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Dynamic response of footing and machine with spring mounting base

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Abstract. The effect of the spring mounting cushion inserted in between a machine base and its concrete footing has been examined experimentally by conducting a number of block vibrations tests. The machine was subjected to steady state vertical harmonic loading. Experiments were performed with two different stiffness values of the spring mounting cushion. The employment of the spring mounting cushion, with the stiffness much smaller than that of soil strata, offers a drastic reduction in the resonant displacement amplitudes of the footing. It also results in a significant decrease in the resonant frequency of the foundation. The resonant displacement amplitudes of both the footing and the machine were found to become lower with the smaller stiffness value of the springs. The resonant frequency for the machine base, in all the experiments, was found to be invariably the same as that of the footing.

Key words. active isolation, damping, machine foundations, stiffness, vibrations.

1. Introduction

Waves generated by machine foundations can adversely affect nearby structures and facilities. Spring mounting cushions sandwiched in between the machine base and its footing are often used to bring down the level of ground vibration around the operating machine. These spring mounting cushions are also sometimes intended to reduce the resonant frequency of the machine foundation. A few studies have been reported in literature on active and passive isolations of wave transmission by means of trench barriers (Woods, 1968; Segol et al., 1978; Ahmad and Al-Hussaini, 1991; Al-Hussaini and Ahmad, 1991, 1996; Ahmad et al., 1996; Muhammad, 1998; Kattis et al., 1999; Srivastava and Kameswara Rao, 2002). However, hardly any information is available which deals with the dynamic response of the machine foundation with the use of the spring mounting system below the machine base. The aim of the present study is to explore explicitly the effect of the employment of the spring mounting base, sandwiched in between the machine and its reinforced concrete footing, on the dynamic response of the footing as well as the machine base. A number of block vibrations tests were carried out for this purpose by using a rotating

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mass type mechanical oscillator both with and without the spring mounting base. Under steady state vibration condition, the variation of the displacement amplitude of both the footing and machine base was obtained with respect to changes in (i) the frequency and eccentricity angle of the oscillator; and (ii) the stiffness of the springs. Experiments were conducted by employing two different stiffness values of the spring mounting system. It is expected that the study will be helpful in understanding the response of the machine foundation with the spring cushioning base.

2. Experimental Program

The spring mounting base was fabricated by attaching the top and bottom ends of five identical vertical open coiled helical metallic springs to two similar square mild steel plates having side 38 cm and thickness 1.2 cm; four springs were placed at the plate corners and the fifth at the center of the plate. The ends of the springs were tightened to the mild steel plates by nuts and bolts. The height of each spring was kept exactly equal to 120 mm. Two different diameter of spring wires, namely 7 and 12 mm, were used for achieving two different stiffness values of the spring mounting base. By placing the spring mounting base on a rigid concrete platform, a static load test was carried out to determine the stiffness (K_s) of the spring mounting system; metallic dead weights were used for the loading purpose and the vertical displacements were monitored by using dial gauges. The stiffness values of the two chosen spring mountings were found to be 0.5317 and 3.955 MN/m, respectively; the stiffness of the spring was found to remain independent with respect to changes in the level of strain. Two different pre-cast concrete blocks of sizes (i) $0.45\ \text{m} \times 0.45\ \text{m} \times 0.235\ \text{m}$ (weight $=$ 1.218 kN), and (ii) $0.60\ \text{m} \times 0.60\ \text{m} \times 0.18\ \text{m}$ (weight $=$ 1.623 kN) were used for modeling the footing. The Lazan rotating mass type mechanical oscillator was used for applying the vertical harmonic loading. The oscillator was connected to an electric motor by means of a flexible shaft as shown in Figure 1. The relationship between the amplitude (F_0) of the applied vertical harmonic force, the circular frequency (ω) and the eccentricity angle (θ) of the oscillator is given by the expression:

$$F_0 = 0.226\omega^2 \sin\,(\theta/2) \tag{1}$$

In the above formula, the units of F_0 and ω are expressed in Newton and rad/sec, respectively; in this paper, the symbol f was also alternatively used for defining the frequency in rotations (cycles) per minute (rpm), that is, $\omega = 2\pi f/60$. For a particular frequency all the measurements were made by obtaining the peak to peak value of the vertical displacement for a time period of 10 s. All the tests were conducted at a site chosen in the campus of Indian Institute of Science. The site comprises of essentially a thick layer of clayey/silty sand consisting approximately 20–25% clay, 20% silt and about 55–60% sand. The maximum dry unit weight of this soil was 17.17 kN/m^3 corresponding to an optimum moisture content of about 15.7%. The site was cleaned and a square pit of size 800 mm and depth 400 mm was excavated in order to remove the top loose soil. The bottom of the pit was perfectly leveled before

