

Seismic Response Analysis of Pile Foundations

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Received: 18 January 2008 / Accepted: 15 February 2010 / Published online: 12 March 2010
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Abstract A quasi-3D continuum method is presented for the dynamic nonlinear effective stress analysis of pile foundation under earthquake excitation. The method was validated using data from centrifuge tests on single piles and pile groups in liquefiable soils conducted at the University of California at Davis. Some results from this validation studies are presented. The API approach to pile response using p - y curves was evaluated using the quasi-3D method and the results from simulated earthquake tests on a model pile in a centrifuge. The recommended API stiffnesses appear to be much too high for seismic response analysis under strong shaking, but give very good estimates of elastic response.

Keywords 3D seismic analysis ·
Effective stress analysis · p - y Curve ·
Piles · Simulation

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1 Introduction

Seismic soil–structure interaction analysis involving pile foundations is one of the more complex problems in geotechnical earthquake engineering. The analysis involves modeling soil–pile interaction, pile-to-pile interaction, inertial interaction and the nonlinear hysteretic behavior of the soil. Most of the current methods for analyzing pile foundations can be categorized into two main groups. One is based on the elastic continuum models and the other on the Winkler springs model. The elastic continuum models are suitable for studying the response under low excitations only when the dynamic response is approximately elastic. They are not suitable for analyses under strong shaking. The reduction in soil stiffness and the increase in damping associated with strong shaking are sometimes modeled crudely in these elastic methods by making arbitrary reductions in shear moduli and arbitrary increases in viscous damping.

The most common approach for the analysis of pile foundations is to use nonlinear Winkler springs with dashpots to simulate soil stiffness and damping. Some organizations such as the American Petroleum Institute (API) give specific guidance for the development of nonlinear pressure-deflection (p - y) curves with depth as a function of soil properties. These recommendations are based on static or slow cyclic loading field tests. The API p - y curves are most widely used in engineering practice. However, their

effectiveness of Winkler models and API (1993) recommended p - y curves in capturing the dynamic response has not been adequately addressed.

Dynamic nonlinear finite element analysis in the time domain using the full 3-dimensional wave equations is not feasible for engineering practice at present because of the time needed for the computations. However, by relaxing some of the boundary conditions associated with a full 3D analysis, it is possible to get reliable solutions for nonlinear response of pile foundations with greatly reduced computational effort. The results are very accurate for excitation due to horizontally polarized shear waves propagating vertically. A full description of this total stress based quasi-3D FEM method, including numerous validation studies, has been presented by Wu and Finn (1997b). The method is incorporated in the computer program PILE3D.

A comprehensive method of dynamic analysis based on the Winkler model was developed to evaluate the effectiveness of the Winkler model and API recommended p - y curves in capturing the dynamic response of a single pile. The method was incorporated in the computer program called PILE-PY. Data from centrifuge tests conducted at the California Institute of Technology on a single pile in dry sand under dynamic loading was analyzed using both PILE-3D and PILE-PY with the API (1993) recommended p - y curves. The results from these analyses are presented in this paper.

The total stress based method was extended for effective stress analysis by incorporating a model for the porewater pressure generation due to earthquake shaking. This effective stress method was also implemented in the computer program PILE3D. The effective stress version of PILE3D-Eff was validated by simulating dynamic centrifuge tests on single piles and pile groups in liquefiable soils. These tests were run at the University of California at Davis and have been reported by Wilson et al. (1995, 1997). This paper presents some of the results from these validation studies.

2 Outline of Simplified 3D Seismic Analysis of Pile Foundations

The basic assumptions of the simplified 3D analysis are illustrated in Fig. 1. Under vertically propagating

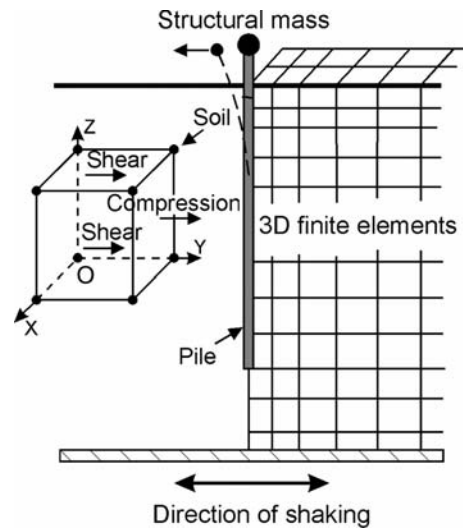


Fig. 1 Quasi-3D model of pile-soil response

shear waves the soil undergoes primarily shearing deformations in xOy plane except in the area near the pile where extensive compressional deformations develop in the direction of shaking. These compressional deformations generate shearing deformations in yOz plane. Therefore, the assumptions are made that dynamic response is dominated by the shear waves in the xOy and yOz planes and the compressional waves in the direction of shaking, Y . Deformations in the vertical direction and normal to the direction of shaking are neglected. Comparisons with full 3D elastic solutions confirm that these deformations are relatively unimportant for horizontal shaking Wu and Finn (1997a). Applying dynamic equilibrium in Y -direction, the dynamic governing equation of the soil continuum in free vibration is written as

$$\rho_s \frac{\partial^2 v}{\partial t^2} = G \frac{\partial^2 v}{\partial x^2} + \theta G \frac{\partial^2 v}{\partial y^2} + G \frac{\partial^2 v}{\partial z^2} \quad (1)$$

where G is the shear modulus, v is the displacement in the direction of shaking, ρ_s is the mass density of soil, and θ is a coefficient related to Poisson's ratio of the soil.

