

Seismic Passive Earth Thrust on Retaining Walls with Cohesive Backfills Using Pseudo-Dynamic Approach

A. H. Shafiee · A. Eskandarinejad ·
M. Jahanandish

Received: 10 January 2010 / Accepted: 1 April 2010 / Published online: 13 April 2010
© Springer Science+Business Media B.V. 2010

Abstract In this paper, the pseudo-dynamic approach is used to estimate seismic passive earth thrust on retaining walls with cohesive-frictional backfills. The time-dependent pseudo-dynamic approach considers the influence of dynamic parameters such as the velocity of primary and shear waves, the period of lateral shaking, and the phase and amplitude variations of horizontal and vertical earthquake accelerations with depth. The failure plane behind the wall is assumed to be planar. The analysis is based on the equilibrium of forces which act within the failure wedge. The obtained results show that the backfill cohesion increases both the seismic passive earth thrust and the failure plane inclination angle with the horizontal plane. It is also observed that both horizontal and vertical seismic accelerations have decreasing effect on seismic passive earth thrust as well as failure plane inclination angle. The results of present pseudo-dynamic analysis propose a lower solution for seismic passive earth thrust compared

to earlier pseudo-static solution available in the literature.

Keywords Seismic passive earth thrust · Retaining walls · Pseudo-dynamic approach · Cohesive backfill

1 Introduction

The estimation of seismic passive earth pressure is vital in design of safe retaining walls. The majority of available solutions in the literature consider the backfill soil to be cohesionless (Soubra 2000; Kumar 2001; Ghahramani and Anvar 2004). The pioneering studies belong to Okabe (1926) and Mononobe and Matsuo (1929) who used the limit equilibrium method and a planar failure surface to compute seismic earth pressure coefficients. Their solution is known as Mononobe-Okabe method. However, a retaining wall may be built to support a cohesive-frictional backfill in practice. Thus, it is very essential to compute seismic earth resistance on such walls. The number of investigations that determine the seismic passive earth pressure on retaining walls with cohesive-frictional backfills is limited. The reason seems to be due to complex equations and cumbersome computational effort when backfill cohesion and the wall adhesion are taken into account. Saran and Prakash (1968) and Saran and Gupta (2003) extended the Mononobe-Okabe solution to walls with c - ϕ soil backfill. They obtained

A. H. Shafiee (✉) · M. Jahanandish
Civil Engineering Department, Shiraz University,
Shiraz, Iran
e-mail: shafiee.amirhossein@gmail.com

M. Jahanandish
e-mail: jahanand@shirazu.ac.ir

A. Eskandarinejad
Civil Engineering Department, University of Hormozgan,
Bandarabbas, Iran
e-mail: alireza.eskandarinejad@yahoo.com

$$\alpha_h(z, t) = \left[1 + \frac{(H - z)}{H}(f_a - 1) \right] \alpha_h g \sin \omega \left(t - \frac{H - z}{V_s} \right) \tag{1}$$

$$\alpha_v(z, t) = \left[1 + \frac{(H - z)}{H}(f_a - 1) \right] \alpha_v g \sin \omega \left(t - \frac{H - z}{V_p} \right) \tag{2}$$

where f_a is the amplification factor, i.e., it designates the variation of shaking amplitude with depth and $\omega = 2\pi/T$ is the angular frequency of lateral shaking.

The weight of the failure wedge ABD is given by:

$$W = 0.5\gamma H^2 \cot(\alpha) \tag{3}$$

The total cohesive force (C) along the failure plane BD can be expressed by:

$$C = cH / \sin \alpha \tag{4}$$

By computing the inertia force on an element of

where $\lambda = TV_s$, $\eta = TV_p$ are the wave lengths of shear and primary waves, respectively and $\xi = t - H/V_s$, $\psi = t - H/V_p$.