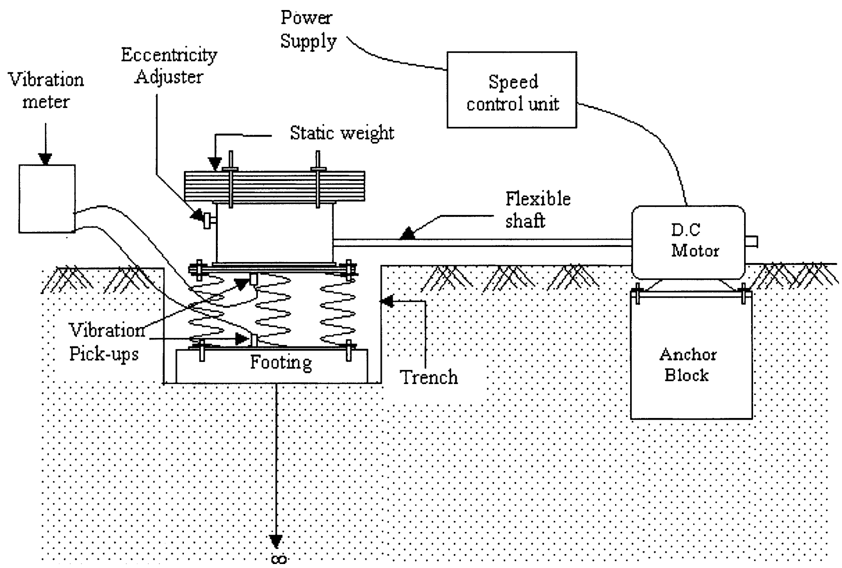

Figure 1. Experimental set-up.

placing the pre-cast reinforced concrete footing block. Sufficient gap between the sides of the pit and the footing block was kept to avoid any development of the frictional resistance from the side surface of the footing. A schematic diagram of the test set up is shown in Figure 1. Two different series of vibrations tests were conducted: (i) Series A: vibrations tests without spring mounting system, and (ii) Series B: vibrations tests with spring mounting system. The necessary details of various tests are given in Tables 1 and 2. In these tables, W = total vibrating weight including the self weight of oscillator for test series A; W_1 = total weight of (i) top plate of the spring mounting base, (ii) oscillator, and (iii) dead weights on the top of the oscillator; and W_2 = total weight of (i) footing block, and (ii) the bottom plate

Table 1. Experiments without spring mounting system: Series A

Test number	Size of the footing	Total weight of the vibrating system, W (kN)	Eccentricity angle, θ (Degrees)
A-I	0.60 m × 0.60 m × 0.18 m	5.862	2.5, 5, 7.5, 10, 12.5, 15
A-II	0.45 m × 0.45 m × 0.235 m	5.456	2.5, 5, 7.5, 10, 12.5, 15

Table 2. Experiments with spring mounting system: Series B

Test number	Size of the footing block	s (MN/m)	W_1 (kN)	W_2 (kN)	Eccentricity angle, θ (Degrees)
B-I	0.60 m × 0.60 m × 0.18 m	0.5317	4.434	1.624	2.5, 5, 7.5, 10, 12.5, 15
B-II	0.60 m × 0.60 m × 0.18 m	3.955	4.434	1.624	2.5, 5, 7.5, 10, 12.5, 15
B-III	0.45 m × 0.45 m × 0.235 m	0.5317	4.434	1.219	2.5, 5, 7.5, 10, 12.5, 15
B-IV	0.45 m × 0.45 m × 0.235 m	3.955	4.434	1.219	2.5, 5, 7.5, 10, 12.5, 15

of the spring mounting base. The magnitude of W (a) for test series A-I was kept closer to the value of $W_1 + W_2$ for test series B-I and B-II, and (b) for test series A-II was kept closer to the value of $W_1 + W_2$ for test series B-III and B-IV.

3. Experimental results

A total number of 36 tests were carried out for test series A and B; for each test, after attaining the steady state of vibration, the variation of the displacement amplitudes of the footing and machine with changes in the frequency of the oscillator was recorded. The results of all the tests are summarized below:

3.1. VIBRATION TESTS WITHOUT SPRING MOUNTING SYSTEM: TEST SERIES A

The variation of the displacement amplitude (Z) of the footing with respect to changes in the frequency (f) of the oscillator is shown in Figures 2(a), (b); the values of θ in these figures are in degrees. It can be seen that an increase in the value of θ leads to (i) decrease in the resonant frequency (f_r), and (ii) increase in the resonant displacement amplitude (Z_r). For a given value of θ, the resonant frequency for the test series A-I ($W = 5.861$ kN) was found to be smaller than that of test series A-II ($W = 5.456$ kN). The resonant displacement amplitude (Z_r) in the case of the test series A-I was found to be a little higher as compared to the test series A-II.