Piles are modeled using ordinary Eulerian elastic beam theory. Bending of the piles occurs only in the yOz plane. Dynamic soil-pile-structure interaction is maintained by enforcing displacement compatibility between the pile and the soil. The finite element code PILE3D and PILE3D-Eff incorporates these concepts of dynamic soil-pile-structure interaction. An 8-node

brick element is used to represent the soil and a 2-node beam element is used to simulate the piles, as shown in Fig. 1. The global dynamic equilibrium equation for the pile soil system is written in matrix form as

$$[M]\{\ddot{v}\} + [C]\{\dot{v}\} + [K]\{v\} = -[M]\{I\} \cdot \ddot{v}_o(t) \quad (2)$$

in which $\ddot{v}_o(t)$ is the base acceleration, $\{I\}$ is a unit column vector, and $\{\ddot{v}\}$, $\{\dot{v}\}$ and $\{v\}$ are the relative nodal acceleration, velocity and displacement, respectively. $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices of the soil-pile system vibration in the horizontal direction.

Direct step-by-step integration using the Wilson- θ method is employed in PILE3D and PILE3D-Eff to solve the equations of motion in Eq. 2. The non-linear hysteretic behavior of soil is modeled by using an incrementally linear method in which properties are varied continuously as a function of soil strain. Additional features such as a tension cut-off and shearing failure are incorporated in the program to simulate the possible gapping between soil and pile near the soil surface and yielding in the near field.

For porewater pressure generation due to shaking, the effective stress model developed by Martin et al. (1975) and modified later by Byrne (1991) was used. During dynamic analysis, the soil properties are changed continuously to reflect the effects of the seismic porewater pressures on moduli and strength.

3 Winkler Model of a Single Pile for Dynamic Analysis

The Winkler model of a single pile is shown in Fig. 2. The near field interaction between pile and soil is modeled by nonlinear springs and dashpots. The near field pile-soil system, together with any structural mass included with the pile, are excited by the free field motions applied to the end of each Winkler spring.

The equation governing the motion of the pile is given in Eq. 3

$$EI \frac{\partial^4 v}{\partial x^4} + \rho A \left(\frac{\partial^2 v}{\partial t^2} + \frac{\partial^2 v_g}{\partial t^2} \right) + c \left(\frac{\partial v}{\partial t} - \frac{\partial v_{ff}}{\partial t} \right) + k_h (v - v_{ff}) = 0 \quad (3)$$

where E , A , I and ρ are Young's modulus, area, second moment of area and mass density of the pile, respectively, k_h is the soil reaction coefficient, c is the

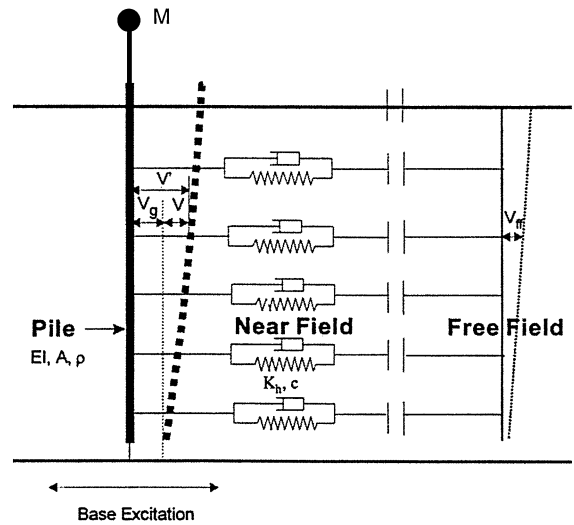


Fig. 2 Winkler model of pile–soil response

equivalent dashpot coefficient, v is the relative displacement of pile with respect to the base excitation v_g , v_{ff} is the relative free field displacement with respect to the base excitation and x is the depth. p – y curves are used for the nonlinear Winkler springs and they are modeled following an incrementally linear elastic tangent stiffness approach. The hysteretic damping is automatically taken into account by this approach and, for the radiation damping, the model proposed by Gazetas et al. (1993) is used. The free field motions at the desired elevations in the soil layer are obtained by conducting a parallel free field analysis using a column of plane strain rectangular elements. The method of determining the single pile dynamic response using a Winkler model with a nonlinear springs and dashpots with a parallel free field analysis is implemented in the computer program PILE-PY.

4 Simulation of Centrifuge Test on a Single Pile in Dry Sand

PILE3D and PILE-PY with API (1993) recommended p – y curves were used to analyze the seismic response of a single pile in a centrifuge test which was conducted at the California Institute of Technology by Gohl (1991). Details of the test may also be found in a paper by Finn and Gohl (1987). Figure 3 shows the soil–pile–structure system used in the test. The

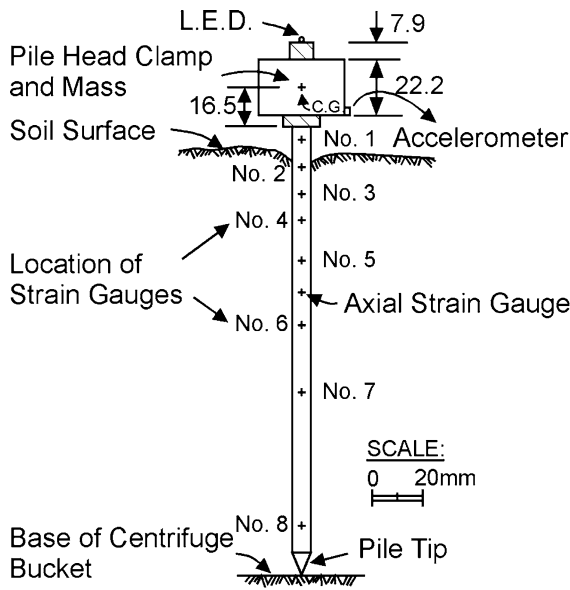


Fig. 3 The layout of the centrifuge test for a single pile

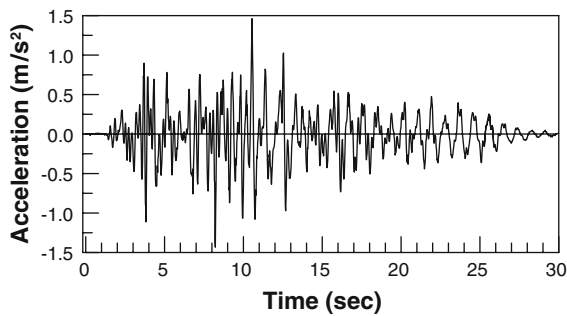


Fig. 4 Input acceleration time history

system was subjected to a nominal centrifuge acceleration of 60 g. A horizontal acceleration record shown in Fig. 4 with a peak acceleration of 0.158 g is input at the base of the system. Figure 5 shows the finite element mesh used in the PILE3D analysis.

