STATE-OF-THE-ART REVIEW

# The Shaft Capacity of Displacement Piles in Clay: A State of the Art Review

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Abstract The rapid expansion of the offshore wind sector, coupled with increasing demand for high rise structures, has placed renewed demand on the driven piling market. In light of this industry growth, this paper reviews the evolution of design approaches for calculating the shaft capacity of displacement piles installed in cohesive soils. The transition from traditional total stress design towards effective stress methods is described. Complex stress-strain changes occur during pile installation, equalisation and load testing and as a consequence, the selection of parameters for use in conventional earth-pressure type effective stress approaches is not straightforward. These problems have led to the development of empirical correlations between shaft resistance and in situ tests, such as the cone penetration tests. However, many of these approaches are limited because they were developed for specific geological conditions. Significant insight into pile behaviour has been obtained from recent model pile tests, which included reliable measurements of radial effective stresses. These tests have allowed factors such as

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School of Architecture, Landscape and Civil Engineering, University College Dublin (UCD), Belfield, Dublin, Ireland e-mail: paul.doherty@ucd.ie friction fatigue and interface friction to be included explicitly in design methods. Whilst analytical methods have been developed to investigate pile response, these techniques cannot yet fully describe the complete stress–strain history experienced by driven piles. The use of analytical methods in examining features of pile behaviour, such as the development of pore pressure during installation and the effects of pile end geometry on pile capacity, is discussed.

**Keywords** Foundation · Piling · Clay · Shear strength · Field testing

#### List of Symbols

Α	Pile cross sectional area.
API	American Petroleum Institute
CEM	Cavity expansion method
COV	Coefficient of variation
CPT	Cone penetration test
D	Pile diameter
Ε	Youngs' modulus of pile material
$F_{\rm tip}$	Correction factor for open-ended piles in the
-	NGI approach
G	Shear modulus
ICP	Imperial college pile
IFR	Incremental filling ratio
<i>I</i> <sub>r</sub>	Rigidity index
Κ	Pile compressibility
$K_{\mathrm{f}}$	Lateral stress coefficient at failure
$K_0$	Lateral stress coefficient at rest

L	Pile length
L/D	Slenderness ratio
LDPT	Large diameter pile tests
$L_{\rm p}$	Plug length
N <sub>diss</sub>	Number of dissipation load cycles
N	Number of shearing load cycles
$N_{\rm und}$	Number of undrained load cycles
$N_{\rm kt}$	Empirical cone strength factor
OCR	Over-consolidation ratio
PI	Plasticity index
PLR	Plug length ratio
$Q_{\rm actual}$	Capacity of a flexible pile
$Q_{c}$	Calculated capacity
$Q_{\rm m}$	Measured capacity
$Q_{ m rigid}$	Capacity of an infinitely stiff pile
R	Radius
R <sub>eq</sub>	Equivalent radius
R <sub>i</sub>	Internal radius
$R_{\rm f}$	Reduction factor for progressive failure
SPM	Strain path method
SPT	Standard penetration test
SSPM	Shallow strain path method
$S_{\mathrm{t}}$	Soil sensitivity
YSR	Yield stress ratio
$f_{\rm s}$	Cone sleeve friction
h	Distance from the pile tip
$q_{ m c}$	Cone tip resistance
$q_{\mathrm{T}}$	Total cone tip resistance
$q_{\rm cnet}$	Net cone resistance
r	Radial distance from pile shaft
s <sub>u</sub>	Undrained shear strength
t	Pile wall thickness
и	Pore pressure
$u_0$	Hydrostatic pore pressure
$u_2$	Pore pressure measured on cone shoulder
z	Depth
$\Delta L_{\rm p}$	Incremental change in plug length
$\Delta L$	Incremental change in pile length
$\Delta u$	Excess pore pressure
$\Delta w_{\rm res}$	Post-peak displacement required to reach
	residual strength
α	Total stress alpha coefficient
$\alpha_{CPT}$	Empirical total cone factor
β	Effective stress beta coefficient
δ	Interface friction angle
$\delta_{\rm r}$	Radial displacement
ξ	Degree of strain softening
λ	Mean stress lambda coefficient

 $\tau_{av}$ Average shaft shear stress Local shaft shear stress  $\tau_{\rm f}$ Peak shaft shear stress  $\tau_{\rm peak}$ Residual shaft shear stress  $\tau_{\rm res}$  $\sigma_{\rm ri}$ Total radial stress during installation Radial effective stress during installation  $\sigma'_{\rm ri}$  $\sigma'_{\rm v0}$ In situ vertical effective stress  $\sigma'_{\rm rc}$ Equalised radial effective stress  $\sigma'_{
m rf}$ Radial effective stress at failure Constant volume friction angle  $\phi'_{\rm cv}$ 

## 1 Introduction

Considerable expansion of the piling industry in recent years has been driven by the development of high-rise structures and the increased exploitation of offshore energy resources. These developments bring new challenges for pile designers as higher capacities and deeper pile penetrations are required in a range of soil types; For example, from soft normally-consolidated clay (Katzenbach et al. 2000) to very stiff overconsolidated glacial till in the North sea, where undrained strength  $(s_u)$  values in excess of 600 kPa are encountered (Overy 2007). Many of the current state of the art design approaches are based on empirical correlations established from databases, mainly populated by onshore piles with relatively small diameters. Therefore, application of these methods to vastly different soil conditions and pile geometries is questionable. Whilst maintained load tests are commonly used to confirm the axial capacity of onshore piles, they can be prohibitively expensive in the offshore environment, particularly when a small number of piles are used to support the structure (for example in the case of an oil or gas platform). Accurate predictive models are therefore essential for economic, efficient and safe foundation design.

This paper reviews the evolution of design practice for estimating the shaft resistance of piles; from the total stress (alpha) approaches, the first effective stress (Beta) methods, analytical approaches such as the Strain Path Method (SPM) and Cavity Expansion Method (CEM) to recent work linking the shaft resistance developed by displacement piles installed in clay to the results of in situ tests such as the Cone Penetration Test (CPT).

#### **2** The Total Stress Approach

The total stress method remains the most popular approach used in design practice to estimate the shaft capacity of piles in clay. The basic form of the approach links the average shaft resistance  $(\tau_{av})$  to the average undrained strength  $(s_u)$  of the clay along the pile shaft, through an adhesion factor,  $\alpha$ :

$$\tau_{av} = \alpha s_u \tag{1}$$

Tomlinson (1957) recognized that the relationship between  $\tau_{av}$  and  $s_u$  was non-linear, with back-figured  $\alpha$  values reducing as the undrained shear strength of the soil increased (see Fig. 1). Many of these initial correlations were developed from static load tests on un-instrumented piles driven through multiple soil strata with variable undrained strengths. This resulted in considerable uncertainty in the estimated alpha coefficient for a given site (Chow 1997).