Equilibrium of forces in the horizontal direction [$\rightarrow +$] yields:

$$P_{pe} \cos \delta - cH \cot \alpha + Q_h - F \sin(\alpha + \varphi) = 0$$

or

$$F = \frac{P_{pe} \cos \delta - cH \cot \alpha + Q_h}{\sin(\alpha + \varphi)} \tag{7}$$

Equilibrium of forces in the vertical direction [$\downarrow +$] gives:

$$W + P_{pe} \sin \delta + P_{adh} - Q_v + cH - F \cos(\alpha + \varphi) = 0 \tag{8}$$

Substituting Eq. (7) into Eq. (8) and following some simplifications yields:

$$P_{pe} = \frac{W \sin(\alpha + \varphi) - Q_h \cos(\alpha + \varphi) - Q_v \sin(\alpha + \varphi) + cH [\sin(\alpha + \varphi)(1 + \tan \delta / \tan \varphi) + \cos(\alpha + \varphi) \cot \alpha]}{\cos(\alpha + \varphi + \delta)} \tag{9}$$

thickness dz in the failure wedge, and then integrating over depth (z), Choudhury and Nimbalkar (2007) derived the following expression for total horizontal inertia force on the failure wedge ABD :

In this type of problem, it is a common practice to present the solutions in a nondimensional form as $P_{pe}^* = P_{pe} / \gamma H^2$ and $c^* = c / \gamma H$. Thus, Eq. (9) can be rewritten as follows:

$$P_{pe}^* = \frac{\frac{1}{\gamma H^2} [W \sin(\alpha + \varphi) - Q_h \cos(\alpha + \varphi) - Q_v \sin(\alpha + \varphi)] + c^* [\sin(\alpha + \varphi)(1 + \tan \delta / \tan \varphi) + \cos(\alpha + \varphi) \cot \alpha]}{\cos(\alpha + \varphi + \delta)} \tag{10}$$

$$Q_h(t) = \frac{\lambda \gamma a_h}{4\pi^2 \tan \alpha} [2\pi H \cos \omega \xi + \lambda (\sin \omega \xi - \sin \omega t)] + \frac{\lambda \gamma a_h (f_a - 1)}{4\pi^3 H \tan \alpha} [2\pi H (\pi H \cos \omega \xi + \lambda \sin \omega \xi) + \lambda^2 (\cos \omega t - \cos \omega \xi)] \tag{5}$$

and the following expression for total vertical inertia force:

$$Q_v(t) = \frac{\eta \gamma a_v}{4\pi^2 \tan \alpha} [2\pi H \cos \omega \psi + \eta (\sin \omega \psi - \sin \omega t)] + \frac{\eta \gamma a_v (f_a - 1)}{4\pi^3 H \tan \alpha} [2\pi H (\pi H \cos \omega \psi + \eta \sin \omega \psi) + \eta^2 (\cos \omega t - \cos \omega \psi)] \tag{6}$$

For the case of fully smooth wall ($\delta = 0.0$), having substituted Eqs. (3), (5), (6) into Eq. (10), P_{pe}^* can be obtained by:

$$P_{pe}^* = \frac{\tan(\alpha + \varphi)}{2 \tan \alpha} - \frac{1}{8\pi^3 \tan \alpha} \left\{ \left[\left(\frac{\lambda}{H} \right) x_1 + \left(\frac{\lambda}{H} \right)^2 x_2 - \left(\frac{\lambda}{H} \right)^3 x_3 \right] a_h + \left[\left(\frac{\eta}{H} \right) y_1 + \left(\frac{\eta}{H} \right)^2 y_2 - \left(\frac{\eta}{H} \right)^3 y_3 \right] a_v \tan(\alpha + \varphi) \right\} + c^* [\tan(\alpha + \varphi) + \cot \alpha] \tag{11}$$

where

$$\begin{aligned}
 x_1 &= 4\pi^2 f_a \cos 2\pi \left(\frac{t}{T} - \frac{H}{\lambda} \right) \\
 x_2 &= 2\pi [(2f_a - 1) \sin 2\pi \left(\frac{t}{T} - \frac{H}{\lambda} \right) - \sin 2\pi \left(\frac{t}{T} \right)] \\
 x_3 &= 2(f_a - 1) [\cos 2\pi \left(\frac{t}{T} - \frac{H}{\lambda} \right) - \cos 2\pi \left(\frac{t}{T} \right)] \\
 y_1 &= 4\pi^2 f_a \cos 2\pi \left(\frac{t}{T} - \frac{H}{\eta} \right) \\
 y_2 &= 2\pi [(2f_a - 1) \sin 2\pi \left(\frac{t}{T} - \frac{H}{\eta} \right) - \sin 2\pi \left(\frac{t}{T} \right)] \\
 y_3 &= 2(f_a - 1) [\cos 2\pi \left(\frac{t}{T} - \frac{H}{\eta} \right) - \cos 2\pi \left(\frac{t}{T} \right)]
 \end{aligned}$$

