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# Effect of Rainfall Intensity on Infiltration into Partly Saturated Slopes

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Abstract This paper describes the development of a model to analyse the rate of infiltration and run-off experienced by a partly saturated soil slope during rainfall. The paper first reviews some of the most popular infiltration models used in geotechnical analysis, and highlights some of the problems associated with their application. One particular model, the Horton Equation is extended to include rainfall intensity directly in its formulation. The new model is shown to predict infiltration responses, which agree with field measurements. In the final section the influence of the rainfall intensity and pattern of rainfall (variation of rainfall intensity) on the infiltration response of a soil is investigated using the new model.

Keywords Water flow  $\cdot$  Unsaturated soil slopes  $\cdot$ Rainfall pattern  $\cdot$  Runoff  $\cdot$  Cumulative infiltration

# 1 Introduction

When rain falls on an unsaturated soil slope, a portion of the total rainfall infiltrates into the soil, whilst the

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K. Gavin e-mail: kenneth.gavin@ucd.ie deficit (total rainfall minus total infiltration) will run-off the surface. Water that percolates into the slope increases the water content of the soil and reduces in-situ suction, thereby decreasing the infiltration capacity of the soil. For this reason, the proportion of the total rainfall that results in infiltration and run-off changes continuously during a rainfall event. The process of the reduction of in-situ suction, results in a decrease in the effective stress (and therefore strength) in the near surface soils, and may result in slope failure (Fourie et al. [1999\)](#page-10-0). Whilst geotechnical engineers are therefore exclusively concerned with predicting the amount of infiltration that will result under a given rainfall intensity, hydrologists are primarily interested in the amount of run-off for catchment flooding studies. This paper considers the effect of rainfall intensity on the proportion of rainfall which will infiltrate into an unsaturated soil slope during a given rainfall event.

# 2 Classical Rainfall-runoff Models

Three infiltration models currently used in geotechnical analyses are reviewed in this section. These include the Green–Ampt [\(1911](#page-10-0)) model, Mein–Larson ([1973\)](#page-10-0) model and Horton's equation (Jury and Horton [2004](#page-10-0)).

# 2.1 Green–Ampt model

The model was first proposed by Green and Ampt [\(1911](#page-10-0)) to describe infiltration through partly saturated

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<span id="page-1-0"></span>soil underlying standing (ponded) water. It is used extensively by hydrologists and geotechnical engineers, and has been extended to predict the infiltration response due to steady rainfall Mein and Larson [\(1973](#page-10-0)), and unsteady Chu [\(1978](#page-10-0)) rainfall events, and has been applied to the assessment of infiltration in slopes by Pradel and Raad [\(1993](#page-10-0)), Fourie et al. [\(1999](#page-10-0)) and Cho and Lee [\(2002](#page-10-0)). The two layer model assumes that a wetting front of downward percolating water results in the formation of a perched water table beneath the ponded water (see Fig. 1). Above the wetting front, the soil is assumed to be completely saturated, whilst the soil below the wetting front is assumed to remain at the initial (pre-infiltration) moisture content.

Applying Darcy's Law, the infiltration capacity of the soil at time  $t$  can be calculated:

$$
i = K_s \left(\frac{Z + H + \Psi}{Z}\right) \tag{1}
$$

In which:  $i$  is the infiltration capacity;  $Z$ , the depth of wetting front; H, the depth of ponded water;  $\psi$ , the suction head at the wetting front;  $K_s$ , saturated soil permeability.