Many of the improvements to total stress design methods from the 1960s onwards, were driven by the rapidly developing offshore oil industry and were incorporated in design guidelines produced by the American Petroleum Institute (API) from 1969. These guidelines provide a useful framework for considering the evolution of total stress methods. McClelland (1974) noted that many of the lowest  $\alpha$ values present in existing databases were mobilized in over-consolidated soils. They suggested that in deep deposits of normally consolidated clay, such as those evident in the Gulf of Mexico, the soil may be stiff as a consequence of the large overburden pressures and in these situations  $\alpha$  values approaching unity could be mobilised on piles founded in soils



Fig. 1 Early alpha correlations developed from load test databases

of high undrained strength. An alternative approach suggested by McClelland (1974) was to consider the stress history or over-consolidation ratio (OCR) of the deposits within the empirical alpha methods. From the late 1960s McClelland Engineers accounted for the influence of OCR indirectly by assuming that the average shaft resistance was equal to the  $s_u$  value (subject to a maximum value of  $\tau_{av} \approx 48$  kPa), for pile penetrations less than 30 m in Normally Consolidated (NC) deposits. In recognition, the first edition of the API design guidelines for offshore piles API-RP2A (1969) suggested that the local shaft resistance ( $\tau_f$ ) mobilised in NC clay be linked directly to the initial in situ vertical effective stress ( $\sigma'_{v0}$ ):

$$\tau_{\rm f} = 0.33 \cdot \sigma_{\rm v0}^{\prime} \tag{2}$$

A natural step toward extending the impact of stress history to over-consolidated (OC) soils was to consider the effect of the in situ mean stress on the undrained strength. Given the inherent difficulty in estimating the horizontal effective stress of OC soils Vijayvergiya and Focht (1972) introduced the lambda  $(\lambda)$  coefficient:

$$\lambda = \frac{\tau_{\rm f}}{\left(2s_{\rm u} + \sigma_{\rm v0}'\right)}\tag{3}$$

Lambda coefficients were back-figured from fortyseven load tests performed on pipe piles with diameters ranging from 155 to 762 mm. The inferred  $\lambda$  values, shown in Fig. 2, were the first design approach to suggest a length effect, with  $\lambda$  decreasing strongly as the pile penetration depth increased.

Such length effects were in keeping with field data reported by Cooke et al. (1979) who presented measurements of the shear stress mobilised during the installation of a 168 mm diameter, closed-ended steel pile in London Clay. The distribution of shear stress, as the pile was driven to 4.5 m below ground level (bgl), is shown in Fig. 3. It is clear that the shear stress mobilised at any depth (z) reduced as the pile penetration depth, or slenderness ratio (L/D) increased. Heerema (1980) introduced the term friction fatigue to describe this facet of behaviour which is not directly incorporated in either Eq. 1 or 2.

Kraft et al. (1981) and Randolph (1983) suggested that progressive failure, which occurs in strain softening soil, was a possible mechanism controlling friction fatigue. The onset of progressive failure from the peak ( $\tau_{peak}$ ) to the residual ( $\tau_{res}$ ) shaft resistance is



Fig. 2 Lambda Coefficient as a function of Pile Length (reproduced from Vijayvergiya and Focht 1972)



Fig. 3 Friction fatigue in London Clay (after Cooke et al. 1979)

shown in Fig. 4. On the basis that strain-softening effects would be greatest on long compressible piles, Randolph (1983) proposed a reduction factor  $(R_{\rm f} = Q_{\rm actual}/Q_{\rm rigid})$  which compares the mobilized



Fig. 4 Progressive Failure in a strain softening soil (modified from Kraft 1981)

resistance  $(Q_{actual})$  to that of an incompressible pile  $(Q_{rigid})$ . The reduction factor incorporates parameters which describe the degree of softening,  $\xi$  and the pile compressibility, *K*:

$$R_{\rm f} = 1 - (1 - \xi) \left( 1 - \frac{1}{2\sqrt{K}} \right)^2 \tag{4}$$

$$\xi = \tau_{\rm res} / \tau_{\rm peak} \tag{5}$$

$$K = \frac{\pi D L^2(\tau_{\text{peak}})/(EA)_{\text{pile}}}{\Delta w_{\text{res}}}$$
(6)

where A is the pile cross sectional area, E is the pile young modulus and  $\Delta w_{res}$  is the post-peak displacement required to mobilise the residual shaft resistance.  $\tau_{peak}$  and  $\tau_{res}$  are the peak and residual shear stress respectively.

A reassessment of Vijayveriga and Focht's database by Drewry et al. (1977) led to an updated bilinear version of the alpha method which was adopted in the 1975 version of the API-RP2A design guidelines in which  $\alpha$  reduced from unity for soft clay to a minimum value of 0.5 for stiff clays ( $s_u > 75$  kPa). This approach known as API (1975) is shown in Fig. 1 to be similar to the original  $\alpha$  methods proposed by Tomlinson (1957) and others. This new design method resulted in significantly longer pile lengths in areas such as the Gulf of Mexico and the rigour of this development was questioned by industry. To address these concerns, the 7th edition of the API method, published in 1976, reintroduced the original method (Eq. 2) now known as Method 1, as an alternative to the 1975 update, (now known as Method 2). Method 1 was recommended for high plasticity clays such as those found in the Gulf of Mexico, whilst Method 2 was suggested for other types of clay.

The poor reliability of total stress design approaches were discussed by Kraft et al. (1981) and Morrison (1984). Both studies identified the treatment of strain softening and stress history as possible contributors to the poor predictive reliability of the methods. Following the pioneering work of Ladd et al. (1977) linking the undrained strength ratio  $(s_u/\sigma'_{v0})$  to the Over-Consolidation Ratio (OCR), parallel studies by Semple and Rigden (1984) and Randolph and Murphy (1985) determined that the predictive performance of total stress design approaches were significantly improved by considering the undrained strength ratio. A correlation proposed by Randolph and Murphy (1985) was incorporated into the API 1987 edition:

$$\alpha = 0.5 \left(\frac{s_{\rm u}}{\sigma_{\rm v0}'}\right)^{-0.5} \quad \text{for} \quad \left(\frac{s_{\rm u}}{\sigma_{\rm v0}'}\right) \le 1$$

$$\alpha = 0.5 \left(\frac{s_{\rm u}}{\sigma_{\rm v0}'}\right)^{-0.25} \quad \text{for} \quad \left(\frac{s_{\rm u}}{\sigma_{\rm v0}'}\right) > 1$$
(7)

Semple and Rigden (1984) proposed a similar approach which included consideration of the pile slenderness (L/D) and is compared to API 1987 in Fig. 5 (wherein linear interpolation can be undertaken for L/D ratios between 50 and 120). Although not explicitly stated in Eq. 7, the commentary for the API 1987 method included four recommended approaches for considering length effects, namely correction factors suggested by Kraft et al. (1981), Randolph and Murphy (1985), Semple and Rigden (1984) and Murff (1980).

Equation 7 was refined in the API 1993 edition where  $\tau_f$  is given as the larger of:

1.2 Semple & Rigden 1.0 ರ Upper Bound for L/D<50 Adhesion Factor, 0.8 API(1987) 0.6 04 Semple & Rigden Lower 0.2 Bound for L/D>120 0.0 0.00 1.00 2.00 3.00 4.00 Undrained Strength Ratio, s<sub>u</sub>/ $\sigma'_{v0}$ 

Fig. 5 Variation of alpha with undrained strength ratio (after Semple and Rigden 1984)

$$\tau_{\rm f} = 0.5 \cdot \left( s_{\rm u} \cdot \sigma_{\rm v0}^{\prime} \right)^{0.5} \tag{8}$$

$$\tau_{\rm f} = 0.5 \cdot s_{\rm u}^{0.75} \cdot \sigma_{\rm v\,0}^{\prime\,0.25} \tag{9}$$

Kolk and van der Velde (1996) proposed an updated version of these expressions which incorporated length effects directly:

$$\tau_{\rm f} = 0.55 \cdot s_{\rm u}^{0.7} \cdot \sigma_{\rm v\,0}^{\prime\,0.3} \cdot \left(\frac{40}{L/D}\right)^{0.2} \tag{10}$$

Recognising that these empirical correlations were developed from databases of relatively short, closedend piles that are unrepresentative of the large diameter open-ended piles used offshore, prompted the UK Department of Energy to instigate the Large Diameter Pile Test (LDPT) research project. This research project, which was jointly funded with industry, involved the installation of two 762 mm diameter, open-ended driven steel piles. The first was installed to a final penetration length of 30 m in overconsolidated clay at Tillbrook Grange, and the second was installed to 55 m in normally consolidated, siltyclay at Pentre. Hobbs (1993) reported that the alpha value of 0.43 predicted using the API methods at Tillbrook Grange compared well with the measured value of 0.4. However, in the low plasticity silty clay at Pentre, the predicted value of 0.95 was a gross over-estimate of the measured alpha value of 0.62.

