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Natural rock joint roughness quantification through fractal techniques

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Abstract. Accurate quantification of roughness is important in modeling hydro-mechanical behavior of rock joints. A highly refined variogram technique was used to investigate possible existence of anisotropy in natural rock joint roughness. Investigated natural rock joints showed randomly varying roughness anisotropy with the direction. A scale dependant fractal parameter, K_v, seems to play a prominent role than the fractal dimension, D_{r1d}, with respect to quantification of roughness of natural rock joints. Because the roughness varies randomly, it is impossible to predict the roughness variation of rock joint surfaces from measurements made in only two perpendicular directions on a particular sample. The parameter $D_{r1d} \times K_v$ seems to capture the overall roughness characteristics of natural rock joints well. The one-dimensional modified divider technique was extended to two dimensions to quantify the two-dimensional roughness of rock joints. The developed technique was validated by applying to a generated fractional Brownian surface with fractal dimension equal to 2.5. It was found that the calculated fractal parameters quantify the rock joint roughness well. A new technique is introduced to study the effect of scale on two-dimensional roughness variability and anisotropy. The roughness anisotropy and variability reduced with increasing scale.

Key words. anisotropy, fractals, rock joints, roughness, scale effects, two dimensions.

1. Introduction

Strength, deformability and flow properties of rock joints depend very much on the surface roughness of joints. These effects arise from the fact that the surfaces composing a joint are rough and mismatched at some scale. The shape, size, number, and strength of contacts between the surfaces control the mechanical properties. The separation between the surfaces or the ''aperture'' determines the hydraulic properties. Therefore, accurate quantification of roughness is important in modeling strength, deformability and fluid flow behaviors of rock joints. Rock mass strength, deformability and fluid flow behaviors in turn depend very much on the properties of joints.

To quantify rock joint surface roughness, several methods have been proposed in the literature. The joint roughness coefficient (JRC) proposed by Barton (1973) has been widely used in engineering practice. Shortcomings of JRC in quantifying rock

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joint roughness have been pointed out by several researchers (Miller et al., 1990; Maerz et al., 1990; Hsiung et al., 1993; Kodikara and Johnston, 1994; Kulatilake et al., 1995). A number of researchers have investigated the applicability of various conventional statistical parameters in quantifying roughness (Tse and Cruden, 1979; Wu and Ali, 1978; Krahn and Morgenstern, 1979; Dight and Chiu, 1981; Maerz et al., 1990; Reeves, 1990). Kulatilake et al. (1995) have pointed out that the values obtained for conventional statistical parameters vary with the measurement scale. Therefore, the surface roughness of rock joints needs to be characterized using scale-invariant parameters such as fractal parameters. Several researchers have suggested using the fractal dimension to quantify rock joint roughness (Brown and Scholz, 1985; Miller et al., 1990; Power and Tullis, 1991; Huang et al., 1992; Poon et al., 1992; Odling, 1994; Den Outer et al., 1995). Kulatilake et al. (1995) pointed out that the fractal dimension itself is not sufficient and at least two fractal parameters are required to quantify rock joint roughness.

A number of methods have been suggested in the literature to estimate fractal dimension of roughness profiles of a rock joint surface. They are the divider (Mandelbrot, 1967), box counting (Feder, 1988), variogram (Orey, 1970), spectral (Berry and Lewis, 1980), roughness-length (Malinverno, 1990), and the line scaling (Matsushita and Ouchi, 1989) methods. Fractals can be either self-similar or self-affine. A self-similar fractal is a geometric feature that retains its statistical properties through various magnifications of viewing. A self-affine fractal remains statistically similar only if it is scaled differently in different directions. Figure 1 illustrates the concepts of self-similarity and self-affinity. In the case of a profile formed by a profilometer trace of a rock discontinuity, controversy has existed over self-similarity and self-affinity. Russ (1994) has asserted that a section taken at any orientation other than parallel to the mean surface orientation would result in a self-affine object. Therefore, it is not appropriate to consider natural rock joint profiles to be self-similar. They are self-affine profiles. The original divider and the original box

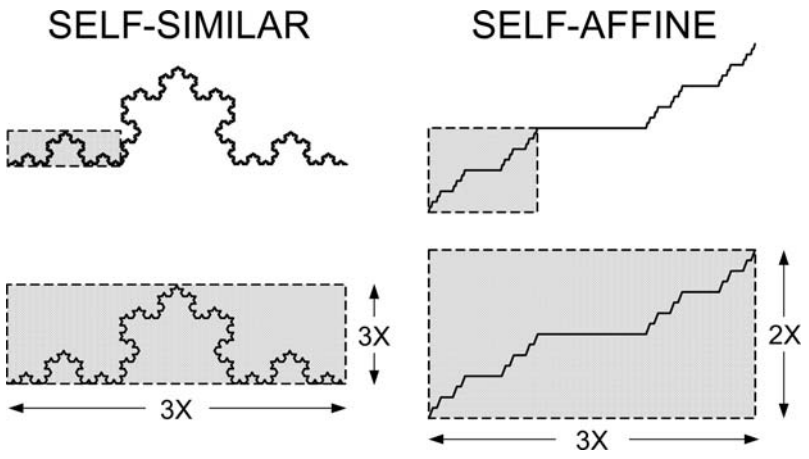

Figure 1. Illustration of self-similar and self-affine fractals.

counting methods are self-similar methods and they provide accurate results only for self-similar profiles. Problems are encountered when self-similar methods are used in the calculation of fractal dimensions for self-affine objects. The next two paragraphs explain the problems associated with the original divider method in calculating correct fractal dimension values for self-affine profiles and how to modify the method to produce correct fractal dimension values.

Linear roughness of natural rock joint profiles can be measured either using a mechanical profilometer or a laser profilometer. Each profilometer has a smallest horizontal step at which the height of the roughness profile can be measured. Therefore, even though the roughness profiles of a natural rock joint surface are continuous, roughness profile data obtained through a profilometer are available only at a certain interval of horizontal spacing. When these roughness data are plotted, they may produce a profile as shown in Figure 2. In this profile, the adjacent data points are connected through linear segments. Even though the horizontal length of each segment is the same, the inclined length (length of the segment) changes from one segment to another, depending on the inclination angle of the segment. Then the minimum feature size of a profile may be defined as the minimum segment length out of all the segment lengths between two adjacent data points on the profile (Figure 2). This minimum distance cannot be less than the horizontal distance at which roughness height data are available. The maximum feature size may be defined as the maximum segment length out of all the segment lengths between two adjacent data points on the profile (Figure 2). The difference between the maximum and minimum feature sizes of a profile reduces, as the profile gets smoother. Also, it is important to realize that both the estimated minimum and the maximum feature sizes of a profile depend upon the resolution of the instrument used in measuring roughness. The concepts mentioned above on the minimum and the maximum feature sizes are equally applicable for generated roughness profiles, because the generated values are available only at a certain interval of horizontal spacing.

