Technical Note

Allowable bearing capacity of shallow foundations based on shear wave velocity

SEMIH S. TEZCAN¹, ALI KECELI², and ZUHAL OZDEMIR³

¹Bogazici University, Bebek, Istanbul, Turkey. e-mail: tezokan@superonline.com ²Istanbul University, Beyazit, Istanbul, Turkey ³ Higher Education Research Foundation, Istanbul, Turkey

(Received 1 March 2004; revised 23 June 2004; accepted 7 July 2004)

Abstract. Firstly, the historical background is presented for the determination of ultimate bearing capacity of shallow foundations. The principles of plastic equilibrium used in the classical formulation of the ultimate bearing capacity are reviewed, followed by a discussion about the sources of approximations inherent in the classical theory. Secondly, based on a variety of case histories of site investigations, including extensive bore hole data, laboratory testing and geophysical prospecting, an empirical formulation is proposed for the determination of allowable bearing capacity of shallow foundations. The proposed expression corroborates consistently with the results of the classical theory and is proven to be reliable and safe, also from the view point of maximum allowable settlements. It consists of only two soil parameters, namely, the in-situ measured shear wave velocity, and the unit weight. The unit weight may be also determined with sufficient accuracy, by means of another empirical expression, using the P-wave velocity. It is indicated that once the shear and P-wave velocities are measured *in-situ* by an appropriate geophysical survey, the allowable bearing capacity is determined reliably through a single step operation. Such an approach, is considerably cost and time-saving, in practice.

Key words. allowable bearing pressure, bearing capacity, foundation design, shallow footings, shear wave.

1. Introduction

The ultimate bearing capacity of a particular soil, under a shallow footing, was investigated theoretically by Prandtl (1921) and Reissner (1924) using the concept of plastic equilibrium as early as in 1921. The formulation however is slightly modified, generalized, and updated later by Terzaghi (1925), Meyerhof (1956), Hansen (1968), De Beer (1970), and Sieffert and Bay-Gress (2000).

The historical bearing capacity formulation, as will be discussed briefly in the next section, is still widely used in geotechnical engineering practice. However, there are various uncertainities in representing the real in-situ soil conditions by means of a few laboratory tested shear strength parameters. The basic soil parameters are c_u = cohesion, undrained shear strength and ϕ = angle of internal friction, which can only be determined by laboratory testing of undisturbed soil samples. It is sometimes impossible to take undisturbed soil samples especially in sandy and gravelly soils.

The *in-situ* measured shear wave velocity, v_s , however as a single field index represents the real soil conditions, much more effectively and reliably than the laboratory tested shear strength parameters. In addition to geophysical refraction seismic survey, there are several other techniques of measuring the shear wave velocity at the site as discussed by Stokoe and Woods (1972), Tezcan et al. (1975). Because, the in-situ measured shear wave velocity, v_{s} , reflects the true photograph of the soil, containing the contributions of the void ratio, effective confining stresses, stress history, shear and compressive strengths, geologic age, etc. As will be seen later in this study, the shear wave velocity, v_s enables the practicing engineer to determine the allowable bearing capacity, q_a in a most convenient, reliable and straight forward manner.

2. Classical Formulation

Using the principles of plastic equilibrium, the ultimate bearing capacity, q_f , of a shallow strip footing, with a depth of D , from the surface and with a width of B and length L, (Figure 1), is given by Terzaghi and Peck (1967) as

$$
q_f = cN_c s_c + \gamma DN_q + 0.5 \gamma BN_\gamma s_\gamma,\tag{1}
$$

where

a) Bearing capacity factors;

$$
N_q = \exp(\pi \tan \phi) \tan^2(45^\circ + \phi/2)
$$

\n
$$
N_c = (N_q - 1) \cot \phi
$$

\n
$$
N_\gamma = 1.8(N_q - 1) \tan \phi
$$
 by Hansen (1968) or
\n
$$
N_\gamma = (N_q - 1) \tan(1.4\phi)
$$
 by Meyerhof (1956)

Figure 1. Failure mechanisms under a strip footing.