In rainfall induced slope failures, the wetting front and the slip surface are coincident, as reduced suction due to infiltrating water is the trigger for failure to occur. Forensic investigations of slope failures caused by rainfall Olivares and Picarelli [\(2003](#page-10-0)), Springman et al. ([2003\)](#page-10-0) reveal that the soil above the slip surface is often partly saturated at failure. The assumption that suction in the wetted zone is zero is therefore questionable. The Green–Ampt model has been shown to greatly under-predict the time for a wetting front depth to form in partly saturated soil Gavin and Xue [\(2007](#page-10-0)). This is predominantly due to

Ponding Depth, H	Ponded water	<b>Ground Surface</b>
<b>Wetting Front</b> Depth, Z	Soil Moisture $\theta$ i= $\theta$ sat	
Soil Moisture $\theta$ i= $\theta$ initial	Wetting Front Suction, Ψ	
		<b>Lower Boundary</b>

Fig. 1 Water distribution and wetting front in the Green– Ampt model

the use of  $K_s$ , in the formulation. Since the soil in a slope fails prior to reaching full saturation, the operational permeability  $(K)$  is lower than  $K_s$ , with Bouwer [\(1966](#page-10-0)) suggesting that  $K \approx 0.5K_s$ .

# 2.2 Mein–Larson Model

At the start of a rainfall event, because of the large suctions present in the unsaturated soil the infiltration capacity of a partly saturated slope is initially high. It is common during the early stages of a rainfall event for the infiltration capacity to exceed the rainfall intensity. Therefore the amount of water, which can infiltrate into the soil, is controlled by the rainfall intensity. As rainfall continues the near surface suctions, and therefore the infiltration capacity, reduce. At some point the rainfall intensity may exceed the infiltration capacity, and the rate of flow into the soil becomes controlled by the infiltration capacity of the soil.

Mein and Larson [\(1973](#page-10-0)) recognized that during rainfall, for given initial soil conditions, the point at which the infiltration rate into a partly saturated soil changed from being supply controlled (i.e. dependent on the rainfall intensity) to being capacity controlled (dependent on the infiltration capacity of the soil) varied with rainfall intensity. Relationships between the infiltration rate and time, for a range of rainfall intensities are shown in Fig. 2.

Assuming that the rainfall intensity is constant throughout a rainfall event, they describe a range



Fig. 2 Infiltration behaviour under different rainfall intensity (after Mein and Larson [1973\)](#page-10-0) (A:  $R_i \lt K_s$ , B:  $i \ge R_i > K_s$ , C (and D):  $R_i \ge i > K_s$ )

of possible soil responses: Case A: In this scenario the rainfall intensity  $(R_i)$  is lower than the saturated permeability  $(K<sub>s</sub>)$  of the soil. All rainfall is assumed to percolate into the soil, and the infiltration rate remains constant (equal to the rainfall intensity) throughout the event. Cases B and C: In these scenarios  $R_i$  is greater than  $K_s$ . During the initial stage of the rainfall event, the infiltration capacity exceeds  $R<sub>i</sub>$ , and all water infiltrates into the soil. At some point  $(T_p)$  the near surface soils become saturated and runoff begins. For a given initial condition, the time  $T_p$ , will depend on the rainfall intensity, with more severe events (higher  $R_i$ ) resulting in rapid saturation of near surface soil. Once the near surface soils become saturated the rainfall intensity exceeds the infiltration capacity  $(R_i \ge i > K_s)$ , ponding begins and is followed by runoff over the ground surface. The infiltration rate starts to decrease when  $t > T_p$ . Case D:  $R_i > I > K_s$ . In situations where the rainfall intensity is higher than the infiltration capacity at the start of the rainfall event, the response will be capacity controlled throughout the rainfall event. Surface run-off and decrease of the infiltration rate begin once rainfall commences.

From Fig. [2](#page-1-0) we note that the key assumptions of the Mein–Larson model are that the minimum infiltration capacity of the soil is equal to the saturated permeability and that run-off will not occur unless the rainfall intensity exceeds the saturated permeability of the soil. Rahardjo et al. [\(2005](#page-10-0)) report measurements of the infiltration response of a residual soil slope subjected to artificial rainfall events. The instrumented slope comprised a 1.5 m deep layer of orange silty clay with a saturated permeability of  $5.18 \times 10^{-6}$  m/s, overlying a 4–5 m deep layer of purple clayey silt with  $K_s = 1.67 \times 10^{-7}$  m/s. A simulated rainfall event, with an intensity of 47 mm/h (flow rate =  $13 \times 10^{-6}$  m/s or 2.5K<sub>s</sub>) and duration of 73.3 min was applied to the slope. The proportion of infiltration and run-off recorded is shown in Fig. 3, where the infiltration rate is seen to decrease to a limiting value of  $2 \times 10^{-6}$  m/s (which corresponds to  $0.4K_s$ ) after approximately 65 min.