The original divider method is best visualized by considering a pair of dividers set to a particular span and then walked along the roughness profile. The number of divider steps required to cover the entire profile is counted, and then multiplied by the divider span, r, to give an estimate of the profile length, L. The divider span is set to another value and the process is repeated several times to produce a discrete

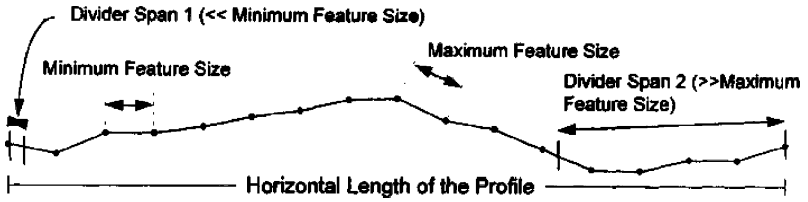

Figure 2. Concept of minimum and maximum feature sizes of a roughness profile.

relation between r and L. For self-similar fractals, the two are related linearly in log–log space according to the following equation (Feder, 1988):

$$\log L = \log a + (1 - D) \log r \tag{1}$$

where $\log a$ is the intercept of the $\log L - \log r$ plot and the slope of the log–log plot equals 1-D in which D is the fractal dimension. If the divider span is considerably shorter than the minimum feature size (for example, see divider span 1 shown in Figure 2), that span will virtually trace the profile without bridging any peaks or valleys of the profile returning the maximum possible value for the length. Therefore, for divider spans that are considerably shorter than the minimum feature size, the returned length L will be more or less the same. Due to this, the $\log L - \log r$ curve gradually flattens as shown in Figure 3 as r decreases beyond the minimum feature size. When the divider span is considerably larger than the maximum feature size (for example, see divider span 2 shown in Figure 2), the returned length will be close to the horizontal length of the profile. Therefore, for the divider spans that are considerably larger than the maximum feature size, the returned lengths will be very close to the horizontal length of the profile. Due to this, the $\log L - \log r$ curve gradually flattens as shown in Figure 3 as r increases beyond the maximum feature size. The aforementioned facts show the difficulty of obtaining a unique slope for the $\log L - \log r$ relation for the whole range of r as shown in Figure 3. The correct slope of $\log L - \log r$ and thus the correct D can be obtained by fitting a regression line to the $\log L - \log r$ data in the non-flattening portion of the curve (i.e. regression line 1 in Figure 3). The above discussion indicates very clearly that there is a need to select a suitable range for r, which is the input parameter in the divider method, taking into account the minimum and the maximum feature sizes in order to obtain accurate D values. Values of r considerably smaller than the minimum feature size or considerably larger than the maximum feature size will produce erroneous D values that are

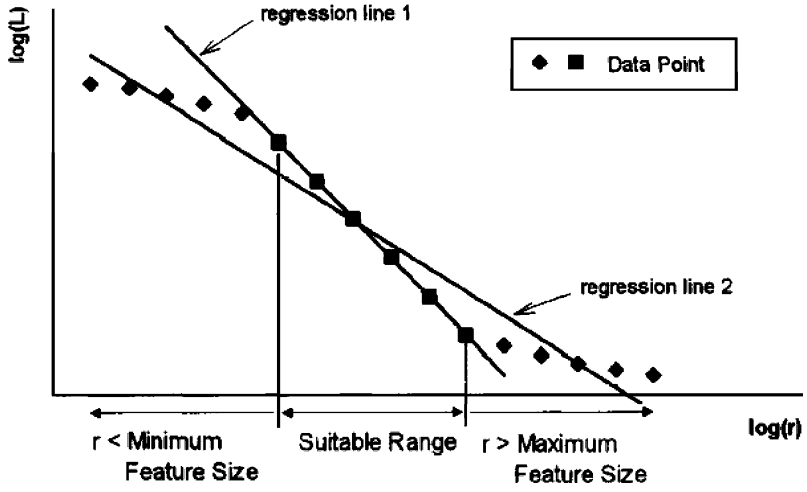

Figure 3. Effect of r on the estimation of fractal dimension with the original divider method.

almost 1. Part of the controversial findings appearing in the literature with respect to application of fractals to quantify roughness has resulted from application of the original divider method with divider lengths that are larger than the feature sizes of the profiles. Usually, in the case of natural rock joint profiles, the chosen divider lengths are much larger than the feature size range of the profile. In a rock joint profile, feature size range can be made comparable to the chosen divider lengths artificially by increasing the feature sizes through application of different magnification factors to the vertical direction only. For each magnified profile, the divider technique then can be applied to estimate the D value according to Equation (1). The D value is expected to increase with the magnification factor and eventually reaching a constant value asymptotically. This constant value is expected to provide the correct fractal dimension value for self-affine profiles. This methodology is known as the modified 1-D divider method (Mandelbrot, 1985).

A set of researchers (Brown and Scholz, 1985; Miller et al., 1990; Power and Tullis, 1991; Huang et al., 1992; Poon et al., 1992; Odling, 1994; Kulatilake et al., 1995; Shirono and Kulatilake, 1997; Kulatilake et al., 1997, 1998; Kulatilake and Um, 1999) have shown the possibility of quantifying roughness of natural rock joints through self-affine fractals using the roughness length, variogram, spectral and line scaling methods. Note that these methods are categorized under self-affine methods. Among the said researchers, some of them (Miller et al., 1990; Huang et al., 1992; Kulatilake et al., 1995; Kulatilake et al., 1997, 1998; Shirono and Kulatilake, 1997; Kulatilake and Um, 1999) have shown that the fractal parameter values calculated by the roughness length, variogram, spectral and line scaling methods may depend significantly on the input parameter values used in each of those methods, as well as on the profile parameters such as stationary/non-stationary nature of the profile, data density (number of data per unit length), profile length etc. It is possible that some of the controversial findings that appear in the literature on application of self-affine techniques to rock joint profiles might have resulted due to the latter mentioned reasons. The second order stationary profiles satisfy the following properties: (a) the mean surface is a constant with respect to the spatial location; (b) the variance of the surface height around the mean surface is a constant with respect to the spatial location; and (c) the covariance function of the surface height depends only on the lag distance irrespective of the spatial location. Profiles that do not satisfy at least one of the above criteria are known as non-stationary profiles. Four relatively recent papers have investigated the accuracy of fractal parameters estimated through the spectral (Shirono and Kulatilake, 1997), line scaling (Kulatilake et al., 1997), variogram (Kulatilake et al., 1998) and roughness length (Kulatilake and Um, 1999) methods and have suggested refined procedures to quantify natural rock joint roughness accurately. In this paper the variogram technique is used with the suggested refined procedures to quantify natural rock joint roughness in different directions and to study the anisotropy of natural rock joint roughness.

Authors are aware of only one study that has looked into estimating two-dimensional roughness parameters for rough planes (Rasouli and Harrison, 2001).

