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Pressure–settlement characteristics of rectangular footings on reinforced sand

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Abstract. A method has been proposed to obtain the pressure–settlement characteristics of rectangular footings resting on reinforced sand based on constitutive laws of soils. The confining effect of the reinforcement provided in the soil at different layers has been incorporated in the analysis by considering the equivalent stresses generated due to friction at the soil–reinforcement interface. The prerequisite of the method is the value of ultimate bearing capacity, which can be obtained from the approaches already available in literature. The value of settlement may be read directly from pressure–settlement curves for the given pressure intensity. Therefore, the rectangular footing resting on reinforced sand can be proportioned satisfying shear failure and settlement criteria.

Key words. constitutive laws of soil; rectangular footings; reinforced soil.

Nomenclature

Symbol	Description
B	width of footing
f_e	lateral coefficient of friction between soil and reinforcement
ϕ	angle of internal friction of soil
ϕ_f	soil-reinforcement friction angle
γ	unit weight of soil
q_u	ultimate bearing capacity of footings on reinforced soil
σ_x	normal stress in x-direction
σ_y	normal stress in y-direction
σ_z	normal stress in z-direction
σ_1	major principal stress
σ_2	intermediate principal stress
σ_3	minor principal stress

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1. Introduction

The decreasing availability of proper construction sites has led to the increased use of marginal ones, where the bearing capacity of the underlying deposits is very low. One new application, the construction of reinforced soil foundation to support a shallow spread footing has a considerable potential as a cost-effective alternative to conventional methods of support. The beneficial effects of using tensile reinforcement to increase the bearing capacity of sands have been clearly demonstrated by several investigators (Binquet and Lee, 1975a, b; Akinmusuru and Akinbolade, 1981; Das and Larbi-Cherif, 1983; Fragaszy and Lawton, 1984; Guido et al., 1986; Huang and Tatsuoka, 1990; Dixit and Mandal, 1993; Khing et al., 1993, 1994; Murthy et al., 1993; Yetimoglu et al., 1994; Adams and Collin, 1997; Kumar and Saran, 2001, 2003a, b).

In general, settlement is the governing criterion for designing a footing resting on weak granular material. Usually, for a given settlement, the load that a footing can carry is obtained either by using plate load test data or standard penetration test. However, a novel method proposed by Sharan (1977) and Prakash et al. (1984) based on constitutive laws of soil, which gives pressure–settlement characteristics of a footing resting on unreinforced soil, can also be adopted. The mathematical model proposed by Kondner (1963) has been used to describe the constitutive laws of soil. This method has two advantages: (i) it eliminates the use of costly field tests, and (ii) it gives directly the pressure–settlement characteristics of actual footing. In the present paper the method proposed by Sharan (1977) and Prakash et al. (1984) based on constitutive laws of soil has been extended for drawing the pressure–settlement characteristics of a rectangular footing resting on reinforced sand. The confining effect of the reinforcement provided in the soil at different layers has been incorporated in the analysis by considering the equivalent stresses generated due to friction at the soil–reinforcement interface. The results of the analysis have been validated with the large-scale model test results of Adams and Collin (1997) and model test results of Kumar (1997, 2003).

2. Theoretical Analysis

The analysis has been developed for studying the pressure–settlement characteristics of rectangular footings resting on sand reinforced with horizontal layers of reinforcement using non-linear constitutive laws of soil. Constitutive laws of soils define the mechanical behaviour of soil and are of prime importance for analysing almost all applied non-linear problems of soil mechanics. The most popular model for describing a constitutive law is a two constant hyperbola suggested by Kondner (1963). The constants of this model can be obtained by performing triaxial tests on the pertaining soil simulating field conditions. The stress–strain behaviour of sands is dependent on confining pressure. This fact has been taken into account in developing the analysis. The analysis is based on the following assumptions:

(i) The effect of weight of soil mass has been considered in determination of stresses. Vertical stress due to weight of soil has been taken equal to γz where γ is unit weight of soil and z is the depth of soil layer. The horizontal stress due to weight of soil has been taken equal to $K_0\gamma z$ where K_0 is the coefficient of earth pressure at rest.
(ii) The base of the footing has been assumed smooth, as the effect of roughness on pressure–settlement characteristics has been found to be negligible from the studies of Sharan (1977).
(iii) Stresses in soil mass have been computed using theory of elasticity. Strains have been computed from the hyperbolic soil model defined by Kondner (1963).
(iv) Soil is assumed to be an isotropic and semi-infinite medium.