Li et al. ([2005\)](#page-10-0) monitored infiltration into an instrumented cut slope in Hong Kong. The cutting was taken in completely decomposed granite, which had a saturated permeability in the range  $1 \times 10^{-6}$  to  $1 \times 10^{-5}$  m/s. The authors recorded the infiltration rate due to rainfall over a number of days in August



Fig. 3 Rainfall, infiltration and runoff rate on a silty clay slope (after Rahardjo et al. [2005](#page-10-0))

and September 2001. The rainfall intensity during this period ranged from  $2.8 \times 10^{-7}$  m/s to  $2.3 \times 10^{-6}$  m/s, i.e. close to, or lower than  $K_s$ . The infiltration coefficient (defined as the ratio of the infiltration rate to rainfall intensity) measured during this period is shown in Fig. 4, together with the ratio of the rainfall intensity to the saturated soil permeability (assuming  $K_s = 1 \times 10^{-6}$  m/s). Both data sets contradict the assumption of the Mein–Larson model in showing that significant run-off occurs when the rainfall intensity is below  $K_s$ .

#### 2.3 Horton Equation

Noting that the infiltration capacity decreases throughout an infiltration event, Jury and Horton [\(2004](#page-10-0)) describe an empirical expression to describe



Fig. 4 The comparison between infiltration rate and rainfall intensity on a completely decomposed granite slope (from Li et al. [2005\)](#page-10-0)

the decay of the infiltration capacity of a soil with time  $(t)$ , where:

$$
i = i_f + (i_0 - i_f)e^{(-\beta t)}
$$
 (2)

where:  $i_0$  is the initial infiltration capacity at  $t = 0$ ;  $i_f$ , the steady state *final* infiltration capacity;  $\beta$ , a constant which describes the rate of decrease of the infiltration capacity;  $t$ , time.

Mishra et al. ([2003\)](#page-10-0) and Linsley et al. ([1975\)](#page-10-0) note that Eq. 2 is only applicable when the rainfall intensity is higher than the infiltration capacity. As the rainfall intensity is likely to vary during a storm, there are periods (see Fig. 5) when the rainfall intensity is below the infiltration capacity, and no run-off is assumed to occur. Although the infiltration capacity continues to decrease during these periods as the infiltrating water reduces the in-situ suctions, it will obviously occur at a variable rate (i.e. the rate of decay is not constant with time). To account for this Green ([1986\)](#page-10-0) suggests using the integrated form of Eq. 2. However, the difficulty in selecting a suitable  $\beta$  value for a given soil highlighted by Mishra et al. [\(2003](#page-10-0)) in a comprehensive review of infiltration models poses a significant challenge.

Despite a comprehensive literature on the topic of infiltration, Diskin and Nazimov [\(1995](#page-10-0)) note that some important aspects of the infiltration response of soils are often not fully considered in most models, these include consideration of the variation of the infiltration capacity during periods of low rainfall intensity, and the effect of variable rainfall rates on



Fig. 5 Infiltration and runoff rate in Horton's infiltration capacity curve (after Viessman and Lewis [1996\)](#page-10-0)

the time taken for runoff to begin. They highlight problems in some models, which relate the variation of infiltration capacity to time, pointing out that for two similar soils, the rate of change of infiltration capacity depends fundamentally on the rainfall intensity. They incorporate rainfall effects into their infiltration capacity model through the following equation:

$$
i = i_0 - R_i(i_0/i_f - 1)[1 - \exp(i_f t/S_m)] \tag{3}
$$

where  $R_i$  is the rainfall intensity and  $S_m$  is the maximum storage capacity of soil in the layer above the location to be examined.

