Geotech Geol Eng (2016) 34:657–669
DOI 10.1007/s10706-015-9975-4

ORIGINAL PAPER

Stability of an Embankment on Soft Consolidating Soil with Vertical Drains

Agrahara Krishnamoorthy · Smit Kamal

Received: 26 June 2014 / Accepted: 30 December 2015 / Published online: 14 January 2016

Abstract Stability of an embankment constructed on soft consolidating soil improved with pre-fabricated vertical drains is investigated. The factor of safety of the embankment is obtained at various time intervals from the end of construction till the end of consolidation in order to check the embankment stability. Finite element method is used to obtain the effective stresses at required points in soil at various time intervals. Critical slip surface is obtained using two methods. In the first method, the critical slip surface is assumed as an arc of a circle selected among various probable slip circles with minimum factor of safety whereas, in the second method, a random walking type Monte Carlo technique is used to predict the critical slip surface. The effects of providing vertical drains on stability of an embankment is investigated by comparing the factor of safety of slope with vertical drains to the factor of safety of slope without vertical drains. It is concluded from the study that the installation of vertical drains enhances the factor of safety of the embankment from the end of construction till the end of consolidation.

Keywords Finite element analysis · Factor of safety · Vertical drains · Time dependent analysis · Embankment stability

A. Krishnamoorthy (✉) · S. Kamal
Department of Civil Engineering, Manipal Institute of Technology, Manipal, Karnataka 576 104, India
e-mail: akmoorthymit@gmail.com

1 Introduction

Embankments constructed on soft soil with high ground water level shows excessive settlements and requires long duration to dissipate excess pore pressure. In such instances, ground improvement measures are often used to enhance stability and to decrease the consolidation period. Prefabricated vertical drains (PVD) is one of the commonly used ground improvement technique to accelerate the consolidation process and have been widely applied in soft soil such as an embankment construction on soft ground in past few decades (Chai and Miura 1999). Soil improved with vertical drains is commonly analysed using one dimensional unit cell solution (Barron 1948; Hansbo 1981) ignoring the effect of vertical drainage. However, in some cases the vertical drainage has a considerable effect on the degree of consolidation of PVD improved subsoil (Chai et al. 2001) and hence a three dimensional analysis considering actual locations of vertical drains and their influence zones is required in such cases. A numerical technique such as finite element method which is a powerful computational tool in engineering has been developed for the analysis of soft soil with vertical drains (Hird et al. 1992; Chai and Miura 1999; Borges 2004; Shen et al. 2005; Yildiz 2009). Since, three dimensional finite element modeling of vertical drain system is very sophisticated and requires large computational effort when applied to a real embankment project with a large number of vertical drains (Biot 1956; Yildiz

2009), two dimensional plane strain finite element analysis using several matching methods (Hird et al. 1992; Chai et al. 2001) to simulate the actual axisymmetric flow condition have been developed and its applicability have been verified through many laboratory as well as field tests on full scale embankments (Indraratna et al. 1994; Chai and Miura 1999; Shen et al. 2005; Yildiz 2009). The major emphasis of these analytical methods as well as laboratory/field tests are to investigate the settlement behaviour of the embankment and pore pressure in underlying soft soil. Stability of embankment is generally expressed in terms of factor of safety and it is the basic result to express the stability of an embankment. Moreover, in the case of an embankment constructed on soft consolidating soils, the factor of safety varies with time due to the change in effective stresses. In addition to settlement and pore pressure, factor of safety of an embankment at various time intervals from the end of construction till the end of consolidation is also needed in order to check its stability.

Many methods to determine the factor of safety of an embankment have been proposed since 1948. Determination of the potential failure surface and the corresponding forces tending to cause slip and to restore or stabilize the sliding mass and the computation of available factor of safety are the essential steps in a stability analysis (Sengupta and Upadhyay 2009). Due to their simplicity, limit equilibrium methods (Bishop 1955; Janbu 1957; Morgenstern and Price 1965; Spencer 1967; Sarma 1979) are the most common and reliable method and can be used with confidence to investigate the slope stability (Alkasawneh et al. 2008). In the last two decades many advanced methods based on optimization techniques such as dynamic programming approach (Baker 1980), simplex method (Bardet and Kapuskar 1989), genetic algorithm (Zolfaghari et al. 2005; Cheng et al. 2007; Sengupta and Upadhyay 2009; Tran and Srokosz 2010) and Monte Carlo based optimization techniques of the random walk type (Greco 1996; Malkawi et al. 2001; Alkasawneh et al. 2008) have also been proposed to predict the critical slip surface. Methods based on finite element method have also been proposed for the problems involving complex boundary and loading conditions (Matsui and San 1992; Kim et al. 1999, 2002; Chen et al. 2014). Limit analysis method which takes advantage of the lower and upper-bound theorems of plasticity theory to provide rigorous bounds of the true solution of a stability problem (Kim et al. 2002) and the strength reduction method in which soil strength parameters are reduced, until the slope becomes unstable, are the common approaches proposed in finite element method. In another approach of finite element method, stresses at required points are obtained by finite element method and the factor of safety is obtained either by using limit equilibrium methods (Duncan and Dunlop 1969; Donald and Giam 1988) or by joining the points of local failure in an embankment (Scott and Yamasaki 1993). Since finite element method is the most preferred numerical technique to solve the consolidation problem such as an embankment on soft soil with vertical drains, a method combined with finite element method to obtain effective stresses and any of the above technique to obtain the factor of safety may be more effective and appropriate.