4.1 Results from PILE3D Analysis

The distribution of shear moduli was measured prior to shaking, while the centrifuge was in flight. Therefore, accurate initial stiffnesses of the pile foundation could be calculated. The dynamic shear strains generated by strong seismic shaking causes changes in shear moduli and damping ratios. This effect is illustrated in Fig. 6 by the distribution of shear moduli at a depth of 2.1 m in the soil around

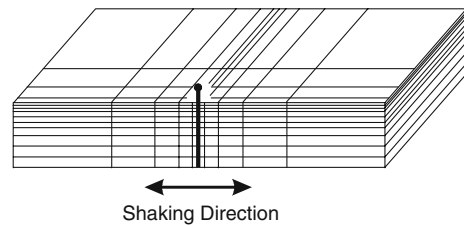


Fig. 5 Finite element mesh for analysis of single pile

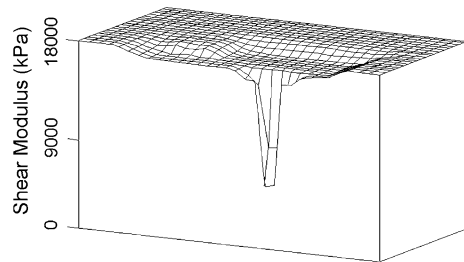


Fig. 6 Distribution of shear moduli around pile at a depth of 2.1 m at time 12.58 s into earthquake

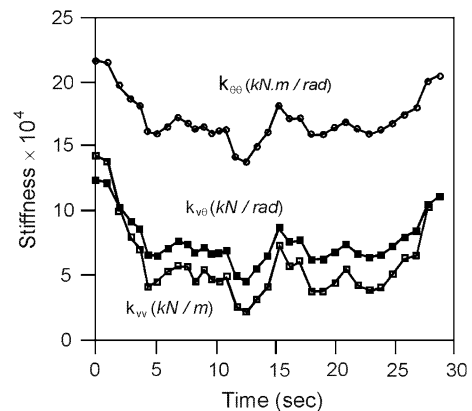


Fig. 7 Dynamic stiffnesses for the single pile

the pile at a time $T = 12.58$ s. The distribution of shear moduli and damping are both time- and space-dependent. This dependence results in a corresponding time-dependence of the stiffnesses of the pile. Dynamic impedances as a function of time were computed using the time- and space-dependent nonlinear shear moduli. Harmonic loads with amplitude of unity were applied at the pile head, and the resulting equations were solved to obtain the complex valued pile impedances. The impedances were evaluated at the surface of the sand. The time histories of the stiffnesses are shown in Fig. 7.

The dynamic stiffnesses (real parts of the impedances) of the single pile during the specified input motions decreased dramatically as the level of shaking increased (Fig. 7). The dynamic stiffnesses experienced their lowest values between about 10 and 14 s, when the maximum accelerations occurred at the pile head. It can be seen that the lateral stiffness component K_{vv} decreased more than the rotational stiffness $K_{\theta\theta}$ or the coupled lateral-rotational stiffness $K_{v\theta}$. The equivalent damping coefficients increased as the level of shaking increased because the hysteretic damping of the soil increased with the level of shaking.

The time histories of stiffnesses in Fig. 7, show clearly the difficulties in selecting a single spring value to represent the lateral or rotational stiffness of a pile foundation. To make a valid selection, one would need to know which segment of the ground motion was most critical in controlling the seismic response of the structure. A spring based on the minimum lateral stiffness would represent the mobilized stiffness during the period of very strong shaking and would be more critical for long period structures. Clearly, the time variation in stiffness and damping provides a useful guide to the selection of the discrete springs and dashpots required by commercial structural analysis software.

The computed and measured moment distributions along the pile at the instant of peak pile head deflection are shown in Fig. 8. The moments computed by PILE3D agree quite well with the measured moments. The peak moment predicted by PILE3D is 344 kNm compared with a measured peak value of 325 kNm.

4.2 Results from PILE-PY Analysis Using API Recommended p - y Curves

Analysis was conducted using PILE-PY and the API (1993) recommended p - y curves to simulate the centrifuge test. Figure 9 shows the comparison of computed free field response by PILE3D and PILE-PY with the measured response. The predicted response by both PILE3D and PILE-PY agree well with the measured response although the PILE3D prediction was slightly better.

The distribution of bending moments was also determined using the p - y curves prescribed by the American Petroleum Institute (API). These p - y curves are defined by the equation,

