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EFFECT OF PORE DISTRIBUTION ON ELASTIC STIFFNESS AND FRACTURE TOUGHNESS OF POROUS MATERIALS

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Abstract. The paper focuses on experimental study of the effect of pore distribution on the mechanical properties of aluminum sheets containing multiple holes. Mechanical behavior of materials of uniform microstructure is compared with that of materials containing pore clusters of circular and elliptical shapes. The overall porosity of all specimens was 0.2. All the experiments were repeated 10 times. Our work demonstrates that overall elastic properties are almost insensitive to the actual distribution of pores - uniform or with distinguishable pore clusters. In contrast, fracture toughness of the specimens is strongly affected by the mutual positions of individual pores. Explicit connection between the fracture stress and minimum pore separation is obtained.

Keywords. Porous material, pore distribution, pore cluster, effective properties, ultimate strength

1. Introduction.

One of the most challenging and, at the same time, most important problems in the mechanics of porous materials is quantitative analysis of the effect of pore distribution on the macroscopic mechanical performance of the material. In the context of effective properties, the main complication appearing is that the ergodic hypothesis does not hold for materials with non-random microstructure (Beran, 1968) and, therefore, volume averaging cannot be used instead of ensemble averaging like in ordinary problems of the mechanics of composites. It leads to a requirement of substantial statistical analysis in the process of solving of such problems. In the 1990s, development of effective computational algorithms (like Fast Multipole Method, Greengard, 1988; or Fast Fourier Transform, Moulinec and Suquet, 1994) provided the possibility for systematic statistical analysis of microstructures and raised discussion about the effect of *non-uniform distribution of inhomogeneities* on the material properties.

Analysis of physical processes in materials with non-uniform microstructures consists of two parts – geometrical description of microstructure, and solution for physical fields (elastic, thermal *etc*.). A general approach to solve the first part - i.e. to describe statistically inhomogeneous two-phase random media - was proposed by Quintanilla and Torquato (1997). They addressed the problem of rigorous geometric description of a two-phase system of non-uniformly distributed fully penetrable spheres that can form clusters. The second part of the analysis depends on the physical content of the problem since different physical properties are characterized in terms of different microstructural parameters (Kachanov and Sevostianov, 2005). Pioneering work in this area was done by Ohno and Hutchinson (1984) where the effect of pore clusters on plastic flow localization was addressed.

Effect of small local fluctuations in volume fractions of the constituents on the effective properties was addressed analytically and numerically by Suquet (2005) and Gărăjeu and Suquet (2007). They have shown, in particular, that the deviations from uniform distribution of pores lead to weakening of the overall carrying capacity of the material (i.e. maximum porosity that material can have without falling to pieces). Bilger et al (2005) discussed the effect of non-uniform distribution of voids on plastic yield. They performed Fast Fourier Transform-based numerical simulation on several microstructures and concluded that materials with uniformly random microstructure are strongest from the point of view of plastic yield, materials with interconnected clusters of pores are the softest ones, while microstructures with isolated clusters are intermediate. The shapes of clusters and their relative density have been discussed in the paper of Sevostianov and Kushch (2009) where quantitative correlation between peak stresses and minimum pore separation was derived for the cases of randomly distributed pores and pores forming well distinguished clusters of circular and elliptic shape.

In the present work we focus on experimental study of the effect of pore distribution. Specimens with randomly distributed pores and with pore clusters were subjected to the uniaxial tension in a controlled environment. The Instron machine used in the experiments was calibrated specifically for these experimental purposes. The software outputted stress-strain data was able to be effectively analyzed and compared among the different pore distributions. In addition, the physical specimens were visually analyzed to validate the data output and crack propagations. To reduce the number of entering parameters all the specimens were of the same size and contained identical numbers of pores of the same radius. The microstructures were first generated on a computer using molecular dynamics algorithm and then drilled in aluminum specimens. We studied materials with randomly distributed pores, with circular pore clusters and with elliptic pore clusters with major axis parallel and perpendicular to the loading direction. All the microstructures were generated ten times and experimental results have been averaged. We observed that the elastic properties are almost insensitive to the type of pore distribution while fracture toughness (ultimate stress) is strongly affected by mutual positions of the pores and is a function of minimum pore separation in a specimen.