The authors show that their equation reduces to the form of Horton's equation under specific conditions, namely:

- (i) The initial moisture content is zero.
- (ii) The infiltration rate is high enough to maintain the rate of infiltration at the infiltration capacity.
- (iii) The rate of percolation is proportional to the moisture content of the near surface soil.

The authors note that assumption (ii), which is common to many infiltration studies, is rarely true.

## 3 Proposed Method

#### 3.1 Infiltration Capacity

The problem of modelling complex material response, such as infiltration into unsaturated soil, arises due to the large number of variables, which affect the solution. The parameter  $\beta$  in Hortons' equation is controlled by many variables, including the soil type, the initial water content, surface conditions (e.g. whether the slope is vegetated), the slope, rainfall characteristics, location on the slope and sub-surface drainage, amongst other factors. It is not possible (or in some cases desirable) to include every possible variable in the formulation of a model because of the lack of experimental data and the need for simplicity.

In this section a model based on an extended form of Horton's equation is proposed. Based on the assumption that the rainfall intensity will affect the rate of decay of the infiltration capacity, this parameter is included in the formulation:

<span id="page-4-0"></span>
$$
i = i_f + (i_0 - i_f) \exp\left[-\left(\frac{R_i}{i_f}\right)\left(\frac{K_s}{i_f}\right)^{1/2} t\right]
$$
 (4)

In the model, four parameters are used to describe the variation of infiltration capacity with time: rainfall intensity  $(R_i)$ , saturated soil permeability  $(K<sub>s</sub>)$ , initial and final infiltration capacity  $(i<sub>0</sub>$  and  $i<sub>f</sub>)$ . The time is in units of hours, while the initial and final infiltration capacity and saturated permeability and are in m/s.

The last three parameters  $(K_s, i_0 \text{ and } i_f)$ , are material parameters, with the saturated soil permeability being unique for the soil, and  $i_0$  and  $i_f$  being dependent primarily on the soil type, water content and the surface condition (Miyazaki et al. [1993](#page-10-0)). Whilst  $K_s$  can be measured using routine laboratory tests,  $i_0$  can be related to the in-situ suction or water content monitored using in-situ devices and  $i_f$  can be established in the laboratory or on site by establishing the residual permeability of a sample of soil with a constant water supply and a head differential equal to that which might reasonably be expected in-situ.

The model was used to predict the reduction of the infiltration capacity of a soil with time during constant intensity rainfall events. The results are shown in Fig. 6. The relative rainfall intensity  $(R_i/i_f)$ which is a measure of the ratio of the rainfall intensity to the final infiltration capacity was varied from 0.5 to 2, whilst two initial conditions were considered. In the first the ratio of the initial to the final infiltration



Fig. 6 Infiltration capacity of soil under different rainfall condition

capacity  $(i_0/i_f)$  was 5, and in the second this ratio was doubled (to model the effect of a prolonged dry period with high initial infiltration capacity). A number of trends are noteworthy:

- (i) The model clearly captures the effect of varying rainfall intensity on the degradation of the infiltration capacity. For example in the case where  $i_0/i_f = 10$ , when the relative rainfall intensity  $R_i/i_f = 2$ , the infiltration capacity equals  $i_f$  after just 2 h. In contrast, when the rainfall intensity is halved  $R_i/i_f = 1$ , the time required for the infiltration capacity to reach equilibrium increases to in excess of 5 h.
- (ii) The exponential form of the infiltration capacity decay curves shown in Fig. 6 highlights that the analysis is more sensitive to assumptions made about the relative rainfall intensity, than it is to the value  $i_0/i_f$  (which reflects the initial condition). For example, comparing the response at a time of 2 h, in the analyses where  $i_0/i_f = 10$ , we note that the range of current infiltration capacity is very sensitive to  $R_i/i_f$ , with  $i/i_f$ increasing from 1 to 5, as  $R_i/i_f$  increases from 0.5 to 2. In contrast if we compare the response of samples at a given relative rainfall intensity  $R_i/i_f = 2$ , and  $i_0/i_f$  of 5 and 10, we see that the predicted response converge quickly.