Krishnamoorthy (2010) developed a procedure to obtain the factor of safety of an embankment on consolidating soil using finite element method to obtain stresses at required points and a Monte Carlo technique (MCT) proposed by Greco (1996) to obtain critical slip surface. The effectiveness of installing vertical drains has also been investigated (Krishnamoorthy 2013) using the proposed technique. In these studies, the behavior of foundation soil is modeled as a linearly elastic material. In order to enhance the effectiveness of the technique further and to represent a realistic behavior of soil, a similar procedure but modeling the soil using a nonlinear relationship is proposed in this study to investigate the stability of an embankment constructed on soft soil with vertical drains. Since, a Mohr–Coulomb yield criterion is used to compare the destabilizing and restoring forces while obtaining a factor of safety, a similar model is also used to model the nonlinear behavior of foundation and embankment soils. In addition to MCT, a critical slip surface obtained from a number of surfaces (CSNS) is also used to obtain factor of safety. In this approach, number of slip circles with different radius and center are generated and the factor of safety is obtained for each slip surface using the stresses obtained by finite element analysis. Among these, a slip surface with minimum factor of safety is chosen as the critical slip surface for a particular time interval. Selection of number of slip surfaces in this method ensures that all the possible critical slip surfaces are investigated and no potential slip surface is left behind. One of the limitation of this search technique compared to the optimization techniques is the computational time. However, it may be noted that, in the consolidation problems using finite element method, the time required to obtain the critical slip surface is negligible compared to the time required to solve the consolidation problem and hence this is not a major issue compared to its effectiveness. The effective stresses in the case of embankment on consolidating soil varies with time, not only due to the dissipation of excess pore pressure but also due to the change in geometry caused by the excessive settlement of foundation soil. Since generation of slip surfaces using number of trials is found to be more stable, this method is also used in the proposed study to obtain the critical slip surface in addition to MCT.

Thus, when finite element method is used to obtain excess pore pressure and effective stresses in soil, the factor of safety may be obtained by limit analysis method, strength reduction method or limit equilibrium method. The major advantage of limit analysis and strength reduction methods compared to traditional limit equilibrium method is the stress and displacement obtained by these methods may lead to a better understanding of the mechanism of slope failure. However, since the equilibrium state of a geotechnical system with reduced soil strength must be repeatedly evaluated, the shear strength reduction finite element method is generally more expensive than the traditional limit equilibrium method (Chen et al. 2014). The limit equilibrium method, on the other hand, possess the advantages of simplicity and effectiveness. Monte-Carlo techniques are very fast, easy-to-implement and can be used to locate the critical slip surface more efficiently (Alkasawneh et al. 2008). Moreover, the critical slip surface from the more advanced limit equilibrium methods compare well with the slip mechanism from limit analysis method (Kim et al. 2002). Hence, for the proposed analysis, effective stresses are obtained by finite element method and factor of safety is obtained using limit equilibrium based MCT and CSNS methods. MCT and CSNS methods are simple and effective, whereas, the finite element method provides a rigorous solution and yields information such as stress, displacement and pore pressure from the beginning of construction till the end of consolidation similar to the approaches based on limit analysis or strength reduction methods.

2 Method of Analysis

The analysis consists of two parts

1. Finding the effective stresses in soil at the required points using finite element method.
2. Locating the critical slip surface and finding the factor of safety using these stresses.

2.1 Determination of Pore Pressure and Effective Stresses in Soil by Finite Element Method

The elemental equation for the consolidation proposed by Zienkiewicz (1977) is expressed in matrix form as

$$\begin{bmatrix} K_s & L \\ 0 & H \end{bmatrix} \begin{Bmatrix} u \\ p \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ L^T & 0 \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{p} \end{Bmatrix} = \begin{Bmatrix} f \\ 0 \end{Bmatrix} \tag{1}$$

K_s is the soil stiffness matrix and H is the flow matrix. u, p and f are the vector of nodal displacements, pore pressure and forces respectively. L is the coupling matrix which is formed from the equation

$$L = \int_s \mathrm{N_s^T} \begin{Bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{Bmatrix} \mathrm{N_f ds} \tag{2}$$

N_s and N_f are the shape functions defining the displacement of the soil element and pore pressure distribution respectively.

Equation (1) in incremental form can be written as

$$\begin{bmatrix} K_s & L \\ 0 & H \end{bmatrix} \begin{Bmatrix} \Delta u \\ \Delta p \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ L^T & 0 \end{bmatrix} \begin{Bmatrix} \Delta \dot{u} \\ \Delta \dot{p} \end{Bmatrix} = \begin{Bmatrix} \Delta f \\ 0 \end{Bmatrix} \tag{3}$$

Δ denotes the variations of each parameter from time t to time $t + \Delta t$ and index i indicates the i^{th} time step. Equation (3) is solved in the incremental form using Newmark's method to obtain displacement and pore pressure at each time interval Δt.

2.2 Modeling of Vertical Drains

Embankment on soft soil with vertical drains is analysed using plane strain finite element method. A matching procedure proposed by Hird et al. (1992) accounting for the effects of the smear zones around the drains is used to simulate the actual axisymmetric flow. The three matching procedures proposed by Hird et al. (1992) are:

1. Geometric matching: the drain spacing is matched while maintaining the same permeability coefficient

$$\frac{B}{R}=\left\{\left(\frac{3}{2}\right)\left[\ln\left(\frac{R}{r_s}\right)+\left(\frac{k_h}{k_s}\right)\ln\left(\frac{r_s}{r_w}\right)-\left(\frac{3}{4}\right)\right]\right\}^{\frac{1}{2}} \tag{4}$$

2. Permeability matching: the coefficient of permeability is matched while keeping the same drain spacing

$$\frac{k_{pl}}{k_{ax}}=\frac{2}{3\left[\ln\left(\frac{R}{r_s}\right)+\left(\frac{k_h}{k_s}\right)\ln\left(\frac{r_s}{r_w}\right)-\left(\frac{3}{4}\right)\right]} \tag{5}$$

3. Combined matching: A convenient value of drain spacing is preselected for plane strain analysis and the corresponding permeability is calculated for this drain spacing

$$\frac{k_{pl}}{k_{ax}}=\frac{2B^2}{3R^2\left[\ln\left(\frac{R}{r_s}\right)+\left(\frac{k_h}{k_s}\right)\ln\left(\frac{r_s}{r_w}\right)-\left(\frac{3}{4}\right)\right]} \tag{6}$$

In the above equations, R, is the radius of axisymmetric unit cell and B is the half width of the plane strain unit cell, k_h/k_s is the ratio of horizontal permeability of the undisturbed and smeared soil and r_s/r_w is the ratio of radius of the smear zone and radius of drain. k_{ax} and k_{pl} are the horizontal permeability of axisymmetric and plane strain conditions respectively. For a square configuration, the unit cell radius, R, is related to the drain spacing, S, using the expression, $R = 0.565\ S$.

2.3 Locating the Critical Slip Surface

As already mentioned, critical slip surface is obtained by two methods. The first method gives the Critical Surface from Number of Surfaces (CSNS) and the second method is based on Monte Carlo Technique (MCT).

