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TECHNICAL NOTE

Seismic Stability of Gravity Retaining Walls Under Combined Horizontal and Vertical Accelerations

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Abstract The seismic sliding limit condition of gravity retaining walls with cohesionless soil backfill is investigated and analytical solutions for the critical acceleration coefficient are provided in this paper. The solutions have been derived in the framework of the upper bound theorem of limit analysis. The retaining walls and the backfill soil are taken as a whole system and the combined action of horizontal and vertical accelerations are considered. For retaining walls with horizontal backfill, the effects of the inclination of the wall internal face and of the soil–wall friction were investigated. The effects of vertical component of the seismic acceleration on the yield horizontal acceleration coefficient were discussed in detailed. Based on a limited parametric study, it is shown that both the roughness and inclination of the internal wall face have some effects to the seismic stability of the wall–soil system. And under some conditions, the effects of vertical acceleration are considerable large and can't be neglected.

Keywords Limit analysis · Gravity retaining wall · Vertical acceleration

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1 Introduction

Earth retaining walls are very common and important geotechnical engineering structures, especially in connection with the protection of transportation facilities and/or nearby structures. Seismic analysis and design of earth retaining walls is a difficult problem, which traditionally requires the determination of the dynamic soil pressures induced by the soil seismic motion on the wall. To compute the active earth thrust acting against retaining walls in seismic conditions, the Mononobe–Okabe method or its extensions are most widely used (Okabe 1924; Mononobe and Matsuo 1929; Richards and Elms 1979; Nadim and Whitman 1983; Zeng and Steedman 2000). The Mononobe–Okabe solution treats earthquake loads as pseudo-static, generated by uniform acceleration in the backfill. The retained soil is considered as perfectly plastic material, which fails along a planar surface, thereby exerting a limit thrust on the wall. The method has prevailed mainly due to its simplicity and the familiarity of engineers with the Coulomb method. However, the Mononobe–Okabe method presents a basic shortcoming: the solution is based on the limit equilibrium of the soil wedge without taking into account the presence of the wall. So Caltabiano et al. (2000) suggested a new solution based on the pseudo-static equilibrium of the soil–wall system and applied it to seismic stability analysis of retaining walls with surcharge.

Limit analysis method can also be used to avoid this shortcoming of Mononobe–Okabe method.

Li et al. (2010) used the category of upper bound theorem of limit analysis to consider the seismic stability of soil–wall system. A method based on limit analysis for calculations of yield acceleration and seismic displacements of multi-block structures (including retaining wall) was suggested by Michalowski (2007). The upper bound approach of limit analysis was also used by Chen (1975), Škrabl and Macuh (2005), Yang (2007) and Ausilio et al. (2000) to consider the problem of seismic retaining structures.

The fact that vertical acceleration has a significant effect on the seismic behavior of retaining walls or reinforced slopes had been pointed out by many researchers (Richards and Elms 1979; Ling and Leshchinsky 1998a; Ling et al. 1997). Ling and Leshchinsky (1998b) investigated the effects of vertical acceleration on the seismic design of geosynthetic–reinforced soil structures. Ingles et al. (2006) conducted the effects of the vertical component of ground shaking on earthquake-induced landslide displacements using generalized Newmark analysis and infinite slope model. Sawicki et al. (2007) proposed a method enabling assessment of seismic-induced movements of gravity block.

Chen and Liu (1990) pointed out that in seismic conditions, active earth pressure coefficients obtained using the M–O method and the limit analysis theorems are in a close agreement since the log-spiral slip surfaces obtained by limit analysis are almost planar. Approximately planar slip surfaces were also observed in dynamic model tests carried out on shaking tables (Watanabe et al. 2003) and in centrifuge (Nakamura 2006).

In this study the whole soil–wall system is investigated using a two-wedge approach and limit analysis to highlight the influence of the wall and of the base friction on the plastic mechanism. The upper bound approach of limit analysis is applied to calculate the yield acceleration of the system undergoing direct sliding failure. Both the horizontal and vertical component of ground motion is considered in the analysis. The computed results of yield acceleration are compared with methods based on limit equilibrium analysis. The closed-form solution may be found useful by engineers in the displacement-based seismic design of retaining walls. In all the analyses the backfill soil is assumed to be homogeneous, dry and cohesionless.