2. Materials and Method.

2.1. Microstructure generation.

The microstructures of the specimens have been first generated on a computer. For this aim, the 2D version of the molecular dynamics (MD) algorithm (see Torquato, 2002 for the details) of growing particles is utilized. The main idea of the algorithm is as follows. We start with fourty tiny circular pores whose initial positions within a specimen and their initial velocities are given by the random number generator. Then the pores move toward each other, collide elastically and grow steadily during a period of 1000 N collisions. Whenever a pore (more exactly, its center) traverses the specimen boundary, it enters the specimen from the opposite side. After the volume content of pores reached 0.20, the system is further equilibrated for a period of 5000N collisions, sufficient to guarantee reproducible thermodynamic properties of the model (Torquato, 2002). To separate pores, we used parameter of *minimum allowable spacing* $\delta_{\min} = \min(|Z_{pq}|/D-1)$ (e.g., Chen and Papathanasiou, 2004), where D is a diameter of pore and $Z_{pq} = Z_p - Z_q$. It is also known as *impenetrability* parameter, in terms of the cherry-pit model (Torquato, 2002). In the present paper, $\delta_{min} = 0.025\,mm$ has been taken. Variation of this parameter can slightly influence the absolute value of the local stress concentrations, but does not change the qualitative behavior of the local and averaged fields (see Sevostianov and Kushch, 2009 for discussion).

Pore clusters of circular and elliptic shape were obtained in the following way. First, the uniform microstructure is generated as described above, with the porosity equal to the prescribed inside the cluster (fifteen pores of diameter of 2 mm). The pores outside the cluster are then removed randomly, one by one, until the overall porosity of 0.20 is reached. A drawback of this simple method is that the structure obtained by the random elimination procedure may lead to a non-uniform, percolation type structure. In the present study, it did not happen because the porosity outside the cluster was sufficiently low. In principle, it is possible to fix this problem by applying the previously described "equilibration" procedure to the area outside the cluster.

2.2. Specimen Preparation.

The specimens for physical experiments were prepared using aluminum alloy 6061-T6. Mechanical properties of this alloy are given in Table 1. The size of the specimens was 220mm x 25mm x 3.175mm. A length of 100mm was used for effective body and the rest was used as support in the grips of the Instron machine. Specimens of five different types have been prepared (see Fig. 1): pore free specimens for control, specimens with random distribution of pores, and specimens with distinguishable clusters of circular and elliptic (of two different orientations) shape. Ten specimens of each type (different realizations of each statistics) were prepared. In porous specimens, forty holes of 2mm diameter were drilled according to computer generated distributions described in the previous subsection, so that the total porosity was 0.2.

Density (kg/m^3)	Young's modulus (GPa)	Yield strength (MPa)	Ultimate strength (MPa)
2710	70.0	240.0	260.0

Table 1. Physical properties of aluminum 6061-T6

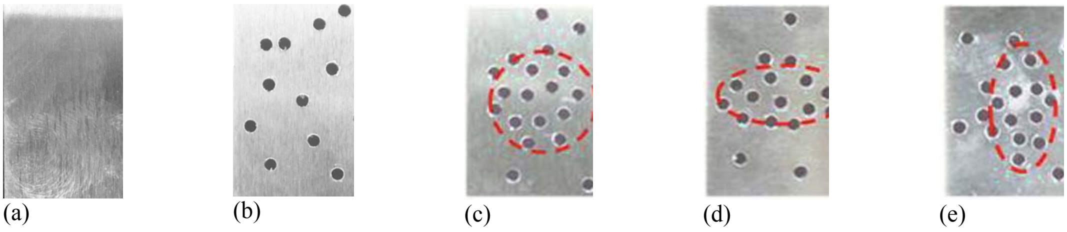

Figure 1. Pore Distributions: a) Control, b) Random, c) Circular Cluster, d) Elliptic cluster with major axis perpendicular to the loading direction, and e) Elliptic cluster with major axis parallel to the loading direction