2.3.1 Critical Slip Surface by Monte Carlo Technique (MCT)

Greco (1996) proposed a new Monte Carlo method of the random walking type, for locating critical slip surface. In this method, each search stage is articulated in two phases: exploration phase and extrapolation phase. A trial slip surface with six or seven trial points is selected. As shown in Fig. 1, A, B, C, D and E is one such trial surface with points A, B, C, D and E as the trial points. In the exploration phase, each of these trial points A, B, C, D and E is then shifted to a new position in eight directions. A, B′, C, D and E is one such surface obtained after shifting B to B′. For each shift the factor of safety is computed. If the factor of safety of the new slip surface (A, B′ C D E) is lesser than the factor of safety of the previous slip surface (A, B, C, D, E) then the point is fixed to this new position (B′). Otherwise, it is returned to the previous position (B). The process of shifting the trial points to new position based on comparison of the old and the new factor of safety is repeated for all trial points A, B, C, D and E. As trial points are shifted to new position a new slip surface is obtained. In the extrapolation phase, the total displacement obtained in the exploration is repeated, and the slip surface is updated if the factor of safety is lesser than that obtained at the end of the exploration phase. The factor of safety of the new slip surface, thus obtained is compared with the factor of safety of the old slip surface. If the factor of safety of the new surface is less than the factor of safety of the old surface then the procedure is repeated for the new surface. Thus shifting of trial points of the slip surface

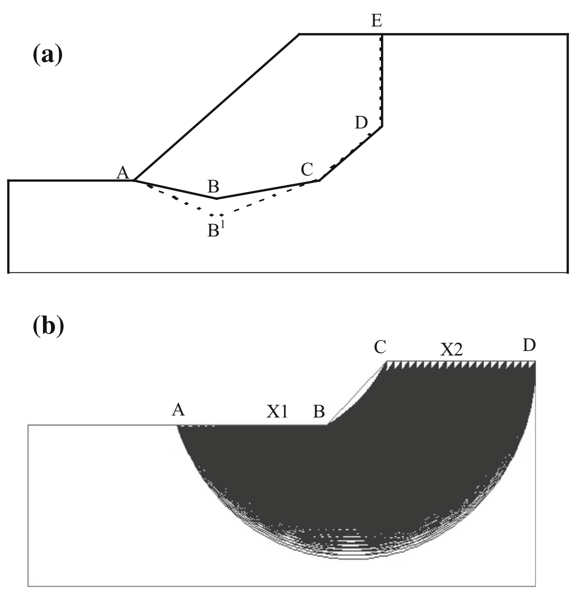

Fig. 1 **a** Trial slip surface ABCDE for MCT. **b** Trial slip circles generated by CSNS method

in eight directions as explained above is repeated until the factor of safety of new slip surface and previous slip surface are same.

2.3.2 *Critical Slip Surface Obtained from Number of Surfaces (CSNS)*

Number of slip circles of different centers and radius are generated in this method using the following steps

1. Two points X_1 and X_2 with coordinates (x_1, y_1) and (x_2, y_2) are selected. As shown in the Fig. 1b, point X_1 is on the ground surface where as point X_2 is on the top surface of the embankment.
2. Different circles passing through the points X_1 and X_2 are generated.
3. Initially, the point X_1 is fixed at point B and point X_2 is gradually shifted from C to D at an increment of 0.25 m.
4. Once the point X_2 reaches the point D, the point X_1 is then shifted towards A at an increment of 0.25 m and for each increment, the step (3) is repeated once again by shifting point X_2 from C to D, till X_1 reaches point A.
5. About twenty-five thousand circles are generated by this procedure as shown in Fig. 1b. Each of the circle thus obtained is divided into arcs of length $\Delta L = 0.5$ m to obtain the factor of safety.

2.4 Determination of Factor of Safety

The trial slip surface is divided into '*n*' number of segments each of length ΔL. The overall factor of safety for a particular slip surface is obtained using the equation

$$\mathrm{F.S} = \frac{\sum T_f \Delta L_i}{\sum T_i \Delta L_i} \tag{7}$$

where τ_i is the mobilized shear stress and τ_f is the shear strength of the material. ΔL_i is the length of ith segment. For the ith segment on a particular slip surface, the values of τ_i and τ_f may be expressed as

$$\begin{aligned} \tau_f &= c' + \sigma'_{ni} tan\phi' \\ \tau_i &= 0.5\,(\sigma'_{yi} - \sigma'_{xi}) sin2\alpha_i + \tau'_{xyi} cos2\alpha_i \\ \sigma'_{ni} &= 0.5\,(\sigma'_{yi} + \sigma'_{xi}) + 0.5\,(\sigma'_{yi} - \sigma'_{xi}) cos2\alpha_i \\ &\quad - \tau'_{xyi} sin2\alpha_i \end{aligned} \tag{8}$$

where c' and ϕ' are the effective cohesion and effective angle of internal friction of the soil respectively. σ'_{ni} is the effective normal stress acting on segment i. σ'_{xi}, σ'_{yi}, and τ'_{xyi} are the effective stresses on ith segment. α_i is the inclination of the ith segment with horizontal.

3 Numerical Applications

Factor of safety obtained from the proposed method of combining the finite element method to obtain effective stresses and the critical slip surface selected from MCT and CSNS method is compared with the factor of safety obtained by Kim and Lee (1997) for a 1:1 homogeneous dry slope of height 5.0 m. The unit weight and modulus of elasticity of soil are 20 kN/m^3 and 15,000 kN/m^2 respectively. The factor of safety obtained by the proposed methods and by Kim and Lee (1997) for the three combinations of c' and ϕ' are presented in Table 1. There is a close agreement between the factor of safety obtained by the proposed methods and by Kim and Lee (1997). This shows that the proposed methods can be used to find the factor of safety of slopes. Moreover, CSNS method is more reliable compared to MCT since all the possible slip surfaces are checked while arriving the critical slip surface. Also, the proposed CSNS method of choosing the critical slip surface will not require any constraints unlike many optimization techniques.