 $s_c = 1 + 0.20 B/L \dots \dots \dots \dots \dots \dots$ ($\phi \neq 0$ conditions) $s_c = [1 + 0.20 B/L][1 + 0.3(D/B)^{0.25}] \dots (\phi = 0 \text{ conditions}, \text{ saturated class})$ $s_{\gamma} = 1 - 0.2 \left(\frac{B}{L} \right) \dots \dots \dots \dots \dots \dots \dots \dots (B/L = \text{footing width to length ratio)}$ s^c ¼ 0:6 : :ðcircular footingÞ

It is customary to take $B/L = 0$ for a strip footing, and $B/L = 1$ for a square footing. The formulation is applicable to 'shallow' foundations in which the depth D, is not greater than the breadth B. The foundation shape factor expression of s_c given above for saturated clays under undrained conditions, where $\phi = 0$, is generated using the N_c curves supplied by Skempton (1951). If the soil is 'weak', or in other words is not fairly dense or stiff, i.e. $D_r < 0.35$, $N_{60} < 8$, $c_u < 100$ kPa, or v_s < 200 m/s, the reduced shear strength parameters c_r and ϕ_r are used in Equation (1) instead of the laboratory determined c and ϕ , as follows (Terzaghi and Peck, 1967):

$$
c_r = 0.67 c \tag{2a}
$$

$$
\tan \phi_r = 0.67 \tan \phi \tag{2b}
$$

3. Sources of Approximations in Classical Approach

The approximations involved in the derivation and use of the ultimate bearing capacity, q_f , given by Equation (1), may be summarized as follows:

- 1. The soil mass is assumed to be purely homogeneous and isotropic, while the soil in nature is extremely heteregenous and tixotropic, further the classical theory is developed only for a planar case, while all footings are 3-dimensional in real behaviour.
- 2. The first term of Equation (1) represents the shear strength, the second term is the contribution of the surcharge pressure due to the depth of foundation, and the third term represents the contribution of the self-weight. It is only an approximation to superimpose the contributions of various load cases in an entirely nonlinear plastic stress–strain environment.
- 3. The contribution of self-weight can be determined only approximately, by numerical or graphical means, for which no exact formulation is available.
- 4. The shear strength of soil within a depth D, from the surface is neglected.
- 5. Depending on the degree of, compressibility of the soil, there may be three types of failure modes; (i) general shear, (ii) local shear, and (iii) punching shear, as shown in Figure 1. The theoretical considerations behind Equation (1), correspond only to the general shear mode, which is typical for soils of low compressibility, such as dense sands and stiff clays. In the local shear failure, only a partial state of plastic equilibrium is developed with significant

compression under the footing. In the punching shear mode, however, direct planar shear failures occur only along the vertical directions around the edges of the footing. Therefore, Equation (1) is no longer applicable for soils of high compressibility, such as loose sand and soft clay, which may undergo, either (ii) the local shear or (iii) the punching shear failures. Consequently, the results of Equation (1) will only be approximate for such soils. In reality, the excessive settlement and not the shear failure is normally the limiting criterion in high compressibility soils.