The last term in Eq. (11) is that part of total passive force corresponds to backfill cohesion. It can be noticed in Eq. (10) that P_{pe}^* is a function of α , t/T , H/λ , and H/η . H/λ is the ratio of time needed for shear waves to pass through the wall height to the period of lateral shaking (T), and H/η is the ratio of time needed for primary waves to pass through the wall height to the period of lateral shaking (T). In the present investigation, the analyses were carried out for $H/\lambda = 0.3, 0.4, 0.5, 0.6$ and $H/\eta = 0.16, 0.21, 0.27, 0.32$, respectively. The minimum value of P_{pe}^* should be obtained by optimizing Eq. (10) with respect to α and t/T . The results of present computations are shown in the next section.

4 Results

The computations were carried out for $\varphi = 5\text{--}50^\circ$, $\delta = 0.0, 0.5$, $\alpha_v = 0, 0.5\alpha_h$, $\alpha_h = 0\text{--}0.5$, $c^* = 0, 0.05, 0.1, 0.2$, and $f_a = 1, 1.2, 1.4, 1.6, 1.8, 2$. It should be mentioned that because of the complexity of Eqs. (5)–(11), the main computations of this study are allocated to $\delta = 0.0$ case. Albeit, the effect of wall friction angle and adhesion on total passive force is shown in a separate table. Similarly to Shukla et al. (2009), the upper limit for horizontal seismic coefficient $\alpha_{h(cr)}$, is taken equal to:

$$\alpha_{h(cr)} = (1 - \alpha_v) \tan \varphi + 2c^* \tag{12}$$

Figure 2 shows the variation of nondimensional total passive force (P_{pe}^*) with horizontal seismic coefficient (α_h) for $c^* = 0.05$, $\varphi = 10^\circ, 20^\circ, 30^\circ$, $\delta = 0.0$, $\alpha_v = 0, 0.5\alpha_h$, $\alpha_h, H/\lambda = 0.3$ and $H/\eta = 0.16$. It is seen that the total passive force decreases with increase in both α_h and α_v . This decreasing effect is in fact due to the negative signs

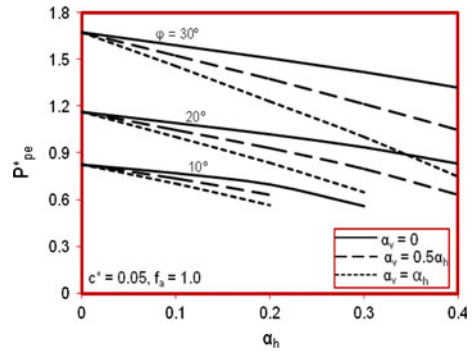


Fig. 2 Variation of nondimensional total passive force (P_{pe}^*) with horizontal seismic coefficient (α_h) for $c^* = 0.05$, $\varphi = 10^\circ, 20^\circ, 30^\circ$, $\delta = 0.0$, $\alpha_v = 0, 0.5\alpha_h, \alpha_h, H/\lambda = 0.3$ and $H/\eta = 0.16$

of horizontal and vertical inertia forces (Q_h, Q_v , respectively) in Eq. (10). Figure 2 also depicts that the influence of α_v is more significant for higher values of α_h .

The variation of P_{pe}^* with c^* for different values of friction angle (φ) and $\delta = 0.0$, $\alpha_h = 0.1$, $\alpha_v = 0.5\alpha_h$, $H/\lambda = 0.3$ and $H/\eta = 0.16$ is shown in Fig. 3. It is observed that c^* has an increasing effect on P_{pe}^* . The reason of this increase is the positive sign of cohesion in Eq. (10). In other words, the backfill soil cohesion improves the shear resistance of the backfill layer and therefore, a greater force is required to induce the passive failure. Figure 3 also depicts that P_{pe}^* is greater for higher friction angles and as nondimensional cohesion (c^*) increases, the curves corresponding to different friction angles move away from each other.