3.1 Stability Analysis of Embankment on Soft Consolidating Soil

Stability of a dry embankment on soft consolidating soil with vertical drains is investigated using the proposed methods. Figure 2 shows an embankment of height 2.0 m, width of crest 10.0 m and side slope 1:1

Table 1 Comparison of factor of safety (F.S) obtained by Kim and Lee (1997) and by proposed methods

c' (kN/m^2)	ϕ' (°)	F.S by Kim and Lee (1997)	F.S by MCT	F.S by CSNS
10	20	1.31	1.30	1.32
6	20	1.01	1.01	1.05
5	30	1.27	1.27	1.32

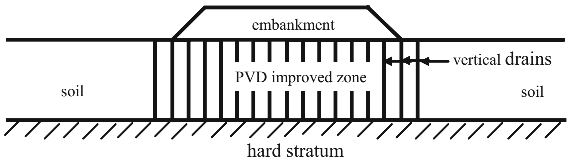

Fig. 2 Embankment on soft soil with vertical drains

Table 2 Matching parameters obtained by various procedures proposed by Hird et al. (1992)

	Geometric matching	Permeability matching	Combined matching
R(m)	0.5	0.5	0.5
B(m)	2.5	0.5	1.25
k_{pl}/k_{ax}	1.0	0.0406	0.253

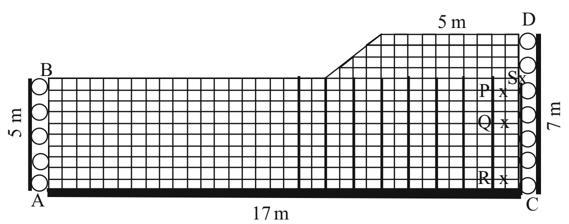

Fig. 3 Finite element descritization of soft soil and embankment

considered for the analysis. Foundation is 5.0 m thick saturated soil laying on a rigid impermeable soil. Prefabricated vertical drains with 98.7 × 6.83 mm^2 section are installed in a square grid at a spacing, S, of 0.9 m underneath the embankment. Smear effect is taken into consideration using $r_s/r_w = 5$ and $k_h/k_s = 10$. Matching parameters for two dimensional plane strain condition for the various matching procedures proposed by Hird et al. (1992) are tabulated in Table 2. Half width of plane strain unit cell, B, is preselected as 1.25 m for combined matching, it is similar to the radius of the axisymmetric unit cell, $R = 0.5$ m, for permeability matching, whereas, it is obtained from the Eq. 4 for geometric matching as 2.5 m. Finite element discretization of the embankment and foundation soil with vertical drain is shown in Fig. 3. Due to symmetry, only half of the embankment and foundation soil is considered in the finite element discretization. Four noded quadrilateral element of size 0.5 m × 0.5 m and triangular element as shown in the figure are used for discretization. The displacement along horizontal and vertical direction is restrained at the bottom surface of the foundation soil (along AC) and displacement in horizontal direction is restrained along the sides AB and CD as shown in figure. Bottom surface AC and side AB are considered as impervious and excess pore pressure is set as zero on the ground level (corresponding to upper drainage surface) and for the nodes corresponding to the location of vertical drains. In the case of soil without vertical drains, excess pore pressure is considered as zero only on the upper drainage surface. It is assumed that the embankment is constructed in sequence. Each layer with 0.5 m height of embankment is constructed in 3.5 days with a pause period of 3.5 days, so that the construction is completed in 24.5 days. The material properties considered for the embankment and the foundation soil are tabulated in Table 3. Since, extensive investigations have been carried out to study the effect of vertical drains on settlement and pore pressure (Borges 2004; Yildiz 2009), parameters that have major influence on factor of safety of embankment are presented and discussed in this study. Also, out of the three matching procedures proposed by Hird et al. (1992), the permeability matching has the drainage spacing similar to the actual axisymmetric case, whereas, Yildiz (2009) proposed a combined matching procedure as the most convenient, as it enables controlling mesh geometry in the finite element analysis. Hence, for the proposed analysis, the vertical drains are modeled using both the permeability matching and combined matching procedure proposed by Hird et al. (1992).

Figure 4 shows the variation of excess pore pressure obtained by permeability matching at points P, Q and R. As shown in Fig. 3, these points are at 0.5, 2.0

Table 3 Properties of embankment and foundation soil

	γ (kN/m^3)	E (kN/m^2)	μ	c' (kN/m^2)	ϕ' (°)	K (m/day)
Embankment	20	8×10^3	0.35	10	30	–
Foundation	18	2×10^3	0.3	0	23	4.86×10^{-4}

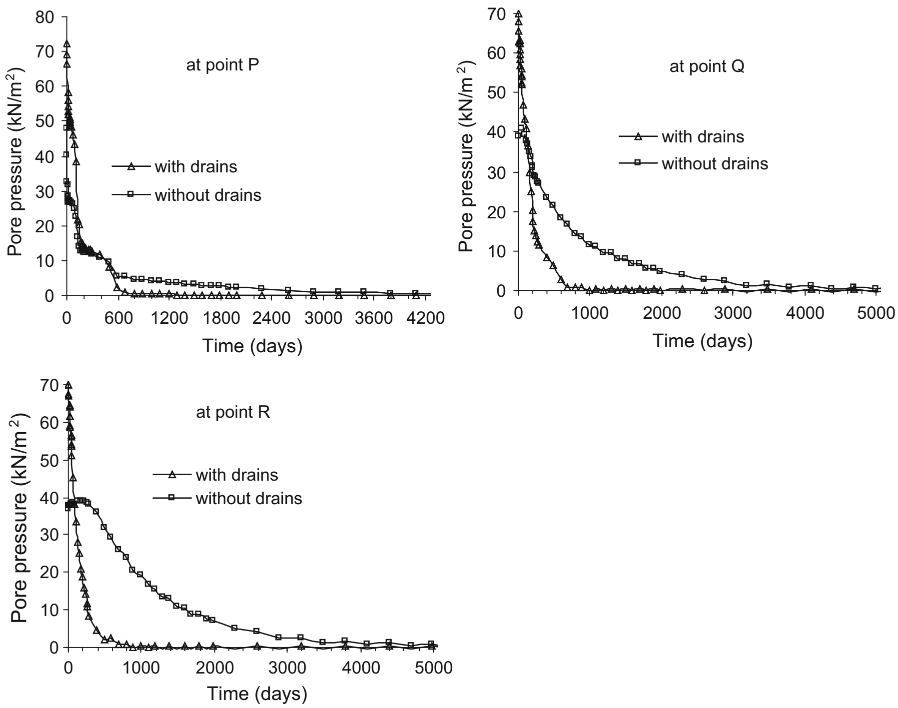

Fig. 4 Variation of excess pore pressure with time at points P, Q and R for the soil with and without drains