2.3 Mechanical Testing.

An Instron 5580 testing machine with mechanical wedge action grips was used for the uniaxial tension experiments on the specimens produced. An extension rate of 1 cm./min. was used during the tensile tests. Bluehill software, standard for Instron machines, along with an extensometer was used to calculate the stresses and strains for the specimens. Pictures were taken before and after the testing and were used for comparison and physical analysis (Fig.2). The ten tests for each pore distribution were averaged and an analysis and comparison of the stress-strain curves were performed.

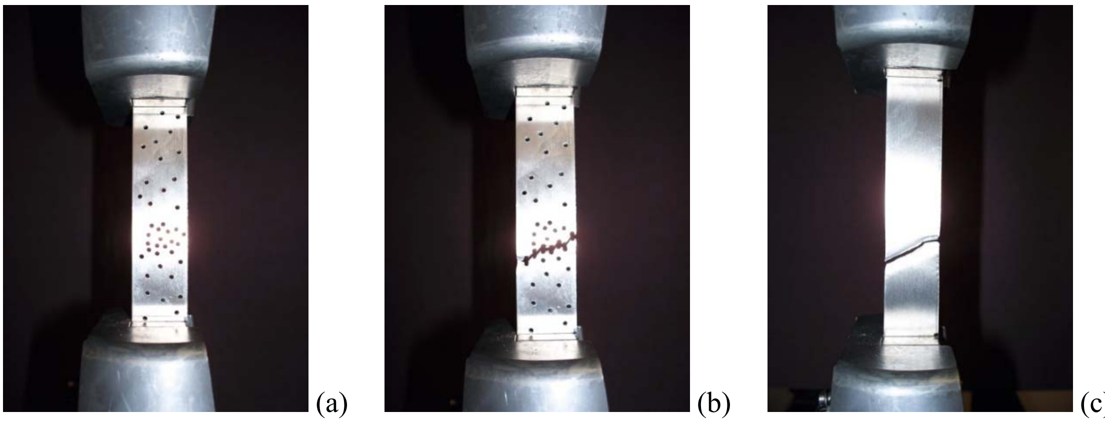

Figure 2. Tensile testing of the specimens: porous specimen with circular cluster before (a) and after (b) loading; (c) fracture of the control specimen.

3. Results and Discussion

The effect of pore distribution on elastic stiffness and fracture toughness of material containing multiple pores is illustrated in Fig.3. Each curve is obtained by averaging of the results over ten specimens. It is seen that the slope of the stress-strain curves – that correspond to the Young's modulus - for porous materials is almost insensitive to the specific pore distribution, while the ultimate strength (stress at which fracture takes place) strongly depends on mutual positions of the pores. The averaged numbers for these two quantities are given in Table 2.

	Pore-Free	**Random pore distribution**	**Elliptic cluster, major axis along the loading direction**	**Circular cluster**	**Elliptic cluster, major axis normal to the loading direction**
$E^*(GPa)$	70.0	39.9	38.2	37.8	37.4
$\sigma_{fr}(MPa)$	326	240	206	185	166

Table 2. Mean Young's modulus and fracture toughness of the specimens with different pore distribution