The Norwegian Geotechnical Institute (NGI) instigated a research programme described by Karlsrud et al. (1993) in which instrumented piles were installed at Pentre, and two Norwegian test sites; a silty clay deposit at Lierstranda and a soft clay deposit at Onsoy. The piles which were 219 mm diameter closed-ended piles and open-ended piles with an external diameter of 812 mm, were driven to final penetration depths ranging from 15 to 37.5 m. The tests showed that the piles installed in low plasticity clays developed very low horizontal effective stresses (and therefore low alpha values). The authors presented new design lines shown in Fig. 6, which were comparable to the API (1993) approach when the Plasticity Index (PI) exceeded 20%. However, for lower PI clay, the proposed alpha values were offset below the API design line.

Karlsrud et al. (2005) described the further development of this design method known as NGI-99:

For 
$$s_u/\sigma'_{v0} < 0.25 : \alpha = 0.32(PI - 10)^{0.3}$$
 (11)

For 
$$s_u/\sigma'_{v0} > 1.0$$
:  $\alpha = 0.5 (s_u/\sigma'_{v0})^{-0.3} \cdot F_{tip}$  (12)

The approach shown graphically in Fig. 7 assumes a constant alpha value which depends on PI for  $s_u/\sigma'_{v0} < 0.25$ , a log-linear variation for  $s_u/\sigma'_{v0}$  up to 1, whilst for higher  $s_u/\sigma'_{v0} > 1$ , a correction factor  $F_{tip}$  is applied to reduce the shaft friction developed on open-ended piles.

Many of the total stress methods discussed above suggest that no distinction exists between the shaft resistance developed by closed and open-ended piles installed in clay. During installation of open-ended piles, soil freely enters the inside of the pile, where initially the soil level inside the pile is the same as the external ground level and the pile is said to be fully coring. As installation continues, high internal shear stresses can develop near the tip of the pile. These shear stresses can act to resist free movement of soil into the pile and result in partial or full plugging



Fig. 6 Correlation of alpha parameter with strength ratio for low plasticity clays (Modified from Karlsrud et al. 1993)



Fig. 7 NGI-99 Pile Design Method showing influence of soil plasticity (after Karlsrud et al. 2005)

occurring. The degree of plugging is quantified through the Plug Length Ratio (PLR) or the Incremental Filling Ratio (IFR), which incorporates the plug length ( $L_p$ ) and incremental change in plug length ( $\Delta L_p$ ), respectively.

$$PLR = L_p/L \tag{13}$$

$$IFR = \Delta L_{\rm p} / \Delta L \tag{14}$$

Gavin and Lehane (2003) and Foye et al. (2009) demonstrated experimentally that plugging increased the shaft resistance of piles installed in sand and proposed correlations between shaft resistance and IFR which have been incorporated into design practice. Karlsrud and Haugen (1981) performed field tests in which they compared the axial resistance developed during installation of open-ended and closed-ended piles. The 153 mm diameter model piles (the open-ended pile had a wall thickness of 4.5 mm) were jacked into over-consolidated clay. They compared the shaft resistance mobilised by the pile and the plugging records and found measured changes in the average shaft resistance developed by the open-ended pile were not well correlated to either PLR or IFR.

Miller and Lutenegger (1997) investigated the effect of pile plugging on the  $\tau_{av}$  values developed during field load tests performed on open and closedended model piles driven or jacked into overconsolidated clay. Consideration of their data in Fig. 8 shows that  $\alpha$  values mobilised by these piles depended on the mode of installation, with jacked



**Fig. 8** Relationship between adhesion factor,  $\alpha$ , and the plug length ratio (PLR) (modified from Miller and Luttenegger 1997)

piles developing much higher  $\tau_{av}$  values. In contrast to the findings of Karlsrud and Haugen (1981), the degree of plugging experienced (quantified through PLR) strongly influenced the mobilised shaft resistance, with  $\alpha$  increasing linearly as PLR reduced. This effect was much more significant for jacked in place piles.

#### 2.1 Summary of total stress approaches

Despite a number of significant contributions being made to the literature on total stress design methods, many of these approaches have a number of inherent drawbacks. Soil behaviour is, of course, governed by effective stresses and complex stress-strain changes occur during the installation of displacement piles which cannot be completely described using the initial undrained strength profile. In addition, the location of the failure surface on which the shear resistance develops during pile loading will depend on the interface roughness, and at least for steel piles, some consideration of the interface friction angle which controls the shear resistance at the soil-steel interface is required. The effects of pile length and stress history are considered in some of the methods (see Table 1). However, a significant drawback in applying any empirical formulation is the extension of such methods to design situations which are outside the scope of the database used to derive the approach. One such problem which arises is the extrapolation of the results from relatively small pile tests to the much larger (and more heavily loaded) piles. This is illustrated in Fig. 9, which compares the

Table 1 Common total stress approaches

Reference	Length effects	Stress history	
Tomlinson (1957)	×	×	
$\lambda$ approach, Vijayvergiya and Focht (1972)	~	×	
API 1(1976–1986)	×	×	
API 2(1976–1986)	×	×	
Semple and Rigden (1984)	~	~	
Randolph and Murphy (1985)	×	~	
API (1987-present)	×	~	
Kolk and van der Velde (1996)	~	~	
NGI-99, Karlsrud et al. (2005)	×	~	

ultimate load developed by the piles in the database compiled by Semple and Rigden (1984) and the large diameter piles tests (LDPT) with the range of capacities required for offshore piles (Schneider et al. 2007). In addition, extrapolation of some total stress methods to consider soils which are significantly different from the deposits used to calibrate the design approach, such as the soft clay in the Gulf of Mexico, or silty deposits (e.g. Lierstranda) result in poor predictions of pile capacity.

## **3** Beta Methods

In an attempt to overcome many of the drawbacks associated with total stress design approaches,



Fig. 9 Comparison of API database capacity (Q) with the range of offshore pile capacities installed offshore

Burland (1973) advocated an effective stress design approach. In Burlands' method  $\tau_{\rm f}$ , which is controlled by the radial effective stress at failure  $\sigma'_{\rm rf}$ , and the interface friction angle,  $\delta$ , is linked to  $\sigma'_{\rm v0}$  through an empirical parameter  $\beta$ :

$$\tau_{\rm f} = \sigma_{\rm rf}' \tan \delta \tag{15}$$

 $\tau_{\rm f} = K_{\rm f} \sigma_{\rm v0}' \tan \delta \tag{16}$ 

$$\beta = K_{\rm f} \tan \delta \tag{17}$$

Assuming that the radial stress coefficient at failure equals the at rest radial stress coefficient, i.e. that  $K_{\rm f} = K_0$ , and that  $\delta = \phi'_{\rm cv}$  (the constant volume friction angle), the  $\beta$  value for normally consolidated soils is given as:

$$\beta = \beta_{\rm NC} = (1 - \sin \phi_{\rm cv}') \tan \delta \tag{18}$$

Interestingly, Burlands' original method was formulated from load tests on bored piles but has since received widespread use for designing driven piles. For example, Pelletier and Doyle (1982) showed the method provided the best prediction of the capacity of a 762 mm diameter pipe pile driven 80 m into stiff clay with interbedded sand lenses.