Using simple mass spring-dashpot model subjected to steady state rotating mass type harmonic loading, with single degree of freedom, the values of the stiffness (K) and the damping ratio (D) of the soil strata for a given value of θ were determined from known values of (i) f_r, (ii) Z_r, and (iii) the amplitude of the applied dynamic force F_0 corresponding to f_r using Equation 1. The following formulae were used in order to determine the values of stiffness (K) and damping ratio (D) of the soil mass:

$$\frac{Z_r W}{W_e e} = \frac{1}{2D\sqrt{1-D^2}} \tag{2}$$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{Kg}{W}}\frac{1}{\sqrt{1-2D^2}} \tag{3}$$

$$F_0 = \frac{W_e}{g} e\omega^2 \tag{4}$$

where W_e is the total eccentric weight in the oscillator and e is the eccentricity (radius) of the rotating mass in the oscillator. The text books of Richart et al. (1970) and Das (1992) can be referred for the derivation of the above different formulae. Using Equations (1) and (4), it can be seen that

$$W_e e = 0.226 g \sin(\theta/2)$$

In the above equation, weight W_e, e and g (acceleration due to gravity) are in N, m and m/sec^2, respectively.

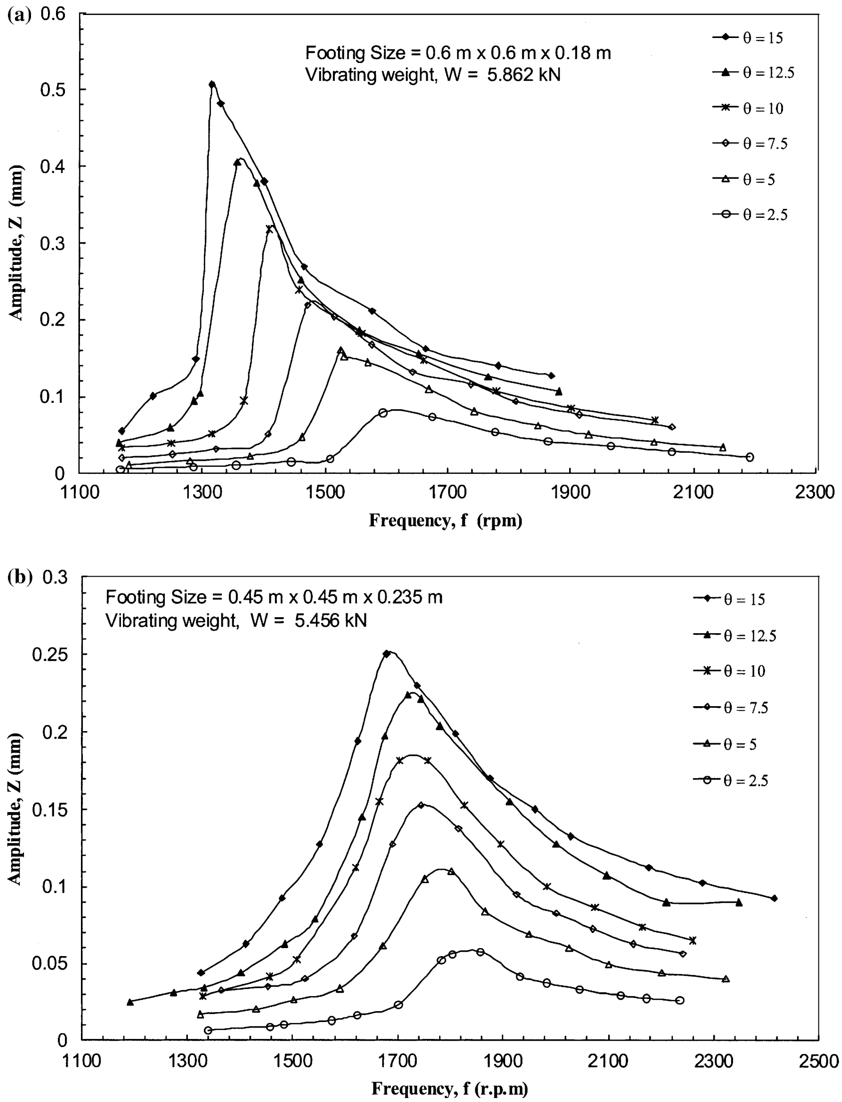

Figure 2. Variation of the displacement amplitude (Z) of footing with changes in f and θ for tests without springs with (a) $W = 5.862$ kN; (b) $W = 5.456$ kN.

The Equations 2, 3 and 5 can be used to find the values of K and D; the values of K and D depend on the level of strain in the material, however, their variation with frequency is still not known. The obtained variation of K and D with changes in θ is shown in Figures 3(a), (b). It can be seen that an increase in the value of θ leads to a decrease in the value of soil stiffness; in other words the stiffness of the soil strata decreases continuously with increases in the magnitude of the resonant displacement amplitude. It demonstrates that a linear elastic model may not be able to provide a