2 Method of Analysis

The kinematic theorem of limit analysis is applied here to analyse the stability of walls retaining backfill soil under seismic loading. This theorem state that a slope will collapse if the rate of work done by external loads and body forces exceeds the energy dissipation rate for any assumed kinematically admissible failure mechanism. Applicability of the theorem requires that soil will be deformed plastically according to the normality rule and the convexity of the soil yield condition.

The forces acting in the soil–wall system considering both horizontal and vertical components of the seismic acceleration are shown in Fig. 1a. The rate of work done by the gravity forces is the vertical component of the velocities multiplied by the weight of the wedges

$$\dot{W}_{\mathrm{g}} = W_{\mathrm{s}} V_0 \sin(\alpha - \varphi) - W_{\mathrm{w}} V_1 \sin \delta_{\mathrm{b}} \tag{1}$$

where α is the angle that planar failure surface makes with the horizontal, φ is angle of internal friction of backfill soil, W_{s} and W_{w} indicate the weight of soil wedge and retaining wall, respectively, V_0 and V_1 are velocity jumps along the slip plane and the base, and δ_{b} is friction angle between retaining wall and the base. Once the system is subjected to seismic loading, the rate of the inertial force needs to be accounted for in the energy balance equation. The rate of work due to horizontal acceleration takes the form (Li et al. 2010)

$$\dot{W}_{\mathrm{sh}} = k_{\mathrm{h}} W_{\mathrm{s}} V_0 \cos(\alpha - \varphi) + k_{\mathrm{h}} W_{\mathrm{w}} V_1 \cos \delta_{\mathrm{b}} \tag{2}$$

where k_{h} is seismic coefficient representing horizontal acceleration as a fraction of the gravity acceleration. And the rate of work due to vertical acceleration is

$$\dot{W}_{\mathrm{sv}} = k_{\mathrm{v}} W_{\mathrm{w}} V_1 \sin \delta_{\mathrm{b}} - k_{\mathrm{v}} W_{\mathrm{s}} V_0 \sin(\alpha - \varphi) \tag{3}$$

where k_{v} is seismic coefficient representing vertical acceleration as a fraction of the gravity acceleration. Herein, the ratio of vertical to horizontal seismic coefficients is expressed by a dimensionless parameter λ (i.e., $\lambda = k_{\mathrm{v}}/k_{\mathrm{h}}$). It should be noted that the value of λ can be positive (which means k_{v} is considered to act upwards) or negative (which means k_{v} is considered to act downwards). Substituting the expression of λ, the Eq. (3) can be rewritten as

$$\dot{W}_{\mathrm{sv}} = \lambda k_{\mathrm{h}} W_{\mathrm{w}} V_1 \sin \delta_{\mathrm{b}} - \lambda k_{\mathrm{h}} W_{\mathrm{s}} V_0 \sin(\alpha - \varphi) \tag{4}$$

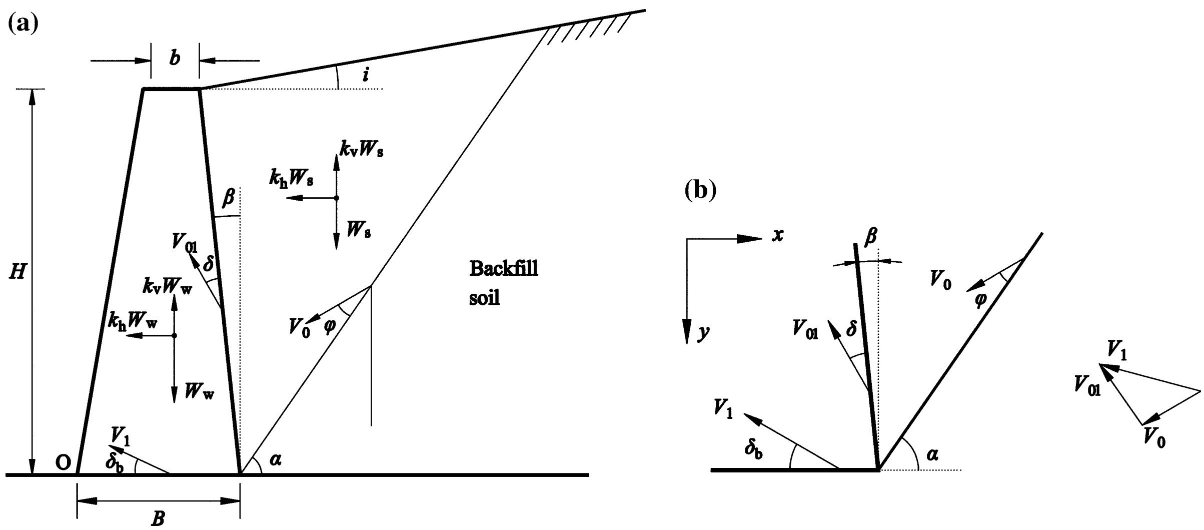

Fig. 1 **a** Translational failure mechanism of retaining wall with backfill soil. **b** Velocity compatibility between adjacent blocks and velocity hodograph