2.1. PROCEDURE FOR ANALYSIS

The procedure has been described in the following steps:

(1) The whole soil mass supporting the footing, of width B, is divided into a large number of thin layers e.g., n layers, up to a depth of $5B$ beyond which the vertical stresses become almost negligible (Figure 1). The thickness of each layer was taken as $0.25B$. The normal and shear stresses at the centre of each layer of soil mass along the vertical section have been computed using theory of elasticity (Poulos and Davis, 1974). The effect of weight of soil has been incorporated by adding γz to σ_z and $K_0\gamma z$ to both σ_x and σ_y, where $K_0 = 1 - \sin\phi$. The value of angle of internal friction ϕ is determined from drained triaxial tests.

(2) For considering the effect of reinforcement at each layer level, stresses σ_x and σ_y have been modified by adding δx and δy to them where

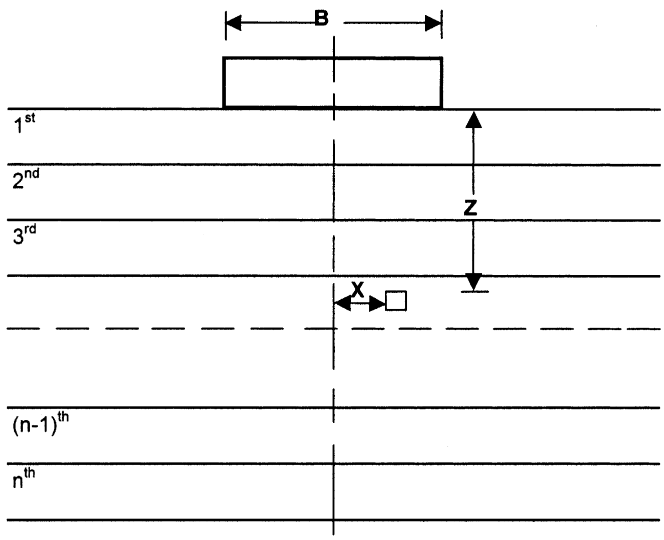


Figure 1. Soil below footing divided into n layers.

$$\delta x = 2f_e(m_{xz}q + \gamma a_{xz}(z + \mathrm{d}f)) \tag{1}$$

$$\delta y = 2f_e(m_{yz}q + \gamma a_{yz}(z + \mathrm{d}f)) \tag{2}$$

where m_{xz} and m_{yz} are non-dimensional frictional resistance factor due to the vertical normal force on the plan area reinforcement in the outward flowing zone of soil, marked as a_{xz} and a_{yz}, which is outside the plane separating the downward and outward flow and f_e is lateral coefficient of friction between soil and reinforcement. The values of m_{xz}, m_{yz}, a_{xz}, a_{yz} can be taken from non-dimensional charts presented by Kumar and Saran (2003b), and $f_e = \tan\phi_f$, where ϕ_f is soil–reinforcement friction angle.

(3) The principal stresses $(\sigma_1, \sigma_2, \sigma_3)$ at a point in the soil mass and their directions $(\theta_1, \theta_2, \theta_3)$ with respect to vertical Z-axis (Figure 2) have been computed using equations of the theory of elasticity (Sekolnikof, 1956; Durelli, 1958; Selby, 1972, presented by Sharan, 1977).

(4) The value of Poisson's ratio has been obtained using the equation

$$\mu = \frac{K_0}{1 + K_0} \tag{3}$$

(5) The ultimate bearing capacity of reinforced soil (q_u) has been calculated using the method suggested by Binquet and Lee (1975b), which was extended to rectangular footings by Kumar and Saran (2003b).

A factor of safety 'F' has been introduced such that at all stress levels the following relationship is satisfied,

$$\frac{q_u}{q} = F \tag{4}$$

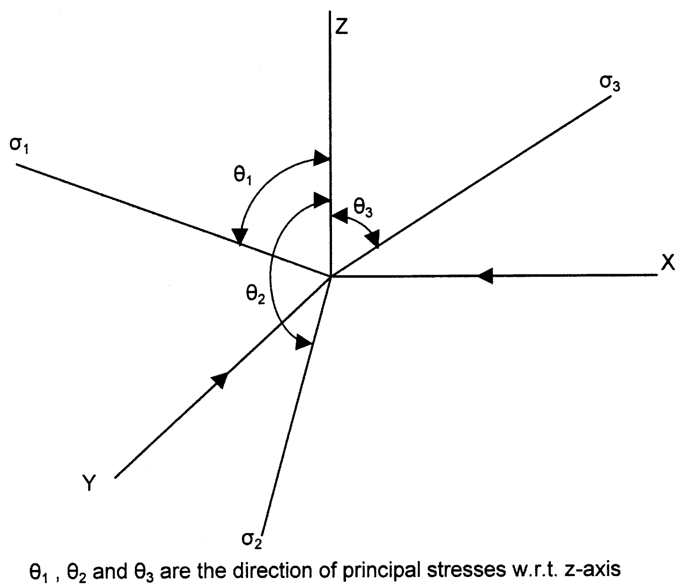


Figure 2. Principal stresses at a point and their directions, three-dimensional.

where q is applied load intensity and q_u is the ultimate bearing capacity of reinforced soil.

