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ORIGINAL PAPER

Analysis and Improvement of Fitting Models for Predicting Subsidence Under High-Speed Railway Lines

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Abstract A number of methods for predicting land subsidence and monitoring deformation under high-speed railway tracks exist, and are divided into three categories: layer-wise summation, numerical calculations based on consolidation theory, and curve fitting. One of these, curve fitting, including the hyperbola, expanded hyperbola, three-point fitting and Asaoka methods, is widely used because it is computationally simple and applicable in many situations. In this paper, we analyze the performance of the four classical curve fitting methods using field data and propose a novel approach to estimate land subsidence. The new method integrates three-point fitting, which is computationally simple whilst stringent in terms of correlation restrictions, with the Asaoka method to significantly improve performance in practical applications. Our experimental results indicate the average relative error of the modified method is reduced by 35.3 % than that of three-point fitting, and the mean correlation coefficient remains within acceptable bounds and even was enhanced by 1.48 %, so that this modified method can substantially improve prediction precision.

Keywords High-speed railway · Settlement · Curve fitting · Prediction model

1 Introduction

The methods used to predict land subsidence can be divided into three main types: classical layer-wise summation, numerical calculation based on consolidation theory, and curve fitting. Curve fitting, which makes use of a mathematical formula to acquire a curve that best fits the field data and precisely reflects real world land subsidence, is used in many projects. In this paper, we analyze the performance of four frequently used curve fitting methods (Zhou et al. 2010; Wang 2009): hyperbola, expanded hyperbola, three-point fitting and the Asaoka method.

The research of Chen Shanxiong suggests that expanded hyperbola matching possesses a higher correlation coefficient yet larger relative error than hyperbola fitting, while the Asaoka method may not apply in certain soil conditions despite strong correlation coefficients. The three-point fitting method is superior to the other three methods when both the correlation coefficient and relative error are taken into account (Zhou et al. 2010; Wang 2009; Xiong et al. 2010). This paper first compares the prediction accuracy of all curves obtained by fitting field data

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sets and then analyzes their advantages and disadvantages. We then propose a modified method for predicting subsidence under high-speed railway tracks.

2 Numerical Modeling

Numerical modeling relies on consolidation theory and structural model of soil, which can be used to compute final subsidence and predict the trend. One of the most commonly used method is Finite Element Method (FEM). It segments solid continuum into several finite discrete units and then combines them into aggregations (Wang 2009).

Generally, we base FEM on a certain foundation model to calculate subsidence. The model parameters are obtained through experiments (Wang 2009). This method is rigorous in theory, but it needs a lot of tests to get corresponding parameters. Besides, the unavoidable disturbance of soil samples in sampling will lead to big differences between calculating and actual parameters, and ultimately influence the computing results. Consequently, FEM is not extensively used in projects for its relatively poor performance (Zhang and Guo 2007; Li 2009a, b).

3 Curve Fitting Methods Commonly Used in High-Speed Railway Projects

3.1 Hyperbola Fitting

Hyperbola is a commonly used and straightforward function for measuring subsidence. It assumes that cumulative land subsidence increases over time while the increment of subsidence gradually approaches zero with a hyperbolical curve proportional to time (Wang 2009). Figure 1 depicts the relationship between subsidence, load and time, using the equation:

$$S_t = S_0 + \frac{t - t_0}{a + b(t - t_0)} \tag{1}$$

In the above figure, S_0 is the initial land subsidence value ($t = 0$), t represents the number of days from the beginning of earthworks, S_t is the land subsidence value in t days, and a and b are the parameters of the function.

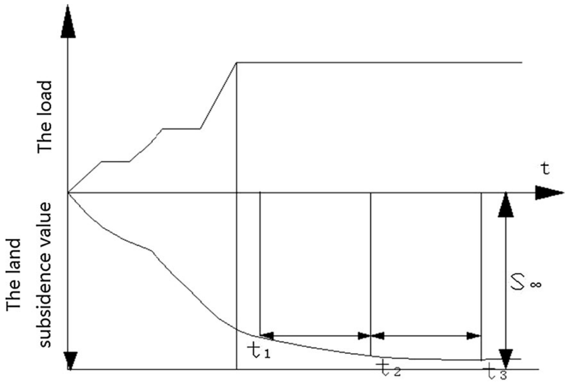

Fig. 1 Subsidence-load-time relations in the hyperbolic method

3.2 Expanded Hyperbola Fitting

Figure 1 indicates that hyperbola cannot make full use of the observational data because all the data before t_0 are abandoned. This results in a lack of understanding of the impact of loads on land subsidence under railway tracks. However, loads can affect the settlement and, therefore, need be taken into consideration. Expanded hyperbola matching is an improved method of hyperbola fitting. It uses the observational data collected before embankment construction by introducing a load factor to extend the time span of the input data (Wang 2009). The expanded hyperbola function is given by:

$$S_t = \frac{1}{a + bt}\xi, \tag{2}$$

where ξ is the load factor, $\xi = \frac{\sigma}{\sigma_{\max}}$, which is determined by σ, the load at time t and $\sigma_{\max}$, the designed maximum load, t represents the number of days from the beginning of the earthwork, and S_t is the land subsidence value at time t.