- 6. The ultimate bearing capacity calculations are very sensitive to the values of shear strength parameters c, and ϕ , which are determined in the laboratory using 'undisturbed' soil samples, which may not necessarily represent the true conditions prevailing at the site. Unrealistically, high bearing capacity is calculated especially, if the shear strength parameter, ϕ , is inappropriately determined to be on the high side in the laboratory. All soil parameters including the real values of internal angle of friction, water content, void ratio, confining pressure, presence of boulders or cavities, etc are not necessarily the same in the soil samples.
- 7. Customarily, after a due geotechnical survey, a single value of allowable bearing capacity q_a , is assigned in practice, to a particular construction site. However, minor variations in sizes, shapes and depths of different foundations at a particular site are overlooked, and the same q_a value is used in foundation design, through out the construction site.
- 8. A factor of safety of 3 is used normally, in order to obtain the allowable bearing capacity, q_a , which contains a significant amount of reserve strength in it, accounting for all the inaccuracies and approximations cited herein. This significantly large factor of safety represents the degree of uncertainties and our 'ignorance' in determining the real soil conditions.
- 9. Last, but not the least, although some quantitative guidance is available as contained in Section 2, there is quite a bit of intuition in determining whether the soil is on the 'strong' or the 'weak' side, for the purpose of using reduced (twothirds) shear strength parameters, in accordance with Equation (2).

4. Practical Recommendations

Based on the practical experiences of the writers, the ranges of allowable bearing capacities for different categories of cohesive and granular soils are summarized in Table 1. For comparisons as well as for quick reference purposes, the values of SPT counts N_{60} shear strength parameters c_u , and ϕ , relative density D_r , and also the shear wave velocity v_s , for each soil category are also given in Table 1. The ranges of allowable bearing pressures q_a , are tested to be in conformity with the empirical recommendations of the Uniform Building-Code (1997), the Turkish Earthquake Code TEC- (1998), and the British Standard 8004 (1986).

of allowable bearing canacities (kPa) Table 1. Recommended ranges of allowable bearing capacities (kPa) $^{\rm b}$ If the foundation base is below the GWT, use the lower values of q_a within the range.

5. Use of Shear Wave Velocity

5.1. FOR CONTROL OF SETTLEMENTS

Based on numerous case studies, as discussed in the subsequent sections, the allowable bearing capacity, q_a , under a shallow foundation in units of kPa, may be obtained from the following empirical expressions:

$$
q_a = 0.024 \gamma v_s \tag{3a}
$$

$$
q_a = 2.4(10^{-4})\rho \, v_s \tag{3b}
$$

where, γ = unit weight (kN/m³), ρ = mass density (kg/m³), and v_s = shear wave velocity (m/sec). Since, a proper foundation design must satisfy not only an assured degree of safety against possible shear failures of the supporting soil, but also the settlements, and in particular the differential settlements, should not exceed the tolerable limits as given by Skempton and MacDonald (1956). Hence, the coefficient of the empirical formula in Equation (3) is so selected to be on the low side, that no settlement problem will necessarily be encountered in relatively soft soil conditions. This point has been rigorously tested and verified for all soft 'weak' soil conditions existing in the case histories given in Table 2.

Although, the empirical expressions of Equation (3) are proposed by the writers, on the basis of extensive geotechnical and geophysical soil investigations at 14 different sites, they should be used with caution. For relatively important buildings, and especially until a stage when the validity of these simple empirical expressions are amply tested and calibrated over a sufficient period of time, the allowable bearing pressure should be determined also by means of conventional methods using Terzaghi's soil parameters.

The proposed empirical expressions are for estimating the allowable bearing pressure only. The settlement calculations however, should be conducted, especially for soft soil conditions and for important structures, using either the elastic theory (Skempton and MacDonald, 1956), or the Skempton–Bjerrum method (1957). Because, settlements sometime may be the dominating factor.

5.2. FOR SETTING AN UPPER CEILING FOR q_a

In order to set a practical upper ceiling for the allowable bearing capacity, q_a , especially for the rocky formations the empirical expression given in Equation (3), is adjusted to yield gradually reduced values through a factor s_v , for shear wave velocities greater than 500 m/s, as follows:

$$
q_a = 0.024 \gamma v_s s_v \geqslant 30.6 \gamma \tag{4}
$$

$$
s_{v} = 1 - 3 \times 10^{-6} (v_{s} - 500)^{1.6}
$$
 (5)

The variation of allowable bearing capacity q_a , with shear wave velocity v_s , is illustrated in Figure 2, where the reduction factor s_v , sets an asymptotic upper limit of $q_a = 30.6$ γ for shear wave velocities $v_s \ge 2000$ m/s.

