Int J Fract (2011) 169:97–104
DOI 10.1007/s10704-010-9582-6

ON USING A DUAL BOUND APPROACH TO CHARACTERIZE THE YIELD BEHAVIOUR OF POROUS DUCTILE MATERIALS CONTAINING VOID CLUSTERS

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Abstract. Two constitutive models for porous ductile materials are employed together to predict the yield behaviour of ductile materials containing void clusters. In this dual bound approach, the upper and lower bound constitutive models of Gurson (1977) and Sun and Wang (1989) are each evaluated in order to obtain upper and lower estimates for the material behaviour. By combining these two solutions, a predictive band can be created to capture the experimental variation in the yielding behaviour. Although these constitutive models have been derived with the assumption of a periodic void distribution, real materials contain void clusters that can significantly alter the onset of yielding and fracture. Therefore it is of great interest to determine if using dual constitutive models can produce an acceptable first-order approximation of the yielding behaviour in these materials. In the present work, the upper and lower bound yield loci are superimposed over numerical data available in the literature for the yielding of materials containing void clusters. It is shown that the dual bound approach is able to capture the material behaviour over a wide range of practically encountered stress triaxialities.

Keywords: lower bound, heterogeneous, void, clusters, ductile fracture

1. Introduction. To account for the affect of microvoids on ductile fracture, constitutive models have been developed to capture the initiation, evolution and coalescence of these voids from the onset of yielding to final fracture. The most widely known of these models was first proposed by Gurson (1977) who derived an approximate yield criterion and void growth rule for a ductile material containing spherical voids. Due to its inherent upper-bound formulation, the Gurson model is rather rigid and tends to overestimate the material behaviour. Calibration parameters were soon introduced into the yield criterion by Tvergaard (1981) and later by Faleskog (1998) and Ragab (2004) to shrink and adjust the yield surface to better reflect the material behaviour obtained from finite-element simulations of voided unit cells.

Recognizing the rigid nature of the Gurson (1977) model, Sun and Wang (1989) applied the lower bound theory of plasticity to the spherical unit cell of Gurson to derive a conservative solution for the material behaviour. If the Gurson and Sun and Wang (SW) yield loci are plotted together as shown in Figure 1, a margin of difference exists between them which creates a band that can capture the experimental variation in the yielding of porous materials. Butcher et al. (2006) demonstrated that the upper and lower bound constitutive models (when equipped with the same void nucleation and coalescence rules) could be used to obtain upper and lower estimations for the fracture strain in order to define a formability band. With this dual bound approach, a good prediction of the material behaviour can be readily achieved while removing the need for extensive calibration or artificial softening of the upper bound approximation.

The objective of the present work is to determine if the dual bound approach of Butcher et al. (2006) can describe the yield behaviour of a porous ductile material containing void clusters. Given that real materials contain heterogeneous void distributions and clusters, it is unlikely that a single bound model could uniquely describe the material behaviour due to the clusters causing a deviation from the assumption of a periodic microstructure. By employing both upper and lower bound-based constitutive models, the variation in the material behaviour due to the presence of the clusters is more likely to be captured between the two bounds. To evaluate the performance of the dual bound approach for heterogeneous void distributions, the Gurson (1977) and Sun and Wang (1989) yield criteria are superimposed over the numerical results of Bilger et al. (2005) who investigated the yielding of materials containing three-dimensional distributions of void clusters.