(6) The constitutive equation for the supporting soil is obtained from triaxial compression test results using Kondner's two constant hyperbola (Figure 3). It gives the relation as below:

$$\varepsilon_1 = \frac{a(\sigma_1 - \sigma_3)}{1 - b(\sigma_1 - \sigma_3)} \tag{5}$$

where ε_1 is the strain in the direction of principal stress σ_1 at deviator stress $(\sigma_1–\sigma_3)$; σ_1, σ_3 are the Major and minor principal stresses; a, b are the Kondner's hyperbola constants (Figure 3(b)).

The modulus of elasticity (E_s) has been calculated from the following equation:

$$E_s = \frac{1 - b\left(\frac{\sigma_u}{F}\right)}{a} \tag{6}$$

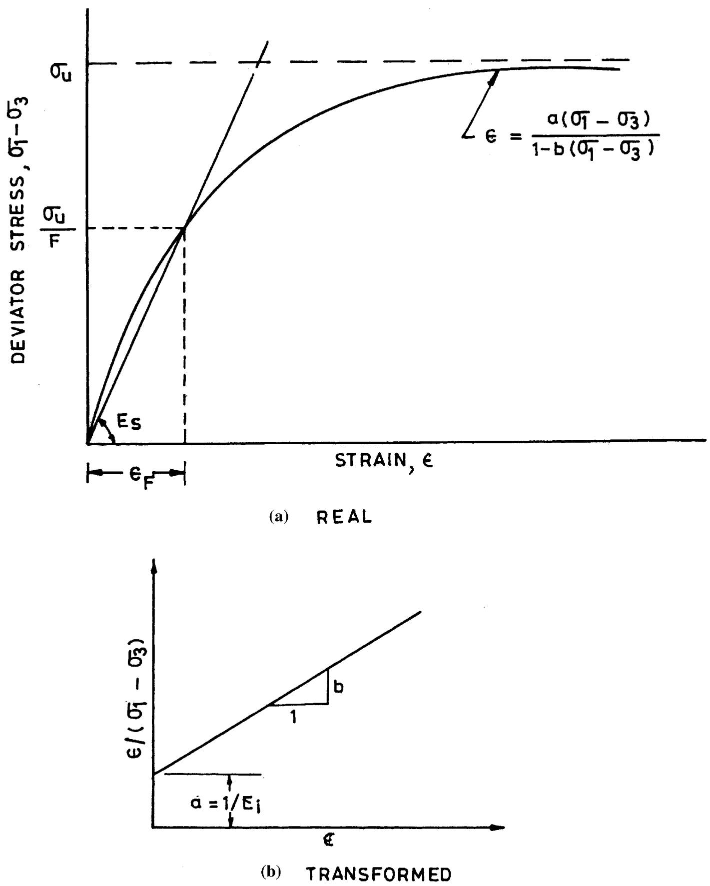


Figure 3. Hyperbolic stress–strain representation (after Kondner, 1963).

where $\sigma_u = (\sigma_1 - \sigma_3)\, F$ and a and b are hyperbola constants which are the functions of confining pressure.

(7) The strain in each layer in the direction of major principal stress has been obtained from the following equation:

$$\varepsilon_1 = \frac{\sigma_1 - \sigma_3^*}{E_s} \tag{7}$$

where $\sigma_3^* = \dfrac{\sigma_2 + \sigma_3}{2}$.