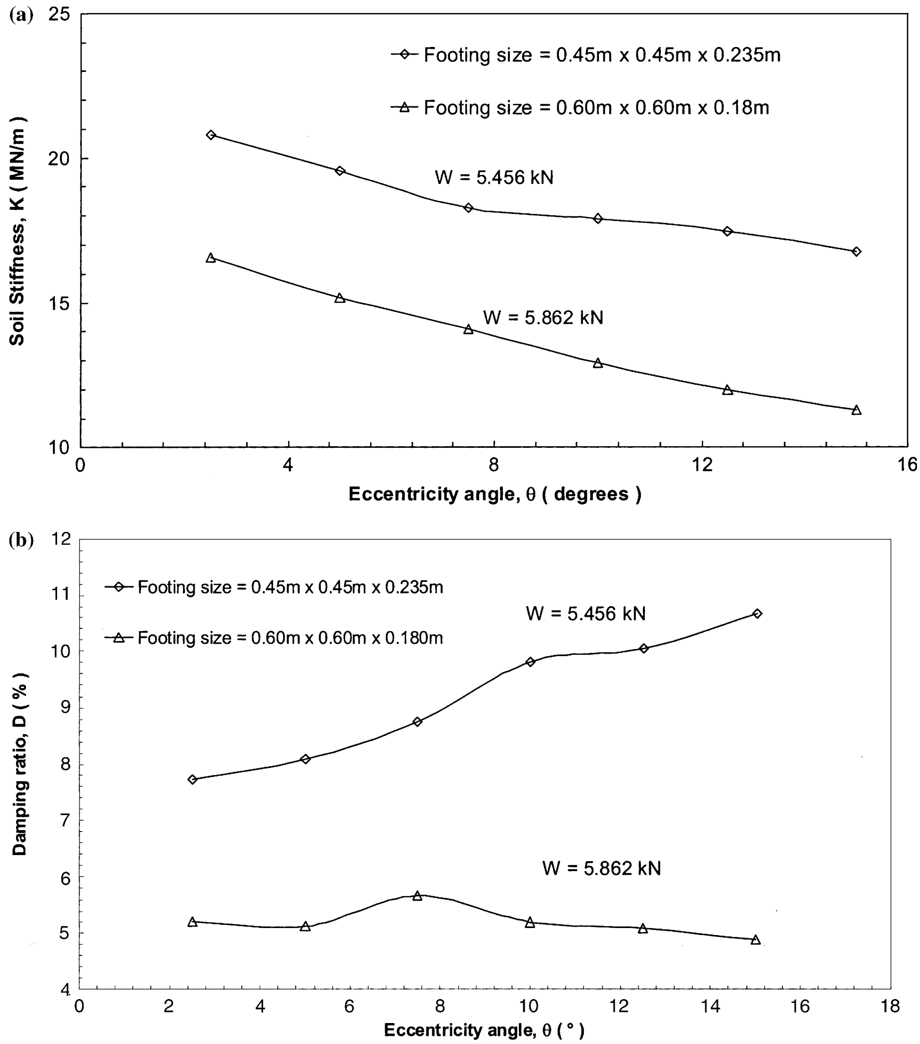

Figure 3. (a) Variation of soil stiffness (K) with changes in θ; (b) Variation of soil damping ratio (D) with changes in θ.

very correct estimate of the dynamic response of the footing. The damping ratio of the soil mass (i) increases with the increase in the value of θ for $W = 5.456$ kN, and (ii) remains almost unchanged with respect to variation of θ for $W = 5.862$ kN.

3.2. VIBRATION TESTS WITH SPRING MOUNTING SYSTEM: TEST SERIES B

The variations of the displacement amplitudes of the machine and footing, Z_1 and Z_2, respectively, with changes in the frequency (f) of the oscillator for different values of eccentricity angle θ are indicated in Figures 4–11; in these figures the values

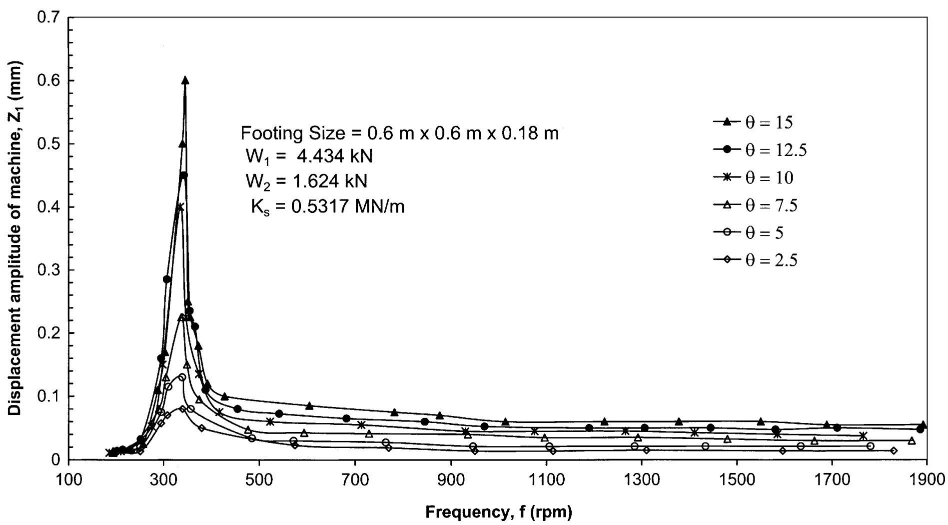

Figure 4. Variation of displacement amplitude of machine (Z_1) with changes in f and θ for $K_s = 0.5317$ MN/m, $W_1 = 4.434$ kN, $W_2 = 1.624$ kN.

