

A generalized methodology for determination of crushing strength of granular materials

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Abstract Determination of crushing strength of granular material is very important for assessing its suitability in various engineering and industrial applications. Laboratory investigations that would yield stress–strain behaviour, and hence, crushing strength of the granular material are extremely cumbersome and time consuming. Also, results obtained from these experiments get influenced by the aspect ratio of the sample, its density, strain rate, size and shape of the grains etc. These difficulties can be overcome by developing a generalized mathematical model, which is primarily based on the physical properties of the granular material such as particle-size and specific gravity, for estimating its crushing strength. With this in view, experiments were conducted on different types of granular materials such as sands, cenospheres (which are found in fly ash and bottom ash) and glass beads, and the results were used for developing such a model. Details of the testing methodology adopted to achieve this are also presented in this paper and validation of the proposed model has been done based on the experimental results and the results reported in the literature.

Keywords Crushing strength · Granular material · Particle-size · Specific gravity · Mathematical model

Nomenclature

δ	deformation of the sample
ρ	density
σ'_{CR}	crushing strength of sample
C_u	uniformity coefficient
C_c	coefficient of curvature
d	diameter of the sample
D_x	particle size corresponding to x % finer
D_{GB}	diameter of glass beads
D_{PG}	diameter of poorly graded granular material
e	void ratio
G	specific gravity
H	height of the sample
H/d	aspect ratio
P	applied load
MF	multiplication factor

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1 Introduction

Crushing of soil grains occurs due to extremely high stresses that may generate during pile driving, construction of high earth or rock fill dams, laying foundations of the offshore gravity structures,

impact of projectiles, drilling at great depths for petroleum extraction etc. (e.g., Miura et al. 1984; Hardin 1985; Zheng et al. 1990; Coop and Lee 1993; Hagerty et al. 1993). Extensive laboratory investigations have been conducted by researchers to assess compression and crushing behaviour of the granular material (e.g., Miura et al. 1984; Hagerty et al. 1993; Schanz and Kitamura 1993; McDowell et al. 1996; Yamamuro et al. 1996; Nakata et al. 2001; Feda 2002; Chuhan et al. 2003). Also, triaxial compression and one-dimensional compression have been carried out by researchers to study the bulk and microscopic response of a variety of granular materials by varying the initial voids ratio e and the aspect ratio H/d , which is the ratio of the height H and diameter d of the sample (e.g., Hagerty et al. 1993; Morland et al. 1993; Ghosal and Self 1995; Anand and Gu 2000; Chuhan et al. 2003; Gupta et al. 2004; Bartake and Singh 2005). Based on these experiments, parameters such as total breakage factor (Hardin 1985), aggregate stability (Oztas et al. 1999), and probability of crushing and particle breakage factor (Nakata et al. 1999) have been defined. These parameters are found to be of immense help for understanding the crushing behaviour of the granular material in a very comprehensive manner. Also, these studies indicate that the factors which primarily influence the crushing of grains are particle-size distribution, grain shape, hardness, density or the voids ratio, the aspect ratio etc. (e.g., Miura et al. 1984; Hardin 1985; Feda 2002; Chuhan et al. 2003).

In a recent study, based on the extensive experimental investigations on different sands, correlation between the yield stress and the grain size, shape and mineralogy of the granular material has been developed (Chuhan et al. 2003). However, this study does not yield empirical or semi-empirical relationships that can be employed for determining crushing strength of the sand, directly. In addition to this, researchers have tried to simulate “single particle crushing” by employing a simple test setup that consists of two parallel plates and a loading arrangement (e.g., Nakata et al. 1999; McDowell and Bolton 1998; McDowell 2001). However, these studies do not simulate the in situ behaviour of the granular material and results are found to be sample specific.

In such a situation, the potential of the methodology proposed by Bartake and Singh (2005) for determining crushing strength of cenospheres, based on the specific gravity G and the load-deformation characteristics, can be extended for determining crushing strength of granular materials, in general. This methodology can also be utilized for developing a generalized mathematical model for determining crushing strength of the granular material, based on its G and the particle-size distribution characteristics. Such a model would also be useful in overcoming the difficulties associated with the experimental investigations, which are quite cumbersome, time consuming and are dependent on the experimental and sample conditions, to a certain extent.

With this in view, experiments were conducted on different types of sands, cenospheres and glass beads and the results are used for developing a generalized model that can be employed for determining crushing strength of the granular material. Validation of the model has been done by comparing the computed results vis-à-vis experimental findings and those available in the literature.