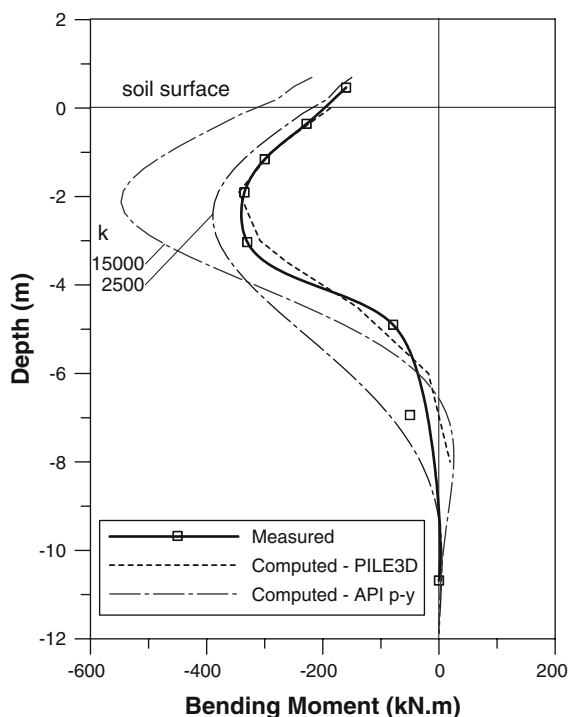


Fig. 8 Comparison of measured and computed bending moment profiles

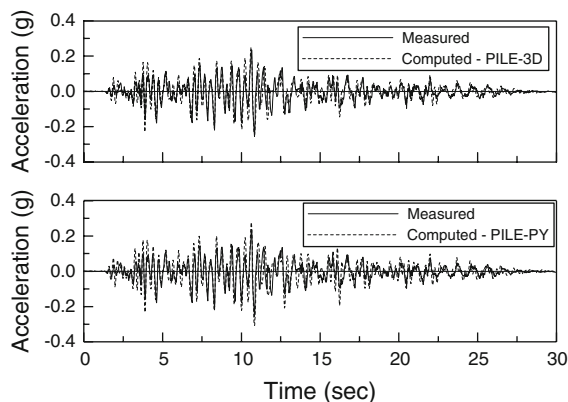


Fig. 9 Comparison of measured and computed free field acceleration time histories

$$P = 0.9 p_u \tanh [(kHy)/(0.9 p_u)] \tag{4}$$

where p_u is the ultimate bearing capacity at depth H , k is the initial modulus of subgrade reaction, y is the lateral deflection, and H is the depth. The relative density of the sand surrounding the pile is 38%. This corresponds to a k of approximately 15,000 kN/m³ according to the API recommendations. The analysis

shows that the p - y curves were much too stiff under strong shaking. The distribution of moments for a value of $k = 15,000 \text{ kN/m}^3$ is shown in Fig. 8. A reasonable approximation to the peak moment in the pile is obtained using $k = 2,500 \text{ kN/m}^3$, which is only 1/6 of the value recommended by API (1993).

In another test in the same sand, run at a very low peak acceleration of 0.04 g, the API stiffness $k = 15,000 \text{ kN/m}$, gives a very good approximation to the measured bending moments (Fig. 10). The response in this case was almost elastic and the initial stiffness controls the response. These results suggest that the initial stiffness of the API p - y curves is reasonable, but that the curves do not head away fast enough from the initial tangent at the origin. Therefore, stiffness at close to the initial value is likely being mobilized over too large a displacement range under strong shaking.

The analysis was repeated following the suggestion of Gazetas and Dobry (1984) that the k value should vary with the depth H , with $kH = E_{\max}$ where E_{\max} is the Young's modulus. The shear wave velocity distribution in centrifuge test was measured during testing and, therefore, accurate measurements of the

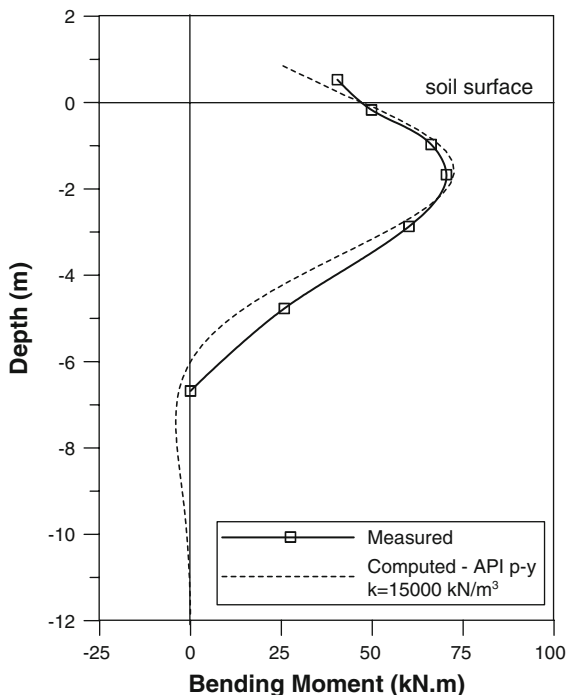


Fig. 10 Comparison of measured and computed pile moments for near elastic response using API procedure

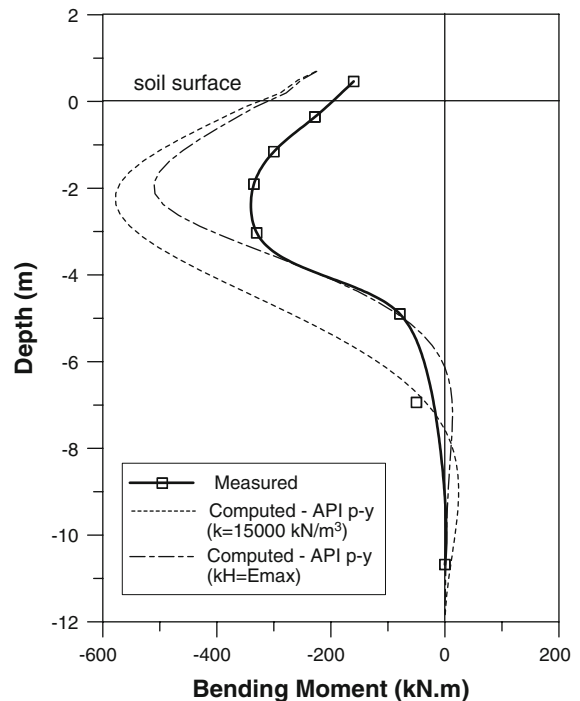


Fig. 11 Comparison of computed and measured moments when $kH = E_{\max}$

shear modulus, G , were available. E_{\max} was estimated based on this G and for a Poisson's ratio of 0.4. The resulting moment distribution is shown in Fig. 11. This approach, to estimating the initial stiffness gives somewhat better approximation to the measured bending moments when using the API p - y curves.

Note that the Fig. 9 shows that the free field response predicted by PILE-PY is good. Therefore, the free field response input to the Winkler model in PILE-PY did not apparently contribute to the poor agreement between the measured and computed pile response under strong shaking.