Meyerhof (1976) incorporated the Over-consolidation Ratio (OCR) and extended Burlands' method to overconsolidated clay:

$$\beta_{\rm OC} = (1.5 \pm 0.5)\beta_{\rm NC}\sqrt{\rm OCR} \tag{19}$$

Based on databases of field load tests both Meyerhof (1976) and Flaate and Selnes (1977) noted a tendency for  $\beta$  to reduce with increasing pile penetration, leading the latter to suggest an empirical length correction factor of the form:

$$\beta = (0.4 \pm 0.1) \cdot \frac{L + 20}{2L + 20} \cdot \text{OCR}^{0.5}$$
(20)

Using the database compiled by Semple and Rigden (1984), Burland (1993) suggested that the degree of over-consolidation could be considered through the undrained strength ratio (see Fig. 10) with  $\beta$  increasing from 0.2 for normally consolidated to lightly over-consolidated soil  $(s_u/\sigma'_{v0} \le 0.4)$  to 0.5 for heavily over-consolidated soils  $(s_u/\sigma'_{v0} \ge 1.0)$ . The data was re-evaluated by Burland in Fig. 11 which shows that  $\beta$  is a linear function of the undrained strength ratio, although notably no length effect was included. It should be noted however, that the development of a



Fig. 10 Variation in  $\beta$  parameter for different strength ratios (after Burland 1993)



Fig. 11 Beta parameter determined by Burland (1993)

reliable effective stress approach at this time was hampered by the dearth of pile test data that included reliable pore pressure and radial stress measurements.

# 4 Methods Correlating Shaft Resistance to CPT Test Results

A large number of semi-empirical design methods linking the shaft resistance developed by displacement piles to in situ test data have been proposed (Powell et al. 2001). The Cone Penetration Test (CPT) is ideally suited to the development of such correlations as it has the following advantages: (1) The installation procedure of the cone is analogous to pile penetration, (2) The high logging rate of cone end resistance, pore pressure and sleeve friction results in the collection of significant data regarding stratigraphic changes and (3) The data, unlike alternatives such as SPT blowcounts, is not operator dependent. These advantages have made CPT based design approaches very attractive to industry (Poulos et al. 2001).

CPT methods have been developed which link  $\tau_{\rm f}$  to the friction sleeve measurement ( $f_{\rm s}$ ), the pore pressure (u) or the cone tip resistance ( $q_{\rm c}$ ). One of the earliest CPT design methods was the Schmertmann and Nottingham approach. This method, which related shaft resistance directly to CPT sleeve friction, was based on the work of Nottingham (1975) and Schmertmann (1978):

$$\tau_{\rm f} = \alpha_{\rm fs} f_{\rm s} \tag{21}$$

 $\alpha_{fs}$  is a reduction factor which is dependent on pile shape, pile material, cone type and embedment ratio and ranges from 0.2 to 1.25. The maximum shaft resistance is limited to 120 kPa.

Based on piling experience in the North Sea, De Ruiter and Beringen (1979) proposed a design approach known as the European method, which followed the traditional total stress framework, relating the shaft shear stress to the undrained strength through an adhesion factor,  $\alpha$  ( $\tau_f = \alpha \cdot s_u$ ). However, in this instance the undrained shear strength was determined using a cone factor,  $N_{\rm kt}$  such that  $s_{\rm u} = q_{\rm c}/$  $N_{\rm kt}$ . The cone factor ranged from 15 to 20 depending on local experience. Despite moves in traditional total stress approaches towards relatively complex relationships between  $\alpha$  and soil strength and/or OCR, DeRuiter and Beringen assumed unique values of  $\alpha = 1$  for normally and  $\alpha = 0.5$  for over-consolidated soils, respectively. The European method mirrors the Schmertmann and Nottingham approach by imposing an upper limit of 120 kPa on the shaft shear stress. It has been suggested that limiting the shaft shear stress reflects an overall reduction in the mobilised stress along the shaft with increasing penetration and is therefore an implicit method of imposing a length effect on the calculated shaft resistance. Interestingly, neither of these methods incorporated an explicit length correction. Poulos et al. (2001) noted that the reason for high OCR values is due to high lateral stresses in the soil at shallow depths, which can give rise to corresponding high shaft shear stresses near the ground surface.

However, as the OCR decreases with depth a parallel decrease in shaft friction can occur. On this basis the limiting shaft friction can only be ignored by adopting a CPT based correlation that explicitly considers the impact of stress history and the corresponding change in shaft friction with depth. Another source of potential uncertainty associated with this application of the European method is the choice of a site specific cone factor.

Tumay and Fakhroo (1981) compiled data from tests performed on piles installed in Louisiana Clay which they used to propose a correlation between  $\tau_{\rm f}$ and  $f_{\rm s}$  which maintained the same general form as the Schmertman and Nottingham method (Eq. 21). The reduction factor  $\alpha_{\rm fs}$  was given as:

$$\alpha_{\rm fs} = 0.5 + 9.5 \mathrm{e}^{-0.09 \mathrm{fs}} \tag{22}$$

The limiting shaft resistance used in this approach is 60 kPa which reflects the relatively low strength of the material used to develop the correlation. This led to the method providing conservative predictions of the capacity of piles installed in over-consolidated clay at Tilbrook Grange (Clarke et al. 1993).

The LCPC method was developed from a comprehensive database of 197 full scale load tests, which were conducted at 48 sites (Bustamante and Gianeselli 1982). Whilst a range of pile types were considered in the database, the tests were predominantly performed on bored and driven piles. The pile diameters ranged from 110 to 1,500 mm and the lengths varied from 6 to 45 m. The ground conditions varied widely across the dataset and included clays, silts, sands, gravels and weathered rock. The LCPC method relates the unit shaft friction mobilised on the pile to the cone tip resistance ( $q_c$ ) through a normalising reduction factor, termed  $\alpha_{CPT}$ :

$$\tau_{\rm f} = \frac{q_{\rm c}}{\alpha_{\rm CPT}} \tag{23}$$

The magnitude of the reduction factor is dependent on the material and pile type and varies from 30 to 120 for driven piles in soft to stiff clay, respectively. The method adopts limiting shaft friction values of 15 kPa for soft clays and 35 kPa for stiff clays. Although these limiting values have been shown to be too conservative at some test sites, in a comparison of the predictive reliability of a number of design methods, Briaud and Tucker (1988) found that the LCPC method outperformed the other design methods considered. The methods relatively good performance is probably a result of including a wide range of soil types in the derivation of the empirical constants.

More recently, Almeida et al. (1996) proposed a CPT based design approach that relied on more accurate piezocone measurements, which allow correction of the cone tip resistance for pore pressures acting on the cone shoulder  $(q_{\rm T})$ . A direct relationship between  $\tau_{\rm f}$  and the net cone resistance  $(q_{\rm cnet} = q_{\rm T} - \sigma_{\rm v0})$  was developed from a database of 43 load tests at eight clay sites. The soil conditions at the test sites ranged from soft normally consolidated clay at Lierstranda to stiff heavily overconsolidated clay at Tilbrook. The empirical  $\alpha_{\rm CPT}$  parameter relating the net cone resistance to the shaft stress is calculated from the normalised cone resistance  $(Q = q_{\rm cnet}/\sigma'_{\rm v0})$ :

$$\tau_{\rm f} = \frac{q_{\rm cnet}}{\alpha_{\rm CPT}} \tag{24}$$

$$\alpha_{\rm CPT} = 11.8 + 14 \log\left(\frac{q_{\rm cnet}}{\sigma'_{\rm v0}}\right) \tag{25}$$

While no length effect was considered directly in the design formula, Almeida et al. (1996) noted that the database contained limited information for piles with high slenderness ratios and suggested using the reduction factor proposed by Semple and Rigden (1984) for piles with L/D greater than 60.