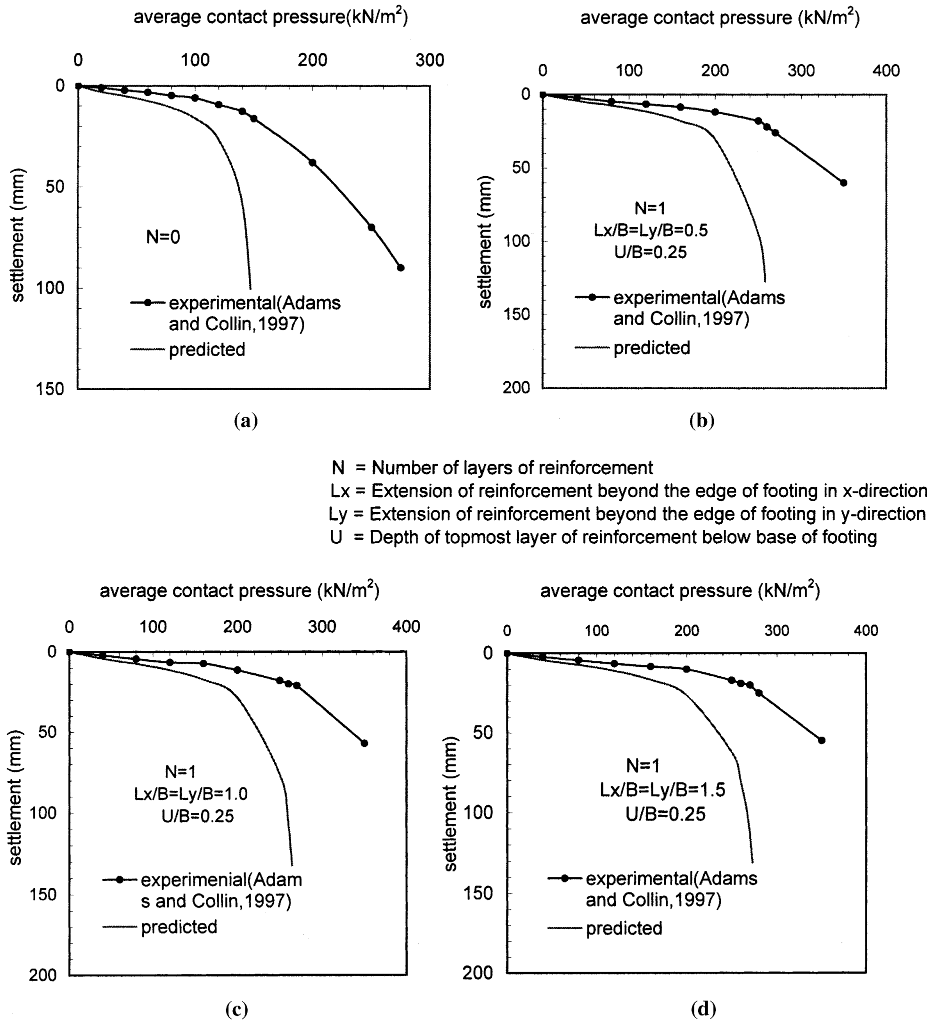
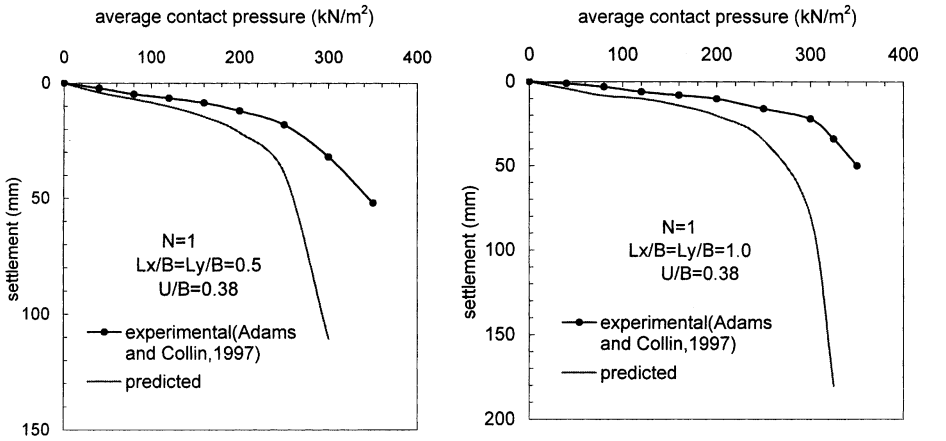


Figure 4. Pressure–settlement curves for square footing on reinforced soil ($B = 0.61$ m, $\gamma = 14.7$ kN/m^3, $\phi = 35°$).



(e) (f)

N = Number of layers of reinforcement
Lx = Extension of reinforcement beyond the edge of footing in x-direction
Ly = Extension of reinforcement beyond the edge of footing in y-direction
U = Depth of topmost layer of reinforcement below base of footing

Figure 4. continuted.