5 Simulation of Centrifuge Tests in Liquefiable Sand

Dynamic centrifuge tests of pile supported structures in liquefiable sand were performed on the large centrifuge at University of California at Davis, California. The models consisted of two structures supported by single piles, one structure supported by a 2×2 pile group and one structure supported by a 3×3 pile group. The typical arrangement of

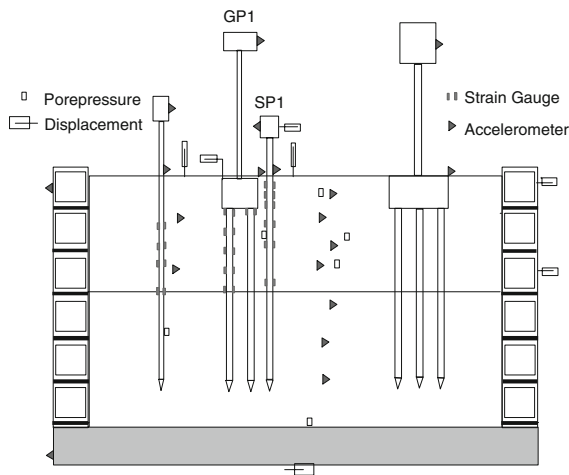


Fig. 12 Layout of models for centrifuge tests

structures and instrumentation is shown in Fig. 12. Full details of the centrifuge tests can be found in Wilson et al. (1997). Only the single pile system (SP1) and the (2 × 2) pile group (GP1) are studied here.

The model dimensions and the arrangement of bending strain gauges in systems SP1 and GP1 are shown in Figs. 13 and 14, respectively. In the (2 × 2) group pile system GP1, the mass and the length of the column that carries the superstructure mass are 233 Mg and 10.9 m, respectively. Model tests were performed at a centrifugal acceleration of 30 g.

Sand was air pluviated to relative densities of 75–80% in the lower layer and 55% in the upper

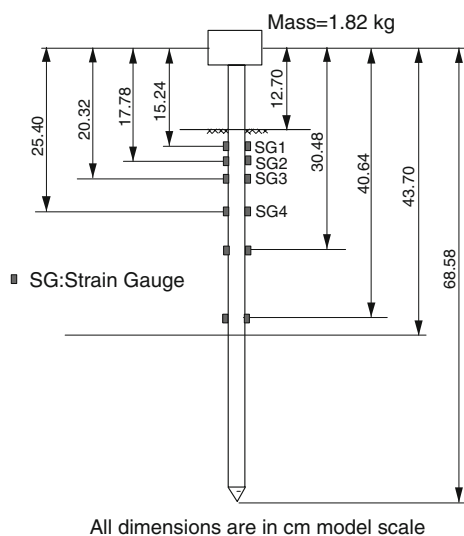


Fig. 13 Instrumented pile for single pile test

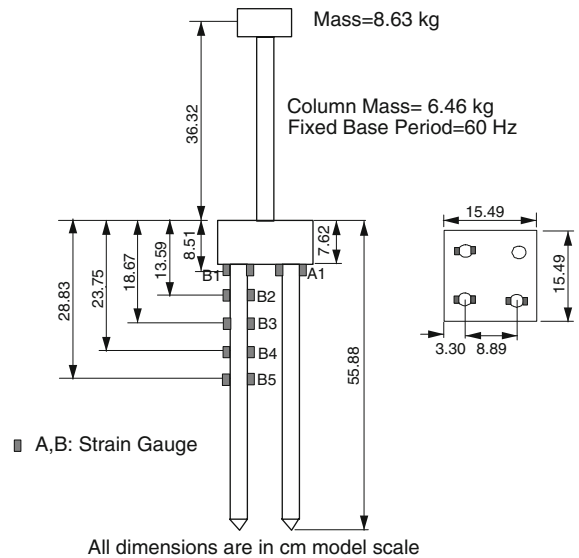


Fig. 14 Instrumented test piles and details of superstructure for 2 × 2 pile group

layer. Prior to saturation, any entrapped air was carefully removed. The container was then filled with hydroxy-propyl methyl-cellulose and water mixture under vacuum. The viscosity of this pore fluid is about ten times greater than pure water to ensure proper scaling. Saturation was confirmed by measuring the p-wave velocity from the top to the bottom of the soil profile near the container center. The measured velocities were high enough (on the order of 1,000 m/s) to indicate that the sample was very close to saturated (Wilson 1998). The responses of the single pile and the 2 × 2 pile group to the Santa Cruz acceleration record obtained during the 1989 Loma Prieta earthquake, scaled to 0.49 g is described and analyzed here.

6 Effective Stress Dynamic Analysis of Single Pile Using PILE3D-Eff

6.1 Finite Element Model of the Single Pile-superstructure System

The finite element mesh used in the analysis is shown in Fig. 15. The finite element model consists of 1,649 nodes and 1,200 soil elements. The upper sand layer which is 9.1 m thick was divided into 11 layers and the lower sand layer which is 11.4 m thick was divided into

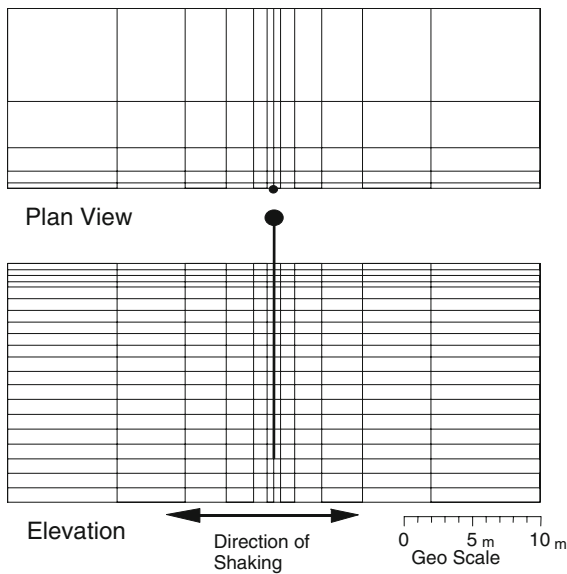


Fig. 15 Finite element mesh for single pile

9 layers. The single pile was modeled with 28 beam elements. 17 beam elements were within the soil strata and 11 elements were used to model the free standing length of the pile above the pile. The superstructure mass was treated as a rigid body and its motion is represented by a concentrated mass at the center of gravity. A rigid beam element was used to connect the superstructure to the pile head. The soil profile consists of two level layers of Nevada sand, each approximately 10 m thick at prototype scale. Nevada sand is a uniformly graded fine sand with a coefficient of uniformity of 1.5 and mean grain size of 0.15 mm.