and 4.5 m depths respectively from ground surface and at a horizontal distance of 0.5 m from the drain below the center of the embankment. It may be noted that half width of plane strain unit cell, B, in the case of permeability matching is similar to the radius, R, of the axisymmetric unit cell in actual three dimensional condition. From Fig. 4 it is observed that the maximum excess pore pressure at the end of construction in the case of soil with drains at points P, Q and R is larger compared to the soil without vertical drains. Time required for almost complete dissipation of excess pore pressure at points P, Q and R for the soil with drains is almost similar and is within 600 days whereas for the soil without drains, the time required for almost complete dissipation of excess pore pressure at points P,Q and R is nearly equal to 3000, 4000 and 5000 days respectively. Also, excess pore pressure at point P dissipates at faster rate up to 300 days compared to dissipation of excess pore pressure beyond 300 days for both the soils with and without drains.

Thus, (1) excess pore pressure dissipates faster at points Q and R (i.e. at depths 2.0 and 4.5 m) in the case of soil with drains compared to the soil without drains (2) rate of dissipation of excess pore pressure after construction is almost similar at all the points P, Q and R in the case of soil with drains, whereas it decreases with increase in depth in the case of soil without drains and (3) for the point P, situated very close to the drainage surface, the period required for complete consolidation is almost similar for both the soils, with and without drains.

Figure 5a–c show the variation of excess pore pressure along the base of the embankment at 0.5 m beneath ground surface, 2.0 m beneath ground surface and 4.5 m beneath ground surface for the soil with drains obtained by permeability and combined matching and for the soil without drains at the end of construction, 300 days after construction and 600 days after construction. Average excess pore pressure below the embankment for the soil with and without drains is tabulated in Table 4. Excess pore pressure profile shows that, as expected, the excess pore pressure is zero at the matched locations of the drains, i.e. at a spacing of 1.0 and 2.5 m respectively for permeability matching and combined matching. However, the excess pore pressure profile predicted by two matching procedures for the soil with drains are different. Immediately after construction, excess pore pressure between the drains is larger for permeability

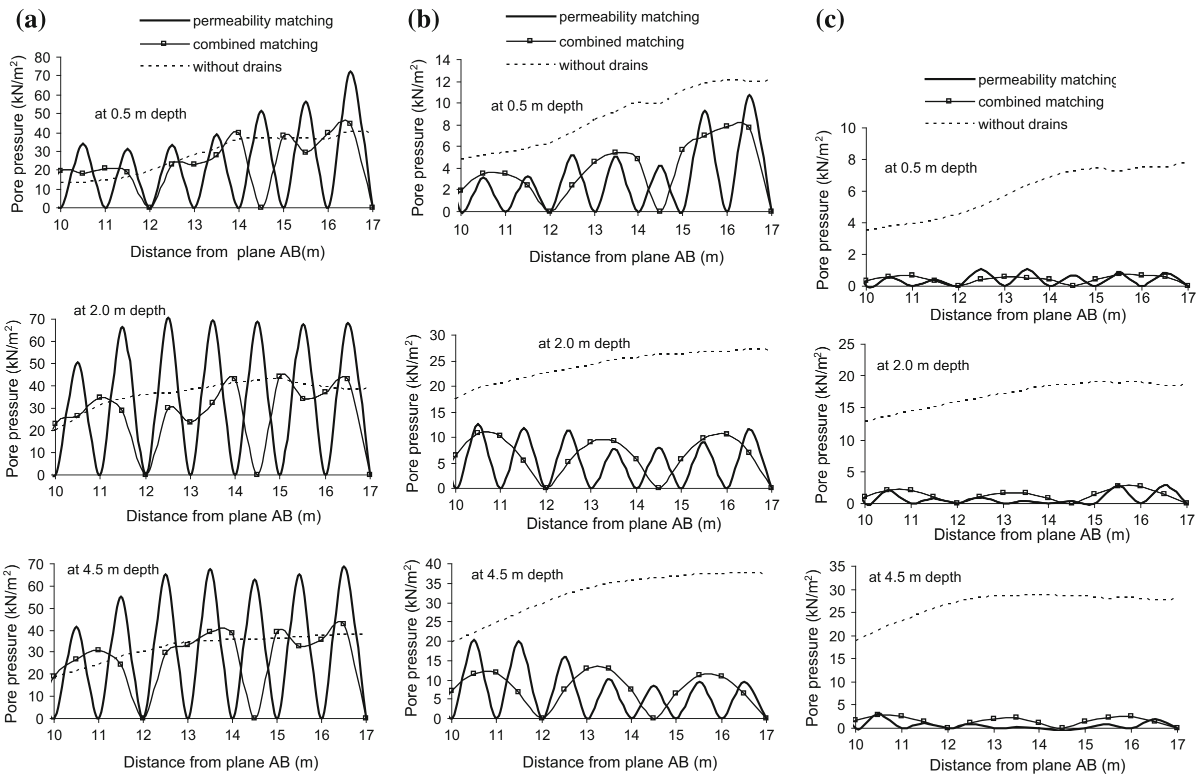

Fig. 5 **a** Excess pore pressure at various depths immediately after construction. **b** Excess pore pressure at various depths 300 days after construction. **c** Excess pore pressure at various depths 600 days after construction

Table 4 Average excess pore pressure (kN/m^2) below the embankment for the soil with and without drains

Time (days)	0.5 m depth			2.0 m depth			4.5 m depth		
	ND	P	C	ND	P	C	ND	P	C
0	28.14	21.15	22.84	36.28	30.72	26.65	32.22	28.48	26.13
300	8.76	2.72	3.79	23.96	4.79	6.32	32.07	6.22	7.51
600	5.59	0.37	0.41	16.94	0.71	1.32	26.44	0.51	1.51