Since there is no cohesion along the slip surface and the retaining wall base, energy dissipation is zero, the energy balance equation yields

$$\dot{W}_g + \dot{W}_{sh} + \dot{W}_{sv} = 0 \tag{5}$$

Submitting (1)–(4) into (5) gives

$$\begin{aligned}(1-\lambda k_y)W_sV_0\sin(\alpha-\varphi) + k_yW_sV_0\cos(\alpha-\varphi) \\ -(1-\lambda k_y)W_wV_1\sin\delta_b + k_yW_wV_1\cos\delta_b = 0\end{aligned} \tag{6}$$

where k_y is the yield acceleration coefficient of the failure mechanism respect to angle α.

For a kinematically admissible failure mechanism, some relationship should be satisfied between the velocity V_0 and the velocity V_1. Let us observe the two adjoining wedges as shown in Fig. 1b. The left and right wedges move with the absolute velocities V_1 and V_0 which incline at angles δ_b and φ to their bases, respectively. The relative velocity of the left wedge with respect to the right one along the interface is represented as V_{01}, which inclines at an angle δ. To allow the velocities assigned to the wedge failure mechanism to be kinematically compatible, the two adjoining wedges must not move to cause overlap or indentation. This implies that the velocity hodograph must be closed, i.e.,

$$V_0 + V_{01} = V_1 \tag{7}$$

From Eq. (7) and Fig. 1b, we obtain:

$$V_0 = V_1\frac{\cos(\delta_b+\delta+\beta)}{\cos(\varphi+\delta+\beta-\alpha)} \tag{8}$$

where δ is friction angle between retaining wall and backfill soil.

Substituting (8) into (6) and rearranging the terms leads to the following expression

$$k_y = \frac{\Gamma\sin\delta_b - \xi\sin(\alpha-\varphi)\Psi}{\Gamma(\cos\delta_b+\lambda\sin\delta_b)+\xi\Psi[\cos(\alpha-\varphi)-\lambda\sin(\alpha-\varphi)]} \tag{9}$$

where for convenience the symbols

$$\begin{aligned}\xi &= \frac{\cos(\delta_b+\delta+\beta)}{\cos(\varphi+\delta+\beta-\alpha)} \qquad \Gamma = \frac{2W_w}{\gamma H^2} \\ \Psi &= \frac{\cos(\beta-i)\cos(\alpha-\beta)}{\cos^2\beta\sin(\alpha-i)} = \frac{2W_s}{\gamma H^2}\end{aligned} \tag{10}$$

are adopted, with Γ denoting the dimensionless weight of the wall; typical values of Γ are in the range 0.6–1.2. The critical seismic coefficient is obtained by minimising k_y, with respect to α. This means taking the first derivatives of k_y and equating them to zero, i.e., $(\partial k_y/\partial\alpha) = 0$. Solving this equation and substituting the value of α, the least upper bound value of yield acceleration factor is calculated. This critical value of k_y is indicated in the following text as k_c.