They have investigated scale effects and anisotropy of two-dimensional roughness through two multivariate parameters. In their method, equilateral tripod of a selected size is kept at different orientations on the rough surface at random locations. For each orientation of the tripod, the directional cosines of the unit normal vector are recorded. Then the variance–covariance matrix of all the unit normal vectors is formed. The normalized eigen value of this matrix is used as a measure of roughness. By repeating the procedure for different tripod sizes, the effect of scale on roughness is evaluated. A test statistic which is a function of the eigen values and eigen vectors of the variance–covariance matrix (Fisher et al., 1987) is used as a measure of roughness anisotropy about the major principal axis. In this paper, a different approach is used to study the effect of scale on anisotropy and variability of two-dimensional natural rock joint roughness. In addition, in this paper, the modified divider technique is extended to two dimensions to calculate two-dimensional roughness parameters for natural rock joints ignoring the anisotropy.

2. Rock Joint Sample Preparation and Roughness Data Collection

Diorite and Grano-diorite block samples, each containing a fracture plane, were collected from an open-pit copper mine that is located in the south of Tucson, Arizona. The samples were then brought to the University of Arizona, Rock Mechanics Laboratory and cut in a manner to make rectangular-prism shaped rock specimens, each having an approximate central horizontal joint. These specimens were then ground (on a surface grinder) to have all 6 faces to be smooth, and be orthogonal to any neighboring face, but parallel to its opposite face. A laser profilometer (Figure 4), which is available at the University of Arizona, was used to perform surface profiling measurements of the rock joints. Linear profiles of the surfaces were obtained at a spacing of 0.3333 mm parallel to the long edge of each specimen. Along each linear profile, surface height was measured at a spacing of 0.3333 mm. The measurement resolution of the laser profilometer was about 3 μm. Both top and bottom fracture surfaces were scanned by the laser profilometer for each rectangular-prism shaped rock specimen.

Figures 5 and 6 show the pictorial views of two samples that were used for the analysis. According to visual perception, the sample grd1-1 btm shows a higher level of roughness compared to that of the sample dr55 btm.

3. Investigation of Joint Roughness Anisotropy Through the Variogram Technique

3.1. MAIN FEATURES OF THE VARIOGRAM METHOD

Let $Z(x)$ be a Gaussian process with stationary increments and mean = 0, and the variogram function is given by $2\gamma(x,h) = E[(Z(x+h)-Z(x))^2]$ where h is the lag distance along the x-axis. If $\gamma(x,h)$ behaves like h^{2H} as $h \rightarrow 0$ (where H is the Hurst

Figure 4. A photograph of the laser profilometer and data acquisition system.

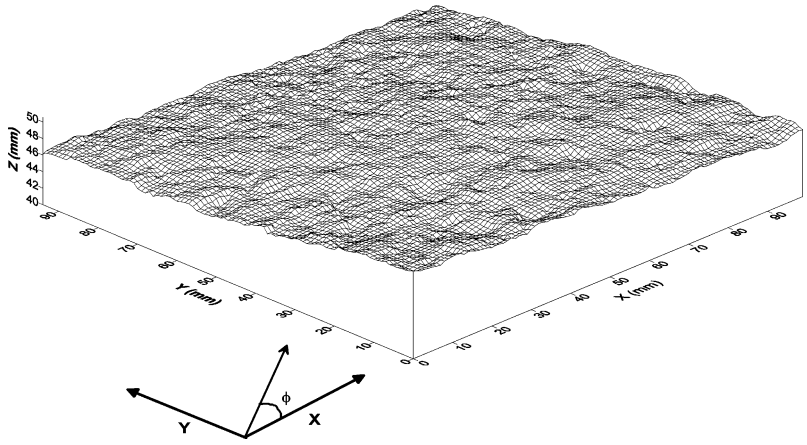

Figure 5. Pictorial view of dr55 btm sample.

exponent), then the fractal dimension, D_{r1d}, of $Z(x)$ is equal to 2-H (Orey, 1970). To estimate D_{r1d}, first H should be estimated. Before H is estimated, it is necessary to check whether the following power law equation holds true:

$$2\gamma(x,h)_{h\to 0} = K_v h^{2H} \tag{2}$$

This can be evaluated by checking the linearity of the plot between $\log_e(\text{variogram})_{h \to 0}$ and $\log_e(h)$. In Equation (2), K_v is a proportionality constant.

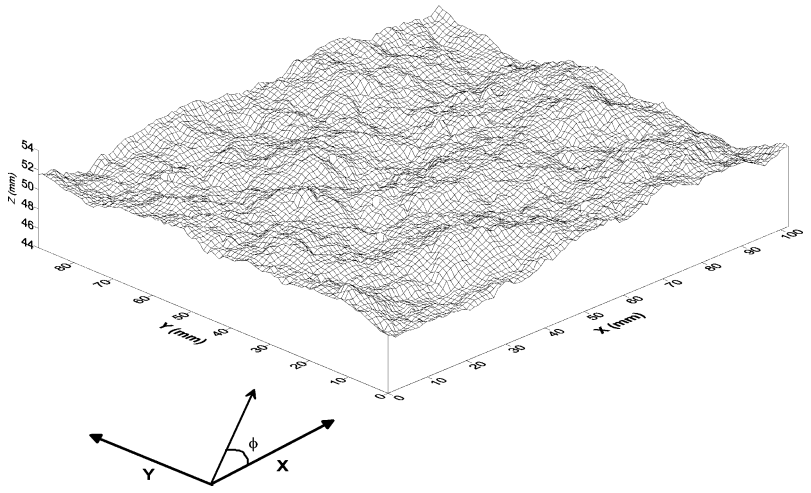

Figure 6. Pictorial view of grd1-1 btm sample.

This can be calculated from the intercept of the linear regression line of the plot between $\log_e$ (variogram) and $\log_e$ (h). However, Equation (2) cannot directly be used to calculate the fractal parameters of a roughness profile. It needs to be expressed in a discretized form so that it can be applied to the digitized roughness profile data. The one-dimensional variogram function, 2γ (x,h), can be expressed in the discretized form as shown in Equation (3), where x is the horizontal distance along a roughness profile and $Z(x)$ be the height of the roughness profile from the datum.

$$2\gamma(x,h) = \frac{1}{M}\sum_{i=1}^{M}[Z(x_i) - Z(x_i + h)]^2 \tag{3}$$

In Equation (3), M is the total number of pairs of roughness heights of the profile that are spaced at a lag distance h. The variogram given by Equation (3) is termed the experimental variogram. Several values of h are selected for which the respective values of γ(x,h) are calculated. $\mathrm{Log_e}(2\gamma(x,h))$ values are then plotted against the $\log_e(h)$ and the D_{r1d} and K_v are calculated from the slope and intercept of the regression line. Note that D_{r1d} and K_v are respectively, measures of the autocorrelation and amplitude of the roughness profile. From Equation (2), it is clear that the variogram is well related to the roughness and it is not only related to D_{r1d}, but also to K_v. This means that both D_{r1d} and K_v are required to quantify roughness. When $h = 1$ unit, $2\gamma(x,h) = K_v$. Unit of h can be changed from mm to km depending on the scale of the roughness profile. Therefore, the value of K_v can change depending on the unit chosen to represent h. This means K_v has the potential to capture the scale effect of roughness.