6.2 Soil and Pile Properties

The small strain shear moduli G_{max} , were estimated using the formula proposed by Seed and Idriss (1970).

$$G_{max} = 21.7 k_{max} P_a (\sigma'_m / P_a)^{0.5} \quad (5)$$

in which k_{max} is a constant which depends on the relative density of the soil, σ'_m is the initial mean effective stress and P_a is the atmospheric pressure. The constant k_{max} was estimated using the approximation suggested by Byrne (1991). The program PILE3D accounts for the changes in shear moduli and damping ratios due to dynamic shear strains at the end of each time increment. The shear strain dependency of the shear moduli and damping of the soil was defined by

the curves suggested by Seed and Idriss (1970) for sand. The friction angles of the upper and the lower layers were taken as 35° and 40° , respectively.

6.3 Porewater Pressure Effects

The seismic porewater pressures were generated in each individual element depending on the current volumetric strain prevailing in that element. The soil properties, the moduli and the strength were modified continuously to account for the effects of the changing seismic porewater pressures.

6.4 Earthquake Input Motion

The Santa Cruz acceleration record was scaled to 0.49 g and used as input to the shake table. The base accelerations of the model were measured at the east and west ends of the base of the model container. Wilson et al. (1997) showed that both accelerations agreed very well. The base input acceleration is shown in Fig. 16.

6.5 Results of Single Pile Analysis

6.5.1 Free Field Response

Figure 17 shows the comparison of measured and computed free field response at three depths and Fig. 18 shows the corresponding acceleration response spectra. They agree very well.

6.5.2 Porewater Pressure Response

Figure 19 shows comparisons between measured and computed porewater pressures at three different

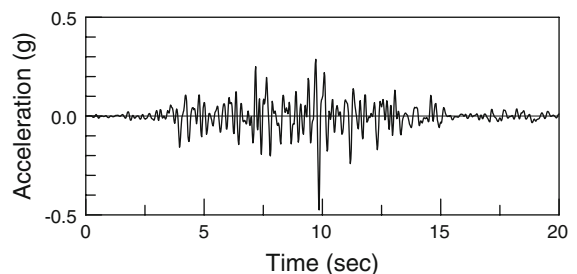


Fig. 16 Input acceleration time history

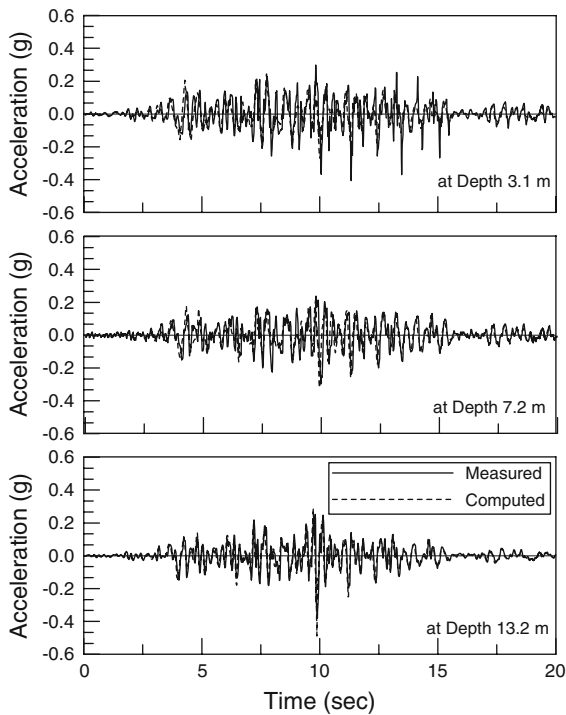


Fig. 17 Comparison of measured and computed free field acceleration time histories at three depths

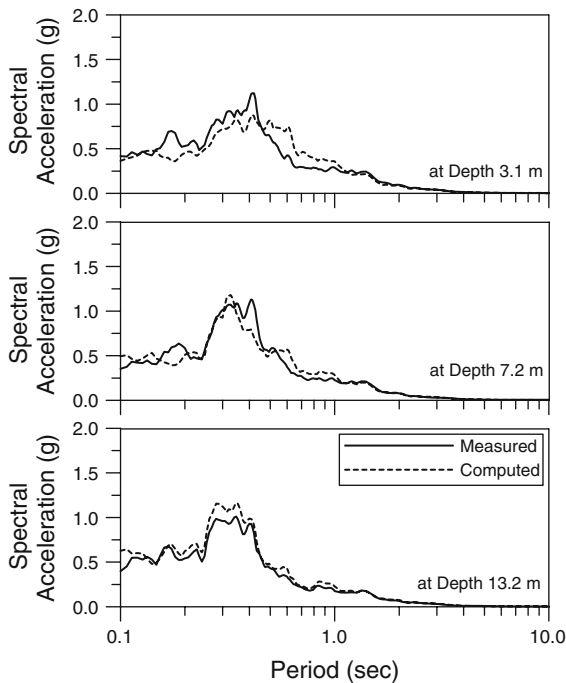


Fig. 18 Comparison of response spectra for the measured and computed free field accelerations at three depths

depths; 1.14, 4.56, and 6.78 m in the free field. There is generally good agreement between the measured and computed porewater pressure responses.

6.5.3 Bending Moment Response

Figure 20 shows the measured and computed bending moment time histories at three different depths; 0.76, 1.52 and 2.29 m. Generally there is a very good agreement between the measured and computed time histories. Figure 21 shows the profiles of measured and computed maximum bending moments with depth. The comparison between measured and computed moments is fairly good, although the maximum moment is overestimated by 10–20% between 1 and 4 m depths.