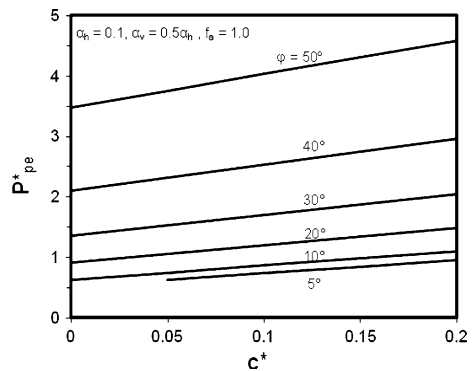


Fig. 3 Variation of nondimensional total passive force (P_{pe}^*) with nondimensional cohesion (c^*) for $\delta = 0.0$, $\alpha_h = 0.1$, $\alpha_v = 0.5\alpha_h, H/\lambda = 0.3$ and $H/\eta = 0.16$

Table 1 Values of (P_{pe}^*) for $f_a = 1.0-2.0$, $\varphi = 5^\circ-50^\circ$, $\delta = 0.0$, $c^* = 0.05$, $\alpha_h = 0.1$, $\alpha_v = 0$, $H/\lambda = 0.3$ and $H/\eta = 0.16$

f_a	φ (degree)					
	5	10	20	30	40	50
1.0	0.648	0.771	1.095	1.593	2.415	3.923
1.2	0.638	0.762	1.086	1.582	2.402	3.906
1.4	0.628	0.753	1.076	1.570	2.388	3.889
1.6	0.617	0.743	1.066	1.559	2.374	3.871
1.8	0.605	0.733	1.056	1.547	2.360	3.854
2.0	0.591	0.723	1.046	1.536	2.346	3.836

Table 1 shows the influence of amplification factor (f_a) on the nondimensional total passive force (P_{pe}^*) for $\varphi = 5^\circ-50^\circ$, $\delta = 0.0$, $c^* = 0.05$, $\alpha_h = 0.1$, $\alpha_v = 0$, $H/\lambda = 0.3$ and $H/\eta = 0.16$. It is noticed that P_{pe}^* decreases very slightly with increase in amplification factor (f_a). In other words, the effect of f_a on nondimensional total passive force (P_{pe}^*) is negligible compared to the effect of other parameters such as φ , c , α_h and α_v . The effect of wall friction angle (δ) on P_{pe}^* is presented in Table 2 for $\varphi = 20^\circ, 30^\circ$, $\delta = 0.0, 0.5\varphi$, φ , $c^* = 0.00-0.20$, $\alpha_h = 0.0, 0.1, 0.2$, $\alpha_v = 0.5\alpha_h$, $H/\lambda = 0.3$ and $H/\eta = 0.16$. It is observed that,

as expected, in all cases the wall friction angle has an increasing effect on P_{pe}^* . Choudhury and Nimbalkar (2005) observed the similar results for the specific case of $c = 0$. Table 3 shows the effect of variations in values of nondimensional parameters H/λ and H/η on P_{pe}^* for $\varphi = 20^\circ, 30^\circ$, $\delta = 0.5\varphi$, $c^* = 0.05$, $\alpha_h = 0.1-0.4$, and $\alpha_v = 0.5\alpha_h$. It is seen that the magnitude of total passive force increases with increase in H/λ and H/η , i.e., by increase in the velocity of primary and shear waves, the magnitude of P_{pe}^* decreases. It is a significant finding that demonstrates the superiority of pseudo-dynamic method over pseudo-static method.

Figure 4 shows the variation of failure plane inclination angle with the horizontal plane (α) with seismic horizontal coefficient (α_h) for $a_v = 0, 0.5a_h$, a_h , $\varphi = 10^\circ, 30^\circ, 50^\circ$, $\delta = 0.0$, $c^* = 0.05$, $H/\lambda = 0.3$ and $H/\eta = 0.16$. It is seen that (1) the inclination angle (α) is lower for greater values of φ ; (2) both the horizontal seismic coefficient (a_h) and vertical seismic coefficient (a_v) have decreasing effect on angle (α). Figure 4 also shows that the rate of decrease in angle (α) with respect to a_h is much more considerable for lower friction angles. For example, for $a_h = 0.2$ and $a_v = 0$ the value of α (in degrees) is

Table 2 Effect of wall friction angle (δ) on P_{pe}^* ($f_a = 1.0$, $\alpha_v = 0.5\alpha_h$, $H/\lambda = 0.3$ and $H/\eta = 0.16$)