### 3.2 Time to Runoff

The review of existing infiltration models highlighted the difficulty in determining the time at which runoff occurs, and noted that the assumptions on which many of these models are based contradict experimental measurements (i.e. that run-off will not occur if  $R_i \lt K_s$ ). At the start of a rainfall event, when the infiltration capacity is at a maximum, run-off will not occur unless the rainfall intensity is extremely high  $(R<sub>i</sub> > i)$ . As percolation of water into the slope continues, the infiltration capacity reduces (as near surface suction reduces). Runoff will occur when the rainfall intensity exceeds the infiltration capacity. The time at which the slope moves from the condition of full infiltration, to when runoff begins is clearly the point at which the rainfall intensity is equal to the infiltration capacity:

$$
R_i = i \tag{5}
$$

Because the new model formulation includes rainfall intensity, we can substitute Eq. 5 into Eq. [4](#page-4-0) to determine the time at which runoff begins  $(T_p)$ :

$$
T_p = \frac{(i_f)^{3/2} \ln [(i_0 - i_f)/(R_i - i_f)]}{R_i (K_s)^{1/2}}
$$
(6)

We see from Eq. 6 that  $T_p$  is a function of the rainfall intensity, saturated soil permeability, initial and final infiltration capacity, and that as the rainfall intensity increases,  $T_p$  reduces. If the rainfall intensity is equal to the initial infiltration capacity, then  $T_p = 0$ , and run-off occurs immediately (such behaviour is observed in severe storm events with high initial rainfall intensities).

# 4 Validation of the Model

# 4.1 Boundary Conditions

Since the model is empirical, some basic boundary conditions should be satisfied.

(i) If the rainfall intensity is equal to, or lower than the final infiltration capacity then runoff will not occur. In this situation the time for runoff to occur is infinite  $(T_p = \infty)$ . Setting  $R_i = i_f$  in Eq. 6, we get:

$$
T_p = \frac{(i_f)^{3/2} \ln [(i_0 - i_f)/(i_f - i_f)]}{i_f (K_s)^{1/2}}
$$
  
=  $\infty$ 

(ii) If the rainfall intensity, at the start of a rainfall event is equal to or exceeds  $i_0$ , runoff starts at the beginning of rainfall and  $T_p$  is zero (T = 0). Accordingly, in Eq. 6, for  $R_i \ge i_0$  we have:

$$
T_p \le \frac{(i_f)^{3/2} \ln [(i_0 - i_f)/(i_0 - i_f)]}{i_0(K_s)^{1/2}}
$$
  
= 0

(iii) During a prolonged dry period where no precipitation occurs, and assuming that changes in moisture content due to evaporation are negligible, the infiltration capacity should remain unchanged. So for  $R_i = 0$  we have:

$$
i = i_f + (i_0 - i_f) \exp(0)
$$
  
=  $i_0$ 

Equations 4 and 6 are seen to satisfy the essential boundary conditions.

#### 4.2 Model Validation using In situ Test Results

## 4.2.1 Infiltration Capacity Decay Curves

Rahardjo et al. ([2005](#page-10-0)) report field measurements of the infiltration response of a cutting, with a slope of 2:1, located on the campus of Nanyang Technological University, Singapore. The soil conditions comprise an upper 1.5 m deep layer of orange silty clay, with the saturated permeability at about  $5.18 \times 10^{-6}$  m/s, underlain by a lower permeability ( $K_s = 1.67 \times 10^{-7}$  m/s), 4–5 m deep layer of purple clayey silt.

Natural and artificial rainfall was applied to an instrumented section of the slope, which covered a plan area of  $20-25$  m<sup>2</sup>. The rainfall intensity was recorded using a tipping bucket rain gauge, whilst the run-off was measured using a flume at the base of the slope. Infiltration was not measured directly but was taken as the difference between the rainfall intensity and run-off.