Kulatilake et al. (1998) pointed out that to calculate accurate fractal parameters through the variogram method h needs to be in a certain range and that the range

depends on the data density, d, and the D_{r1d} value of the profile. Because the D_{r1d} value of the profile need to be estimated is unknown, they came up with a conservative equation of $hd = 1.76$ to estimate the initial h value by knowing the d value of the profile to apply for profiles having D_{r1d} values between 1.0 and 1.7. It was suggested to compute six more h values using an increment factor of 1.2 starting from the estimated initial h value, and then to use these seven h values in computing the corresponding $\gamma(x.h)$ values and hence to plot $\log_e(2\gamma(x,h))$ vs. $\log_e(h)$ to estimate fractal parameters. It was also suggested to compute the linear correlation coefficient (R) of the resulting regression line and only if it is greater than 0.8, then to use the slope and intercept of the resulting regression line to compute D_{r1d} and K_v. These suggestions were strictly followed in calculating the fractal parameters D_{r1d} and K_v included in this paper. Also, they stated that for the calculated fractal parameters to be accurate the roughness increments of the profiles should satisfy second order stationarity requirements.

3.2. COMPUTATION OF ONE-DIMENSIONAL ROUGHNESS PROFILES IN DIFFERENT DIRECTIONS USING THE MEASURED ROCK JOINT ROUGHNESS DATA

Because the available computer program to apply the variogram fractal technique (Kulatilake et al., 1998) was developed to deal with the profiles of one dimensional roughness data, and the analysis had to be carried out along different directions on the rock joint surfaces to investigate the anisotropy pattern of natural rock joints, several one-dimensional roughness data profiles had to be computed using the measured natural rock joint roughness data incorporating an interpolation technique.

The data, that were measured using a laser profilometer, were available in a grid pattern in the form of $z = f(x, y)$, where x–y is a rectangular coordinate system on the horizontal plane and the z value at each point is the height of the rough surface from the x–y-plane in the vertical direction. The height of the rough surface is thus a point wise distribution of values ranging from the minimum surface height to the maximum surface height. The grid points on the x–y-plane were spaced at 0.3333 mm intervals along the x and y directions. The plan view of the x–y-plane of the sample is shown in Figure 7. Since the data were available in a grid pattern, an interpolation technique had to be used to create the one-dimensional roughness profiles along the required directions. These one-dimensional roughness profiles were computed at a distance of 10 mm apart and parallel to each other for every 15-degree increment with the x-axis as shown in Figure 7.

The computer program, which was developed to generate the roughness profiles along different directions using the measured rock joint roughness data, uses the inverse distance weighted (idw) interpolation technique [see Equation (4)] to compute the roughness data points along the profile directions. For the analysis, the data points were generated at a spacing of 0.3333 mm apart along each of the considered profiles.

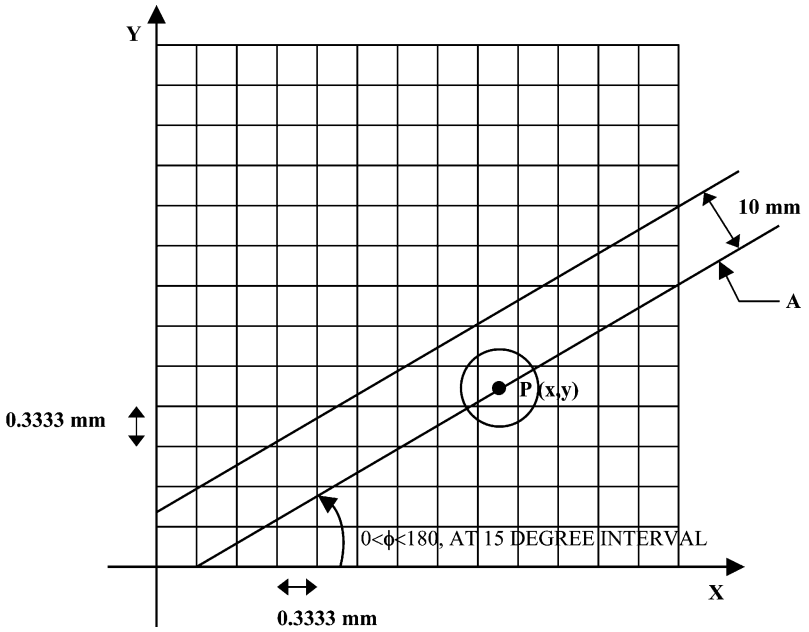

Figure 7. An illustration showing the interpolation procedure.

The following example explains the way in which the algorithm in the computer program works to compute the rough surface height along each of the considered profile directions. To find the rough surface height at a location $p\ (x, y)$ in profile A (Figure 7); first, the program looks around for the rough surface height data points within a radius of 0.3333 mm from $p\ (x, y)$. If it finds any point within the specified radius, then it uses those rough surface height data points to calculate the rough surface height at $p\ (x, y)$ according to the following equation:

$$z(x, y) = \frac{\sum_{i=1}^{n} \frac{z_i}{r_i}}{\sum_{i=1}^{n} \frac{1}{r_i}} \tag{4}$$

where z_i is the ith point within the radius of 0.3333 mm from $P\ (x, y)$ and r_i is the distance to the point from $P\ (x, y)$. This procedure is repeated over and over again until the program finishes interpolating all necessary data points along the line A at the spacing of 0.3333 mm.

3.3. CALCULATED FRACTAL PARAMETERS USING THE VARIOGRAM METHOD

In this study, the minimum acceptable initial lag distance for the variogram method was calculated using the equation $hd = 1.76$, which was suggested by Kulatilake et al. (1998). The calculated one-dimensional roughness data profiles had the data density of 3. If this d value is substituted in the aforesaid equation, then h becomes 0.586. In

this study, the initial h value of 0.6 was used. The computer program uses an increment factor of 1.2 along with the initial h value to calculate the other 6 consecutive lag distance values. These seven lag distance values were used as stated in Section 3.1 to calculate the fractal parameters D_{r1d} and K_v for the calculated roughness profiles stated in Section 3.2, which are available at every 15-degree increment with the x-axis as shown in Figure 7. Note that in each direction calculations were made along several roughness profiles that are spaced 10 mm apart.