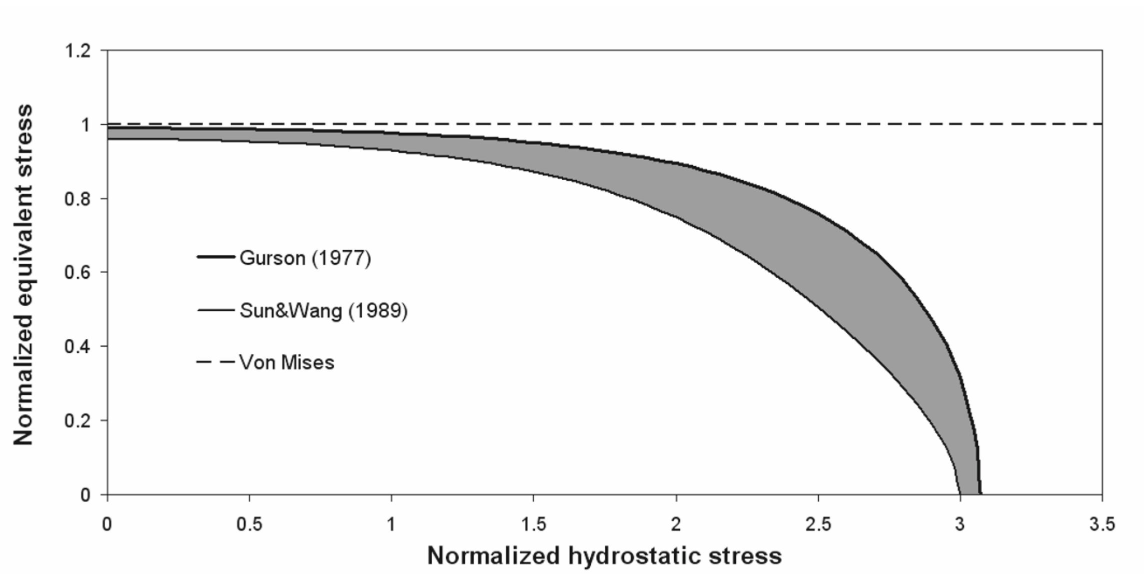


Figure 1. Yield loci of the Von Mises, Gurson (1977) and Sun and Wang (1989) models for a porosity of 1%. The experimental yielding behaviour of a material should lie on or between the two bounds (in the shaded band). The macroscopic equivalent stress and hydrostatic stress are normalized by the yield stress of the material.

2. Constitutive modeling of ductile fracture

2.1. Upper bound material model. Gurson (1977) was the first to propose an approximate yield criterion and flow rules for a porous ductile material based on the upper bound theory of plasticity. The upper bound formulation is used to determine the maximum macroscopic stresses required to sustain plastic flow. To simplify the analysis, the material is assumed to have a periodic distribution of spherical voids with each void located at the center of a spherical unit cell. Tvergaard (1981) later modified the Gurson model by introducing the calibration parameters, $q_{1\text{-}3}$, into the Gurson yield criterion to create the widely known Gurson-Tvergaard (GT) variant which is expressed as

$$\Phi = \frac{\Sigma_{\mathrm{e}}^2}{\bar{\sigma}^2} + 2 f q_1 \cosh\left(q_2 \frac{3}{2} \frac{\Sigma_{\mathrm{hyd}}}{\bar{\sigma}}\right) - 1 - q_3 f^2 = 0 \tag{1}$$

where f is the porosity or void volume fraction, $\bar{\sigma}$ is the equivalent tensile flow stress of the matrix material and Σ_{e} and Σ_{hyd} are the macroscopic equivalent and hydrostatic stress, respectively. The Gurson criterion reduces to the von Mises yield criterion for a damage-free material by setting $f = 0$ in Eq. (1). Unless otherwise specified, $q_1=q_2=q_3=1$ in the present work to retain the original Gurson formulation, ensuring a meaningful comparison with the lower bound solution of Sun and Wang (1989) which has not been modified with calibration parameters. Only spherical voids are considered in the present work since the SW model has not been extended to include void shape effects like more recent Gurson-based models have been (Benzerga et al. 1999, Pardoen and Hutchinson 2000, Ragab 2004).

2.2. Lower bound material model. Sun and Wang (1989) were the first to use the lower bound theory of plasticity to provide a conservative constitutive model for porous ductile materials. Although the upper and lower bound models share the same unit cell geometry, the resulting solution depends on the cell boundary conditions. The upper bound approach of Gurson (1977) was based on kinematically admissible velocity fields while the lower bound used statically admissible stress fields. The SW material model obeys the same void growth, nucleation and coalescence rules employed in the Gurson model. The SW yield criterion is expressed as

$$\Phi = \frac{\Sigma_{\mathrm{e}}^2}{\bar{\sigma}^2} + b_1 f \cosh\left(\frac{3\Sigma_{\mathrm{hyd}}}{2\bar{\sigma}}\right)\left[1 + b_3 f^2 \sinh^2\left(\frac{3\Sigma_{\mathrm{hyd}}}{2\bar{\sigma}}\right)\right]^{-\frac{1}{2}} - b_2 = 0 \tag{2}$$