2 Experimental investigations

Three grades of sands and cenospheres (denoted as S1, S2, S3 and CS1, CS2, CS3, respectively), were used in this study. These samples were characterized for their mineralogical composition by X-ray diffraction (XRD) spectrometer (Phillips, Holland), which is fitted with a graphite monochromator and Cu- $k\alpha$ radiation source. For identifying various minerals in these samples JCPDS (1994) files were used. Sand samples exhibit presence of quartz as the most predominant mineral while cenospheres are noticed to consist of quartz (as the most predominant), mullite, sillimanite and hematite minerals. These sands were mixed in different proportions, as listed in Table 1, to create 12 different samples of sand.

For establishing particle-size distribution characteristics of these samples, dry sieving was conducted (ASTM D 422-63). Glass beads used in this study are of uniform diameter D_{GB} (= 0.5, 1.0, 1.5, 2.0, 2.5, 3.0 mm).

Table 1 Designation of the sand mixtures used in the study

Sample	(% by weight)		
	S1	S2	S3
A	30	30	40
B	30	40	30
C	40	30	30
D	75	25	0
E	50	50	0
F	25	75	0
G	0	75	25
H	0	50	50
I	0	25	75
J	75	0	25
K	50	0	50
L	25	0	75

G of sands, cenospheres and glass beads was obtained by using an Ultra-Pycnometer (Quantachrome, USA), which employs Helium gas (ASTM D 5550).

2.1 Test setup

The test setup depicted in Fig. 1 was fabricated to study the crushing characteristics of the granular material (sands, cenospheres and glass beads). As depicted in the figure, the test setup consists of a stainless steel (SS) mold of inner diameter 33 mm, outer diameter 51 mm and height 110 mm, in which granular material can be filled

to achieve a sample of desired aspect ratio *H/d* and density ρ . Two stainless steel pistons (top and bottom) of 32 mm diameter were used for crushing the sample. A collar was provided at the bottom of the mold on which two removable stainless steel clips of thickness 10 mm can be fixed. These clips were removed before commencing the test to achieve loading of the sample from both the ends. With this setup, influence of *H/d* on the measured crushing strength σ_{CR} of the samples can also be studied.

2.2 Testing methodology

A 5 T capacity servo hydraulic universal testing machine UTM (AIMIL, India) was employed for establishing crushing characteristics of the sample. These tests were performed at a strain rate of 1.25 mm/min (Karner et al. 2004). A 5000 kg load cell, attached to a digital readout unit was used for recording the load *P* transmitted to the sample. Prior to the loading of the sample, silicon grease was applied on the two pistons to minimize the side friction. The sample was poured into the mold by adopting the rainfall technique (i.e., pouring the sample using a funnel by maintaining a constant height of fall) to achieve a certain *H/d* and ρ equal to 1.74 g/cm³. The *P* transmitted to the sample was recorded, at an interval of 10 s, until it reaches 3500 kg.

3 Results and discussion

Figure 2 depicts the particle-size distribution characteristics of the sand and cenosphere samples considered in this study. Gradational characteristics of the samples of cenospheres CS1, CS2 and CS3 are also superimposed in the figure to highlight the range of particle-size of the granular materials considered in this study. From the figure, particle sizes *D_x* corresponding to *x* percent finer, uniformity coefficient *C_u*, and the coefficient of curvature *C_c*, were computed and are listed in Table 2. The value of *G* for these samples is also presented in the table. *G* for all types of glass beads is found to be 2.46.

Load-deformation (*P-δ*) characteristics of different sands and cenospheres corresponding to

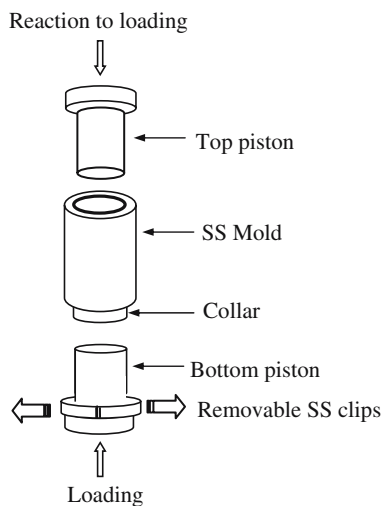


Fig. 1 The test setup

different H/d values, were developed. However, for the sake of brevity, results pertaining to Sample S1 and CS3 (with $H/d = 1.0$) only are presented in Figs. 3 and 4, respectively. It can be noted from Fig. 3 that followed by the initial elastic compression of the grains (depicted by the portion **AB** of the P - δ characteristics), the sample transforms to a new state of the material. Hence, intersection of the tangents drawn to the initial (**AB**) and final (**DE**) portions of the P - δ characteristics would intersect at **C** ($= 703$ kg), which would yield σ_{CR} ($= 87.4$ kg/cm²) of the sample. Incidentally, the results obtained are found to be consistent with those reported in the literature (e.g., Hardin 1985; Yamamuro et al. 1996; Chuhan et al. 2003).