Table 1 lists the mean, maximum, minimum, and coefficient of variation of the fractal parameters (D_{r1d}, and K_v) that were calculated for natural rock joint roughness data of different samples along $\phi = 0$-degree direction. From Table 1, it is clear that even along a single direction, the roughness significantly varies from place to place. The values of coefficient of variation clearly indicate that the relative variation of K_v is higher than that of D_{r1d}. That means K_v plays a prominent role than D_{r1d} with respect to quantification of roughness of natural rock joints. In Table 1, from top to bottom, the samples are arranged in the order of increasing roughness level based on visual observation of the samples. D_{r1d} captures the autocorrelation of the roughness profile and K_v is a measure of amplitude of the profile. Since, D_{r1d} and K_v capture different and somewhat independent properties of the roughness profiles, a combination of both D_{r1d} and K_v should be used to quantify the roughness of rock joints. Kulatilake et al. (1995) suggested the parameter $D_{r1d} \times K_v$ for accurate quantification of roughness using fractals. This parameter is used here for the study. Table 1 lists the variation of $D_{r1d} \times K_v$ for different samples. The parameter $D_{r1d} \times K_v$ increases with the roughness level of the samples. Hence, the parameter $D_{r1d} \times K_v$ may be used to quantify the roughness of rock joint profiles accurately.

3.4. DIRECTIONAL VARIABILITY OF ROUGHNESS THROUGH VARIOGRAM METHOD

The variogram method was applied to the one-dimensional roughness profiles that were calculated using the IDW interpolation technique in different directions for each of the two rock joint surfaces. Mean values of fractal dimension D_{r1d}, scale dependent fractal parameter K_v and $D_{r1d} \times K_v$ were estimated in different directions. Finally these average values were plotted to see for any directional variability in the roughness in the rock joint surfaces. The observed variations are shown in the Figures 8 and 9. The figures clearly show the existence of anisotropy in the rock joint samples. However there is no definite pattern in which the roughness varies with the direction.

Table 1. Statistical properties of the fractal parameters D_{r1d} and K_v of rock joint samples in 0-degree direction

Sample name	Mean			Maximum		Minimum		Coeff. Var.	
	D_{r1d}	K_v	$D_{r1d} \times K_v$	D_{r1d}	K_v	D_{r1d}	K_v	D_{r1d}	K_v
Dr55 btm	1.295	0.022	0.029	1.409	0.027	1.190	0.016	0.045	0.182
Grd1-1 btm	1.382	0.057	0.079	1.514	0.095	1.269	0.038	0.053	0.281

If a regular roughness anisotropic pattern exists, then D_{r1d} and K_v figures should show elliptical symmetry. Roughness varies in a random manner with the direction. Random roughness variation was even more pronounced in several other samples authors investigated that are not shown in this paper. The two rock joint samples shown in the paper are most probably formed through tensile fracturing. It is not possible to expect a regular roughness anisotropic structure on tensile fracture surfaces. However, a possibility exists to observe a roughness anisotropic structure on rock joint surfaces formed by shear fracturing. Thus, finding roughness parameters only along two perpendicular directions would not be sufficient to predict the roughness parameters along other directions. Sample dr55 btm shows highest values of D_{r1d} and K_v along 90-degree direction with the x-axis, presumably the highest roughness direction of that sample. The variation of D_{r1d} value with direction of grd1-1 btm is not significant. However, it has lowest values of K_v between 0 and 30-degree directions and highest along the 90-degree direction. These values seem to agree with the visual observation of roughness of the surface. Directional variation of $D_{r1d} \times K_v$ is much

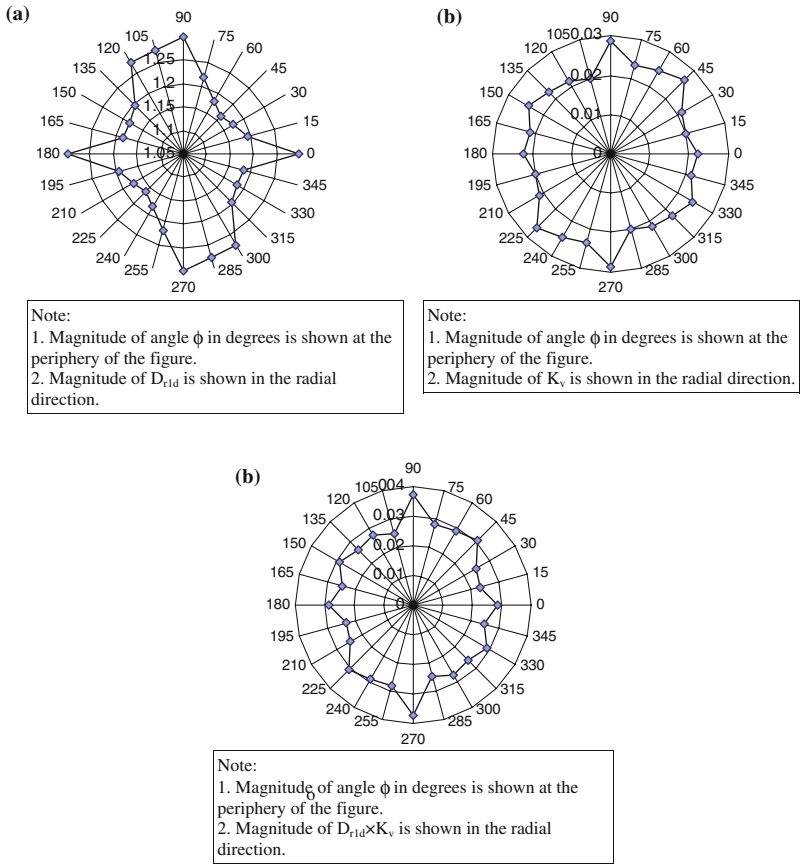

Figure 8. Variation of (a) D_{r1d}, (b) K_v and (c) $D_{r1d} \times K_v$ with direction for rock joint surface dr55 btm.

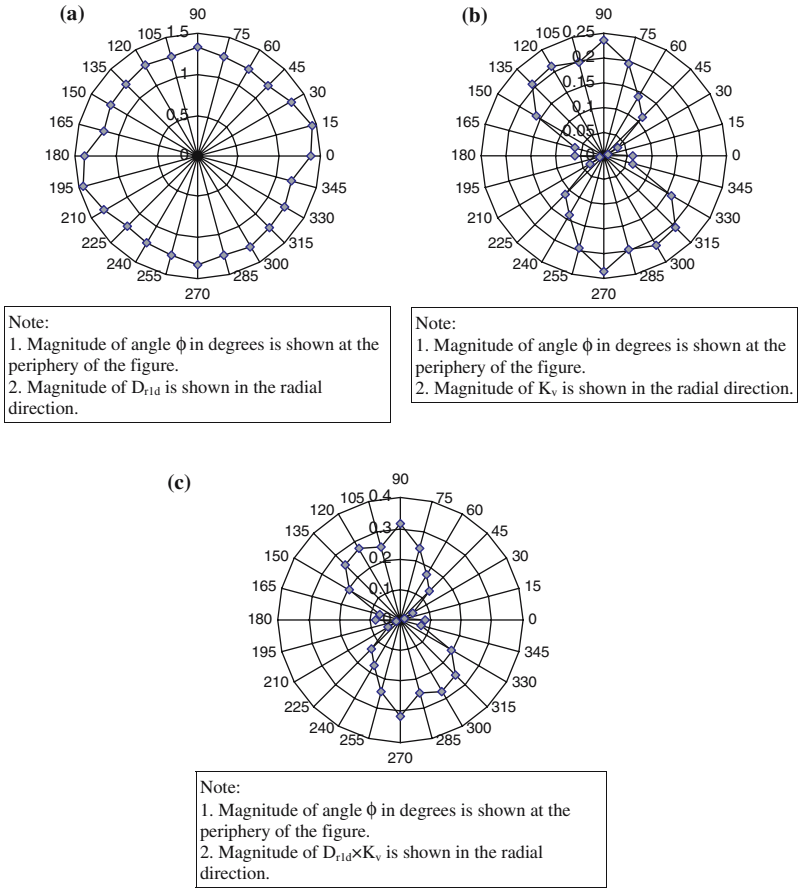

Figure 9. Variation of (a) D_{r1d}, (b) K_v and (c) $D_{r1d} \times K_v$ with direction for rock joint surface grd1-1 btm.