3.3 The Three-Point Method

The mathematical expression of the three-point method (Wang 2009; Zhang and Guo 2007; Li 2009a, b) is expressed as:

$$S_t = S_\infty (1 - \alpha e^{-\beta t}) + S_i \alpha e^{-\beta t}, \tag{3}$$

where, S_t is land subsidence at any time, S_i and S_∞ represent the instantaneous subsidence and final subsidence, respectively, and α, β are coefficients calculated using three given points on the curve. The coefficient β can be deduced as:

$$\beta = \frac{1}{t_2 - t_1} \ln \frac{S_2 - S_1}{S_3 - S_2}. \tag{4}$$

Generally, α is given as a theoretical value $\frac{8}{\pi^2}$, thus the instantaneous land subsidence value can be written as:

$$S_t = \frac{S_1 - S_\infty (1 - \alpha e^{-\beta t})}{\alpha e^{-\beta t}}. \tag{5}$$

3.4 Asaoka Method

Under the condition of one dimensional consolidation, the initial mathematical expression can be described as:

$$S_t = S_\infty - (S_\infty - S_0) e^{\frac{t - t_0}{a_1}}. \tag{6}$$

Here a_1 is a constant depending on coefficient of consolidation and conditions of soil boundary. Then we can obtain the settlement value at time t, that is:

$$S_t = \int_0^H \varepsilon(t, z) dz \tag{7}$$

where, $\varepsilon(t, z)$ represents the vertical strain and H represents the depth of the soil layers.

4 Analysis of Curve Fitting Models

We compared the applicability of these models for predicting land subsidence using 15 sets of field data. First, we used the models, except for the three-point method, to fit observational data using Matlab and acquire the corresponding parameters. Then we recalculated the models using preliminary data to predict land subsidence values at time t.

The predictions from the three-point method were directly obtained by plotting the points rather than using a calculation. Figures 2, 3, 4, 5 and 6 illustrate the results of the different models in predicting land settlement in five soil profiles.

According to railway track laying guidance in China,[1] the minimum threshold of the correlation coefficient in curve fitting is 0.92, and this figure is the criterion for assessing the accuracy of methods for predicting land subsidence (You 2007; You and Li 2005). The correlation coefficient between $\bar{X}$ and $\bar{Y}$ can be described as:

$$R = \frac{\mathrm{COV}(\bar{X}, \bar{Y})}{\sqrt{D(\bar{X})}\sqrt{D(\bar{Y})}}. \tag{8}$$

Given that $\bar{X}$, $\bar{Y}$ is the sample mean of two arrays, $\mathrm{COV}(\bar{X}, \bar{Y})$ is the covariance of $\bar{X}$, $\bar{Y}$, and $D(\bar{X})$, $D(\bar{Y})$ represents the variance of two arrays, respectively. The precision of the models is shown by the relative error K which is defined as the ratio of the absolute error and measured value (Li 2009a, b), as follows:

$$K = \frac{|S_\alpha - S_\beta|}{S_\alpha}. \tag{9}$$

Here S_α is the measured land subsidence value, and S_β is the calculated value of the fitting formula.

In practice, the prediction of land subsidence under high-speed railway tracks mainly focuses on the final soil settlement. Therefore, we analyzed the subsidence predicted by the various models and compared this with the observational values using their correlation coefficients and relative errors [according to Eqs. (8) and (9), respectively]. The results for the hyperbolic and extended hyperbolic methods are shown in Table 1 and those for the three-point and Asaoka methods in Table 2.

Figures 2, 3, 4, 5 and 6 indicate that all fitting models including hyperbola, expanded hyperbola, three-point and Asaoka methods can roughly reflect the land subsidence trend and approximate the observational data. However, Tables 1 and 2 also show that correlation coefficients of several of the tested data sets do not meet the correlation coefficient criterion. The hyperbola, expanded hyperbola, three-point and Asaoka methods fail to meet the standard in four or five groups out of the total 15 sets of data. The Asaoka method possesses a lower maximum relative error with 4.76 %, compared with the hyperbola (7.45 %), expanded hyperbola (7.52 %) and the three-point methods (5.93 %).