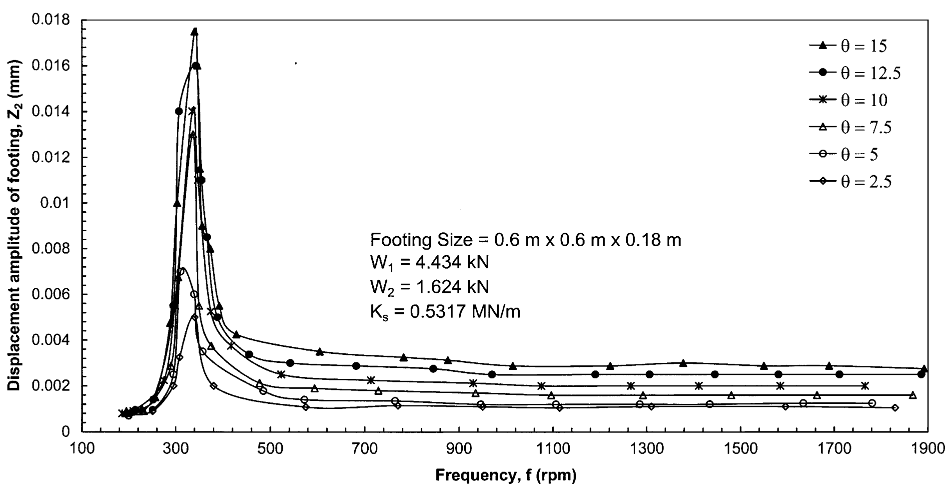

Figure 5. Variation of displacement amplitude of footing (Z_2) with changes in f and θ for $K_s = 0.5317$ MN/m, $W_1 = 4.434$ kN, $W_2 = 1.624$ kN.

of θ are in degrees. These figures provide the results corresponding to two different values of spring stiffness and two different footing blocks. Figures 4, 6, 8 and 10 provide the response of the machine base (Z_1), on the other hand, Figures 5, 7, 9 and 11 present the response of the footing base (Z_2). The displacement amplitudes of all the results at the resonant frequencies have also been provided in Tables 3 and 4. Tables 5 and 6 provide the magnitudes of the resonant frequencies for all the results. The following observations were made:

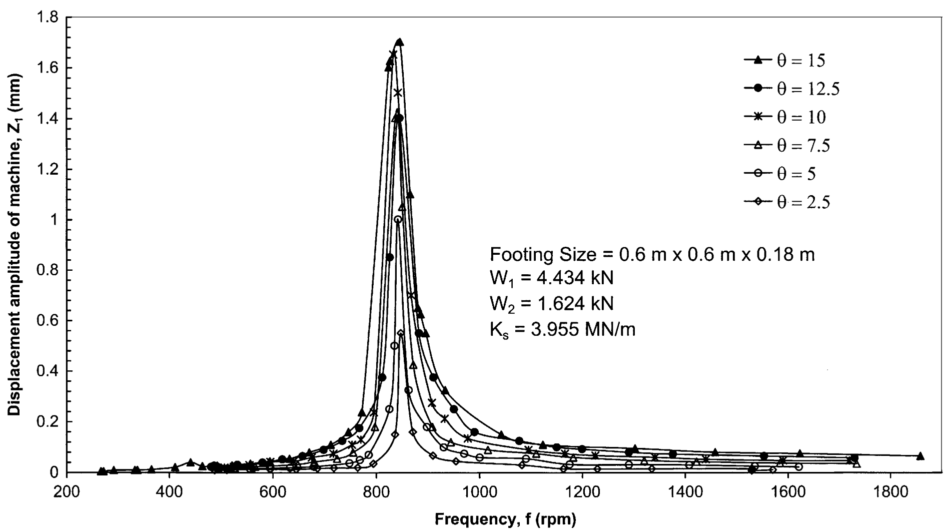

Figure 6. Variation of displacement amplitude of machine (Z_1) with changes in f and θ for $K_s = 3.955$ MN/m, $W_1 = 4.434$ kN, $W_2 = 1.624$ kN.

