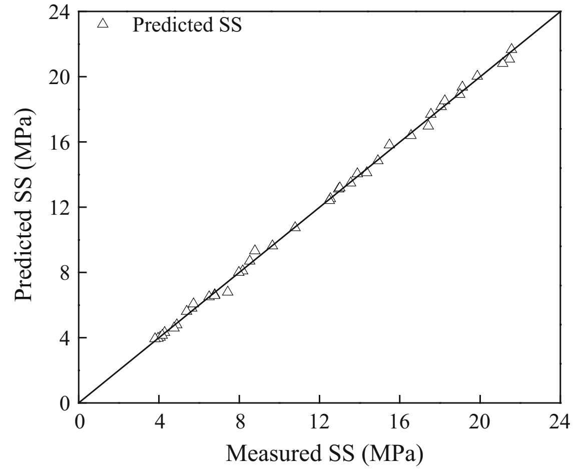

Fig. 6 Relation between predicted and measured SS values

Table 4 Validation results of rock strength prediction models

Model	Performance index			
	R^2	VAF	RMSE	MAPE
UCS	0.9997	99.98	0.5276	0.916
SS	0.9985	99.98	0.2234	1.754

$$R^2 = 1 - \frac{\sum_{i=1}^{N} (y_i - \widehat{y}_i)^2}{\sum_{i=1}^{N} (y_i - \bar{y})^2} \quad (9)$$

$$VAF = \left(1 - \frac{\mathrm{var}(\mathrm{y_i} - \widehat{\mathrm{y}}_\mathrm{i})^2}{\mathrm{var}(y_i)}\right) \times 100 \quad (10)$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left(y_i - \widehat{y}_i\right)^2} \quad (11)$$

$$MAPE = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{y_i - \widehat{y}_i}{y_i} \right| \times 100 \quad (12)$$

4 Discussion

ANN prediction models of rock strengths have been developed by Sarkar et al. (2010) based on these raw data. The results of ANN models show that the predicted and determined values indicate a very good correlation ($R = 0.99$) and confirm the applicability of ANN for estimation of rock strength parameters (Sarkar et al. 2010; Singh and Verma 2012; Mert 2014; Mohamad et al. 2015). By comparing the models built using LS-SVM and ANN, it shows that these two soft computing techniques can both used for prediction and have an equivalent prediction performances. All these support the applicability of soft computing techniques for the estimation of strength parameters of rock.

5 Conclusion

Aimed at estimation of strength parameters of rock, a soft computing technique, which is least squares support vector machine (LS-SVM), is used for the establishment of rock strength prediction models. The prediction results based on LS-SVM prediction models of rock UCS and SS are both extremely close to the measured values, and the squared correlation coefficients (R^2) of rock UCS and SS prediction models are 0.9997 and 0.9985, respectively, which indicates the prediction models based on LS-SVM could be accepted for prediction. By introducing the prediction results based on another soft computing technique (i.e. ANN), a conclusion that the applicability of soft computing techniques for estimation of strength parameters of rock is further supported.

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