5 A Modified Curve Fitting Model

On the basis of this analysis of model precision, we proposed a novel curve fitting model by integrating the

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[1] Passenger Dedicated Ballastless Railway Track Laying Condition Assessment Techniques Guideline.

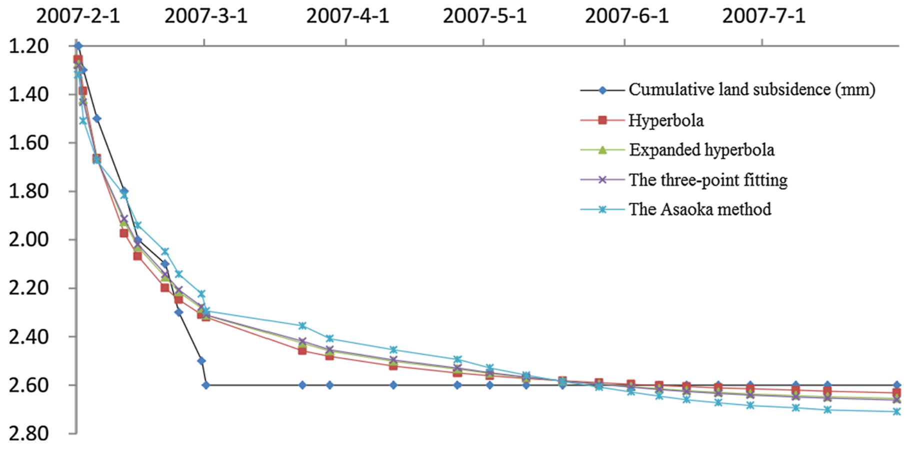

Fig. 2 Settlement predictions in the first profile

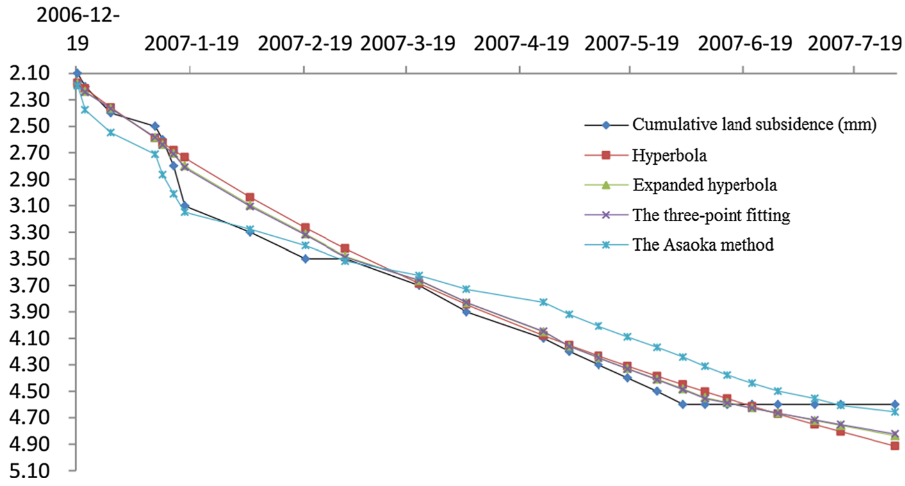

Fig. 3 Settlement predictions in the second profile

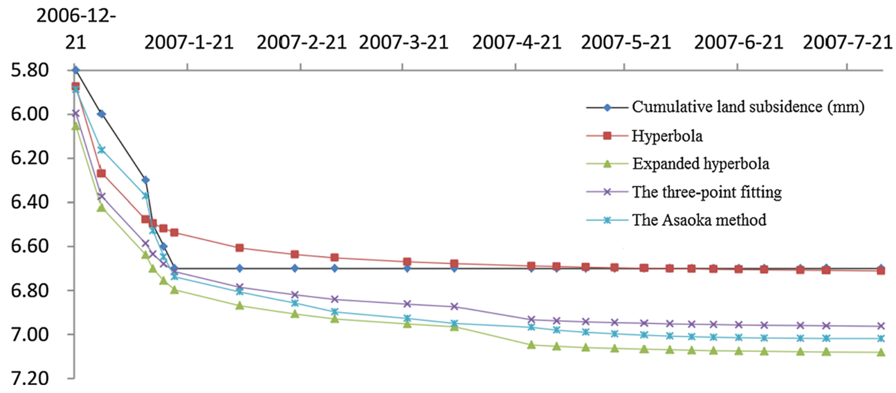

Fig. 4 Settlement predictions in the seventh profile

three-point and Asaoka methods to improve performance in predicting land subsidence. We based our new model on the three-point method, which is both accurate and easy to integrate with other methods.