ALLOWABLE BEARING CAPACITY OF SHALLOW FOUNDATIONS 209

aseismic refraction surveys, bgeophysical soil layers, c ^aseismic refraction surveys, ^bgeophysical soil layers, ^cthe classical Terzaghi approach Equation (1), ⁴the shear wave velocity approach Equation (1), ^oOil tanks belong to the Turkish Petroleum Office Co., Ankara, Turkey.

5.3. FOR CALCULATING UNIT WEIGHTS

There is a direct relationship between the average unit weight γ , and the P-wave velocity of a soil layer. Based on extensive case histories of laboratory testing, a convenient empirical relationship in this regard, is proposed by the writers as follows:

$$
\gamma_p = \gamma_0 + 0.002 \,\nu_p \tag{6}
$$

Figure 2. Allowable bearing capacity of soils based on v_s .

where, γ_p = the unit weight in kN/m³ based on P-wave velocity, v_p = P-wave velocity in m/s, and γ_0 = the reference unit weight values given as follows:

- $\gamma_0 = 16$ for loose sandy, silty and clayey soils
- $\gamma_0 = 17$ for dense sand and gravel
- $\gamma_0 = 18$ for mudstone, limestone, claystone, conglomerate, etc.
- $\gamma_0 = 20$ for sandstone, tuff, graywacke, schist, etc.

As seen in Figure 3, the unit weights calculated by the empirical expression given in Equation (6) are in excellent agreement with those determined in the laboratory. In the absence of any bore hole sampling and laboratory testing of soil samples, the above empirical expression provides a reliable first approximation for the unit weights of various soils, once the *in-situ* measured P-wave velocities are available. In fact, the speedy evaluation of unit weights, prior to any soil sampling, enables the practicing engineer to calculate the allowable bearing capacity q_a , readily from Equation (3).

Figure 3. Unit weights based on v_p - velocities.

6. Case Histories

6.1. FIELD INVESTIGATIONS

In order to establish a sound and reliable relationship between the allowable bearing capacity q_a , and the shear wave velocity v_s , a series of case histories have been studied as summarized in Table 2. For each case, in-depth geotechnical and geophysical site investigations have been conducted and a comprehensive set of in-situ and laboratory tested soil parameters have been determined. Most of the basic soil parameters, for each typical soil layer, are shown in Figures 4–6. If however, for any particular soil parameter in any typical soil layer, multiple values were available, from various bore hole and seismic survey measurements, only the average of these multiple values have been indicated.

6.2. ALLOWABLE BEARING CAPACITIES BY THE CLASSICAL THEORY

The first column in these figures contain the *in-situ* measured SPT data, N_{30} , the laboratory tested values of c_u = undrained shear strength (kPa), ϕ' = effective internal angle of friction, $\gamma_n = \text{unit weight } (kN/m^3)$, and also the $q_f = \text{ultimate}$ bearing capacities (kPa), and $q_a =$ allowable bearing capacities (kPa) calculated using the classical approach of Equation (1). If, a particular soil layer is considered to be 'weak' in accordance with Terzaghi's and Peck (1967) recommendations, twothirds of shear strength parameters have been utilized in the bearing pressure capacity calculations, as given in Equation (2).

Figure 4. Allowable bearing capacities for case histories No.1 through No.4 (Units are; $\gamma = kN/m^3$, $c = kN/m^2$, $q_a = kN/m^2$, v_s , $v_p = m/s$).

Figure 5. Allowable bearing capacities for case histories No.5, 6, 7, and 9 (Units are; $\gamma = kN/m^3$, $c = kN/m^2$, $q_a = kN/m^2$, $v_s, v_p = m/s$).