φ	δ	α_h	P_{pe}^*					
			$c^* = 0.00$	$c^* = 0.05$	$c^* = 0.10$	$c^* = 0.15$	$c^* = 0.20$	
20°	0	0	–	1.163	1.305	1.448	1.591	
		0.1	0.907	1.051	1.194	1.337	1.480	
		0.2	0.784	0.932	1.077	1.222	1.366	
	0.5 φ	0	1.318	1.541	1.765	1.988	2.212	
		0.1	1.145	1.370	1.594	1.818	2.042	
		0.2	0.959	1.189	1.416	1.642	1.867	
	φ	0	1.762	2.100	2.439	2.777	3.115	
		0.1	1.500	1.839	2.178	2.516	2.855	
		0.2	1.219	1.566	1.909	2.249	2.589	
	30°	0	0	–	1.673	1.846	2.020	2.193
			0.1	1.353	1.527	1.700	1.873	2.047
			0.2	1.200	1.376	1.550	1.724	1.898
0.5 φ		0	2.488	2.832	3.178	3.522	3.867	
		0.1	2.193	2.538	2.883	3.227	3.572	
		0.2	1.889	2.236	2.582	2.928	3.274	
φ		0	3.053	3.493	3.933	4.373	4.813	
		0.1	2.669	3.110	3.551	3.991	4.431	
		0.2	2.278	2.721	3.162	3.604	4.045	

Table 3 Effect of variations of H/λ and H/η values on P_{pe}^* ($f_a = 1.0$, $\alpha_v = 0.5\alpha_h$, $\delta = 0.5$, φ , $c^* = 0.05$)

φ	α_h	P_{pe}^*			
		$H/\lambda = 0.3$	$H/\lambda = 0.4$	$H/\lambda = 0.5$	$H/\lambda = 0.6$
20°	0.1	1.370	1.385	1.404	1.425
	0.2	1.189	1.222	1.262	1.305
	0.3	0.991	1.047	1.111	1.180
	0.4	0.744	0.843	0.945	1.048
30°	0.1	2.538	2.564	2.595	2.631
	0.2	2.236	2.290	2.354	2.426
	0.3	1.925	2.009	2.109	2.219
	0.4	1.597	1.716	1.856	2.007

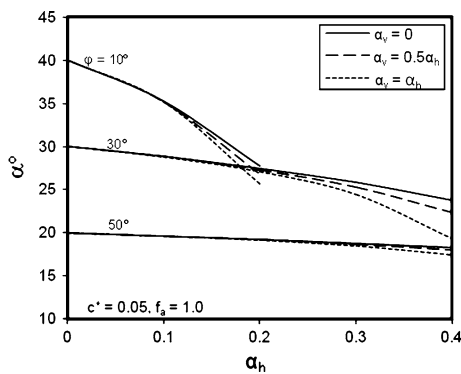


Fig. 4 Variation of failure plane inclination angle (α) with seismic horizontal coefficient (α_h) for $\varphi = 10^\circ, 30^\circ, 50^\circ$, $\delta = 0.0$, $c^* = 0.05$, $H/\lambda = 0.3$ and $H/\eta = 0.16$

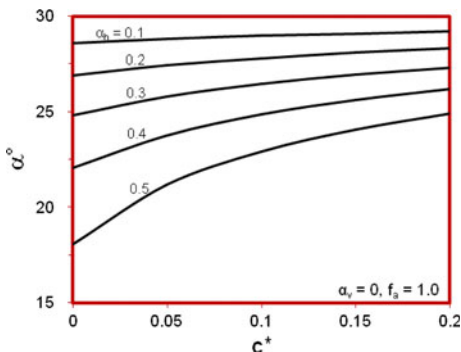


Fig. 5 Influence of nondimensional cohesion (c^*) on failure plane inclination angle (α) for $\varphi = 30^\circ$, $\delta = 0.0$, $\alpha_v = 0.0$, $H/\lambda = 0.3$ and $H/\eta = 0.16$

69% of its static value for $\varphi = 10^\circ$, whereas for $\varphi = 50^\circ$ and the same set of parameters, α (in degrees) is just about 96% of its static value.

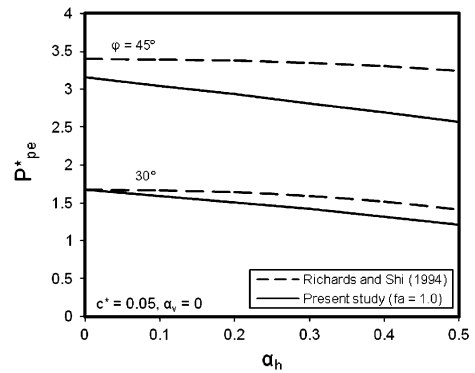


Fig. 6 Comparison of variation of nondimensional total passive force (P_{pe}^*) with horizontal seismic coefficient (α_h) obtained by the present study with earlier investigations