In the integrated method, select three points (t_1, S_1), (t_2, S_2), (t_3, S_3) in the field data (define $\Delta_t = t_2 - t_1 = t_3 - t_2$), given $t_1 = t_0$. When substituting the three points into (10), the original formula of the

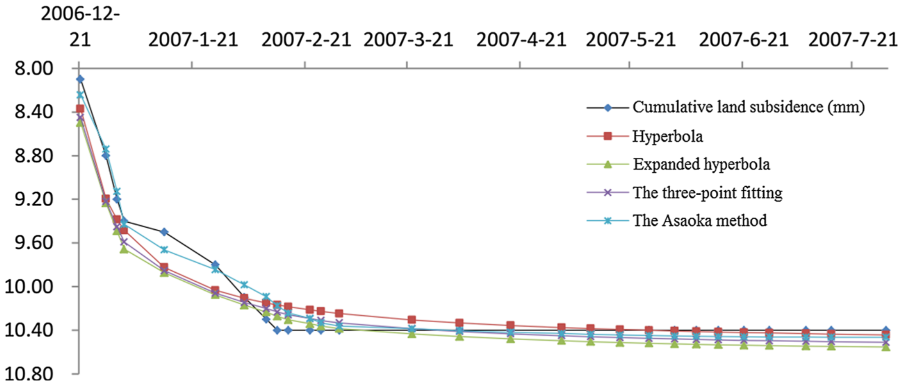

Fig. 5 Settlement predictions in the eighth profile

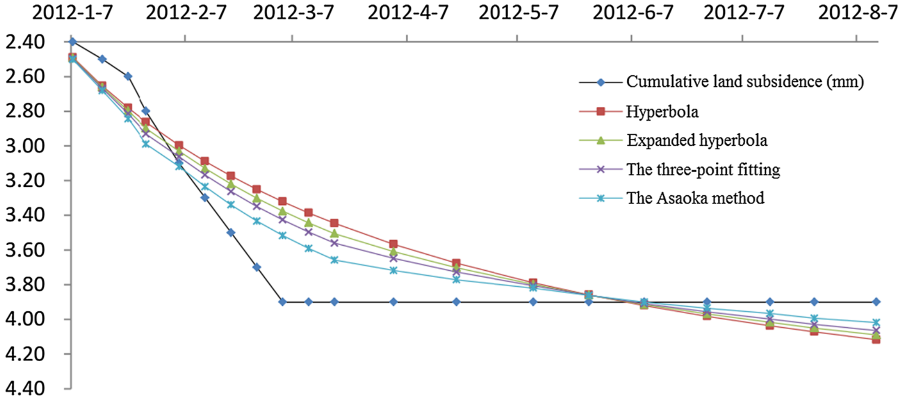

Fig. 6 Settlement predictions in the twelfth profile

Asaoka method, the new method is expressed as (11) and (12), as follows:

$$\frac{s_t - s_\infty}{s_0 - s_\infty} = \ell^{\frac{t-t_0}{\partial}}, \tag{10}$$

$$\frac{s_2 - s_\infty}{s_1 - s_\infty} = \ell^{\frac{t_2-t_1}{\partial}}, \tag{11}$$

$$\frac{s_3 - s_\infty}{s_1 - s_\infty} = \ell^{\frac{t_3-t_1}{\partial}}. \tag{12}$$

Using the above equations, s_∞ and ∂ can be calculated and then substituted into (6) to obtain the final results. To verify the precision of the modified model, the 15 sets of field data were used to test its accuracy and the results compared with the traditional three-point method were described in Table 3. The average relative error is reduced by from 2.95 to 1.91 %. In the meantime, the mean correlation coefficient is over 0.92 and even enhanced by 1.48 %. The new method can therefore significantly improve its performance for predicting land subsidence.

6 Conclusion

In this paper, we carried out a detailed analysis of four frequently used curve fitting models in predicting land subsidence under high-speed railway lines, and compared their applicability and deficiencies. On the basis of the results, we proposed a modified model for predicting land subsidence and verified its performance using field data.

We found the hyperbola, expanded hyperbola, three-point fitting, Asaoka method and the modified method are all capable of providing dependable solutions for land subsidence prediction. And they are much simpler and easier in calculation compared to numerical modeling.