It can be noted from Fig. 4 that after an initial elastic response (depicted by the portion **AB**),

rearranging and repacking of the particles starts (depicted by the portion **BC**). Incidentally, point **C** corresponds to the point on the P - δ characteristics with the maximum curvature. Beyond point **C**, cenospheres lose their integrity and alter to a new state of the material. Hence, a tangent drawn to the P - δ characteristics, at point **C**, would intersect its linear portion (represented as **DE**) at **F**, which would yield σ_{CR} of the sample. As depicted in Fig. 4, the crushing load of the sample is 1280 kg, which would yield σ_{CR} equal to 159.2 kg/cm². This crushing strength falls in the range of the values reported in literature (Kruger 1996, <http://www.sinosi.com>), which does not present details of the testing methodology adopted.

As a practice, researchers have used e versus logarithm of effective stress σ' relationship, rather than P - δ characteristics, to study the crushing

Fig. 2 Particle-size distribution characteristics of the granular materials

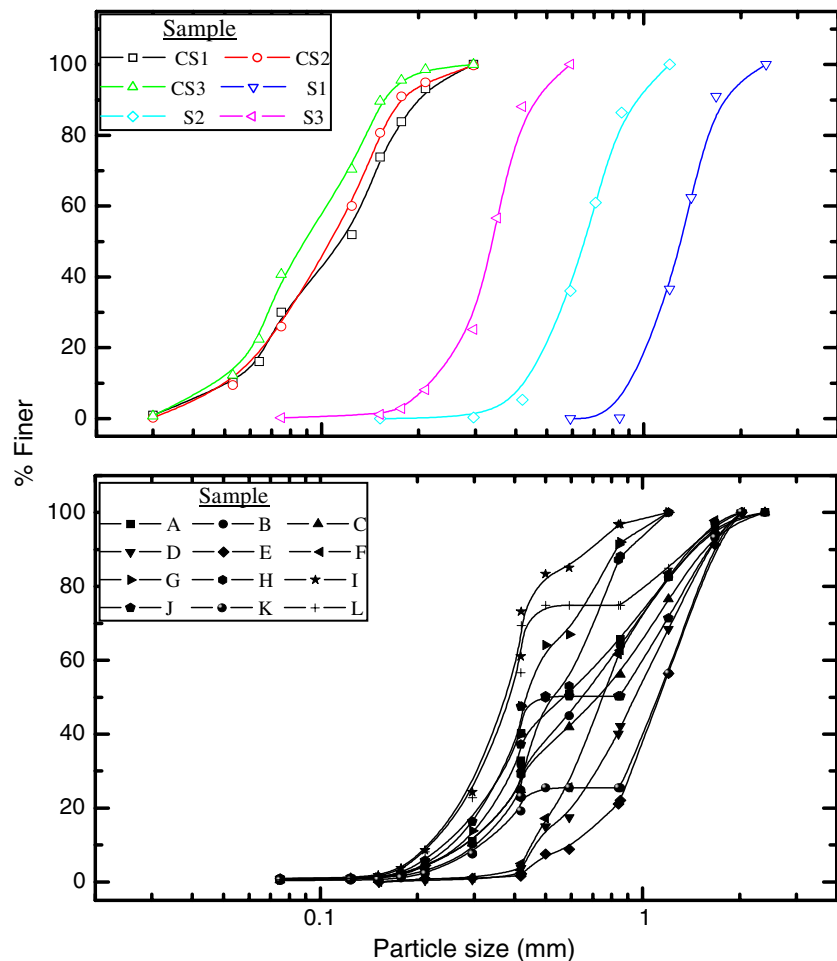


Table 2 Physical characteristics of the granular materials used in the study

Sample	G	D_x (mm)										C_u	C_c
		$x = 10$	15	30	40	45	50	60	80	85	90		
S1	2.65	0.70	0.98	1.11	1.21	1.26	1.30	1.38	1.57	1.65	1.72	1.97	1.28
S2		0.42	0.47	0.53	0.60	0.63	0.66	0.70	0.82	0.88	0.94	1.67	0.95
S3		0.25	0.27	0.30	0.32	0.33	0.34	0.35	0.42	0.43	0.46	1.41	1.01
A	2.65	0.28	0.31	0.39	0.43	0.50	0.56	0.72	1.15	1.27	1.43	2.61	0.76
B		0.28	0.32	0.42	0.52	0.58	0.66	0.80	1.15	1.27	1.43	2.83	0.77
C		0.28	0.32	0.42	0.55	0.63	0.73	0.89	1.27	1.43	1.55	3.13	0.72
D		0.60	0.71	0.93	1.02	1.08	1.12	1.21	1.52	1.61	1.75	2.03	1.19
E		0.47	0.53	0.72	0.82	0.87	0.93	1.07	1.39	1.51	1.59	2.29	1.02
F		0.45	0.49	0.53	0.67	0.71	0.75	0.82	1.14	1.28	1.39	1.83	0.75
G		0.31	0.34	0.44	0.46	0.50	0.53	0.63	0.79	0.84	0.92	2.02	1.00
H		0.25	0.29	0.38	0.40	0.42	0.44	0.49	0.71	0.76	0.82	1.93	1.14
I		0.22	0.24	0.31	0.34	0.36	0.38	0.41	0.48	0.57	0.69	1.88	1.08