The strains in the direction of other principal stresses are calculated from the following relationships

$$\varepsilon_2 = \frac{\sigma_2 - \mu(\sigma_1 + \sigma_3)}{\sigma_1 - \mu(\sigma_3 + \sigma_2)} \varepsilon_1 \tag{8}$$

$$\varepsilon_3 = \frac{\sigma_3 - \mu(\sigma_1 + \sigma_2)}{\sigma_1 - \mu(\sigma_3 + \sigma_2)} \varepsilon_1 \tag{9}$$

(8) Evaluation of vertical strain

$$\varepsilon_z = \varepsilon_1 \cos^2 \theta_1 + \varepsilon_2 \cos^2 \theta_2 + \varepsilon_3 \cos^2 \theta_3 \tag{10}$$

(9) The vertical settlement (S_e) of any layer along a vertical section is computed by multiplying the strain ε_z with the thickness of each layer δZ

$$S_e = \varepsilon_z \, \delta Z \tag{11}$$

The total settlement along any vertical section is computed by numerically integrating the expression

$$S_t = \int_0^n \varepsilon_z \, dZ \tag{12}$$

(10) The surface load intensity is varied and steps 1–9 are repeated. The pressure-settlement curves are obtained.

3. Validation of analysis

The validation of this analysis is made with the following:

(1) The model test results conducted by Kumar (1997) on square footing $0.175 \times 0.175\,\mathrm{m}$ resting on sand ($D_{10} = 0.12$, $Cu = 2.08$, $Cc = 1.6$, $RD = 60\%$,

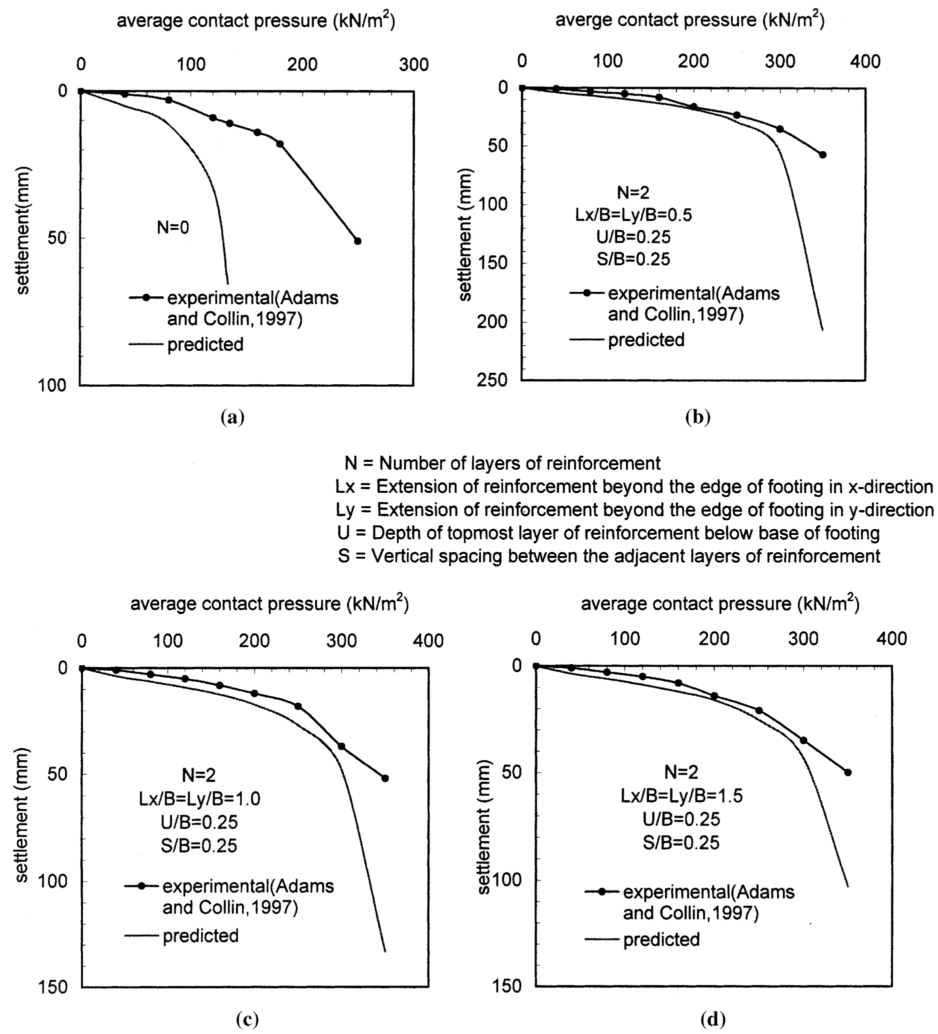
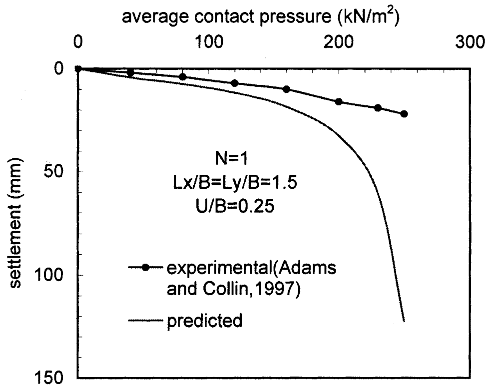