Four different rainfall intensities 47, 57, 72 and 83 mm/h were applied to the slope. The field measurements revealed that:

- (i) At the start of each test the infiltration rate was equal to the rainfall intensity, and no run-off occurred.
- (ii) The time  $(T_p)$  at which runoff began, decreased with increasing rainfall intensity. Once runoff began  $(t > T_p)$ , the infiltration rate decreased, and run-off increased with time in all tests.
- (iii) With the exception of the first test (47 mm/h rainfall intensity), run-off continued after the end of the application of rainfall to the slope, indicating that subsurface (lateral) flow was occurring. This suggests that the assumption that the infiltration can be calculated by subtracting the run-off (which occurred during rainfall) from the rainfall intensity, resulted in an overestimate of the actual infiltration for the three high intensity rainfall events.

Two rainfall patterns were studied using the new model. In the first 47 mm/h rainfall intensity was applied to the slope for a period of 73.3 min, whilst in the second, the rainfall intensity was increased to 72 mm/h. This latter high intensity rainfall was applied for 43.3 min. The infiltration curves predicted using the new model are compared with the measurements in Fig.  $7a$  and b. Rahardjo et al.  $(2005)$  $(2005)$ report values for  $K_s$  and  $R_i$  which were used in the analysis. The value of  $i_f$  was taken directly from measured field data for the 47 mm/h rainfall event shown in Fig. 7a, where no run-off was measured after the end of the rainfall event. However, the continuation of run-off after the end of the heavier rainfall (shown in Fig. 7b), suggests that the shape of the measured infiltration capacity decay curve, and particularly the  $i_f$  value measured is incorrect (the  $i_f$ value would be expected to be too high). Rahardjo et al. ([2005\)](#page-10-0) note that the initial and final volumetric content near the toe of the slope always appeared to



Fig. 7 Predicted infiltration decay curves of the Silty clay slopes with comparison to the measured one

be higher than those at the crest, suggesting that some water that infiltrates near the crest of the slope may then flow downslope, parallel to the slope surface, thus resulting in higher water content and lower suction near the toe of the slope than near the crest. The same  $i_f$  value was therefore adopted for both analyses. The final parameter needed was  $i_0$ . Although this was not quoted in the paper (and was naturally variable as it is affected by antecedent rainfall), it was noted that at the start of each experiment the infiltration rate was equal to the rainfall intensity, the initial infiltration capacity exceeded the rainfall intensity in all cases. Therefore, analyses were carried out for a range of assumed  $i_0$ values varying from 80 (which is just above the highest rainfall intensity) to 200 mm/h.

The prediction for the 47 mm/h rainfall event is seen to be quite accurate, and as noted, is relatively insensitive to the value of  $i_0$  (which was the only unknown parameter) assumed in the analysis, although the time calculated for runoff to occur time is quite sensitive to the  $i_0$  value assumed. For the higher rainfall intensity rainfall pattern shown in Fig. 7b, the model clearly over-predicts the rate of decay of the infiltration capacity, however, this is primarily due to the differences between the assumed and measured  $i_f$  value.

## 4.2.2 Total Infiltration

Pradel and Raad [\(1993](#page-10-0)) and Fourie et al. ([1999\)](#page-10-0) show that the stability of unsaturated slopes is critically influenced by the depth of wetting front and the suction values at this depth. The wetting front depth and suction are influenced by the total infiltration of water  $(I_c)$  into an unsaturated soil. The total infiltration can be calculated by integrating Eq. [4:](#page-4-0)

$$
I_c = R_i T_p + \int_{T_p}^{T_D} i dt
$$
\n(7)

in which  $T_D$  is the total duration of the rainfall event and  $i$  is the infiltration capacity from Eq. [4](#page-4-0).

The total infiltration (the difference between total rainfall and total runoff) for the 47 and 72 mm/h rainfall patterns calculated using Eq. 7 are shown in Fig. [8](#page-7-0). We see that the model provides good estimates of the measured data, with predicted total

<span id="page-7-0"></span>

Fig. 8 Comparison of measured total runoff with the predicted values

run-off increasing as the assumed initial infiltration capacity  $(i_0)$  decreases.

# 5 Effect of Rainfall Intensity and Pattern on Infiltration Response

In this section the effects of rainfall intensity and rainfall pattern on the infiltration response of a partly saturated slope are examined using the new model.