Eslami and Fellenius (1997) proposed a CPT method, which used both the pore pressure measured at the cone shoulder  $(u_2)$  and the total cone end resistance  $(q_T)$ , to estimate shaft resistance from the effective cone resistance,  $q_E$ .

$$\tau_{\rm f} = C_{\rm s} \cdot q_{\rm E} \tag{26}$$

$$q_{\rm E} = q_{\rm T} - u_2 \tag{27}$$

The empirical parameter  $C_{\rm s}$  was shown to vary from 0.08 to 0.25 for soft to stiff clay. The method proposed by Eslami and Fellenius (1997) was based on 104 case histories across a broad variety of site conditions from very stiff clay/mudstone at a Japanese site (Matsumoto et al. 1995) to very soft Norwegian clays (Almeida et al. 1996). The broad geological and geographical spread of this database probably contributes to the good predictive performance across a range of site conditions (Cai et al. 2009).

## 4.1 Summary of CPT Based Approaches

A brief summary of the CPT methods which are used in design practice was presented. Since the CPT test is in essence a miniature pile, correlations between cone resistance and shaft resistance show great promise. However, a number of caveats should be considered before the empirical design approaches reviewed above are used in design practice:

- Many of the methods were developed and calibrated over 20 years ago. Modern electric cones provide significantly more reliable cone profiles.
- 2. The methods tend to be derived for specific regions and their application to more widespread geological profiles needs to be carefully considered. Extension of these methods to conditions that are significantly different to those on which they were calibrated is questionable.
- 3. Many deal with friction fatigue effects indirectly through the inclusion of limiting maximum shaft friction values. This tends to result in over-conservatism.
- 4. The European method uses the cone data to determine an undrained strength profile and is essentially a total stress method with many of the problems inherent in such approaches.
- 5. Many of the pile tests used in the databases did not measure local friction and pore pressure values, the absence of which preclude thorough understanding of the effective stress conditions controlling pile behaviour.

## 5 Enhanced Understanding of Pile Behaviour from Instrumented Pile Tests

Randolph (2003) noted that any scientific approach to determining the shaft resistance of a displacement pile should consider the complex stress–strain history experienced, which includes; the initial in situ condition, pile installation, equalisation and loading (see Fig. 12). Given the dearth of such information in the database of field tests used to develop total stress design approaches, a number of leading geotechnical research institutes including NGI (Karlsrud and Haugen 1985), Massachusetts Institute of Technology (Morrison 1984), Oxford University (Coop 1987) and Imperial College London independently initiated



Fig. 12 Changes in pile stress regime over time

programmes of field testing using instrumented piles. The research conducted at Imperial College, employed the heavily instrumented Imperial College Pile (ICP) which was installed in a wide range of clay types (see Table 2). The ICP contained total stress, porewater pressure and shear stress transducers at a number of locations on the pile shaft identified by their distance from the pile tip (*h*) normalised by the pile diameter (*D*). The particular advantage of the ICP was that by simultaneously measuring radial stress ( $\sigma_r$ ), porewater pressure (*u*) and local shear stress ( $\tau_f$ ), the development of shaft resistance during installation, equalisation and loading could be considered in a fundamental manner using stress paths measured in radial effective stress—shear stress

Table 2 Test Sites used by Imperial College

Site	Description	Reference
Canons Park	Stiff Eocene London Clay	Bond and Jardine (1991)
Cowden	Stiff glacial till	Lehane and Jardine (1994a)
Bothkennar	Low OCR shallow marine clay	Lehane and Jardine (1994b)
Pentre	Low OCR Glacio- Lacustrine silty clay	Chow (1997)

space. In addition, load cells allowed the distribution of residual load on the pile to be determined. Many of the load tests used in the derivation of empirical design approaches ignored the presence of residual loads, with the consequence that the inferred shear stress distributions were most likely incorrect. Fellenius et al. (2004) clearly demonstrated the effect of ignoring residual load effects when considering the load distribution measured on a closed-ended pile driven into soft compressible clay.

Lehane and Jardine (1994b) reported data from the ICP installation into lightly over-consolidated marine clay at Bothkennar (see Fig. 13). They demonstrated that during jacked installation of the ICP, the end bearing resistance  $(q_b)$  mobilised during each jacking stroke was approximately equal to the CPT  $q_c$  end resistance at that depth. The shape of the radial total stress ( $\sigma_{\rm ri}$ ) profiles mirrored the  $q_{\rm b}$  profile suggesting that  $\sigma_{ri}$  was controlled by the soil state. Friction fatigue was evident with  $\sigma_{ri}$  reducing as h/D increased from 4 to 14. Although the ICP contained an additional radial stress sensor at h/D = 25, the readings at this sensor level during pile installation at Bothkennar were virtually indistinguishable from those at h/D = 14. The normalised radial total stresses developed at Bothkennar are compared to those mobilised in overconsolidated glacial till deposits at Cowden and



Fig. 13 Installation total stress at Bothkennar (after Lehane 1992)

heavily over-consolidated London Clay in Fig. 14. Whilst friction fatigue is evident at all sites (where  $\sigma_{ri}/q_c$  values were seen to decrease as h/D increased), it is clear that the effects are more pronounced for the heavily over-consolidated clays. By contrast, Chow (1997) found that the  $\sigma_{ri}/q_c$  profile developed during installation of the ICP into two sand deposits, a loose to medium dense dune sand at Labenne, and dense sand at Dunkirk, was unique and did not depend on soil state.

Noting the lack of a unique direct correlation between  $\sigma_{ri}/q_c$  and h/D for the three clay sites at which the ICP was installed, Lehane (1992) observed that the normalised radial total stress developed during installation of the ICP (see Fig. 14) increased in proportion to the over-consolidation ratio (OCR). He compiled a database of high quality instrumented pile tests which included data from the ICP tests and nine other sites (including tests performed by NGI, MIT and Oxford). Based on this database Lehane proposed the following best fit expression to describe the radial total stress parameter,  $H_i$ :

$$H_{\rm i} = \frac{(\sigma_{\rm ri} - u_o)}{\sigma'_{\rm v0}} = 3.92 \,{\rm OCR}^{0.41} (h/R)^{-0.2}$$
(28)

A feature of all the ICP pile tests was that during pore pressure equalisation, relaxation of the radial total



Fig. 14 Relationship between the installation radial total stresses and distance from the pile tip (after Lehane and Jardine 1994)

stress set-up during installation occurred. This reduction was quantified using the relaxation coefficient  $K_c/H_i$ :

$$\frac{K_{\rm c}}{H_{\rm i}} = \frac{\left(\sigma_{\rm rc}' - u_o\right)}{\sigma_{\rm ri} - u_0} \tag{29}$$

The equalised radial effective stresses ( $\sigma'_{rc}$ ) mobilised on the ICP were seen to exhibit a similar dependence on OCR as seen for installation radial total stresses (see Fig. 15), with the effective stress offset by an amount which Lehane (1992) suggested was controlled by the clay sensitivity ( $S_t$ ). The  $\sigma'_{rc}$  values could thus be described using a conventional earth pressure approach with a correlation originally proposed by Lehane (1992) being updated by Chow (1997):

$$\sigma_{\rm rc}' = K_{\rm c} \sigma_{\rm v0}' \tag{30}$$



Fig. 15 Radial stress relaxation as a function of soil sensitivity (after Randolph et al. 2003)

$$K_{\rm c} = [2.2 + 0.016 \,{\rm YSR} - 0.87 \log 10S_{\rm t}] {\rm YSR}^{0.42} h/R^{-0.2}$$
(31)

The radial effective stress regime surrounding a displacement pile depends on the soil yield stress ratio, sensitivity and the geometric (h/R) term, which accounts for the effects of friction fatigue in reducing the radial effective stress. The yield stress ratio (YSR), (which is the ratio of effective vertical yield stress to the in situ vertical effective stress) was suggested as a more comprehensive measure of the stress history by Jardine et al. (2005). All correlations produced by Chow (1997) replaced the OCR term with YSR as shown in Eq. 31.