Figure 5. Pressure–settlement curves for square footing on reinforced soil ($B = 0.61\,\mathrm{m}$, $\gamma = 14.5\ \mathrm{kN/m^3}$, $\phi = 34.5°$).



(e)

N = Number of layers of reinforcement
Lx = Extension of reinforcement beyond the edge of footing in x-direction
Ly = Extension of reinforcement beyond the edge of footing in y-direction
U = Depth of topmost layer of reinforcement below base of footing

Figure 5. continued.

$\gamma = 15.8\,\text{kN/m}^3$) reinforced with horizontal layers of Tensar SS20 geogrids having tensile strength of 20 kN/m in both the directions.

(2) The model test results conducted by Adams and Collin (1997) on square footing 0.61×0.61 m in size, resting on sand ($\gamma = 14.7$ and $14.5\,\text{kN/m}^3$) reinforced with biaxial geogrids having tensile strength of 20 and 25 kN/m in the longitudinal and transverse directions respectively.
(3) The model test results conducted by Kumar (2003) on square footing 0.20×0.20 m resting on sand ($\gamma = 16.5\,\text{kN/m}^3$, $\phi = 38.5°$) reinforced with Tensar SS20 geogrids.

The comparison of experimental and predicted values of settlement at different contact pressures is shown in Figures 4–7. It is evident from these figures that the predicted values of settlement tally reasonably well up to approximately two-third of the ultimate bearing pressure. As usually in design the settlements are for the working pressure intensity (≈ultimate bearing capacity/2 or 3), this approach may be used for their estimation with confidence.

For the calculation purposes the soil–reinforcement friction angle ϕ_f has been taken from the following relation:

$$\tan \phi_f = K \tan \phi$$

where the value of K lies between 0.33 and 0.53 (Binquet and Lee, 1975; Huang and Tatsuoka, 1990; Kumar, 1997). The parameters of constitutive laws of soil have been