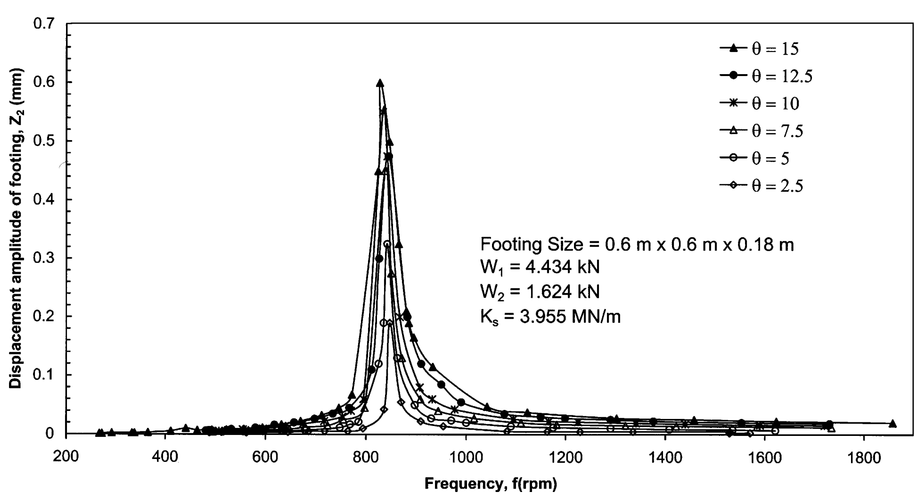

Figure 7. Variation of displacement amplitude of footing (Z_2) with changes in f and θ for $K_s = 3.955$ MN/m, $W_1 = 4.434$ kN, $W_2 = 1.624$ kN.

- The resonant frequency for the response of the machine base is found to be the same as that of the footing (refer Figures 4–11). Unlike test series A, the magnitude of the resonant frequency for the test series B does not vary with changes in θ.
- Like test series A, an increase in the value of θ for the experiments in the test series B leads to an increase in the resonant displacement amplitudes, Z_{1r} and Z_{2r}, of the machine and footing, respectively (refer Figures 4–11).

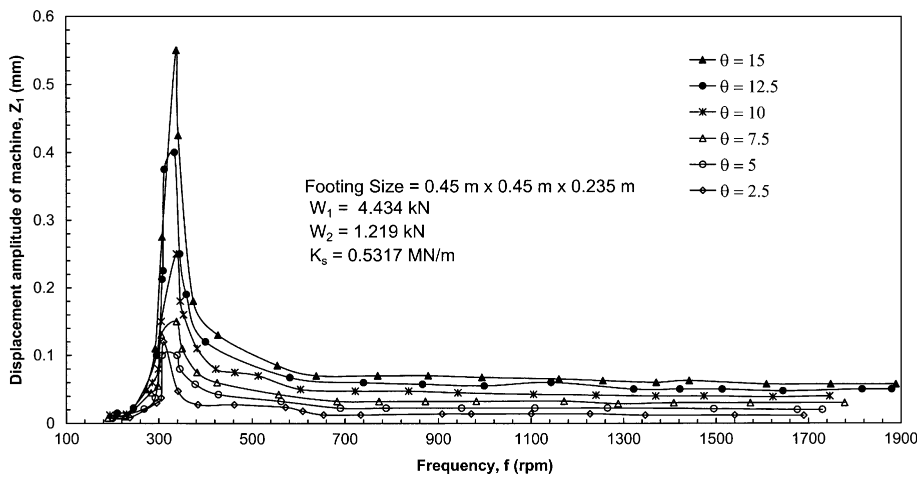

Figure 8. Variation of displacement amplitude of footing (Z_1) with changes in f and θ for K_s = 0.5317 MN/m, W_1 = 4.434 kN, W_2 = 1.219 kN.

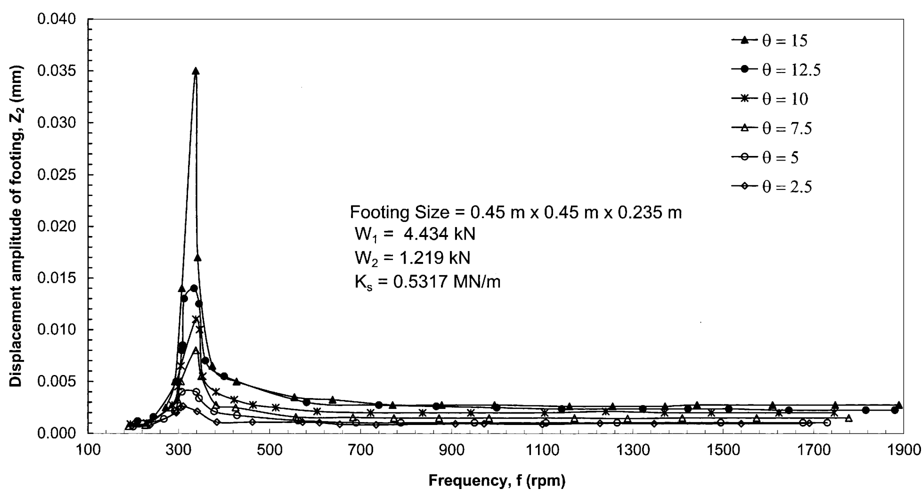

Figure 9. Variation of displacement amplitude of footing (Z_2) with changes in f and θ for K_s = 0.5317 MN/m, W_1 = 4.434 kN, W_2 = 1.219 kN.