closer to the directional variation of K_v than that of D_{r1d}. This shows that K_v plays a prominent role than D_{r1d} with respect to overall roughness of natural rock joints. Based on all the aforementioned results one can say; in rock engineering designs, it is very important to give proper consideration to the directional variability of roughness to estimate rock joint mechanical and hydraulic properties.

4. Application of a Two-dimensional Version of the Modified Divider Technique to Calculate the Two-dimensional Roughness of Natural Rock Joint Surfaces

4.1. EXTENSION OF THE MODIFIED DIVIDER TECHNIQUE TO TWO DIMENSIONS

As explained in Section 1, the modified divider method can be used to calculate the fractal dimensions of self-affine roughness profiles (Mandelbrot, 1985). This method

was extended to two dimensions to quantify the two-dimensional fractal dimensions of roughness surfaces of natural rock joints.

In this method, first the roughness surface area was discretized into 4-sided polygons of a certain size. The area of these polygons on x–y plane would be a square with an elemental area A_0 and A_0 was varied systematically. Then these polygons were further subdivided into four triangles. After that the surface area formed by the triangular elements on the rock surface were calculated to estimate the total surface area A of the roughness surface. This process was repeated for different A_0 values. The relation between ln (A) and ln (A_0) is given in Equation (5) as:

$$\ln(A) = (2 - D_{r2d}) \ln(A_o) + C \tag{5}$$

The gradient of the linear regression line of Equation (5) was used to compute the fractal dimension, D_{r2d}, of the rock joint roughness surface. In Equation (5), the constant C is a scale dependant fractal parameter. Then the z coordinates of the roughness data were multiplied by a magnification factor and the aforementioned area measurement process was repeated and a graph was drawn between ln (A) and ln (A_0) to find the fractal dimension of the magnified roughness surface area. This process was repeated for different magnification factors.

4.2. VALIDATION OF THE DEVELOPED TWO DIMENSIONAL MODIFIED DIVIDER TECHNIQUE

Figure 10 shows a generated Fractional Brownian Motion (FBM) surface with fractal dimension 2.5. The developed two dimensional modified divider technique was applied to the said surface to perform a validation. The variation of the computed fractal dimension of this surface with the magnification factor is shown in Figure 11. The calculated fractal dimension turned out to be 2.48 for magnification factors over 100. That means the developed method calculates the fractal dimension of the surface to an accuracy of 99.2%.

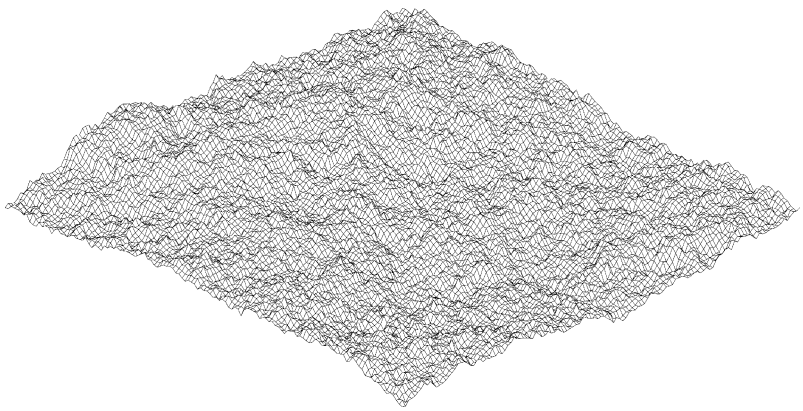

Figure 10. Generated fractional Brownian surface with fractal dimension equal to 2.5.

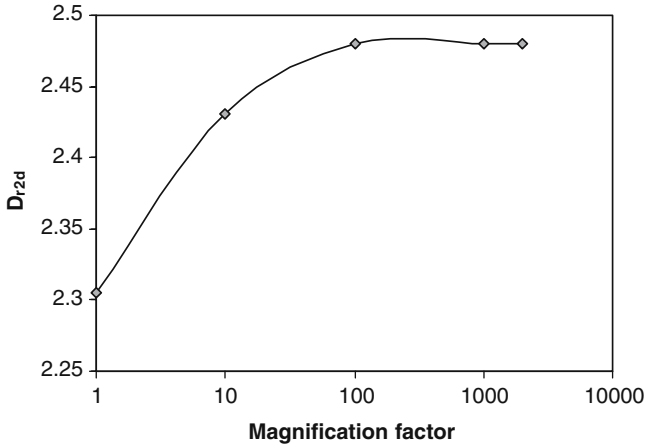

Figure 11. Variation of calculated fractal dimension D_{r2d} with the magnification factor for the generated FBM surface with $D_{r2d} = 2.5$.

4.3. APPLICATION OF THE TWO DIMENSIONAL MODIFIED DIVIDER TECHNIQUE TO NATURAL ROCK JOINT SURFACES

As an example, Figures 12–15 show the ln (A) vs. ln (A_0) plots obtained for dr55 btm rock joint surface for different magnification factors. The calculated fractal dimension of the rock joint surface was close to 2 for the original roughness data without magnification (see Figure 12). For the magnified roughness data, the fractal dimension was found to increase with the magnification factor and reaches an asymptotic value (see Figure 16). This asymptotic value can be considered as the real fractal dimension of the roughness surface dr55 btm. The Figure 17 shows the

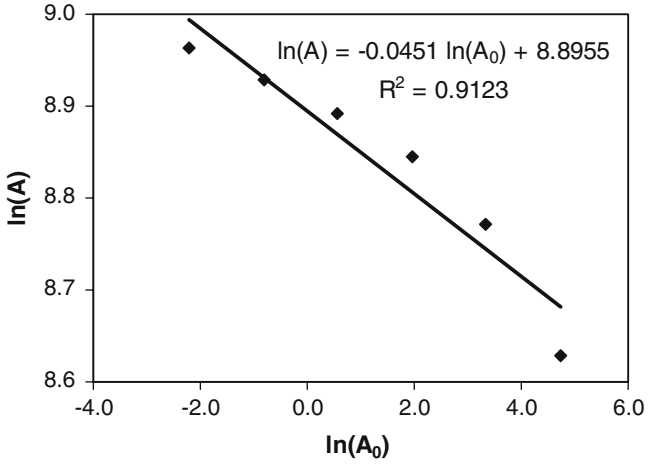

Figure 12. Variation of ln (A) with ln (A_0) for the roughness data of dr55 btm with the magnification factor 1.