J		0.33	0.37	0.90	0.98	1.05	1.11	1.21	1.45	1.55	1.67	3.72	2.05
K		0.24	0.28	0.38	0.42	0.43	0.83	1.01	1.37	1.47	1.61	4.20	0.57
L		0.22	0.25	0.31	0.35	0.37	0.39	0.42	1.03	1.20	1.35	1.88	1.06
CS1	0.82	0.05	0.06	0.08	0.09	0.11	0.12	0.13	0.17	0.18	0.20	2.42	0.80
CS2	0.81	0.05	0.06	0.07	0.09	0.10	0.11	0.12	0.15	0.16	0.18	2.23	0.82
CS3	0.75	0.05	0.06	0.07	0.08	0.08	0.09	0.10	0.14	0.14	0.15	2.21	0.95

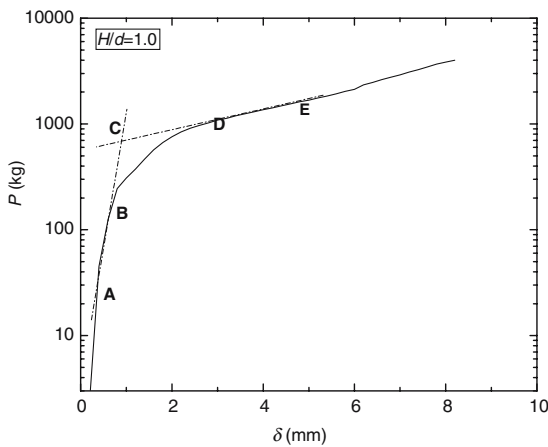


Fig. 3 Load-deformation characteristics of Sample S1

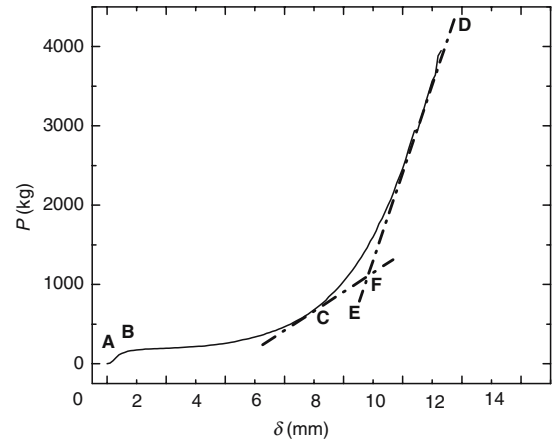


Fig. 4 Load-deformation characteristics of Sample CS3

characteristics of the granular materials (e.g., Hagerty et al. 1993; Nakata et al. 2001). It must be noted that e - $\log(\sigma')$ relationship would yield a point of maximum curvature, which has been termed as the “yield point” (DeSouza 1958) and the “break point” (Roberts 1964) and is representative of the stress required to initiate crushing of the grains. However, determination of the yield point involves uncertainty and error, mainly, due to improper identification of the point of maximum curvature on e - $\log(\sigma')$ relationship. In such a situation, the methodology described in this

paper, for determining crushing strength of granular materials, seems to be quite useful.

Further, the influence of H/d on σ_{CR} of different samples of sands was investigated. For the sake of brevity, P - δ characteristics of the Sample S1, corresponding to different H/d , are depicted in Fig. 5. It can be noted that the variation in crushing strength values of the sample with respect to H/d is almost insignificant. In addition to this, for checking the reproducibility of the P - δ characteristics, tests were performed on Sample S1 by maintaining $H/d = 1.0$ and the

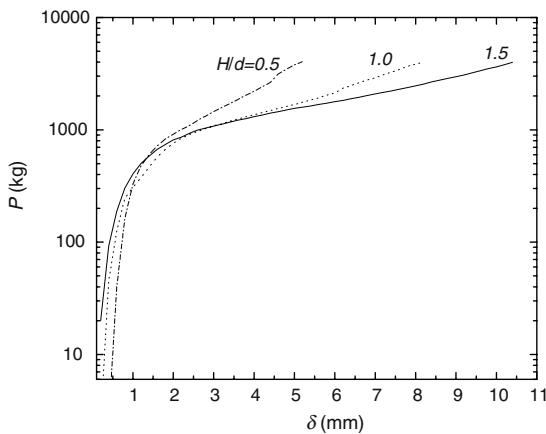


Fig. 5 Load-deformation characteristics of Sample S1 for different H/d values

results are depicted in Fig. 6. It can be noted from the figure that, the three trials yield practically same response.