where

$$b_1 = 2 - \frac{1}{2}\ln f \quad b_2 = 1 + f(1+\ln f) \quad b_3 = \left(\left(\frac{b_1}{b_2}\right)^2 \coth^2\left(\frac{3\sigma_{\mathrm{mt}}}{2\bar{\sigma}}\right) - \left(f^2 \sinh^2\left(\frac{3\sigma_{\mathrm{mt}}}{2\bar{\sigma}}\right)\right)\right)^{-1}$$

$$\sigma_{\mathrm{mt}} = -0.65\bar{\sigma}\ln f$$

Similar to the Gurson model, the lower bound formulation reduces to the Mises yield criterion for a damage-free material by setting $f = 0$ in Eq. (2). Sun and Wang (1989) plotted the lower and upper bound approximations over the experimental results of Shima and Oyane (1976) and found that the data fell between the two limits, while appearing to favour the lower bound solution. More recently, Francescato et al. (2004) performed a numerical limit analysis to determine the strength of porous materials having long cylindrical cavities and observed that the SW model best described the material behaviour at low porosities.

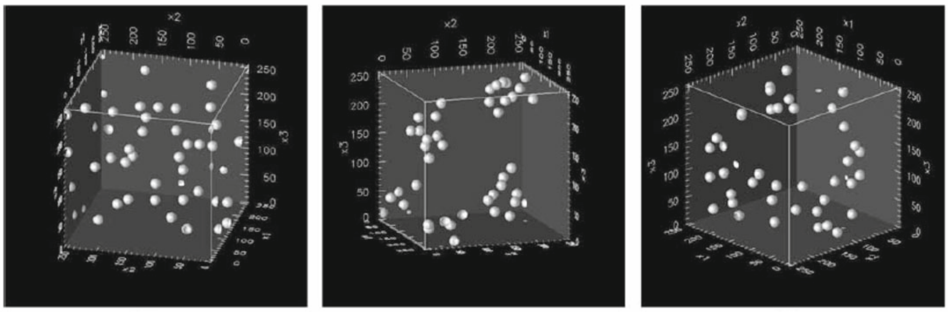
3. Evaluation of the dual bound approach

3.1. Previous applications of the dual bound. Previous investigations which apply the dual bound approach to ductile fracture have been performed by Butcher et al. (2006, 2010). In these studies, the upper-bound and lower-bound constitutive models were used to predict fracture in both a stretch flange forming and a tube hydroforming operation. It was observed that neither model could uniquely describe the experimental fracture strains but that the data could be faithfully captured between the Gurson and SW predictions. While the results of Butcher et al. (2006, 2010) are appealing, it should be noted that the upper and lower forming limits were obtained through calibration of the material parameters used to describe void nucleation. As such, it is of interest to evaluate the performance of the dual bound approach in a more rigorous manner where void nucleation, evolution and coalescence are neglected and only the onset of yielding is predicted. By applying the dual bound approach to a yielding situation, no adjusting or calibration parameters in the void sub-models are required and the actual performance of this approach can be evaluated.

3.2. Present work. To obtain the required yielding data in order to evaluate the dual bound approach, we turn to the work of Bilger et al. (2005) who investigated the overall and local response of porous media composed of a perfectly plastic matrix weakened by voids. Bilger et al. (2005) employed the Fast Fourier transform (FFT) method to numerically determine the onset of yielding in materials having microstructures characterized by one of three void spatial distributions:

i. a random void distribution without void clustering,
ii. connected clusters of voids, or
iii. disconnected clusters of voids.

The study considered two-dimensional and three-dimensional arrangements, each under two loading types (type-A, which combined pure shear with hydrostatic tension, and type-B which did not have a shear component) over a large range of stress triaxiality, T (the ratio of hydrostatic to equivalent stress). In the present study, we will only focus on the three-dimensional arrangements because the Sun and Wang (1989) model does not have an analogous 2-D solution like the Gurson (1977) model does. The void volume fraction was held constant at 0.6% for all of the three-dimensional cluster arrangements which are presented in Figure 2.