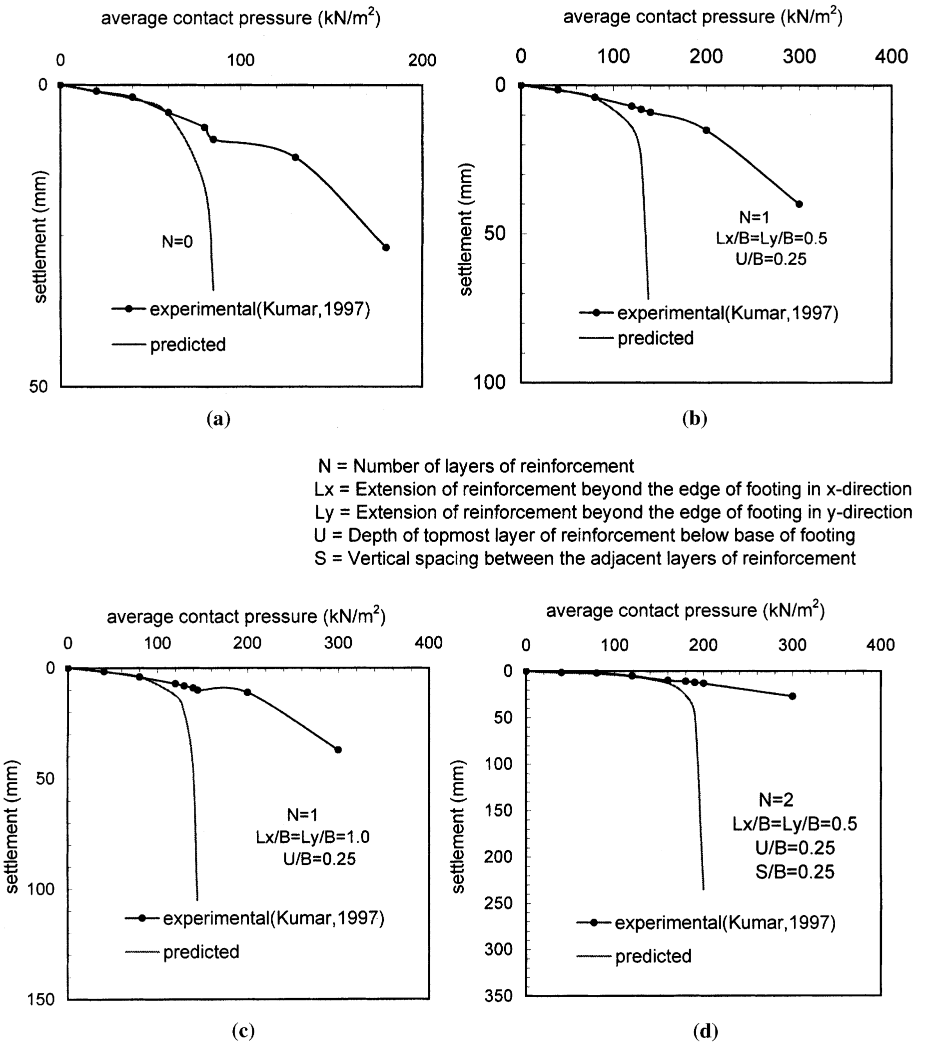


Figure 6. Pressure–settlement curves for square footing an reinforced soil ($B = 0.175\,\text{m}$, $\gamma = 15.8\ \text{kN/m}^3$, $\phi = 37°$).

calculated after conducting triaxial shear tests on sand samples of similar characteristics as has been used by investigators for experimental studies.

4. Conclusions

(1) An analytical procedure has been proposed to predict the pressure–settlement of rectangular footings resting on reinforced cohesionless soils, using the non-linear constitutive laws of such soils. This approach, however, requires