Similarly, the influence of H/d on σ_{CR} of different cenosphere samples was investigated. For the sake of brevity, P - δ characteristics of Sample CS3, corresponding to different H/d , are depicted in Fig. 7. Using the above-mentioned methodology, σ_{CR} of Samples CS1, CS2 and CS3, corresponding to different H/d , was computed and is depicted in Fig. 8. From the figure, it can be noticed that initially, σ_{CR} increases non-linearly with H/d and thereafter attains a constant value. The transition point between these two trends corresponds to

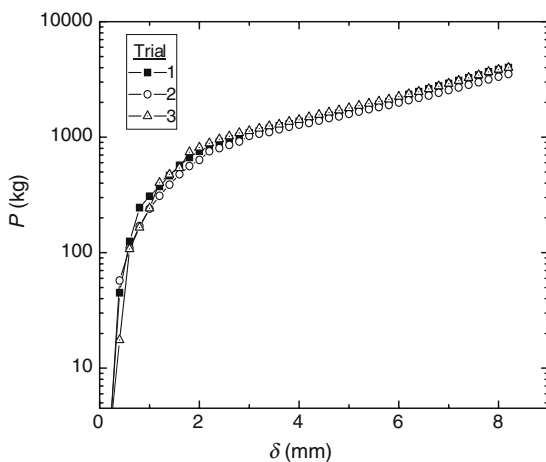


Fig. 6 Reproducibility of the load-deformation characteristics of sand (Sample S1, $H/d = 1.0$)

$H/d = 1$. Hence, for determining σ_{CR} of the cenosphere sample, tests must be conducted with $H/d = 1$. For checking the reproducibility of the P - δ characteristics, tests were performed on Sample CS3 by maintaining $H/d = 1$ and the results are depicted in Fig. 9. It can be noticed from the figure that the four trials yield practically same trends.

Further, σ_{CR} for different samples of sands and cenospheres, corresponding to $H/d = 1.0$, was determined and is tabulated in Table 3.

As samples used for this study exhibit a very wide range of the particle-sizes (refer Fig. 2) and specific gravity, attempts were made to correlate their σ_{CR} values with a parameter σ_{CR}/G , which is indicative of crushing strength per unit mass of the material. Hence, by using the optimization software Mathematica 4 (Mathematica 2000) and the results of 11 samples (S1, S2, S3, E, F, G, I, K, L, CS1 and CS3), relationships represented by Eqs. 1–6 were developed. As stated earlier, D_x appearing in these relationships corresponds to x percent finer fraction of the granular material. Mathematica 4 has a built-in ‘Nonlinear-fit’ function, which is based on the least-square fit concept. It fits the data to the fitting function and yields the best-fit parameters.

$$\sigma_{CR}/G = \frac{25 \cdot D_{45} \cdot D_{80}}{(D_{10} \cdot D_{30})^{0.7} \cdot (D_{90})^{1.5}} \tag{1}$$

$$\sigma_{CR}/G = \frac{20 \cdot D_{15} \cdot D_{45} \cdot D_{50} \cdot D_{80}}{(D_{10})^{1.6} \cdot D_{30} \cdot D_{60} \cdot (D_{90})^{1.2}} \tag{2}$$

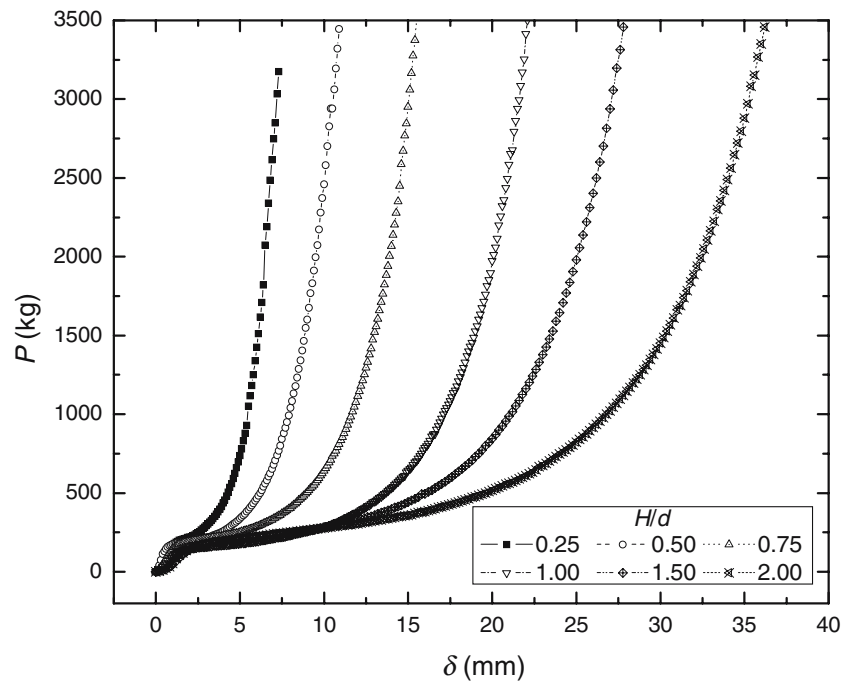
$$\sigma_{CR}/G = \frac{25 \cdot (D_{15}) \cdot (D_{45})^{1.5} \cdot (D_{60})^{0.5} \cdot (D_{85})^{2.5}}{(D_{10} \cdot D_{30})^{1.5} \cdot (D_{80})^{0.3} \cdot (D_{90})^3} \tag{3}$$