- The comparison of the results of test series A (refer Figures 2a, b) and test series B (refer Figures 4, 5, 8, 9 and Tables 3–4) reveals that the employment of the spring mounting system having stiffness 0.5317 MN/m brings down considerably the resonant displacement amplitudes of the footing. However, the resonant displacement amplitudes of the machine base for K_s = 0.5317 MN/m remains almost of the same order as for the experiments without spring cushioning. On the other hand for the experiments with K_s = 3.955 MN/m (refer Figures 6, 7, 10, 11 and Tables 3 and 4), the displacement amplitudes of the footing remain

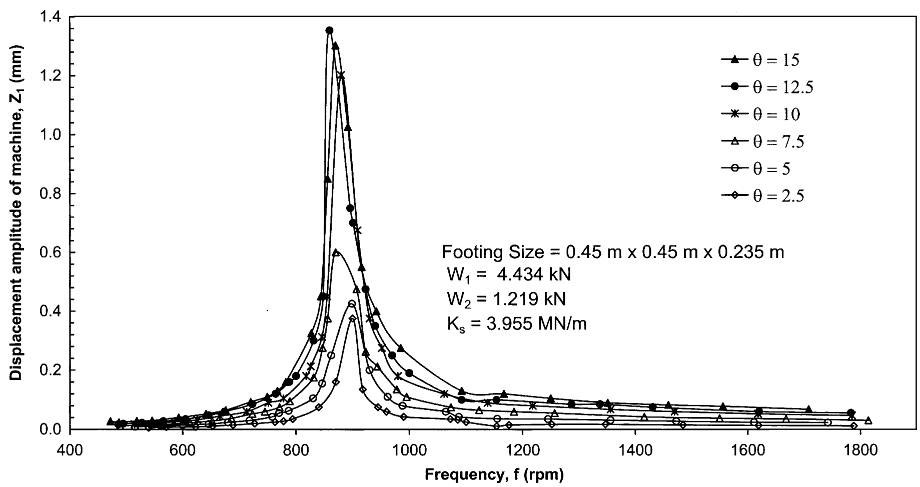

Figure 10. Variation of displacement amplitude of machine (Z_1) with changes in f and θ for $K_s = 3.955$ MN/m, $W_1 = 4.434$ kN, $W_2 = 1.219$ kN.

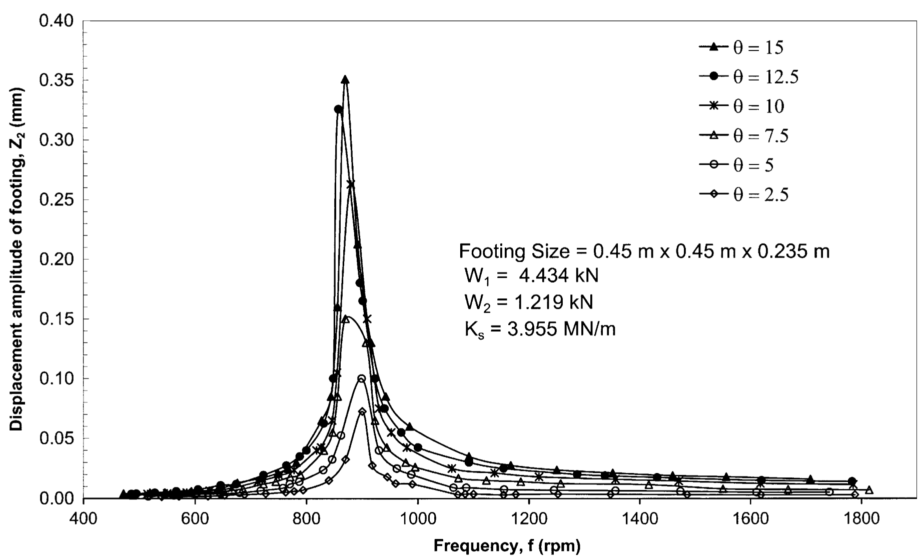

Figure 11. Variation of displacement amplitude of footing (Z_2) with changes in f and θ for $K_s = 3.955$ MN/m, $W_1 = 4.434$ kN, $W_2 = 1.219$ kN.

almost the same as without the spring mounting cushion; however, the displacement amplitude of the machine base becomes even greater as compared to the experiments without the spring mounting system. This is due to the fact that the damping of the chosen spring mounting system is negligible. It is expected that if the damping of the spring cushion is increased, it will bring down the displacement amplitude of the footing as well as the machine base (Kameswara Rao, 1998).