# 5.1 Influence of Rainfall Intensity

One of the drawbacks identified with the original Horton Equation was that the infiltration capacity decay curve was not influenced by the rainfall intensity in a transparent way. The  $\beta$  term, which controls the rate of decay of the infiltration capacity, is usually determined based on the measured field response. A value derived in this way will reflect only the average rainfall during the field-monitoring period. Horton's equation cannot, therefore account for variations in rainfall intensity, during an infiltration event.

In the modified model Eq. [4,](#page-4-0) the rate of change of the infiltration capacity can be expressed as:

$$
\frac{\partial i}{\partial t} = -(i_0 - i_f) \left(\frac{R_i}{i_f}\right) \left(\frac{K_s}{i_f}\right)^{1/2} \exp\left[-\left(\frac{R_i}{i_f}\right) \left(\frac{K_s}{i_f}\right)^{1/2} t\right]
$$
\n(8)

The curvature of the infiltration capacity decay curve is related not only to the time since the start of infiltration, but also to rainfall intensity. This is illustrated in Fig. [9](#page-8-0), where during the initial period of a rainfall event, the rainfall intensity  $(R_1)$  is high. At time  $t_1$ , the rainfall intensity reduces  $(R_2 \lt R_1)$  and the slope of the infiltration capacity decay curve reduces.

# 5.2 Complex Rainfall Patterns

During a natural rainfall event, the rainfall intensity will not be constant. Given that the rate of decay of the infiltration capacity decay curve is affected by the rainfall intensity, analyses were carried out to study the infiltration and runoff characteristics of soil subjected to complex rainfall patterns.

Smith ([1972\)](#page-10-0) describes the infiltration properties of two soils, Nibley Silty Clay Loam (NSCL) and Colby Silt Loam (CSL). The soils were chosen to represent a relatively high permeability soil (where  $i_f$ and  $K_s$  for NSCL = 10 mm/h), and a lower permeability soil ( $i_f$  and  $K_s$  for CSL = 5 mm/h). In the analyses, three different rainfall intensities: 10, 20 and 40 mm/h were used. The total precipitation in all analyses was equal to 40 mm. The following rainfall patterns were considered:

- 1: Heavy-Light (H-L) pattern—Rainfall starts at an initial high intensity (20 mm/h), for a period of 1 h, then becomes lighter (10 mm/h), for a 2-h period.
- 2: Light-Heavy-Light (L-H-L) pattern—Rainfall starts at an initial low intensity (10 mm/h) for a period of 1 h, then becomes heavier (20 mm/h), for 1 h, followed by low intensity (10 mm/h) rainfall for the final hour.
- 3: Light-Heavy (L-H) pattern—Rainfall starts at an initial low intensity (10 mm/h), for a period of 2 h, this is followed by a 1-h period of high intensity rain (20 mm/h).
- 4: Heavy (H) pattern—Rainfall is applied at a very high intensity (40 mm/h) for a period of 1 h.

In the first three cases the total rainfall intensity and duration (3 h) of each event are identical, only the sequence of application is altered, whilst in the final case, the total rainfall intensity is the same as cases 1–3, and only the duration is altered. The infiltration behaviour of the two soils under these rainfall

<span id="page-8-0"></span>

Fig. 9 The changing rate of infiltration capacity of soil under complex rainfall conditions

patterns was assessed using the new model. Smith [\(1972](#page-10-0)) did not measure values for the initial infiltration capacity  $(i_0)$  for the soils and an  $i_0$  value of 0.29 cm/min (equivalent to 174 mm/h) was assumed for all cases in the original paper.