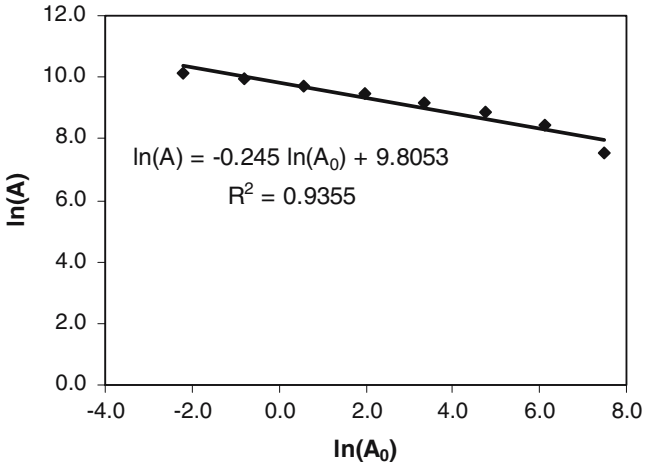

Figure 13. Variation of ln (A) with ln (A_0) for the roughness data of dr55 btm with the magnification factor 10.

variation of calculated fractal dimension with the magnification factor for grd1-1 btm sample.

Table 2 lists the values obtained for two dimensional fractal parameters of the rock surfaces. The values calculated for parameters D_{r2d} and C_x for the data coming from samples dr55 btm and grd1-1 btm agree with the intuitively expected roughness levels of those samples (see Table 2).

5. A New Method to Investigate Scale Effects and Anisotropy of Two-dimensional Roughness of the Rock Joints Surfaces

In this method, first the roughness surface is discretized into square cell elements of a certain size. Each cell is then divided into two triangular elements in the two possible ways. The orientation of each triangular element in three-dimensions is represented by its outward unit normal vector. A computer program was written to find the unit normal vectors of these elemental planes and then to plot the outward unit normal vectors on a stereographic projection. The methodology was repeated for different increasing cell sizes. The statistical distribution of the unit normal vectors of all the triangles of the rock joint surface on a stereographic projection (Figure 18) represents the directional distribution of rock joint roughness. Figure 18 shows that even though the mean roughness plane is almost the same, the spatial variability of the roughness plane direction decreases with increasing cell size. In addition, Figure 18 shows how the roughness anisotropy changes with the scale. Figure 19 shows how the spherical variance, which is given by Equation (6), a measure of variability of the statistical distribution of the unit normal vectors, decreases with increasing cell size.

$$\text{Spherical Variance, S} = \frac{N - |\vec{R}|}{N} \tag{6}$$

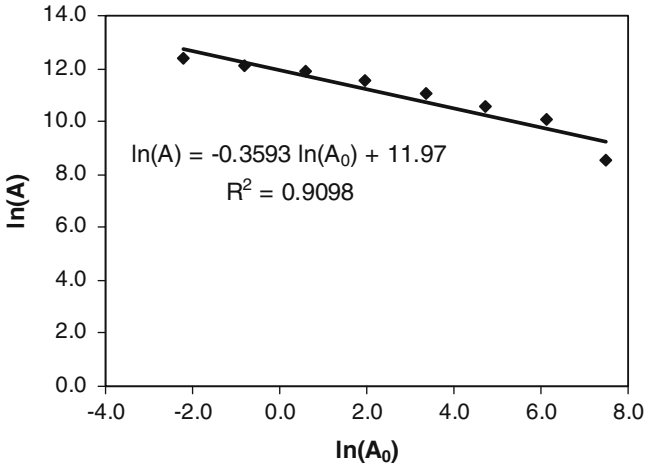

Figure 14. Variation of ln (A) with ln (A_0) for the roughness data of dr55 btm with the magnification factor 100.

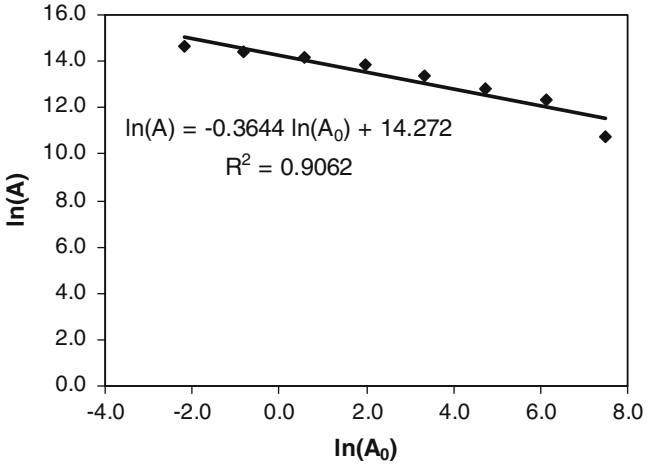

Figure 15. Variation of ln (A) with ln (A_0) for the roughness data of dr55 btm with the magnification factor 1000.

In the above equation, N is the number of unit normal vectors on the rock joint surface, and $|\vec{R}|$ is the magnitude of the resultant normal vector of the said unit normal vectors. Figure 19 basically shows the effect of scale on the variability of roughness direction.

6. Conclusions

The parameter $D_{\mathrm{r1d}} \times K_{\mathrm{v}}$ seems to capture the overall roughness characteristics of natural rock joints well. Therefore instead of using the highly subjective JRC

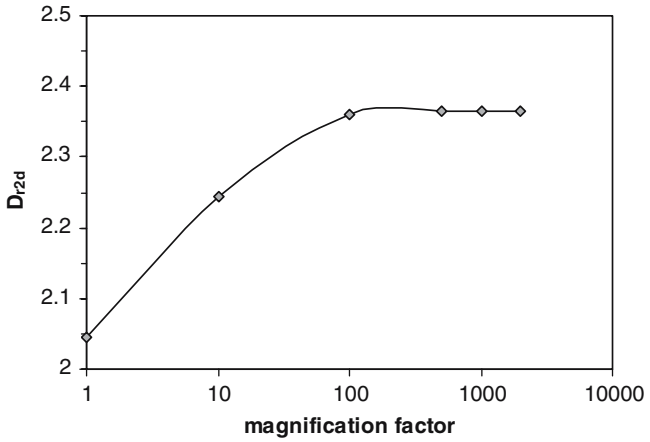

Figure 16. Variation of fractal D_{r2d} with the magnification factor for sample dr55 btm.