The four classical methods possess both strengths and weaknesses. Hyperbola fitting is simple but inaccurate and applies only under conditions with constant loads. Expanded hyperbola is the best fit to the data in the embankment period, but shows higher errors than the other methods. The three-point method

Table 1 Data analysis of hyperbolic and extended hyperbolic methods

Profile number	Field data	Hyperbola			Expanded hyperbola		
		Predicting value	Correlation coefficient	Relative error	Predicting value	Correlation coefficient	Relative error
1	2.60	2.63	0.96	1.15	2.65	0.97	2.11
2	4.60	4.91	0.86	6.85	4.84	0.89	5.13
3	3.30	3.50	0.87	5.93	3.45	0.92	4.48
4	3.90	4.09	0.93	4.89	4.05	0.93	3.75
5	4.10	4.22	0.93	2.90	4.19	0.94	2.19
6	7.30	7.42	0.94	1.68	7.40	0.94	1.41
7	6.70	6.71	0.95	0.17	7.08	0.86	5.69
8	10.40	10.44	0.96	0.41	10.55	0.93	1.48
9	9.60	9.69	0.98	0.96	9.72	0.94	1.23
10	8.90	8.93	0.97	0.37	8.93	0.97	0.33
11	2.80	2.89	0.93	3.13	2.90	0.91	3.43
12	3.90	4.12	0.86	5.58	4.09	0.89	4.87
13	6.60	6.85	0.92	3.75	6.86	0.93	3.87
14	5.40	5.80	0.76	7.45	5.81	0.74	7.52
15	6.20	6.41	0.93	3.39	6.39	0.93	3.13

Table 2 Data analysis of the three-point and Asaoka methods

Profile number	Field data	The three-point fitting			The Asaoka method		
		Predicting value	Correlation coefficient	Relative error (%)	Predicting value	Correlation coefficient	Relative error (%)
1	2.60	2.66	0.95	2.35	2.71	0.96	4.21
2	4.60	4.82	0.92	4.85	4.66	0.95	1.22
3	3.30	3.39	0.95	2.69	3.38	0.96	2.46
4	3.90	4.07	0.88	4.45	4.02	0.89	3.02
5	4.10	4.20	0.93	2.52	4.15	0.92	1.30
6	7.30	7.73	0.84	5.93	7.34	0.94	0.55
7	6.70	6.96	0.89	3.92	7.02	0.83	4.76
8	10.40	10.51	0.96	1.07	10.46	0.98	0.61
9	9.60	9.73	0.95	1.35	9.79	0.94	1.94
10	8.90	8.93	0.98	0.35	8.92	0.99	0.25
11	2.80	2.91	0.90	3.77	2.92	0.87	4.12
12	3.90	4.06	0.91	4.21	4.02	0.92	3.04
13	6.60	6.87	0.93	4.12	6.88	0.91	4.27
14	5.40	5.41	0.97	0.25	5.57	0.92	3.12
15	6.20	6.35	0.95	2.47	6.34	0.96	2.21

is easy to calculate but requires high correlation. The Asaoka method is also relatively simple to calculate but is not applicable in unsaturated soil layers. Therefore, we should choose proper methods in unknown in situ soil and ballast conditions.

The new land subsidence prediction model, which integrates the three-point and the Asaoka methods, requires complex calculation procedures, but nevertheless can achieve significant improvement in correlation coefficients and relative errors. We believe it

Table 3 Results comparisons of three-point and modified methods

Profile number	Field data	The three-point fitting			Modified method		
		Predicting value	Correlation coefficient	Relative error (%)	Predicting value	Correlation coefficient	Relative error (%)
1	2.60	2.66	0.95	2.35	2.55	0.95	1.92
2	4.60	4.82	0.92	4.85	4.67	0.96	1.52
3	3.30	3.39	0.95	2.69	3.24	0.94	1.82
4	3.90	4.07	0.88	4.45	4.12	0.92	5.64
5	4.10	4.20	0.93	2.52	4.05	0.91	1.22
6	7.30	7.73	0.84	5.93	7.38	0.93	1.10
7	6.70	6.96	0.89	3.92	6.91	0.93	3.13
8	10.40	10.51	0.96	1.07	10.32	0.99	0.77
9	9.60	9.73	0.95	1.35	9.44	0.94	1.67
10	8.90	8.93	0.98	0.35	8.98	0.95	0.90
11	2.80	2.91	0.90	3.77	2.85	0.94	1.79
12	3.90	4.06	0.91	4.21	3.98	0.94	2.05
13	6.60	6.87	0.93	4.12	6.79	0.95	2.88
14	5.40	5.41	0.97	0.25	5.47	0.93	1.30
15	6.20	6.35	0.95	2.47	6.14	0.94	0.97

will be highly applicable in the operational phase of high-speed railway lines.

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