$$\sigma_{CR}/G = 15 \cdot \left[\left(\frac{(D_{15})^{0.5}}{(D_{30})^{1.55}} \right) + \left(\frac{(D_{60})^2}{D_{90}} \right) \right] \tag{4}$$

$$\sigma_{CR}/G = \frac{15}{(D_{10})^{0.8} \cdot D_{60}} \tag{5}$$

$$\sigma_{CR}/G = \frac{25 \cdot (D_{10})^{0.15}}{D_{30}} \tag{6}$$

Fig. 7 Load-deformation characteristics of cenospheres (Sample CS3)



Using Eqs. 1–6, σ_{CR}/G for these 11 samples was computed, as listed in Table 4, and its percentage error with respect to the experimentally obtained σ_{CR}/G , as depicted in italics, was also determined. The average percentage error for the relationships presented by Eqs. 1–6 is found to vary from 4 to 20%. However, the minimum value of the average percentage error (= 4.31) is obtained by using the relationship presented by Eq. 2. This indicates that the crushing strength of

the granular material can be estimated by using Eq. 2, with the least amount of error. Further, for validating this equation, parameter σ_{CR}/G was computed; (i) using Eq. 2 and (ii) from the experimental results available in the literature (Been et al. 1991; Pestana and Whittle 1995; Yamamuro et al. 1996; Bopp and Lade 1997; Nakata et al. 2001), as listed in Table 5. For most of the soils, the percentage difference between these two values of σ_{CR}/G , with respect to the

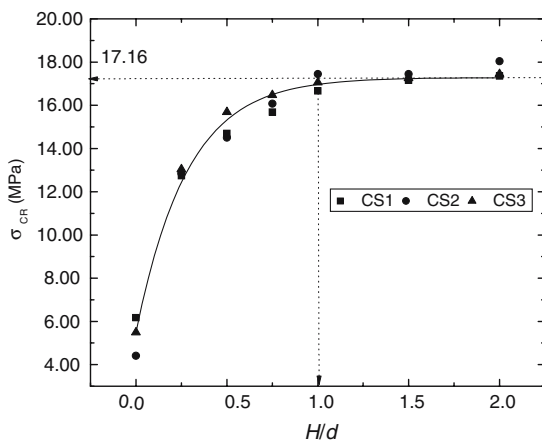


Fig. 8 Variation of σ_{CR} with H/d for different cenospheres

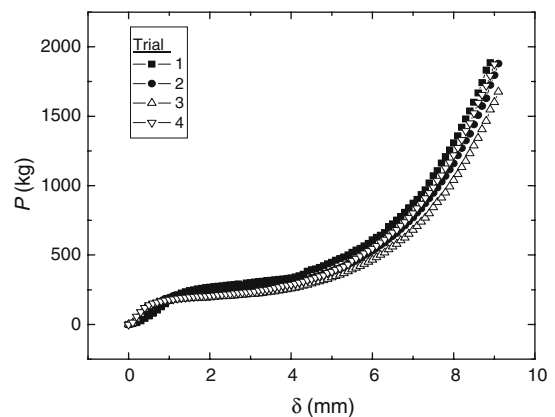


Fig. 9 Reproducibility of the load-deformation characteristics of cenospheres (Sample CS3, $H/d = 1$)

Table 3 Experimental results

Sample	<i>P</i> (kg)	σ_{CR} (kg/cm ²)
S1	703	87.40
S2	905	112.62
S3	1208	150.25
A	906	112.68
B	922	114.67
C	900	111.94
D	630	78.25
E	710	88.19
F	810	100.62
G	920	115.29
H	1000	124.32
I	1020	126.85
J	700	86.94
K	920	114.29
L	1055	131.06
CS1	1364.70	169.74
CS2	1426.21	177.39
CS3	1479.92	184.07

experimental value (depicted in italics), is found to be less than 10. This demonstrates the usefulness and efficiency of Eq. 2 for computing crushing strength of the granular material, in general.