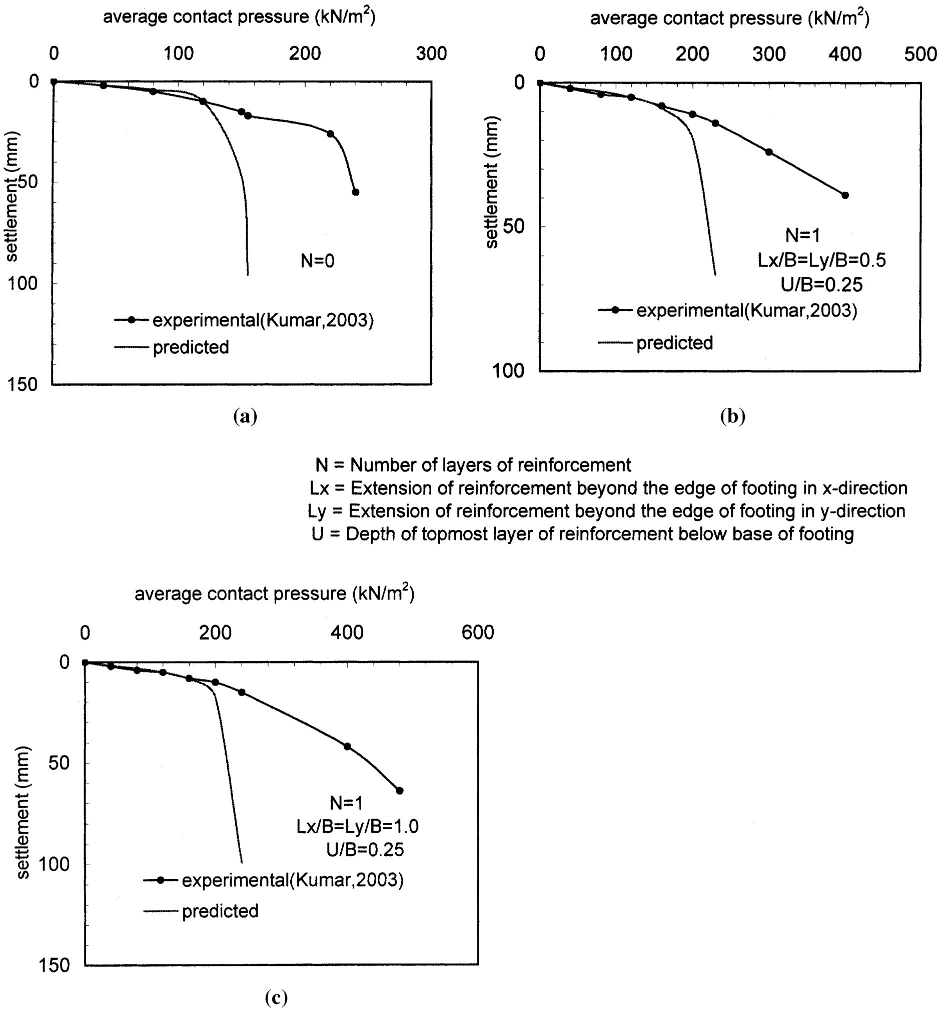


Figure 7. Pressure–settlement curves for square footing on reinforced soil ($B = 0.20\,\text{m}$, $\gamma = 16.5\ \text{kN/m}^3$, $\phi = 38.5°$).

predetermination of the ultimate bearing capacity of footing on reinforced soil, which can be obtained from the approaches already available in literature. In this investigation, an approximate method suggested by Kumar and Saran (2003b) has been used for the determination of ultimate bearing capacity of footing on reinforced soil.

(2) Predicted and model test results match well up to two-third of ultimate bearing pressure. As usually in design the settlements are for the working pressure intensity ($\approx$ultimate bearing capacity/2 or 3), this approach may be used for their estimation with confidence.

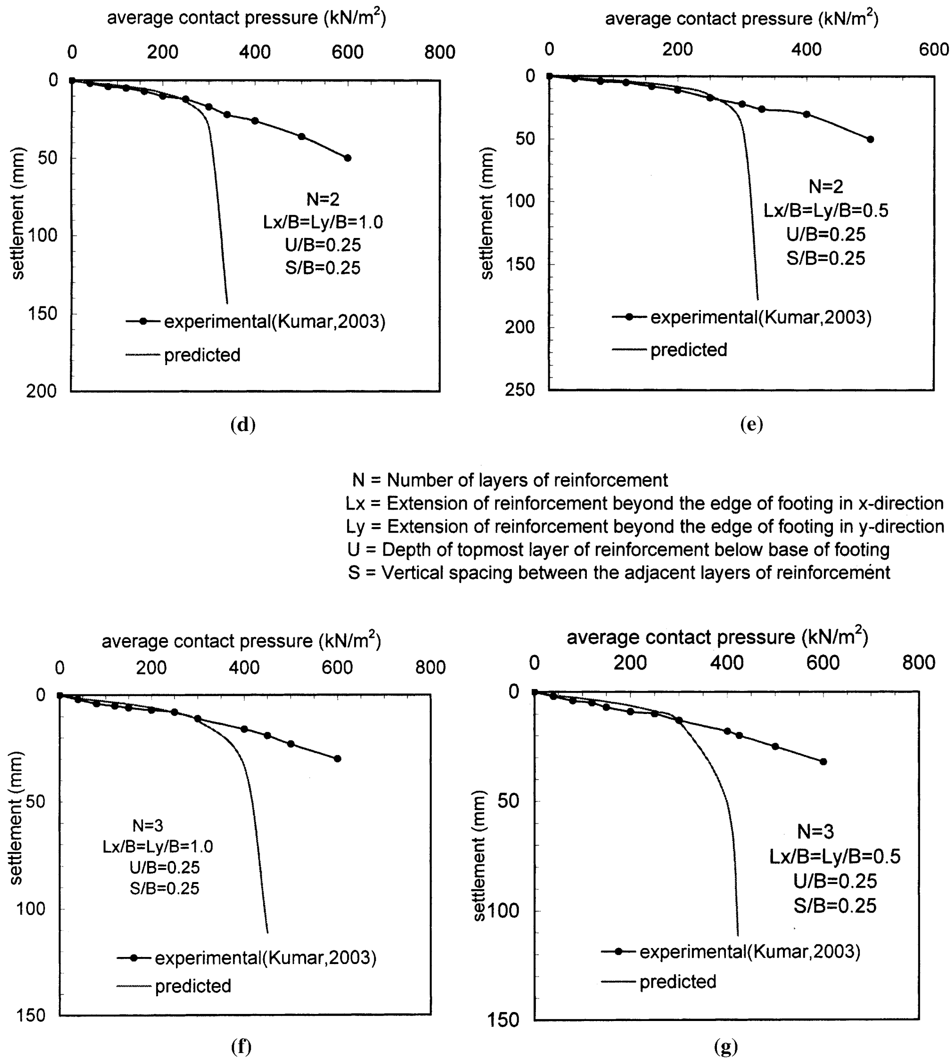


Figure 7. continued.

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