Chow (1997) considered many possible mechanisms which could contribute to friction fatigue including (1) heave—with upward soil displacements resulting from pile installation causing a reduction in radial stress, (2) pile whip—in which lateral movement of the pile head results in loss of contact between the pile wall and the surrounding soil, (3) stress concentration at the pile tip caused by the large end bearing resistance generated during pile installation and (4) the effects of extreme cyclic loading. Whilst mechanism (1) and (2) would affect the radial effective stress profile at relatively shallow pile penetrations, (3) and (4) are likely to be dominant for typical pile geometries.

Coop and Wroth (1989) presented measurements of  $\sigma_r$  made during the installation of the Oxford University Instrumented Model Pile (IMP) in Over-Consolidated Clay at Huntspill. The IMP had sensors at two locations

and they noted an h/R effect which they attributed to reconsolidation occurring during the 10 min time lag required for the trailing transducer to reach a given depth. Randolph (2003) argued that pore pressure dissipation which occurred during installation of the ICP was at least partly responsible for the reduction in  $\sigma_{ri}/q_c$  with increasing h/D, noted in Fig. 14.

Measurements of the normalised excess porewater pressure ( $\Delta u/s_u$ ) mobilised during the installation of the LDPT piles into lightly over-consolidated silty clay at Pentre and in heavily over-consolidated clay at Tilbrook Grange are shown in Fig. 16a and b respectively. Data from both sites include  $\Delta u$  values measured by sensors on the pile shaft and by piezometers embedded in the soil at radial distance (*r*) of 1 m and 2 m from the pile shaft. The measurements are plotted against normalised distance from the pile tip, with negative h/D values indicating that the pile tip is above the piezometer level. The following trends are noteworthy:

- 1. At both sites the maximum  $\Delta u/s_u$  was recorded as the pile tip passed at or close to the sensor (within the region h/D = 0 to h/D = 3).
- 2. The normalised increase in porewater pressure  $\Delta u/s_u$  was much greater (approximately double) in the low YSR soil. This is compatible with high measurements of radial total stress mobilised during pile installation at other low YSR soils such as Bothkennar and Belfast Sleech (Gavin et al. 2010), where despite radial effective stress  $(\sigma'_{ri})$  values approaching zero during installation,

Excess Pore Pressure Ratio,  $\Delta u/s_u$ (a) **(b)** Excess Pore Pressure Ratio, Au/s<sub>n</sub> -20 -20 Normalised Distance from Pile Base, h/D -10 r=1mr=1m4 1.5 2.5 10 10 Shaft Measurments 2m 20 20 Shaft Measurments 2m 30 30

Fig. 16 Normalised Excess Pore Pressures measured in the soil mass and on the pile shaft at a Pentre and b Tilbrook respectively the generation of large excess pore pressures resulted in high normalised  $\sigma_{\rm ri}/q_{\rm c}$  values (where  $\sigma_{\rm ri} = \sigma'_{\rm ri} + u_0 + \Delta u$ )

- 3. Values of  $\Delta u/s_u$  reduced rapidly as the h/D value increased to h/D = 10, thereafter the rate of reduction slowed considerably. This rapid reduction of  $\Delta u/s_u$  as the highly stressed pile base passes, suggests that the relatively high rates of friction fatigue noted for  $\sigma_{ri}$  values measured during installation of the ICP, may have resulted from unloading as the distance to the pile base increased to 10 pile diameters.
- 4. For *h/D* values in excess of 10, the rate of reduction of excess porewater pressures slowed considerably, with the time for consolidation being controlled by the pile diameter and permeability of the soil.

Bond (1989) compared the axial capacity of the ICP when driven or jacked into the London Clay at Canons Park. During driving, the pile was subjected to 4,500 rapid undrained load cycles ( $N_{und}$ ). The standard installation procedure adopted for the ICP tests, wherein the pile was jacked in 200 mm jacking strokes resulted in 20 installation load cycles at Canons Park. Despite the extreme range of installation load cycles applied, the piles developed similar axial load capacities following equalisation.

Lehane (1992) investigated the effect of the number of installation load cycles on the normalised radial effective stresses  $(\sigma'_{\rm ri}/q_{\rm c})$  developed during installation of the ICP, (see Fig. 17). At each test site, the 102 mm diameter closed ended pile was installed using a 200 mm jacking stroke, which caused a minimum of two installation load cycles for the sensor nearest the pile tip (h/D = 4) and a maximum of twelve load cycles at h/D = 25. The data suggests that much higher normalised radial effective stresses were mobilised in the high YSR soils at Canons Park and Cowden (with values approaching 25% of the CPT  $q_c$  resistance near the pile tip). It is possible that the much higher installation effective stresses developed in these soils resulted in more pronounced cyclic shearing effects, and consequently friction fatigue was also much greater for these high YSR soils.

Chow (1997) varied the jacking stroke length used to install the ICP at Pentre in order to consider a wider range in the number of installation load cycles.



Fig. 17 Impact of shearing cycles on normalised effective stress

In addition to the standard undrained load cycles  $N_{und}$  experienced during ICP installation, she considered load cycles where partial dissipation of the excess porewater pressure (termed  $N_{diss}$  cycles) was allowed to occur in the highly laminated silty clay at Pentre. Chow found that the  $\sigma'_{rc}$  values mobilised were relatively unaffected by the number of undrained load cycles. In contrast, the number of dissipation cycles had a more significant effect, with  $\sigma'_{rc}$  reducing as  $N_{diss}$  increased, as shown by Fig. 18.

Xu et al. (2006) reported radial total stress and pore pressure measurements made during the fast monotonic vibratory installation of 1.02 m diameter steel tubes to depths of 12 and 13.5 m in soft clay. She noted a clear tendency for both  $\sigma_r$  and  $\Delta u$  values



Fig. 18 Influence of dissipation cycles (adapted from Chow 1996)

to decrease as h/D increased. In this case friction fatigue was evident in the absence of any shearing or dissipation load cycles which suggests that the proximity of the pile base is an important feature of observed friction fatigue.

Gavin et al. (2010) presented the results of a series of field experiments performed to study the effect of installation method on the shaft resistance developed by a pile installed in soft silty clay. A series of tests were performed on piles which experienced different levels of cyclic loading during installation. The test results indicated that the radial total stress, pore water pressure and shear stress on the pile shaft during installation, were strongly affected by the installation procedure; all three were found to increase when the jacking stoke length used during installation increased (or the number of cyclic load applications decreased). The dominant feature which caused large stress increases during installation were the relatively high pore pressures developed during the installation of the pile into soft clay. These excess porewater pressures which set-up during installation, exhibited friction fatigue which could be explained by partial dissipation of excess porewater pressure which occurred as points remote from the pile tip experienced unloading as the distance from the pile tip increased. An interesting finding from these experiments was that despite the significant effect of installation method on the installation resistance of the piles, the equalised radial effective stresses measured after the dissipation of excess porewater pressure were found to be insensitive to the installation method and all piles mobilised similar shaft resistance when load tested.

Doherty and Gavin (2010) reported tests where open-ended and closed-ended piles were installed in soft clay. They found that the pile base resistance, radial total stress and pore pressure recorded at the pile shaft were significantly affected by the degree of plugging experienced during open-ended pile installation. However, because increases in radial total stress mirrored increases in porewater pressure, radial effective stresses and therefore shaft resistance were unaffected by the degree of plugging. The authors noted that the relatively high pore pressures mobilised during pile installation in the soft clay may have constrained volume change in the interface shear zone during pile installation.