Figure 6. Allowable bearing capacities for case histories No.10 through No.14 (Units are; $\gamma = kN/m^3$, $c = kN/m^2$, $q_a = kN/m^2$, v_{s} , $v_p = m/s$).

6.3. ALLOWABLE BEARING CAPACITY BY v_s

The second column contains, the *in-situ* measured v_s and v_p -velocities (m/sec), $v =$ Poisson ratio, $\gamma_p =$ unit weights (kN/m³) determined on the basis of P-wave velocities given in Equation (6), q_a = allowable bearing capacities (kPa) based on shear wave velocities, in accordance with Equation (3). In all case histories, the shear wave velocity, v_s , and the P-wave velocity, v_p , have been measured *in-situ* by means of seismic refraction method, using low level explosives. The propagating waves have been recorded by means of a 12-channel *Smart Seis Geometrics* instrument, which is capable of producing very high resolution of signal/noise ratio, due to its instant analogue and/or digital signal analyses and automatic filtering process.

In practice, the geophysical explorations are not daily business in foundation engineering, therefore, there is a necessity for experienced technical staff for such a purpose. The shear wave velocities may be measured, through impact energy methods, during the bore hole drilling, or using the cross-hole technique (Stoke and Woods, 1972; Tezcan et al., 1975)

Realizing that, the bearing capacity is correlated with large strains at failure, while the shear wave velocity is associated with 'zero strain' levels, the proposed empirical expressions are adjusted effectively in order to accommodate the differences in strain levels.

For each case history, the allowable bearing capacities obtained by the classical theory have been compared in Figure 7, with those determined by Equation (3), using the shear wave velocities. It is seen that there is a very good agreement between these two different sets of values. The allowable bearing capacities q_a , based on the shear wave velocities are more uniform in distribution, exhibiting no erratic variation and further, they provide an inherent factor of safety against shear failure and intolerable settlements. The empirical allowable bearing pressure expression given in Equation (3), ensures for all foreseeable soft soil conditions, including those of the case studies that, the maximum allowable settlement is not exceeded.

7. Conclusions

- 1. The determination of adequately safe allowable bearing capacity of a soil layer under a shallow foundation is a problem of vital importance in geotechnical engineering. The classical approach is not only costly and time consuming due to extensive in-situ and laboratory testing required, but also involves significant approximations and intuitive judgements. Despite the 'exact' nature of the classical theory, a huge factor of safety, on the order of 300 percent, is utilised in order to account for the unexpected inaccuracies and our 'ignorance' of the real soil conditions.
- 2. The proposed empirical shear wave velocity approach however, is surprisingly cost effective, and time saving. The *in-situ* measured shear wave velocity, v_s , as an indispensable single field index, is capable of representing the real soil conditions at the

Figure 7. Comparisons of allowable bearing capacities (Numerals beside the data points are the case study numbers).

site, including the true influence of a family of soil parameters like water content, confining pressure, relative density, void ratio, nonuniformity, discontinuity, nonhomogeneity, shear and compressive strength, etc. The complications and misrepresentations associated with soil sampling, sample disturbance, accurate simulations in the laboratory testing, etc. are all avoided. Shear wave velocity measurement at a site however, calls for additional cost and expert geophysical personnel.

- 3. The depth, width and length of a foundation plays a significant role especially in granular soils, in the derivation of mathematical formulation when following the classical approach. In cohesive soils, the geometry of foundation does not play a significant role anyhow. Nevertheless in classical theory, the soil is idealized into an isotropic, homogeneous and uniform elasto-plastic planar geometrical medium. In the shear wave velocity approach however, there is absolutely no need to consider the foundation size and depth, even in granular soils, since the influence of all these parameters are inherently incorporated in the in-situ measured v_s -values. The classical approach is further handicapped by the layered conditions. In shear wave velocity approach however, the bearing capacity of a single layer, immediately under the foundation, is directly determined, as a one step operation.
- 4. The empirical formulations proposed for calculating both the allowable bearing capacity q_a , and the unit weight γ , are proven to be safe and reliable as verified consistently by 14 different laboratory tested case histories. The validity and reliability of the proposed scheme will be better established however, as the

proposed empirical method is constantly calibrated by conventional method at more and more sites.