The infiltration capacity decay curves (predicted using Eq. [4](#page-4-0)) for the four rainfall events are shown in Fig. [10](#page-9-0) (a–d for NSCL, e–h for CSL). The effect of the variation in rainfall intensity on the infiltration response observed can be summarised as follows:

- (i) From Fig. [10a](#page-9-0)–c, which describes the response of NSCL during the 3-h rainfall event, the amount of run-off generated is low. Run-off is only predicted in for the L-H pattern (Fig. [10c](#page-9-0)), when  $t > 2.5$  h.
- (ii) The responses of CSL during the 3-h rainfall event are shown in Fig. [10e](#page-9-0)–g. It is clear that the amount of run-off is significantly greater than that generated for the NSCL, this is because of the lower final infiltration capacity of CSL. The total run-off predicted for the two rainfall patterns (L-H and L-H-L), when applied to the CSL are similar, at 34% and 35% of the total rainfall. However, the time at which the run-off occurs is quite different, with the majority of run-off occurring during the period 1–2 h for the L-H-L pattern, and during the 2–3 h period for the L-H pattern. The amount of run-off predicted for the H-L pattern is significantly lower at the period where the rainfall intensity is highest,

which coincides with the time when the infiltration capacity of the soil is greatest.

(iii) Comparing the response of both soil types to the heavy rainfall pattern (Fig. [10d](#page-9-0) and h), we see that although the total rainfall, rainfall intensity and initial infiltration capacity are identical, the final infiltration capacity, which is higher for the NSCL, significantly affects both the time to runoff and the proportion of run-off which occurs.

#### 5.3 Average Rainfall Intensity

The analysis in the previous section suggests that the rainfall pattern, rather than the average rainfall intensity during a storm, will control the amount of infiltration (and run-off) during a given rainfall event. In practice it is difficult to describe rainfall patterns with discontinuous functions such as those used in Figs. 9 and [10](#page-9-0), and the average rainfall intensity for the whole event is used in the assessment of infiltration. The first three rainfall patterns described in Fig. [10,](#page-9-0) have an average rainfall intensity calculated over the entire time period of 13.3 mm/h. The infiltration capacity decay curve determined for the CLS soil using the average rainfall intensity is compared to the curves for the complex rainfall patterns in Fig. [10e](#page-9-0)–g. Although the total runoff predicted (28%) is similar to the values predicted under the complex rainfall patterns (see Fig. [11\)](#page-9-0), it is clear that run-off occurs during different time periods, for all events.

# 6 Conclusion

A model to predict the infiltration response of partly saturated soil slopes is proposed. The model suggests that the amount of infiltration, and run-off, which result from a given rainfall intensity, can be described using the initial and final infiltration capacity, the saturated permeability of the soil and the rainfall intensity. Whilst, the final infiltration capacity and the saturated permeability can be easily obtained in the laboratory, and the rainfall intensity can be assigned based on statistical analysis of rainfall records in the

<span id="page-9-0"></span>Fig. 10 Infiltration rate curves of Nickel gravely sandy loam  $(a-d)$  and Colby silty loam (e–h) under four sets of different rainfall patterns





Fig. 11 The infiltration capacity curve of Colby silty loam under average rainfall intensity

area, the initial infiltration capacity depends on the in-situ conditions and will be strongly affected by antecedent rainfall conditions. The new model considers one-dimensional vertical infiltration only and does not consider the effects of lateral flow or downslope subsurface drainage.

Although there is a dearth of reliable data in the literature with which to compare the performance, the model provided reasonable and consistent predictions of the response of an instrumented slope to varying rainfall intensities. Sensitivity analyses performed using the model suggest that:

- (i) Because of the exponential form of the proposed model, predictions were relatively insensitive to  $i_0$  values used in analyses. Although  $i_0$  does affect the accuracy of the prediction of the time when run-off begins. However, the accurate modelling of run-off response is not critical to geotechnical analysis of infiltration problems.
- (ii) The amount of infiltration (or run-off), which occurs, is particularly sensitive to the rainfall pattern. If the total infiltration is known, the change of the volumetric water content of the near surface soil (reduction in suction) can be calculated and an assessment of the effect of these changes on the stability of the slope can be made. For a given total rainfall, rainfall events which begin at high intensity and end at low intensity generate less run-off than those which start at low intensity and end high. This has significant implications for slope stability analyses. In the case where runoff is low, a greater portion of the total rainfall enters the soil, thereby reducing near surface suction and leading to a greater risk of failure.

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