(a) (b) (c)

Figure 2. Three-dimensional clustered microstructures: (a) random (b) disconnected clusters and (c) connected clusters (Bilger et al. 2005)

Bilger et al. (2005) compared their results with the Gurson-Tvergaard (GT) model which proved to be overly stiff and overestimated yielding in the different microstructures. Due to the Gurson model overestimation, it is of interest to see if the combined upper and lower bound solutions can capture yielding of the material containing clusters. The SW model is inherently more sensitive to porosity than the Gurson model and experiences earlier yielding and additional material softening. In the present study, we have plotted the lower bound solution of SW along with the Gurson model in Figures 3-5 to assess if the dual bound approach can capture yielding in the different microstructures. For clarity, Figure 5 shows a close-up view of the results at low stress triaxialities. To provide a contrast with the traditional Gurson model, the more-commonly employed GT solution with fitting parameter q=1.5 is shown for comparison with the original Gurson and SW models.

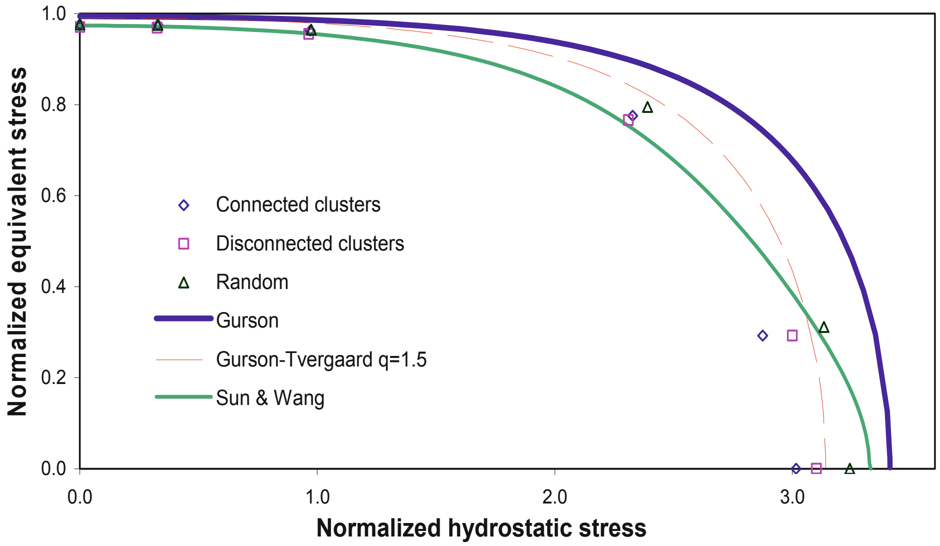


Figure 3. The Gurson and SW yield surfaces have been superimposed onto the numerical results of Bilger et al. (2005) for type-A loading (hydrostatic tension and shear).

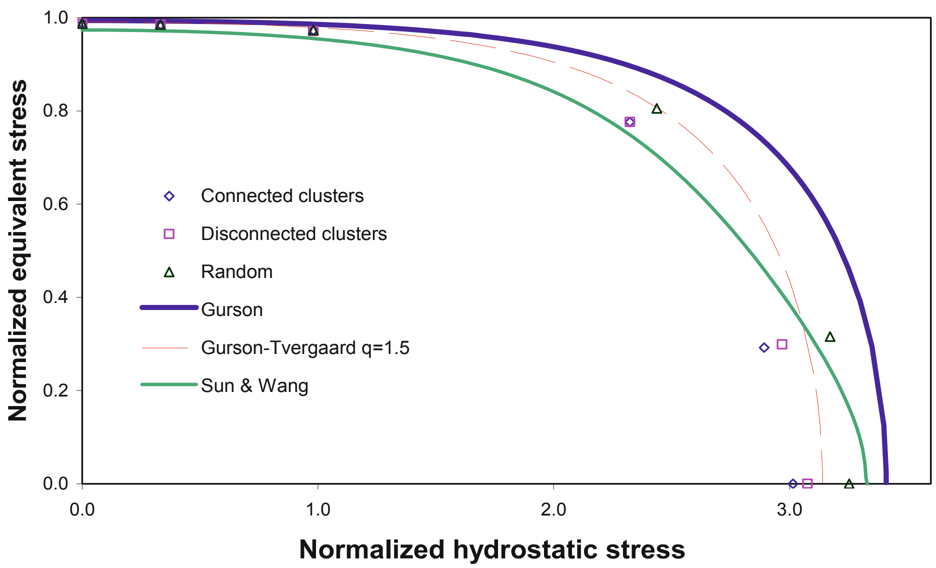


Figure 4. The Gurson and SW yield surfaces have been superimposed on the numerical results of Bilger et al. (2005) for type-B loading (hydrostatic tension, no shear).