Acknowledgements

We gratefully acknowledge the assistance and cooperation extended by Mr. Tufan Durgunoglu, and Mr. Abdullah Calisir of the Geotechnics Co., Istanbul, who conducted the geotechnical and geophysical soil investigations of all the case studies discussed herein. Sincere thanks are also due to both Professor Osman Uyanik, of Suleyman Demirel University, Isparta, for his invaluable criticisms and corrections of the manuscript, and Mr. Mustafa Cevher, Chief Geophysical Engineer, Municipality of the Greater City of Izmit, Turkey, for his constant supply of field data and encouragement.

References

- British Standard 8004 (1986) Code of Practice for Foundations, British Standards Institution, London.
- DeBeer, E. E. (1970) Experimental determination of the shape factors and the bearing capacity factors of sand, Geotechnique, 20, 387–411.
- Hansen, J. B. (1968) A Revised Extended Formula for Bearing Capacity, Danish Geotechnical Institute Bulletin, No. 28.
- Meyerhof, G. G. (1956) Penetration tests and bearing capacity of cohesionless soils, Proceedings $ASCE$, 82, (SM1), Paper 866, 1–19.
- Prandtl, L. (1921) Über die Eindringungsfestigkeit (Härte) plastischer Baustoffe und die Festigkeit von Schneiden, (On the penetrating strengths (hardness) of plastic construction materials and the strength of cutting edges), Zeit. Angew. Math. Mech., 1(1),15–20.
- Reissner, H. (1924) Zum Erddruckproblem (Concerning the earth-pressure problem), Proc. 1st Int. Congress of Applied Mechanics, Delft, pp. 295–311.
- Sieffert, J. G. and Ch. Bay-Gress (2000) Comparison of the European bearing capacity calculation methods for shallow foundations, Geotechnical Engineering, Institution of Civil Engineers, 143, 65–74.
- Skempton, A. W. (1951) The bearing capacity of clays, Proceedings, Building Research Congress, 1, 180–189.
- Skempton, A. W. and Bjerrum, L. (1957) A contribution to the settlement analysis of foundation on clay, Geotechnique, 7, 168–178.
- Skempton, A. W. and MacDonald, D. H. (1956) Allowable settlement of buildings, Proceedings ICE, 5(3), 727–768.
- Stokoe, K. H. and Woods, R. D. (1972) In situ Shear Wave Velocity by Cross-Hole Method, Journal of the Soil Mechanics and Foundation Divison, ASCE, 98, No. (SM5), 443–460.
- Terzaghi, K. (1925) Structure and volume of voids of soils, Pages 10, 11, 12, and part of 13 of Erdbaumechanik auf Bodenphysikalisher Grundlage, translated by A. Casagrande in From Theory to Practice in Soil Mechanics, John Wiley and Sons, New York, 1960, pp. 146–148.
- Terzaghi, K. and Peck, R. B. (1967) Soil Mechanics in Engineering Practice, 2nd edn, John Wiley and Sons, New York.

Tezcan, S. S., Erden, S. M. and Durgunoğlu, H. T. (1975) In situ Measurement of Shear Wave Velocity at Boğaziçi University Campus, Proceedings of International Conference on Soil Mechanics and Foundation Engineering, Vol. 2, Istanbul Technical University, Ayazağa, Istanbul, Turkey, pp. 157–164.

Turkish Earthquake Code (TEC) (1998) (www. koeri.boun.edu.tr).

Uniform Building Code (UBC) (1997) International Conference of Building Officials, 5360 Workman Mill Road Whittier, California, USA.