ND without drains, *P* permeability matching, *C* combined matching

matching as compared to combined matching. However, it is interesting to observe that the average excess pore pressure tabulated in Table 4 obtained by permeability matching and combined matching is almost similar. Thus, while the excess pore pressure in between the locations of drains, immediately after construction, in the case of soil with drains is larger compared to the soil without drains, the average excess pore pressure for the soil with drains is slightly lesser compared to the soil without drains. Large excess pore pressure will develop in between the location of drains in order to balance the zero excess pore pressure at the locations of the drains. Also, the excess pore pressure is maximum at the center of embankment and decreases towards the toe of the embankment for the soil without drains at all the depths. For the soil with drains, the excess pore pressure decreases towards the toe at 0.5 m depth whereas, it will not vary much with distance and is more or less uniform beneath the base of the embankment at 2.0 and 4.5 m depths. Further, the average excess pore pressure is almost similar at all the depths at the end of construction for both the cases of with and without drains and it is almost equal to the weight of

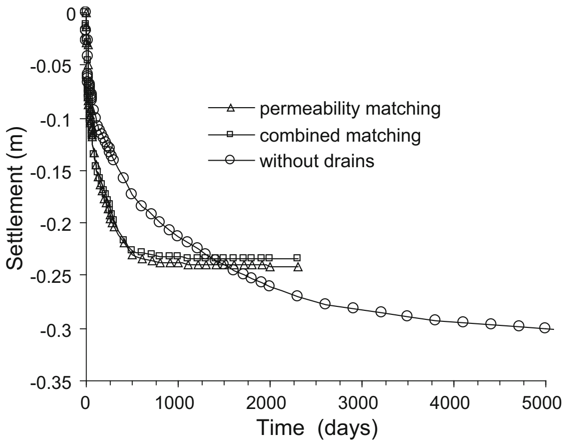

Fig. 6 Variation of settlement with time at point S for the soil with and without drains

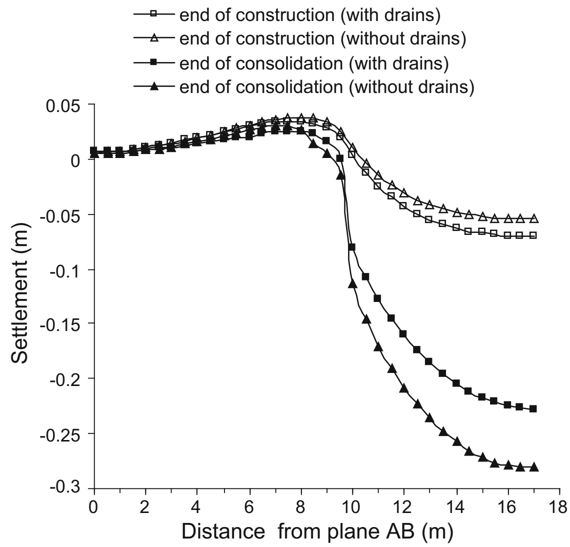

Fig. 7 Settlement at the base of the embankment for the soil with and without drains

the embankment. However 300 and 600 days after construction, the average excess pore pressure is almost similar for all the depths only in the case of soil with drains where as it increases with depth for the soil without vertical drains. This clearly shows that the water drains in horizontal direction in the case of soil with drains since drainage distance is similar at all depths (i.e. 0.5 m) whereas, it drains in vertical direction in the case of soil without drains and due to this the drainage distance increases with depth. Thus, as already shown by many investigators, one of the major advantage of vertical drains is the significant decrease in drainage distance. Due to this, the excess pore pressure dissipates at faster rate after construction in the case of soil with vertical drains and results in lesser excess pore pressure compared to that of the soil without drains as observed from Fig. 5b, c.

To study the change in profile of the embankment during consolidation, variation of settlement with time at point S (beneath the center of the embankment), and variation of settlement along the base of the embankment is investigated. Variation of settlement with time at point S is shown in Fig. 6. As it can be seen, the settlement is more or less stabilized about 600 days after construction for the soil with drains and 3500 days after construction for the soil without drains. Also, the long term settlement (settlement at the end of consolidation) is larger for the soil without drains compared to the soil with drains. In addition, the soil without drains takes much longer time to stabilise compared to the corresponding settlement of soil with drains. When the settlement with time predicted by permeability matching procedure is compared with that of combined matching in Fig. 6, it may be observed that the settlement predicted by both the matching procedures are almost similar. Thus, as against two matching procedures predicting different excess pore pressure profile, the settlement predicted by two matching procedures are almost similar. Similar observations were also drawn by Yildiz (2009) after comparing the results of three dimensional unit cell predicted by three matching procedures proposed by Hird et al. (1992). For the present study the settlement at the end of consolidation predicted by two matching procedures are 0.24 m (predicted by permeability matching) and 0.234 m (predicted by combined matching).

Variation of settlement along the base of embankment at the ground surface is shown in Fig. 7. Settlement profile shows that maximum settlement occurs at the center of the embankment and gradually decreases towards the toe of the embankment. Settlement profile also shows heave that starts from toe, increases first, reaches a maximum value at 1.0 m from toe and then gradually decreases till it reaches zero value at point B. It can also be seen from figure that the settlement at the end of construction at point S for the soil with drains is larger compared to the soil without drains, whereas, the settlement at the end of consolidation is lesser for the soil with drains compared to the soil without drains. This clearly shows

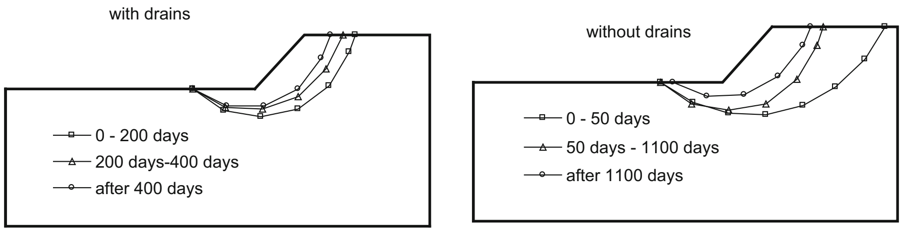

Fig. 8 Critical slip surfaces for an embankment on soft soil with and without drains

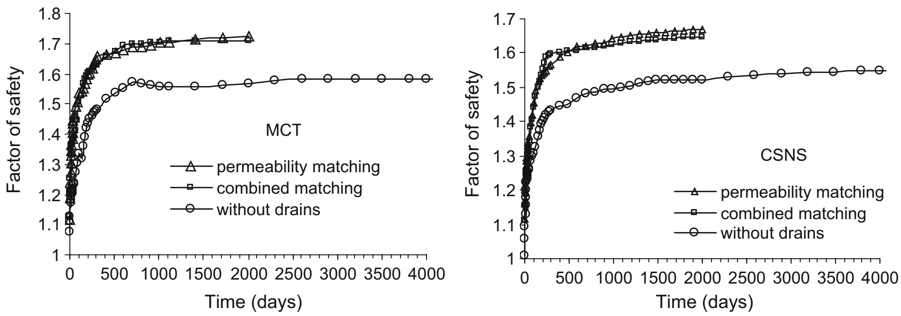

Fig. 9 Variation of factor of safety for an embankment on soft soil with and without drains

that the soil with vertical drains settles lesser after construction compared to the soil with out drains. Moreover, the maximum heave, both at the end of construction as well as at the end of consolidation is also less for the soil with drains compared to the soil without drains, indicating that the soil with drains settle more uniformly compared to the soil without drains. The above observations, clearly shows that the embankment on soft soil with drains is more stable than that of the embankment on soft soil without drains.