The influence of nondimensional cohesion (c^*) on failure plane inclination angle (α) is clearly shown in Fig. 5 for various values of horizontal seismic coefficient (α_h). It is observed that the angle (α) increases with increase in c^* , i.e., the area of the failure zone decreases. The reason seems to be due to the fact that the increase in backfill cohesion enhances the shear resistance of the backfill soil. Thus, the smaller portion of the backfill soil is subjected to collapse. The rate of the increase in angle (α) is much more considerable for higher values of α_h and lower values of c^* . For example, the value of α for $c^* = 0.2$ is 6.8° greater than $c^* = 0.0$ when $\alpha_h = 0.5$; whereas it is just as small as 0.6° greater when $\alpha_h = 0.1$ for the same values of c^* .

5 Comparison of Results

Fig. 6 shows the variation of P_{pe}^* with horizontal seismic coefficient (α_h) for $\varphi = 30^\circ, 45^\circ$, $\delta = 0.0$, $c^* = 0.05$, $\alpha_v = 0.0$, $H/\lambda = 0.3$ and $H/\eta = 0.16$ obtained by the present study compared to the results of Richards and Shi (1994). It is noticed that the results of present study propose the lower results. i.e., the present study is more conservative than Richards and Shi (1994) solution. Figure 6 also depicts that for higher values of α_h , the difference between present study and those of Richards and Shi (1994) increases.

6 Conclusions

The pseudo-dynamic approach was used to compute seismic passive earth thrust on the retaining walls

with cohesive-frictional backfills. It is possible to consider time-dependent nature of earthquake acceleration in this method. The slip surface behind the wall was assumed to be planar. The results of present analysis shows that the total passive force (P_{pe}^*) decreases with increase in both horizontal seismic coefficient (α_h) and vertical seismic coefficient (α_v). The influence of α_v is more significant for higher values of α_h ; the backfill cohesion has an increasing effect on P_{pe}^* ; the effect of amplification factor (f_a) on nondimensional total passive force (P_{pe}^*) is negligible; the wall friction angle has an increasing effect on P_{pe}^* ; by increase in the velocity of primary and shear waves, the magnitude of P_{pe}^* decreases; the inclination angle (α) is lower for greater values of φ and both the horizontal seismic coefficient (a_h) and vertical seismic coefficient (a_v) have decreasing effect on angle (α); the angle (α) increases with increase in backfill cohesion. The rate of this increase is much more considerable for higher values of a_h and lower values of cohesion. The comparison of results of present study with the results of earlier study shows the similar trend of total passive force variations with available parameters.

References

- Choudhury D, Nimbalkar SS (2005) Seismic passive resistance by pseudo-dynamic method. *Geotechnique* 55(9):699–702
- Choudhury D, Nimbalkar SS (2007) Seismic rotational displacement of gravity walls by pseudo-dynamic method: passive case. *Soil Dyn EQ Engrg* 27(3):242–249
- Das BM (1993) Principles of soil dynamics. PWS-KENT Publishing Company, Boston
- Ghahramani A, Anvar SA (2004) Dynamic earth pressure simulation by single degree of freedom system. *Proceedings, 13th World Conf EQ Engrg, Vancouver*
- Kumar J (2001) Seismic passive earth pressure coefficients for sand. *Can Geotech J* 38(4):876–881
- Mononobe N, Matsuo H (1929) On the determination of earth pressures during earthquakes. *Proceedings, World Engr. Conf 9, Tokyo*: 274–280
- Okabe S (1926) General theory of earth pressure. *J Japanese Soc Civ Eng Tokyo Japan* 12(1)
- Richards R, Shi X (1994) Seismic lateral pressures in soils with cohesion. *J Geotech Eng ASCE* 120(7):1230–1251
- Saran S, Gupta RP (2003) Seismic earth pressures behind retaining walls. *Indian Geotech J* 33(3): 195–213
- Saran S, Prakash S (1968) Dimensionless parameters for static and dynamic earth pressures behind retaining walls. *Indian Geotech J* 7(3):295–310
- Soubra AH (2000) Static and seismic passive earth pressure coefficients on rigid retaining structures. *Can Geotech J* 37(2):463–478
- Steedman RS, Zeng X (1990) The influence of phase on the calculation of pseudo-static earth pressure on a retaining wall. *Geotechnique* 40(1):103–112