# 6 Analytical Methods for Assessing Pile Behaviour

Analytical approaches which have been developed to predict pile behaviour include the Cavity Expansion Method (CEM) and Strain Path Method (SPM). In the CEM method, pile installation is simulated by expanding a cylindrical cavity in a soil mass (with a volume equal to that of the pile). See Kirby and Esrig (1979) and Randolph et al. (1979). During undrained installation, the radial displacement  $\delta_r$ , at a distance *r* from the centre of a closed-ended pile with a radius *R* is given by:

$$\frac{\delta r}{R} = \sqrt{1 + \left(\frac{r}{R}\right)^2} - \frac{r}{R} \tag{32}$$

Because this one-dimensional approach ignores vertical deformations, shearing around the pile tip, and the influence of the ground surface, it does not properly model the complex strain histories of elements close to the shaft of displacement piles. It therefore provides poor estimates of shaft stress (Xu et al. 2006). However, Lehane and Gill (2004) show that it provides reasonable predictions of radial displacement. The stresses developed during cavity expansion can be predicted using closed form solutions (Butterfield and Bannerjee 1970; Randolph and Wroth 1979) or through FEM solutions (Randolph et al. 1979; Whittle 1987). The excess pore pressure  $\Delta u$  can be estimated from the shear modulus, *G* and undrained strength (Gibson and Anderson 1961):

$$\frac{\Delta u}{s_{\rm u}} = \ln \frac{G}{s_{\rm u}} - \frac{r}{R} \tag{33}$$

Although G is a strain dependent parameter, Randolph (2003) suggests that for lightly over-consolidated clay, the maximum pore water pressure at the pile-soil interface is in the range 4–6  $s_u$ . This expression has been shown to provide reasonable predictions for  $\Delta u$  values measured during instrumented pile tests in soft clay (McCabe 2002; Doherty 2010).

The Strain Path Method (SPM) considers a twodimensional strain field caused by pile installation which is modelled as the flow of an ideal fluid around the tip (Baligh 1985, 1986). The resulting flow streamlines are used to determine strain paths, with stresses obtained by employing a suitable constitutive model. When compared with the CEM approach, the SPM method which is described in detail by Lehane (1992) provides improved predictions of the soil response in the region of the pile shaft.

Azzouz and Morrison (1988) compared measurements of the radial total stress measured during installation of a model pile, referred to as the piezo lateral stress (PLS) cell, in Empire Clay, with predictions using both CEM and SPM methods. The CEM approach was applied using the Modified Cam Clay (MCC) model, the SPM method was also applied with the MIT-E2 model. The authors concluded that no method provided consistently good predictions, and the predictive performance was highly dependent on the soil model chosen. In discussing the relative performance of both approaches, Jardine (1985) and Bond (1989) noted that the 2D nature of the SPM model was required to properly model the friction fatigue phenomenon noted in their field tests using the ICP. This is illustrated in Fig. 14, in which SPM predictions of the normalised radial total stress at Bothkennar are compared to measured values from the ICP installation. Whilst the SPM method over-predicts the mobilised stresses, critically it captures the rapid decay in stress in the vicinity of the pile tip.

Chin (1986) used the SPM method to investigate difference between the strains caused by both closed and open-ended pile installation. The open-ended pile geometry modelled was typical of an offshore pile with a pile diameter, *D* to wall thickness *t*, *D/t* ratio of forty. Comparisons of the Octahedral Shear Strains developed by the piles, illustrated in Fig. 19, shows that the strains are comparable when the equivalent radius  $R_{eq}$  factor is used to normalise the open-ended pile measurements.

$$R_{\rm eq} = \sqrt{R^2 - R_{\rm i}^2} \tag{34}$$

where  $R_i$  is the internal radius of the open-ended pile.

On this basis, Chow (1997) suggested that the Imperial College design approach for piles in clay, which was originally developed from the results of closed-ended pile tests, could be extended to open-ended piles by substituting  $R_{eq}$  for R:

$$K_{\rm c} = [2.2 + 0.016 \,{\rm YSR} - 0.87 \,\log 10 \,S_{\rm t}] \\ \times \,{\rm YSR}^{0.42} \,h/R_{\rm eq}^{-0.2} \tag{35}$$

Sagaseta and Whittle (2001) provided an updated SPM model known as the Shallow Strain Path



Fig. 19 Strain contours for an open-ended pile and a solid closed-ended pile of an equivalent radius calculated using the Strain Path Method (SPM) (adapted from Chin 1986)

Method (SSPM) which accounts for ground surface effects on the stress and strain response during initial penetration of the pile. In the SSPM method the radial displacements are predicted from:

$$\delta_{\rm r} = \frac{R^2}{2} \left( \frac{L}{r\sqrt{r^2 + L^2}} \right) \tag{36}$$

Xu et al. (2006) compared the radial displacements measured during the installation of a 1 m diameter open-ended pile in soft clay, with those predicted using both CEM and SSPM methods. Measurements taken from inclinometer tubes at radial distances of 2 and 7 radii from the pile shaft, when the pile tip depth was 9 m bgl are compared to predicted values in Fig. 20. The CEM method estimates a constant  $\delta_r$ profile, which unlike the measured profile, is independent of the position of the pile tip and generally overestimates soil movement. By contrast, the SSPM method provides much more realistic predictions, although it fails to capture the large increase in  $\delta_r$ , measured at R = 2 radii, at a depth of 4.5 m.

Randolph (2003) proposed a simple generalized form of the cavity expansion method for determining



**Fig. 20** Radial movements adjacent to an OE Pile Installation at a depth of 9 m and **a** 2 radii and **b** 7 radii from the pile (adapted from Xu et al. 2006)

the radial effective stress acting on the pile shaft both immediately after installation and following complete consolidation.

$$\sigma_{\rm ri} = u_0 + \frac{1}{S_{\rm t}} \frac{s_{\rm u}}{\tan \delta_{\rm r}} + \left(1 - \frac{1}{S_{\rm t}}\right) \frac{1 + 2K_0}{3} \sigma_{\rm v0}' + s_{\rm u} \ln(\rho I_{\rm r})$$
(37)

where the first component on the right hand side represents the initial pore pressure,  $u_0$ , the second reflects the external radial effective stress,  $\sigma'_{ri}$ , the third accounts for shear induced excess pore pressure and the fourth is the expansion induced excess pore pressure.  $\rho$  is the area ratio of the pile, which for a fully coring pile is approximately 4t/D, whilst for a fully plugged or closed-ended pile is one. The rigidity index,  $I_{r}$ , is the ratio of the shear modulus to the shear strength (i.e.  $G/s_u$ ). This method is therefore the first to explicitly consider the transition from fully coring to fully plugged installation for open-ended piles.

Following consolidation, the equalized radial effective stress can be estimated from the earth pressure coefficient  $K_c$ :

$$K_{\rm c} = \frac{\sigma_{\rm rc}'}{\sigma_{\rm v0}'} = \frac{\sigma_{\rm o}'}{\sigma_{\rm v0}'} + \frac{\rm YSR}{\mu} \ln\left(1 + \frac{\lambda\mu}{\rm YSR}\frac{\Delta u_i}{\sigma_{\rm v0}'}\right)$$
(38)

Randolph (2003) suggested values of 1 and 5 respectively for the empirical parameters  $\lambda$  and  $\mu$  in order to match radial effective stresses profiles on the ICP reported by Chow (1997). Chen and Randolph (2007) showed that this approach provided a slight over-prediction of the external radial stress changes during suction caisson installation in centrifuge model tests. However, it predicted reasonable estimates of the external shaft friction following complete equalization, although under-predicting the result for the sensitive clay (Chen and Randolph 2007). The method was applied to predict the radial total stress, porewater pressure and radial effective stress mobilized during installation of the NGI closed-ended test pile at Haga (see Fig. 21), where it is seen to provide reasonable estimates of values measured near the pile tip at a depth of 5 m. However, at points remote from the pile tip, the stresses are overestimated.