Finally, in order to investigate the variation of stability of the embankment with time, the factor of safety is obtained at various time intervals starting from the end of construction till the end of consolidation for the soil with and without drains. Figure 8 shows the critical slip surfaces predicted by the CSNS method for the soil with and without vertical drains. Both the soils with and without drains shows three typical critical slip surfaces from the end of construction till the end of consolidation. The first for consolidation period up to 200 days, the second from 200 to 400 days and the last for consolidation beyond 400 days for the soil with drains whereas for the soil without drains the three critical slip surfaces are at the time intervals from 0 to 50, 50 to 1100 days and after 1100 days. For both the soils, with and without drains, decrease in size of critical surface due to the movement of surface towards the toe of the embankment with progress in consolidation can be observed from Fig. 8. Also, the size of the critical slip surface is larger and extends deeper into the soft soil for the soil without drains compared to the soil with drains. However, in both the cases, the critical slip surface does not extend beyond 1.0 m depth from the ground surface and hence, for the embankment and soft soil considered for the study, variation of excess pore pressure in the soil situated below a depth of 1.0 m may not have much influence on variation of factor of safety of the embankment with time.

Variation of factor of safety with time for the embankment constructed on soft consolidating soil with and without vertical drains is shown in Fig. 9. Following observations can be made from Fig. 9.

1. Factor of safety of embankment on soil without drains obtained by MCT shows four distinct parts. They are:
   (a) Rapid increase in factor of safety (1.07–1.48) with time for the first part from the end of construction up to about 300 days
   (b) Moderate increase in factor of safety (1.48–1.57) with time for the second part from 300 to 600 days

(c) Marginal decrease in factor of safety (1.57–1.55) with time for the third part from 600 to 1400 days
(d) Marginal increase in factor of safety (1.55–1.58) with time again from 1400 days till the end of consolidation

Factor of safety obtained by CSNS method, for the soil without drains however shows only three parts. i.e.:

(a) Rapid increase in factor of safety (1.01–1.44) from the end of construction to about 300 days
(b) Moderate increase in factor of safety (1.44–1.48) with time for the second part from 300 to 600 days
(c) Marginal increase in factor of safety (1.48–1.55) beyond 600 days till the end of consolidation

Thus it may be noted that while the factor of safety increases rapidly up to 300 days, it may either increase or decrease marginally with time beyond 300 days. Rapid increase in factor of safety till 300 days is due to the attribution of express dissipation of excess pore pressure within top 1.0 m depth of foundation soil (as observed from Fig. 4). After 300 days till the end of consolidation, variation of factor of safety with time is influenced by the dissipation of excess pore pressure as well as the change in profile of the embankment caused by the settlement of foundation soil. Factor of safety predicted by MCT decreases marginally from 600 to 1400 days may be due to significant settlement of the foundation soil during this period without much change in excess pore pressure within 1.0 m depth of foundation soil.

2. Thus factor of safety of the embankment on consolidating soil varies not only due to the dissipation of excess pore pressure but also due to the change in profile of the embankment with time as consolidation progresses. Factor of safety of the embankment on soft soil may either increase or decrease with time due to dissipation of excess pore pressure in the soft soil and due to change in profile of embankment.
3. Similar to embankment on soil with out drains, the factor of safety of soil with drains also shows rapid increase in factor of safety with time for initial part of consolidation up to 300 days, gradual increase in factor of safety from 300 to 600 days and little or no increase in factor of safety beyond 600 days till the end of consolidation.
4. When the factor of safety predicted by two matching procedures for the soil with drains are compared, it can be observed that the factor of safety predicted by both the matching procedures are almost similar. However, comparison of MCT with CSNS method to predict the factor of safety indicates that the factor of safety obtained by MCT is slightly larger compared to the factor of safety obtained by CSNS method.
5. When the factor of safety of the embankment on soil with drains is compared with that of the embankment on soil without drains, it can be observed that the factor of safety of the embankment on soil with drains is larger compared to the embankment on soil without drains at the end of construction due to rapid dissipation of excess pore pressure in the soil with drains during construction. Moreover, at the end of consolidation, despite the excess pore pressure dissipates completely in both the soils with and without drains, the factor of safety is not similar for these two cases. The factor of safety at the end of consolidation for the embankment on soil with drains is also larger compared to that of the embankment on soil without drains. This is due to relatively lesser and uniform settlement of foundation soil with drains compared to that of the soil without drains. Thus, provision of drains enhances the factor of safety (stability) of an embankment both at the end of construction and at the end of consolidation. While the factor of safety increases due to rapid dissipation of excess pore pressure at the end of construction, the increase in factor of safety at the end of consolidation is due to uniform and lesser settlement of foundation soil with drains compared to the soil without drains.

4 Summary and Conclusions

A numerical procedure using plane strain finite element method to find effective stresses and a critical slip surface obtained from (1) number of probable slip surfaces and (2) Monte Carlo technique to obtain the factor of safety of an embankment on soft consolidating soil with vertical drains is proposed. In the proposed analysis, the foundation and embankment soil is modeled using Mohr–Coulomb model. The stability of an embankment on soft soil with vertical drains is investigated using the proposed analysis.