However, the preceding discussion is applicable for the granular material with a wide range of particle sizes and hence can not be employed for determining σ_{CR} of the material which is poorly-graded, such as glass beads, which are of uniform diameter size D_{GB} . Hence, in order to prove the applicability of Eq. 2 even for the poorly-graded granular material, the load-deformation characteristics of the glass beads (with different D_{GB}) were established, by maintaining $H/d = 1$. For the sake of brevity, results of only three glass beads ($D_{GB} = 1, 2$ and 3 mm) are depicted in Fig. 10. σ_{CR} for the glass beads was computed (corresponding to the peak failure load), normalized with specific gravity G and is plotted with respect to D_{GB} , as depicted in Fig. 11.

The trend presented in Fig. 11 can be represented as:

$$\sigma_{CR}/G = 47 \cdot (D_{GB})^{-0.54} \tag{7}$$

Further, using Eq. 2, σ_{CR}/G for glass beads was computed by assuming $D_x = D_{PG}$, where D_{PG} corresponds to the diameter of poorly-graded

Table 4 A comparison of the experimental and computed value of σ_{CR}/G (in kg/cm²) for different samples

Sample	Experimental	Computed from Equation					
		1	2	3	4	5	6
S1	33.0	26.82	31.63	34.24	30.61	19.27	16.86
		<i>25.65</i>	<i>6.55</i>	<i>1.59</i>	<i>10.09</i>	<i>74.93</i>	<i>99.84</i>
S2	42.5	40.23	39.12	41.2	38.44	31.72	34.75
		<i>5.65</i>	<i>8.63</i>	<i>3.14</i>	<i>10.58</i>	<i>33.96</i>	<i>22.32</i>
S3	56.7	60.92	57.72	57.8	57.63	52.23	59.58
		<i>6.93</i>	<i>1.77</i>	<i>1.91</i>	<i>1.62</i>	<i>8.57</i>	<i>4.83</i>
E	33.78	34.8	32.04	35.67	31.06	27.67	25.51
		<i>4.36</i>	<i>3.87</i>	<i>6.69</i>	<i>7.16</i>	<i>20.28</i>	<i>30.46</i>
F	38.0	35.79	35.35	36.32	38.2	29.31	35.07
		<i>6.17</i>	<i>7.48</i>	<i>4.63</i>	<i>0.52</i>	<i>29.63</i>	<i>8.35</i>
G	43.5	45.53	40.66	41.32	40.99	41.2	40.55
		<i>4.46</i>	<i>6.98</i>	<i>5.28</i>	<i>6.14</i>	<i>5.59</i>	<i>7.27</i>
I	47.87	50.93	50.77	47.69	53.5	58.05	56.74
		<i>6</i>	<i>5.71</i>	<i>0.37</i>	<i>10.52</i>	<i>17.53</i>	<i>15.63</i>
K	43.12	45.38	44.93	42.44	48.85	47.4	46.14
		<i>4.97</i>	<i>4.02</i>	<i>1.61</i>	<i>11.72</i>	<i>9.03</i>	<i>6.55</i>
L	49.0	46.2	48.24	32.25	51.25	56.83	56.92
		<i>6.06</i>	<i>1.57</i>	<i>51.92</i>	<i>4.39</i>	<i>13.78</i>	<i>13.92</i>
CS1	207.0	208.96	207.13	206.57	205.25	202.2	208.16
		<i>0.94</i>	<i>0.06</i>	<i>0.21</i>	<i>0.85</i>	<i>2.37</i>	<i>0.56</i>
CS3	233.0	230.45	231.2	231.3	235.61	238.91	232.94
		<i>1.11</i>	<i>0.78</i>	<i>0.73</i>	<i>1.11</i>	<i>2.47</i>	<i>0.02</i>
Average % error		<i>6.57</i>	4.31	<i>7.09</i>	<i>5.88</i>	<i>19.83</i>	<i>19.06</i>