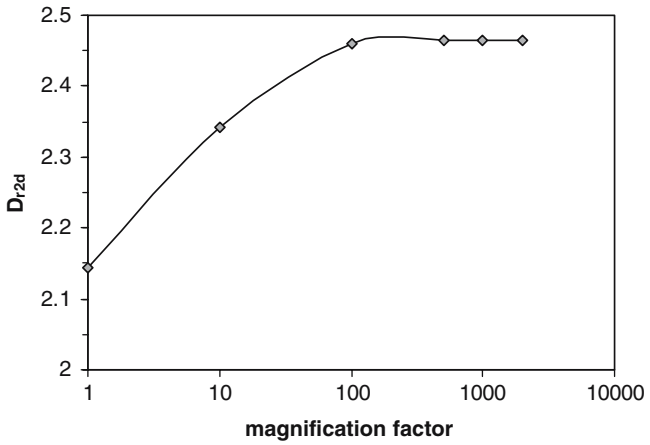

Figure 17. Variation of fractal D_{r2d} with the magnification factor for sample grd1-1 btm.

Table 2. Two-dimensional fractal dimension and the intercept of ln (A) vs. ln (A_0) plot for different roughness surfaces

Sample	D_{r2d}	C_{1000}	C_{2000}
Dr55 btm	2.3645	14.272	14.965
Grd1-1 btm	2.4654	15.128	15.821

Note: In Table 2, C_x is the intercept value when magnification factor of x is used.

parameter, the parameter $D_{r1d} \times K_v$ may be used to quantify the roughness of rock joints. The latter mentioned parameter should be quantified for more rock joint roughness data in the future to see whether it would be possible to increase the

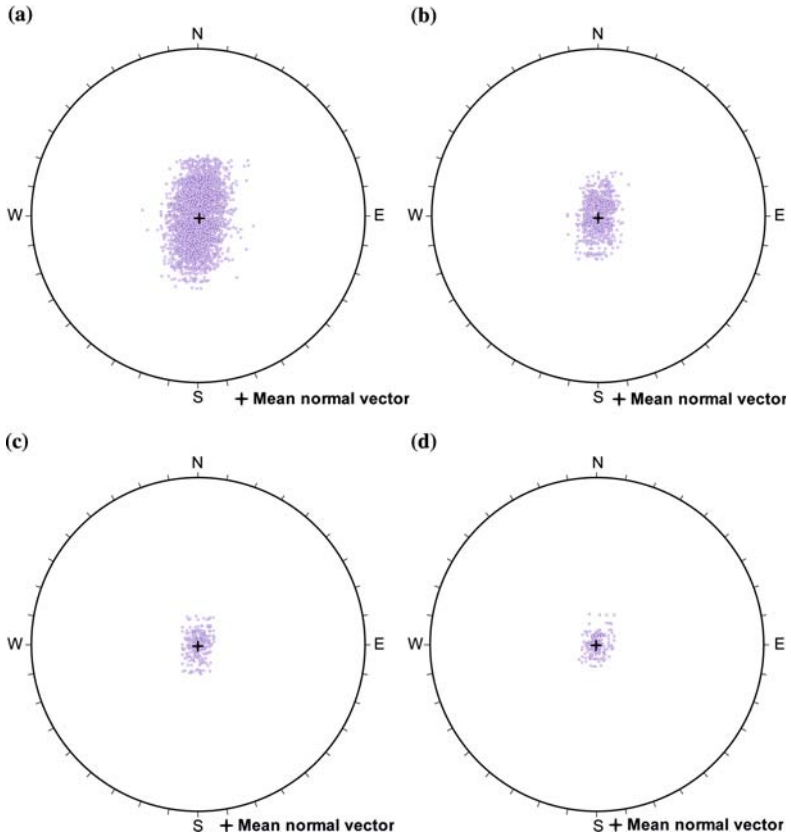

Figure 18. Statistical distributions of normal vectors of triangular elements with cell size (a) 3 mm, (b) 6 mm, (c) 10 mm and (d) 12 mm for grd1-1 btm sample.

confidence of using this parameter in rock engineering practice to quantify rock joint roughness. It was observed that K_v plays a more prominent role than D_{r1d}, in quantifying the roughness of natural rock joints.

Based on the studies carried out to investigate the rock joint roughness anisotropy patterns, it can be concluded firmly that the rock joints do show roughness anisotropy and in general it does vary randomly with the direction. Again the scale dependant parameter K_v seems to be prominent than D_{r1d} with respect to roughness anisotropy of natural rock joints. Since the roughness varies randomly, it is impossible to predict the roughness variation of rock joint surfaces from measurements made in only two perpendicular directions on a particular sample.

The one dimensional modified divider technique was extended to two dimensions. First, the accuracy of this modified divider technique was tested using a generated fractional Brownian surface. The method computed the fractal dimension of the fractional Brownian surface very accurately and it was applied to the natural rock joint roughness surfaces to quantify the two-dimensional fractal dimension, D_{r2d},

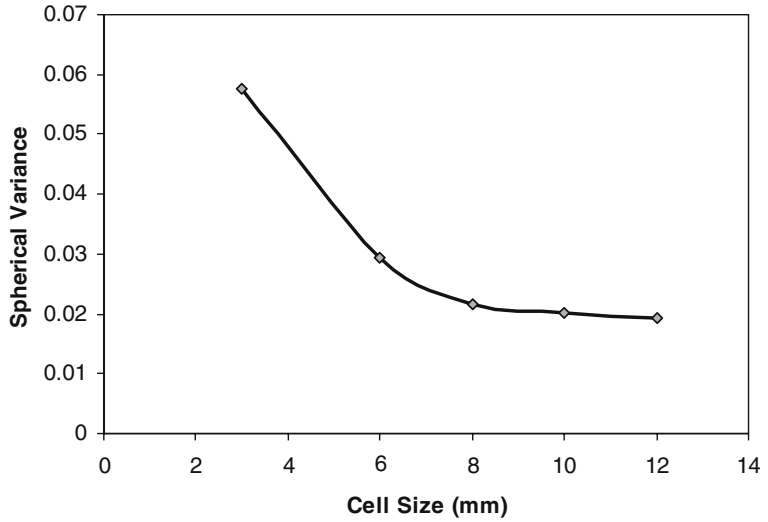

Figure 19. Variation of spherical variance with the cell size for rock joint surface of grd1-1 btm sample.

and the scale dependent fractal parameter C of those roughness surfaces. It was found that the parameter D_{r2d} quantifies the roughness well.

A new methodology was introduced to study the effect of scale on two-dimensional roughness variability and anisotropy of rock joints. Here, the surface of a rock joint was discretized into elemental planes and the unit normal vectors of these elemental planes were used to find the mean normal vector direction and the variability of the unit normal vectors of the rock joint surface. The mean normal vector of the roughness surface changed only slightly when the element size (scale) was varied. However, the spherical variance, which is a measure of the variability of the unit normal vectors of the rock surface, reduced drastically with the increasing scale. The roughness anisotropy also reduced with increasing scale.

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