It is shown theoretically that the proposed technique is reliable and easy to implement in finite element method to obtain the factor of safety of an embankment on soft consolidating soil when nonlinear constitutive model is used to represent its behavior. Moreover, the factor of safety obtained from the proposed methods matches well with the factor of safety obtained from other optimization techniques such as nonlinear programming. The installation of vertical drains accelerates the consolidation process during and after the construction period and hence the maximum factor of safety can be achieved in lesser time due to vertical drains. Installation of vertical drains also decreases the post construction settlement of the embankment and helps to settle the embankment more uniformly. During initial stage of consolidation, the variation of factor of safety with time is primarily influenced by the rate of dissipation of excess pore pressure with in the depth of soil up to which the critical slip surface passes. The variation of factor of safety at later stage of consolidation is influenced by the change in profile of the embankment caused by the dissipation of excess pore pressure at the other part of the soft soil beneath the critical slip surface. The factor of safety may increase during initial stage of consolidation due to express dissipation of excess pore pressure in soft soil near the ground surface while in the later stage of consolidation, the factor of safety may either increase further or decrease gradually due to the change in profile of the embankment as the consolidation progresses. The installation of vertical drains enhances the factor of safety at the end of construction due to rapid dissipation of excess pore pressure and at the end of consolidation due to relatively lesser and uniform settlement of the foundation soil.

References

Alkasawneh W, Malkawi AIH, Nusairit JH, Albataineh N (2008) A comparative study of various commercially available programs in slope stability analysis. Comput Geotech 35(3):428–435

Baker R (1980) Determination of the critical slip surface in slope stability computations. Int J Numer Anal Methods Geomech 4(4):333–359

Bardet JP, Kapuskar MM (1989) A simplex analysis of slope stability. Comput Geotech 8(1):329–348

Barron RA (1948) Consolidation of fine-grained soils by drain wells. Trans ASCE 113:718–742

Biot MA (1956) General solutions of the equations of elasticity and consolidation for a porous material. J Appl Mech 78:91–96

Bishop AW (1955) The use of the slip circle in the stability analysis of slopes. Geotechnique 5(1):7–17

Borges JL (2004) Three-dimensional analysis of embankments on soft soils incorporating vertical drains by finite element method. Comput Geotech 31(8):665–676

Chai JC, Miura N (1999) Investigation of factors affecting vertical drain behavior. J Geotech Geoenviron Eng ASCE 125(3):216–226

Chai JC, Shen SL, Miura N, Bergado DT (2001) Simple method of modeling PVD-improved subsoil. J Geotech Geoenviron Eng ASCE 127(11):965–972

Chen X, Wu Y, Yu Y, Liu J, Xu XF, Ren J (2014) A two-grid search scheme for large-scale 3-D finite element analyses of slope stability. Comput Geotech 62:203–215

Cheng YM, Li L, Chi SC (2007) Performance studies on six heuristic global optimization methods in the location of critical slip surface. Comput Geotech 34(6):462–484

Donald IB, Giam SK (1988) Application of the nodal displacement method to slope stability analysis. In: Proceedings of the 5th Australia—New Zealand conference on geomechanics, Sydney, pp 456–460

Duncan JM, Dunlop P (1969) Slopes in stiff-fissured clay and shales. J Soil Mech Found ASCE 95(2):467–492

Greco VR (1996) Efficient Monte Carlo technique for locating critical slip surface. J Geotech Eng ASCE 122(7):517–525

Hansbo S (1981) Consolidation of fine-grained soils by prefabricated drains. In: Proceedings of the tenth international conference on soil mechanics and foundation engineering, Stochholm, Swedan, pp 677–682

Hird CC, Pyrah IC, Russell D (1992) Finite element modeling of vertical drains beneath embankments on soft ground. Geotechnique 42(3):499–511

Indraratna B, Balasubramaniam AS, Ratnayake P (1994) Performance of embankment stabilized with vertical drains on soft clay. J Geotech Eng ASCE 120(2):257–273

Janbu N (1957) Earth pressures and bearing capacity calculations by generalized procedure of slices. In: Proceedings of the 4th international conference on soil mechanics and foundation engineering, London, UK, pp 207–212

Kim JY, Lee SR (1997) An improved search strategy for the critical slip surface using finite element stress fields. Comput Geotech 21(4):295–313

Kim J, Salgado R, Yu HS (1999) Limit analysis of soil slopes subjected to pore-water pressures. J Geotech Geoenviron Eng 125(1):49–58

Kim J, Salgado R, Lee J (2002) Stability analysis of complex soil slopes using limit analysis. J Geotech Geoenviron Eng 128(7):546–557

Krishnamoorthy A (2010) Factor of safety of a consolidating slope by finite element method. Indian Geotech J 40(2):116–123

Krishnamoorthy A (2013) Factor of safety of slope on consolidating soil with vertical drains by finite element method. Int J Geotech Eng 7(3):225–231

Malkawi AIH, Hassen WF, Sarma SK (2001) Global search method for locating general slip surface using Monte Carlo techniques. J Geotech Geoenviron Eng ASCE 127(8): 688–698

Matsui T, San KC (1992) Finite element slope stability analysis by shear strength reduction technique. Soils Found 32(1):59–70

Morgenstern NR, Price VE (1965) The analysis of the stability of general slip surfaces. Geotechnique 15(1):79–93

Sarma SK (1979) Stability analysis of embankments and slopes. J Geotech Eng ASCE 105(12):1511–1524

Scott LH, Yamasaki K (1993) Slope failure analysis using local minimum factor of safety approach. J Geotech Eng ASCE 119(12):1974–1987

Sengupta A, Upadhyay A (2009) Locating the critical failure surface in a slope stability analysis by genetic algorithm. J Appl soft Comput 9(1):387–392

Shen SL, Chai JC, Hong ZS, Cai FX (2005) Analysis of field performance of embankments on soft clay deposit with and without PVD—improvement. Geotext Geomembr 23(6): 463–485

Spencer E (1967) A method of analysis of the stability of embankments assuming parallel interslice forces. Geotechnique 17(1):11–26

Tran C, Srokosz P (2010) The idea of PGA stream computations for soil slope stability evaluation. CR Mec 338(9):499–509

Yildiz A (2009) Numerical modeling of vertical drains with advanced constitutive models. Comput Geotech 36(6): 1072–1083

Zienkiewicz OC (1977) The finite element method. McGraw-Hill, London, pp 540–544

Zolfaghari AR, Heath AC, McCombie PF (2005) Simple genetic algorithm search for critical non circular failure surface in slope stability analysis. Comput Geotech 32(3):139–152