Table 5 Validation of the Eq. 2

Sample/Soil type	G	σ_{CR}/G^a (Experimental)	Reference	σ_{CR}/G^a (Computed)
A	2.65	42.5	Present study	40.36
B		43.27		5.2
C		42.18		43.23
D		29.53		0.077
H		46.87		3.67
J		32.81		28.49
CS2	0.81	219.0		3.65
Erksak (330/0.7) ^b	2.66	33		51.04
Uniform Ottawa sand (20–40) ^b	2.66	32.5		8.17
Uniform Ottawa sand (40–80) ^b		44.5		38.54
Uniform Ottawa sand (80–140) ^b		65	8.70	
Graded Ottawa sand (20–40, 40–80) ^b		34	218.89	
Graded Ottawa sand (20–40, 40–80, 80–140) ^b		35.5	0.05	
Graded Ottawa sand (20–40, 80–140) ^b		33	35.3	
Cambria sand	2.69	18	Been et al. (1991)	6.5
Silica sand	2.65	76	Pestana and Whittle (1995)	34.1
				4.69
				49.0
				9.1
				75.0
				13
				30.82
				9.1
				39.0
				8.5
				36.2
				8.3
			Yamamuro et al. (1996); Bopp and Lade (1997)	16.5
				9.09
			Nakata et al. (2001)	71.5
				6.29

^a in kg/cm² ; ^b grade of the sand

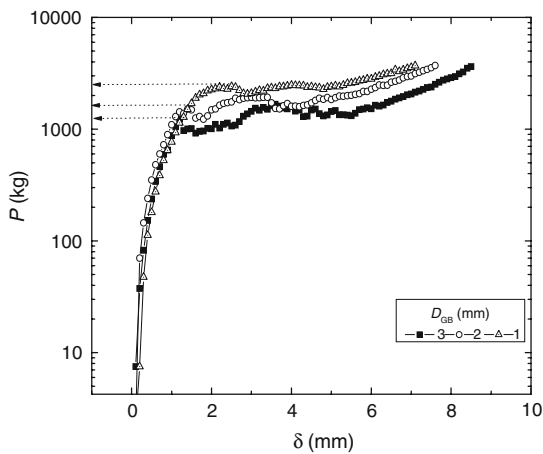


Fig. 10 Load-deformation characteristics of the samples of glass beads

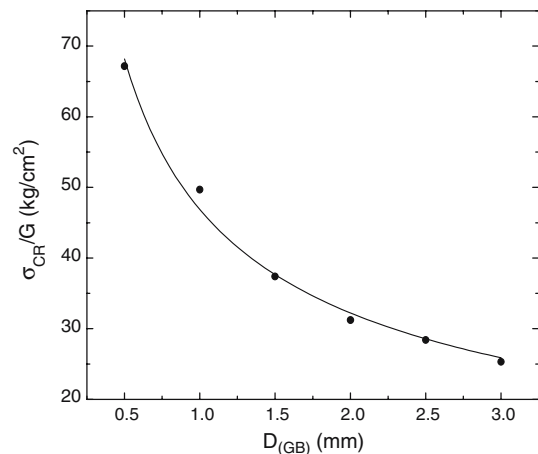


Fig. 11 Variation of crushing strength of glass beads with their diameter

granular material. The results obtained from this analysis are compared with those obtained from Eq. 7, as depicted in Fig. 12.

From Fig. 12, it can be noted that, results obtained from Eqs. 2 and 7 exhibit almost similar trends. However, with respect to Eq. 2, Eq. 7 is found to over-predict values of σ_{CR}/G for the same D_{PG} . With this in view, a multiplication factor, MF, which is the ratio of σ_{CR}/G obtained from Eq. 7 and Eq. 2 was computed and its variation with D_{PG} was established:

$$MF = 2.5 \cdot (D_{PG})^{0.3}. \quad (8)$$

As such, σ_{CR} of any granular material can be obtained by employing the following guidelines:

- For uniformly-graded material, Eq. 2 should be employed, and
- For poorly-graded material, results from Eq. 2 should be multiplied by MF.

4 Concluding remarks

A methodology for determining crushing strength of the granular material, based on its load-deformation characteristics has been developed and reported in this paper. Based on the rigorous experimentation on different samples of sands,

cenospheres and glass beads, a mathematical relationship between the crushing strength, particle-size characteristics and specific gravity of the granular material has been proposed. This relationship is found to be quite efficient and useful for determining crushing strength of granular materials.

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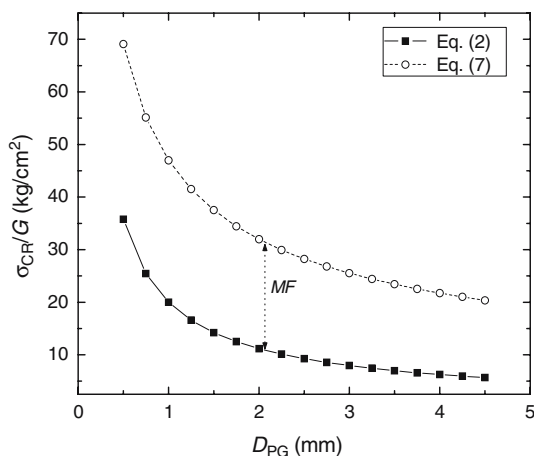


Fig. 12 Comparison of the results obtained from Eq. 2 and Eq. 7

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