#### 7 Discussion

McClelland et al. (1969) noted that the evolution of pile design was based on "judgement, intuition and fragments of experience". Forty years on and this



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observation could still be said to hold true. Advances in analytical approaches such as CEM and SSPM methods and semi-empirical design methods correlated to extensive experimental programmes such as the IC-05 design approach are welcome steps towards more rational design methods. Many of the advances were obtained through the use of instrumented model piles which measured the radial effective stresses during the complete stress history of the pile. However, Randolph (2003) demonstrated the gap between analytical predictions and experimental observation and uncertainty remains in a number of key areas including; (1) the reliability of existing predictive methods, (2) the effect of friction fatigue, (3) the relative resistance of open and closed ended piles and (4) the effect of loading rate on the measured shaft resistance.

The API total stress design method remains the industry standard although the Imperial College (IC-05) approach is gaining significant traction with Overy and Sayer (2007) demonstrating its predictive reliability. Jardine et al. (2005) compiled a database of load tests on piles in clay and compared the predictive capability of IC-05 and API-93. The results (see Table 3) revealed that the IC method was significantly more reliable than API with a mean value of the predicted to measured resistance  $(Q_c/Q_m)$ 1.03 for IC-05 compared to 0.85 for the API. In addition, lower scatter was observed around the IC-05 predictions, demonstrated by the lower COV (coefficient of variation = standard deviation/mean) in Table 3. Significantly, several sources of bias were noted for the API approach, with the ratio  $Q_c/Q_m$ increasing with pile slenderness (L/D) resulting in capacity estimates being conservative for short piles and being unconservative for long piles. In addition, a bias with respect to OCR was observed with the API method generally over-predicting the resistance of normally consolidated deposits and being overconservative in heavily over-consolidated deposits. These observations suggest that the evolution of the API approach to include the effects of stress history and friction fatigue were unsuccessful .

Clausen and Aas (2001) produced an independent review of the IC-05 and API methods, and found that whilst the IC method provided a marginally more accurate mean prediction of the shaft resistance than the API, the COV was much higher than those reported by Jardine et al. (2005), see Table 3. They found that the IC-05 method over-estimated the resistance developed in low plasticity clays with low OCR values. When these soils were excluded from the database, the variability of both the API and IC methods decreased substantially and both methods were conservative (with  $Q_c/Q_m < 1$ ). Ridgway and Jardine (2007) identified a range of 'problem' clays whose capacity was over-predicted using the IC-05 method. These were predominantly low plasticity clay, which were characterised by low cone resistance and skin friction values measured using the CPTu. The resistance of piles at these 'problem' sites was also poorly predicted using the API approach, with the NGI method offering the only reasonable predictions by explicitly considering the plasticity index.

Jardine and Chow (1996) recommend that the key input parameters required for the successful application of the IC design approach are (1) site specific measurements of the interface friction angle  $\delta$ , (2) information on the soil sensitivity and (3) a reliable profile of OCR. The importance of accurate measurement of  $\delta$  was highlighted by Saldivar and Jardine (2005) who report predictions of the shaft resistance of piles installed in Mexico City clay, which has a plasticity index (PI) of 160%. Ring shear tests on the clay revealed an unusually high  $\delta$  value of 36°. Using existing empirical correlations between  $\delta$ and PI (Jardine et al. 2005) would result in a  $\delta$  value

Table 3 Comparison of API and IC-05 reliability

Method	IC-05	API-93	IC-05	API-93	IC-05 <sup>a</sup>	API-93 <sup>a</sup>
Reference	Jardine et al. (2005)	Jardine et al. (2005)	Clausen an	d Aas (2001)	Clausen an	nd Aas (2001)
$Q_{\rm c}/Q_{\rm m}$	Shaft	Shaft	Total	Total	Total	Total
Mean, $\mu$	1.03	0.99	1.03	1.1	0.81	0.93
COV	0.2	0.33	0.69	0.49	0.34	0.3

<sup>a</sup> Ignores 8 pile tests in 'problem' soils

of 8–12° for this high plasticity clay. The difference between the interface friction value inferred from the empirical correlation and the actual values would result in underestimates of the pile resistance and excessively long pile lengths would be required (approximately 4 times longer).

The difficulty of using average  $Q_c/Q_m$  statistics to assess the reliability of pile design methods is illustrated in Fig. 22a and b, where the shear stress profiles measured on the LDPT piles installed in Pentre and Tilbrook respectively are compared to design profiles predicted using IC-05, API-93, NGI-99 and the LCPC method.

The following can be observed:

- 1. The API method overestimated the shear stress profile over the entire pile length at Pentre and thus overestimated the overall shaft capacity, whereas at Tillbrook Grange it provided an excellent prediction of the overall resistance, despite significantly over-predicting the shaft resistance developed on the upper portion of the pile shaft and under-predicted the resistance developed near the pile tip.
- 2. The LCPC method accounts for friction fatigue by introducing a limiting value of shaft friction which can be developed in a given deposit. This limiting resistance is seen to grossly underestimate the shaft resistance at both sites.
- Whilst the NGI-99 method provided a very good prediction of the distribution of shear stress at Pentre, the profile predicted at Tilbrook Grange

was very similar to that predicted using the APImethod.

4. The IC method produced good overall predictions of the total shaft resistance at both sites. However, at Pentre, the shear stress profile was over-predicted near the pile tip and underestimated along the remainder of the shaft.

Much of the uncertainty associated with the methods set out above results from poor treatment of the distribution of shear stress on the pile shaft. Whilst significant research effort has identified the critical role of volume change in the interface shear zone during cyclic shearing as a key mechanism controlling friction fatigue in sand, uncertainties remain as to how to quantify these effects in clay, where pile size, soil permeability, installation method and soil state are likely to contribute to friction fatigue. As a result, approaches to deal with this aspect of behaviour vary from the use of conservative upper bound shaft resistance values to the use of a geometric reduction factor.

Another feature where design methods differ is in the treatment of possible differences between the shaft resistance mobilised by open and closed-ended piles. The API method assumes that there is no difference in shaft resistance for piles driven in clay, whilst assuming in sand, open-ended piles develop only 80% of the shaft resistance of a closed-ended pile. The LCPC approach accounts for pile end condition by using higher reduction factors for lowdisplacement piles. The NGI method assumes no





Fig. 23 Pile Rate Dependence (adapted from Jaime et al. 1990)

difference in the behaviour of piles in normally consolidated soil and a reduction factor that increases as the degree of over-consolidation increases. The IC approach includes the same reduction factor (which assumes that piles are fully coring with IFR = 100%) in all soil types. This reduction factor assumes that friction fatigue occurs at a higher rate for open-ended than closed-ended piles.

It is worth noting the methods described in this paper were developed to calculate the static axial shaft capacity of piles in clay. Because of the high cost and considerable time required to verify the pile capacity using static load tests, the use of faster and cheaper dynamic load test methods are becoming increasing popular within industry. It should be remembered that the resistance of the majority of soils considered herein are rate dependent (Brown et al. 2006). An example of the effect of loading rate on the pile resistance mobilised during maintained static loading and constant rate of penetration (CRP) loading of piles installed in Mexico City Clay are shown in Fig. 23, where the higher rate CRP test procedure resulted in stiffer pile response and over a 50% increase in pile resistance.

## 8 Conclusions

This paper described the evolution of design methods used to calculate the shaft resistance of displacement piles in clay. Because of their simplicity, total stress design approaches remain popular in practice. However, the empirical parameters linking shaft resistance to the undrained strength of the soil are affected by multiple factors and the treatment of important facets of pile behaviour such as the effects of stress history and friction fatigue are often contradictory. Analytical approaches such as CEM and SPPM offer considerable promise. However, Randolph (2003) noted that some of the aspects of pile behaviour observed in instrumented pile tests were not modelled by these methods and as result he suggested some developments to the CEM approach. Until further calibration and validation of these methods is conducted they are unlikely to be used widely in industry. It is apparent that additional testing of large diameter, high capacity piles, where radial effective stresses are measured throughout installation, equalisation and load-testing, is required in order to address many of the uncertainties that remain in current pile design methods.

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