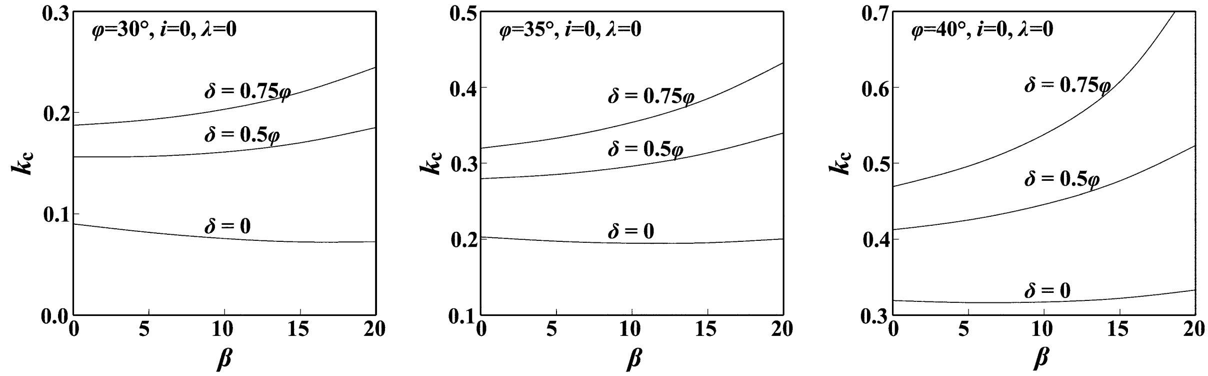

Fig. 2 Effects of of β and δ on lateral acceleration coefficient k_c for different values of φ

3 Results and Discussions

3.1 Effect of β and δ

From the point of a theoretical analysis, the internal face of the wall can be vertical ($\beta = 0$) or inclined at a positive or negative angle to the vertical ($\beta \neq 0$). However, negative batters is almost never implemented in practice, whereas larger batters (more than 10°) may be more relevant. So a range of 0°–20° is adopted in the plot showing the effect of β on k_c. Further, in most cases the wall is not perfectly smooth and when failure develops, wall sliding shear stresses are mobilized at the contact surface between the soil wedge and the wall. This frictional interaction has been considered by introducing an angle δ to the velocity interval between the wall and soil wedge.

Figure 2 shows the variation of k_c versus β for the dimensionless weight $\Gamma = 0.8$ and for different values of φ and the ratio δ/φ. Generally, when φ increases, corresponding to more stable systems, higher values of k_c are needed to bring the soil–wall system to limit condition. Further a remarkable dependence on δ is also observed. For the case of $\varphi = 30°$ and $i = \lambda = 0$, when $\delta = 1\varphi/2$ is adopted k_c is about 0.16 while when $\delta = 0$ (the internal face of the wall is smooth) the wall can only sustain a seismic horizontal acceleration of about 0.09 g. The effects of β are also very impressive except the case that the wall is smooth ($\delta = 0$). When the wall faces are rough ($\delta = 1\varphi/2$ and φ), the effect of β is very remarkable and the system is more stable with larger angles of β. Furthermore, it is shows that when the value of φ and δ is considerable large the effect of β on k_c is apparently more remarkable than when angles φ and δ take smaller values. These results indicate that in the design of retaining walls, the wall geometry should be determined according to the smoothness of the internal face especially when the internal friction angle of backfill soil is high.

3.2 Effect of k_v

The vertical component of the seismic acceleration affects the limit condition of the soil–wall system. Caltabiano et al. (2012) presented a method based on seismic limit equilibrium analysis of sliding retaining walls under different surcharge conditions. The effect of the vertical component of the seismic acceleration on the yield horizontal acceleration coefficient was investigated. Under the case of retaining wall with vertical and smooth face ($\beta = \delta = 0$), they came to the conclusion that the effect of the vertical component of the seismic acceleration on the limit equilibrium condition is small and can be neglected. Herein, the vertical smooth wall and rough wall with inclined internal face are investigated.

Figure 3 shows the variation of k_c versus λ for the dimensionless weight $\Gamma = 0.8$ and for different values of φ, β and the ratio δ/φ. This ratio is assumed to range from −0.5 to 0.5 as suggested by seismic codes (e.g., Eurocode8 [37]). It can be seen from the figure that, for the case of $\beta = \delta = 0$, as expected, the effect of k_v on k_c is rather small. But it cannot be neglected that the varying ranges of k_c shows a remarkable dependence on φ and that angles β and δ also have some effects on it. It is observed from Fig. 3 that when the values of φ, β and δ are considerably large the effect of k_v on k_c is rather large and cannot be neglected. Take the case of