6.5.4 Acceleration Response

Figure 22 shows the measured and computed acceleration response of the superstructure and Fig. 23 shows the corresponding response spectra. The measured and computed responses agree very well.

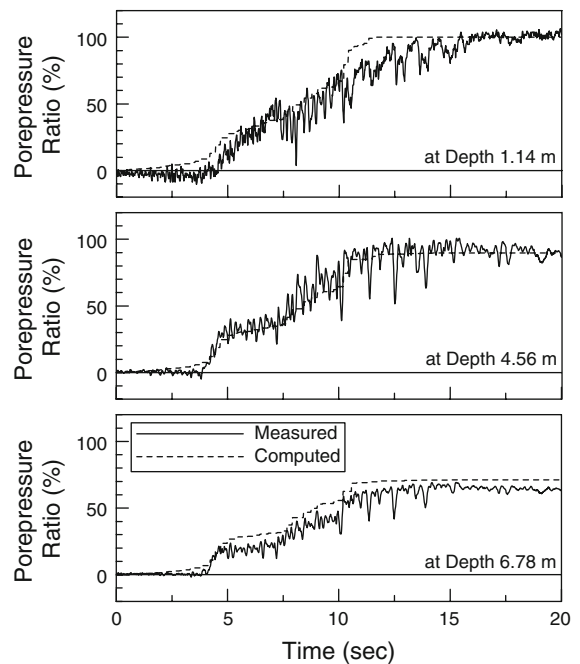


Fig. 19 Comparison of measured and computed porewater pressure time histories at three depths

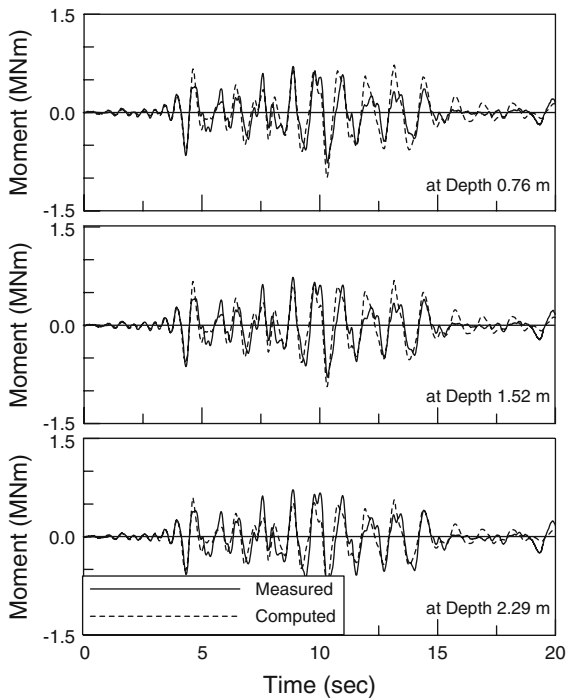


Fig. 20 Comparison of measured and computed bending moment time histories at three depths

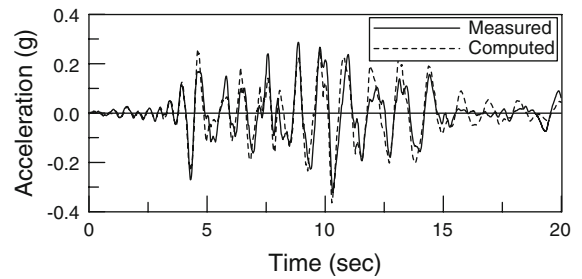


Fig. 22 Comparison of measured and computed superstructure acceleration time histories

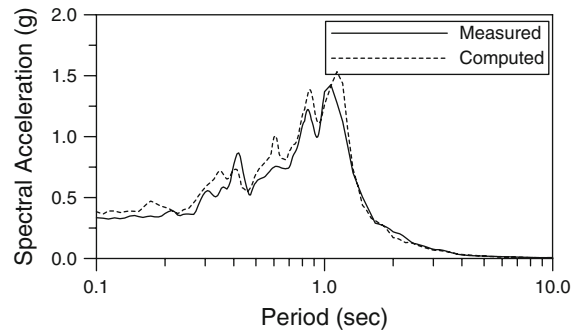


Fig. 23 Comparison of acceleration response spectra of the measured and computed superstructure accelerations

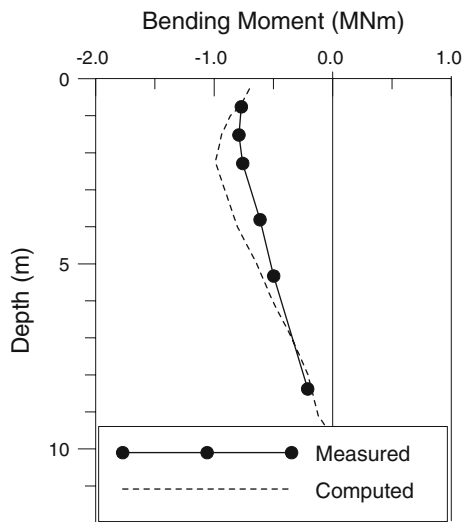


Fig. 21 Comparison of measured and computed maximum bending moment profiles along the pile

7 Analysis of 2 × 2 Pile Group

Effective stress analyses were also carried out to simulate the response of the (2 × 2) pile group-

superstructure system. The finite element mesh is similar in type to that in Fig. 15 except for the presence of the pile cap.

The pile cap was modeled with 16 brick elements and treated as a rigid body. The superstructure mass was treated as a rigid body and its motion was represented by a concentrated mass at the center of gravity. The column carrying the superstructure mass was modeled using beam elements and is treated as a linear elastic structure. As the stiffness of this column element was not reported, it was calculated based on the fixed base frequency of the superstructure reported by Wilson et al. (1997) as 2 Hz.