At low stress triaxialities the results are neatly bounded by the dual bound. The excellent agreement of the dual bound with the numerical results over the low to moderate triaxiality regime is fortuitous from a practical perspective since the stress triaxiality in a typical sheet metal forming operation is less than unity. For example, uniaxial stretching gives rise to a triaxiality of 1/3, while equal-biaxial stretching renders a value of 2/3. In the low stress triaxiality regime shown in Figure 5, none of the yield criteria were able to independently reproduce the results. However, despite the different loading conditions, stress triaxialities and microstructures, the dual bound approach is able to provide very good upper and lower estimates for the yield behaviour. This is an advantage of the dual bound approach because although neither model was derived for void clusters, a good representation of the material behaviour can still be captured between the two bounds. By obtaining upper and lower estimates for the material behaviour, the variation in the material behaviour due to deviations from the assumption of a periodic void distribution can be better captured than if using a single model. From a practical perspective, it is reasonable to expect the material response to usually fall within the upper and lower limits as predicted by the approximate models of Gurson and Sun and Wang. This result is very attractive to industry because the original Gurson and Sun and Wang models can be quickly implemented in a commercial finite-element code and employed to obtain a first order prediction of the material behaviour in a forming process of interest.

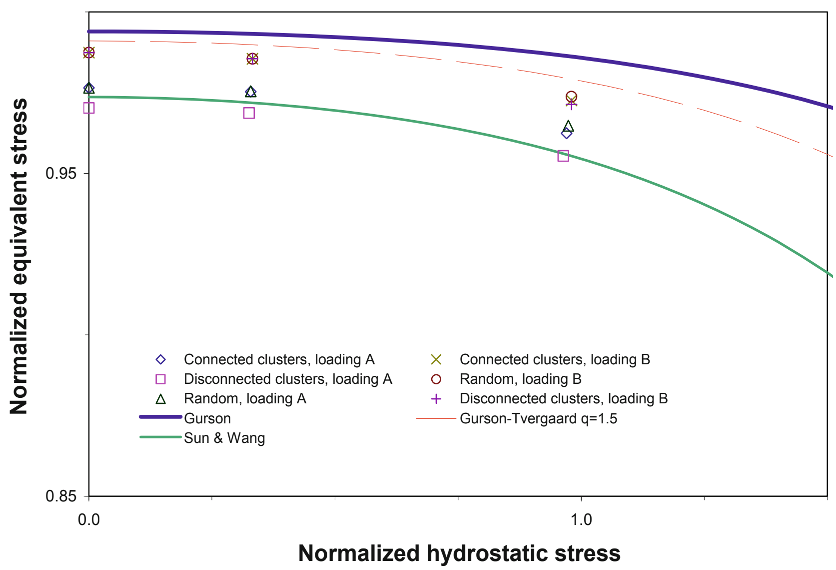


Figure 5. Comparison of the three-dimensional macroscopic yield surfaces with the numerical results of Bilger et al. (2005) for two different loading conditions at low triaxialities.

4. Conclusions. The band which exists between the Gurson (1977) and Sun and Wang (1989) yield surfaces is a useful tool for manufacturing engineers who need to understand the yield behaviour of porous ductile materials containing heterogeneous damage during metal forming operations. In the present work, the numerical results of Bilger et al.

(2005) were compared with the Gurson and SW models which together form a dual bound. The upper and lower bound models enveloped the results at all but very high stress triaxialities. Despite the variation in loadings, microstructures, and stress triaxialities, the dual bound was able to provide good representation of the results. This is significant because real materials contain heterogeneous void distributions, and the two constitutive models considered were not originally designed to account for void clusters. The dual bound approach appears to be an attractive and simple method when compared to complicated variants of the Gurson model which require calibration. Since both the Gurson and SW models are approximations based on the same unit cell geometry, and use complimentary formulations, it is advantageous to consider their results in conjunction.

Acknowledgements. Financial support for this work has been provided by the Natural Sciences and Engineering Research Council of Canada (NSERC), the New Brunswick Innovation Foundation (NBIF) and the Auto 21 Network of Centers of Excellence, and is gratefully acknowledged.

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