Table 3. A comparison of displacements amplitudes at resonant frequencies

	Without springs with $W = 5.862$ kN	With springs $K_s = 0.5317$ MN/m $W_1 = 4.434$ kN $W_2 = 1.624$ kN		With springs $K_1 = 3.955$ MN/m $W_1 = 4.434$ kN $W_2 = 1.624$ kN	
θ (°)	Z_r (mm)	Z_{1r} (mm)	Z_{2r} (mm)	Z_{1r} (mm)	Z_{2r} (mm)
15	0.506	0.600	0.016	1.700	0.500
12.5	0.405	0.450	0.016	1.400	0.475
10	0.318	0.400	0.014	1.650	0.550
7.5	0.219	0.225	0.013	1.400	0.450
5	0.161	0.130	0.006	1.000	0.325
2.5	0.079	0.080	0.005	0.550	0.190

Table 4. A comparison of displacements amplitudes at resonant frequencies

	Without spring $W = 5.456$ kN	With springs $K_1 = 0.5317$ MN/m $W_1 = 4.434$ kN $W_2 = 1.219$ kN		With springs $K_1 = 3.955$ Mn/m $W_1 = 4.434$ kN $W_2 = 1.219$ kN	
θ (°)	Z_r (mm)	Z_{1r} (mm)	Z_{2r} (mm)	Z_{1r} (mm)	Z_{2r} (mm)
15	0.250	0.550	0.035	1.300	0.350
12.5	0.224	0.400	0.014	1.350	0.325
10	0.181	0.250	0.011	1.200	0.263
7.5	0.153	0.150	0.008	0.600	0.150
5	0.110	0.100	0.004	0.425	0.100
2.5	0.058	0.120	0.003	0.375	0.073

Table 5. A comparison of resonant frequencies for footing block having size 0.60 m × 0.60 m × 0.18 m

	Resonant frequency in rpm		
	Without springs (f_r)	With spring mounting system for case-I ($f_{1r} = f_{2r}$)	
θ (°)	$W = 5.862$ kN	$K_s = 0.5317$ MN/m $W_1 = 4.434$ kN $W_2 = 1.624$ kN	$K_1 = 3.955$ MN/m $W_1 = 4.434$ kN $W_2 = 1.624$ kN
15	1315	346	846
12.5	1357	343	845
10	1408	335	833
7.5	1471	337	837
5	1526	339	842
2.5	1594	340	847
	f_{n1}	327.5	893.3

- As compared to the experiments in test series A, the employment of the spring mounting system causes a considerable decrease in the magnitude of the resonant frequency (refer Tables 5 and 6). It should be noted that the measured value of the resonant frequency for the test series B remains invariably closer to natural undamped frequency (f_{n1}) of the weight W_1 resting directly on the spring having stiffness K_s;

Table 6. A comparison of resonant frequencies for footing block having size 0.45 m × 0.45 m × 0.235 m

	Resonant frequency in rpm		
	Without springs (f_r)	With Spring Mounting system for case-II ($f_{1r} = f_{2r}$)	
θ(°)	$W = 5.456$ kN	$K_s = 0.5317$ MN/m $W_1 = 4.434$ kN $W_2 = 1.219$ kN	$K_s = 3.955$ MN/m $W_1 = 4.434$ kN $W_2 = 1.219$ kN
15	1678	338	870
12.5	1719	334	858
10	1704	338	880
7.5	1744	338	870
5	1802	339	899
2.5	1858	310	900
	f_{n1}	327.5	893.3

where

$$f_{n1} = \frac{1}{2\pi}\sqrt{\frac{K_s g}{W_1}} \tag{6}$$

- It can be noticed from Tables 5 and 6 that an increase in the value of W_2 from 1.219 to 1.624 kN) brings not much significant change in the resonant frequency. As mentioned earlier the resonance occurs at the frequency close to the natural frequency f_{n1} as defined above. The magnitude of f_{n1} does not depend on W_2 and, therefore, a change in the value of W_2 hardly affects the frequency at the resonance. It can also be seen from Tables 3 and 4 that the effect of an increase in the value of W_2 on the values of Z_{1r} and Z_{2r} is not very clear. It is difficult to explain this observation as not only the weights of the two footings blocks were different but also their base areas were also not the same.

4. Discussion

The study deals only with the response of machine and its footing to steady state vibration. However, no attempt has been made to monitor the response during the transient state vibrations when the machine is either started or switched off. The foundations for the machines are designed such that the resonance is always avoided; that is, the resonant frequency of the foundation-soil system is kept either greater or smaller than the operating frequency of the machine. The resonant frequency of the foundation-soil system, without springs, can be made smaller by increasing the mass of the footing. On the other hand, rather than increasing the mass of the footing, a flexible spring mounting system can be placed below the machine so as to reduce the resonant frequency of the machine foundation. Therefore, a spring cushion inserted in between the machine base and its footing can be employed not only to bring down the displacement amplitude of the ground vibrations surrounding the footing but also it can be utilized to avoid the occurrence of resonance. In the present study, no attempt has been made to examine the effect of the damping parameter of the spring mounting cushion on the dynamic response of the footing and the machine base.

5. Conclusions

The effect of the spring mounting base on the response of the rigid footing and machine subjected to steady state harmonic vertical loading has been explicitly brought out by conducting a number of block vibration tests. The employment of the spring mounting cushion, having stiffness much smaller than that of soil strata, results in a drastic reduction in the resonant displacement amplitude of the footing. It also causes a significant reduction in the resonant frequency of the machine foundation. For all the experiments with the spring mounting base, the resonant frequency for the machine base was found to be invariably same as that of the footing.

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