7.1 Results of (2 × 2) Group Pile Analysis

7.1.1 Acceleration Response

Figure 24 shows computed and measured pile cap acceleration time histories and Fig. 25 shows the corresponding response spectra. There is generally a good agreement between the measured and computed values.

7.1.2 Bending Moment Response

Figure 26 shows time histories of measured and computed moments at depths of 2.55 and 4.08 m. The measured and computed time histories compare quite

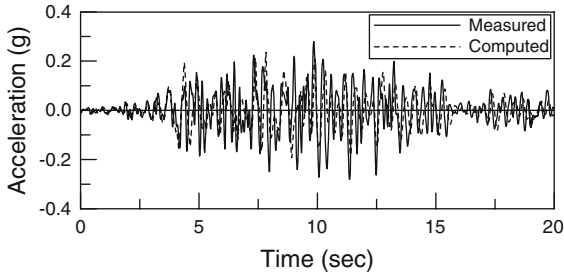


Fig. 24 Comparison of measured and computed pile cap acceleration time histories

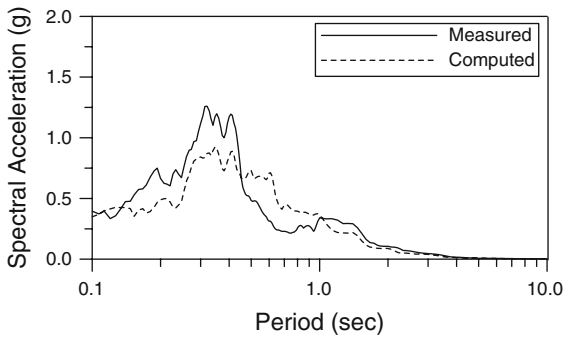


Fig. 25 Comparison of acceleration response spectra of the measured and computed pile cap accelerations

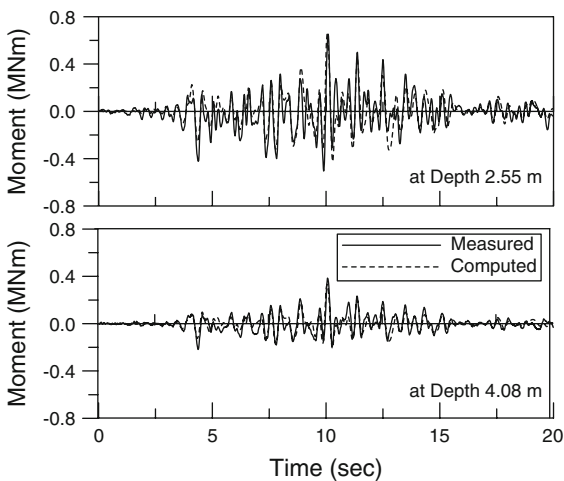


Fig. 26 Comparison of measured and computed bending moment time histories at two depths

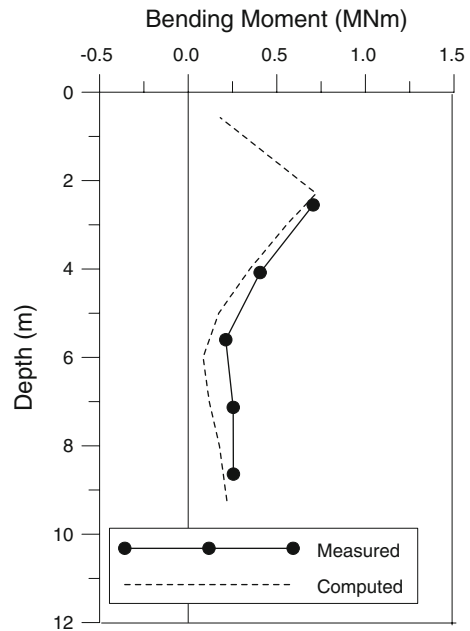


Fig. 27 Comparison of measured and computed maximum bending moments along the pile

well. Residual moments were removed from the time history of measured moments before the comparison was made. Figure 27 shows the measured and computed bending moment profiles with depth. They also compare well.

8 Conclusions

Methods for nonlinear dynamic analysis of pile foundations based on a simplified 3D model of the half space called PILE3D and PILE3D-Eff are presented for the total and effective stress analyses of pile foundations, respectively. These methods can take into account not only the strain dependence of the moduli and damping but also the effects of porewater pressures on pile response. The PILE3D and PIL3D-Eff can calculate the time variation of pile head stiffness components for both dry and saturated soils for the first time.

Centrifuge test of a single pile in dry sand under strong shaking was analyzed using PILE3D and was shown to simulate the seismic response very well. It was also demonstrated that the time variation of the stiffness of pile foundation under strong shaking could be obtained from PILE3D that would allow a more

realistic selection of the representative discrete stiffness required by structural analysis programs than the rather arbitrary procedures often used in practice.

The p - y method which is commonly used for analyzing the seismic response of pile foundations was evaluated using data from a centrifuge model test and the results from PILE3D. In this study, the p - y curves recommended by the American Petroleum Institute were used. The evaluation suggests that these p - y curves are much too stiff for the analysis of strong shaking during earthquakes. When these stiffnesses were used to analyze low levels of shaking, the agreement between measured and computed moment distributions in the pile was very good.

Simulation studies conducted on the University of California centrifuge tests in potentially liquefiable sand including the two representative tests reported here suggest that the effective stress based PILE3D-Eff has the capability to analyze pile foundations in potentially liquefiable soils with sufficient accuracy for engineering purposes. In potentially liquefiable soils, progressive build up of pore water pressure under earthquake shaking and the consequent reduction in strength and stiffness of the soil may induce large bending moments and shear forces in the piles. It may also reduce the stiffness of the pile foundation significantly. The effective stress approach presented here can capture the effects of the seismically induced pore water pressures and capturing these effects is an important part of seismic design of pile foundation in engineering practice.

Acknowledgments The authors are thankful to D. W. Wilson and B. Gohl for the centrifuge tests data. Financial support provided by UBC through University Graduate Fellowship and by NSERC through Research Assistantship to the first author is gratefully acknowledged.

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