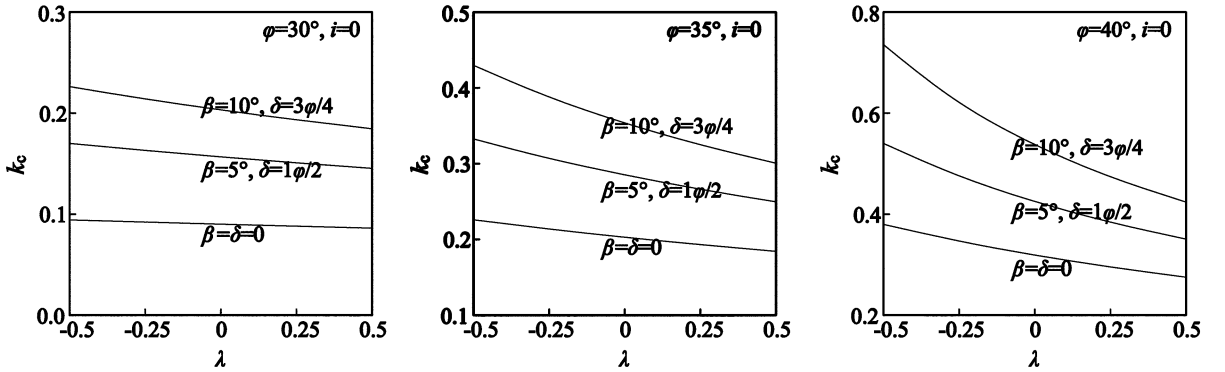

Fig. 3 Effects of vertical acceleration on k_c under different values of φ, β and the ratio δ/φ

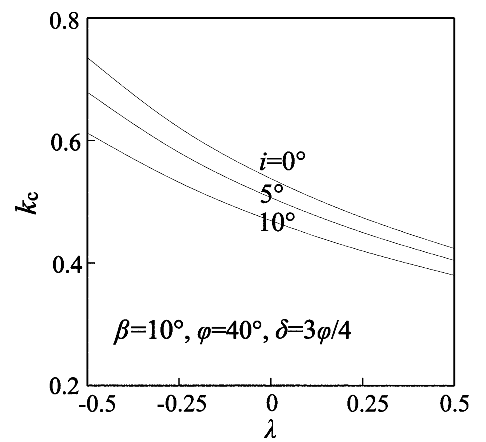

Fig. 4 Effects of vertical acceleration on k_c under different values of backfill soil slope i

$\varphi = 35°$, $\delta = \varphi$ and $\beta = 10°$ as an example: starting from $\lambda = 0$, positive increasing values of λ produce a reduction of k_c of 15 % when $\lambda = 0.5$. And this reduction get to 21.2 % for the case with $\varphi = 40°$, $\delta = 3\varphi/4$ and $\beta = 10°$. Conversely negative decreasing values of λ produce an increase of k_c of 18 and 27 % when $\lambda = -0.5$ for the above two cases, respectively. These results also show that positive (upward, as assumed in Fig. 1) vertical accelerations are disadvantage for the stable of wall–soil system, while negative (downward) vertical accelerations improve the stability condition.

3.3 Effect of i

The slope of backfill soil will have some effects on the overall stability of the system under both static and seismic conditions. Figure 4 shows the variation of k_c versus λ for the dimensionless weight $\Gamma = 0.8$, $\varphi = 40°$, $\delta = 3\varphi/4$, $\beta = 10°$ and for different values of i. As expected, a sloped backfill soil has some destructive effects on the stability of the wall–soil system which can be reflected from the position of k_c curves for different angles of i. It can also be seen from Fig. 4 that for different slope conditions of backfill soil, the effects of vertical accelerations are in same patterns.

4 Conclusions

This work attempts to develop a method to analyze the seismic stability of gravity retaining walls with backfill soil under the category of upper bound theorem of limit analysis. Considering the action of both horizontal and vertical ground accelerations, closed-form solutions for the critical acceleration factor are derived based on the pseudo-static analysis of soil–wall system.

The study showed that both the roughness and inclination of the internal wall face have some effects to the seismic stability of the wall–soil system and the inclination of the wall should be determined according to the smoothness of the wall face. The effects of vertical acceleration on the yield horizontal acceleration coefficient can be different with different conditions of φ, β and δ. When the values of φ, β and δ are relative small the effects of vertical acceleration are rather small and may be neglectable. While the effects of vertical acceleration are considerable large and can't be neglected if values of φ, β and δ are large. It also found that for different slope conditions of backfill soil, the effects of vertical accelerations are in same patterns. Though the preceding simplified analysis was carried out under the conservative assumption, hardly verified in real cases, that peak horizontal acceleration and peak vertical acceleration occur simultaneously and are in phase it is expected that this limit analysis method can be useful because of its simplicity and reasonability.

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