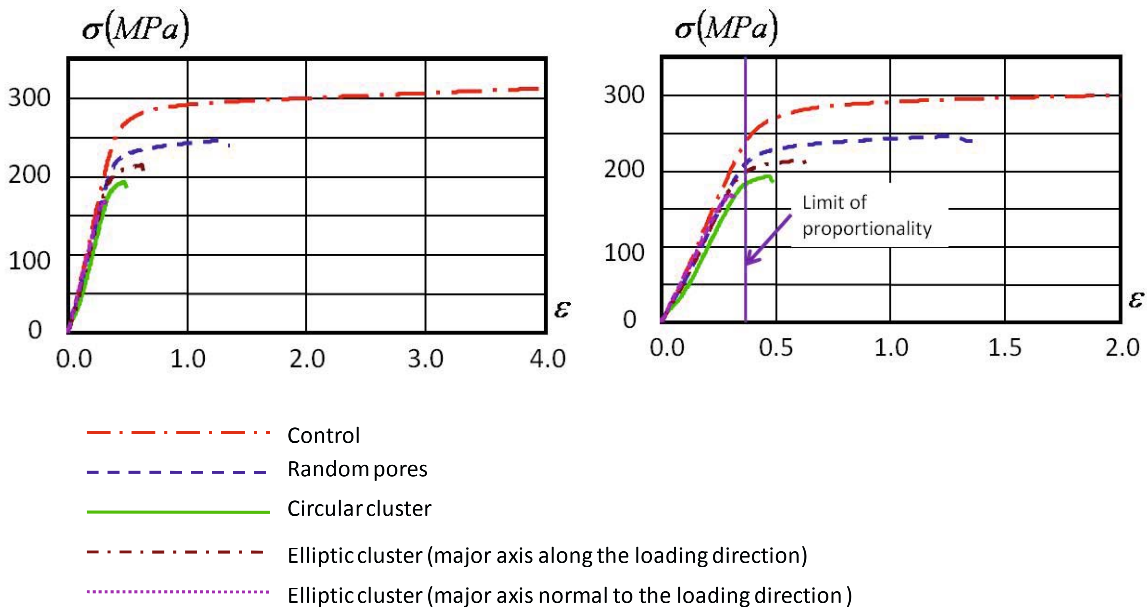

Figure 3. Stress-strain for specimens with various distribution of pores (averaged over ten patterns).

Remark. An interesting observation can be made from analysis of curves in Fig 3 regarding the onset of plastic yield. The yield starts at the about the same level of strain for both porous and dense materials (purple line in fig 3b), in agreement with numerical simulations of Zohdi and Kachanov (2005). The exact numbers are given in Table. In particular, it provides an experimental support for yield stress estimates of porous materials via elastic and conductive properties developed by Sevostianov and Kachanov (2001, 2006).

It is interesting to compare the experimental results with predictions of various micromechanical schemes for porosity 0.2. For this aim, we use non-interaction approximation, differential scheme and Mori-Tanaka scheme (see, for example, Markov, 2000 for details). According to these schemes, effective Young's modulus of a two-dimensional solid containing circular pores is expressed as follows.

- Non-interaction approximation (with prescribed stress)

$$E^* = \frac{E_0}{1+3p}; \tag{3.1}$$

- Differential scheme

$$E^* = E_0(1-p)^3; \tag{3.2}$$

- Mori-Tanaka scheme

$$E^* = \frac{E_0(1-p)}{1+2p}; \tag{3.3}$$

where p is the total porosity. The mentioned schemes are insensitive to the pore distributions. For $p = 0.2$, the mentioned three schemes yield the following values of E^*/E_0 : 0.625 (non-interaction approximation), 0.512 (differential scheme), and 0.571 (Mori-Tanaka scheme). The experimentally measured values vary between 0.534 and 0.569 that is in-between the values predicted by the differential scheme and by Mori-Tanaka's scheme.

In contrast with the Young's modulus, the fracture properties show strong dependence on the mutual positions of the pores. Cracks start to propagate at pores that were at closer distances to each other and to the edge of the specimen and grew to next the closest hole in the relatively horizontal plane and shown in Figure 4. Holes that were not along the crack but were near the high stress concentrated areas of the specimen (such as the clusters) deformed significantly and elongated with the uniaxial stress. It can be seen, the path of the other cracks forming by the deformation kinks in the metal as shown in Figure 4.

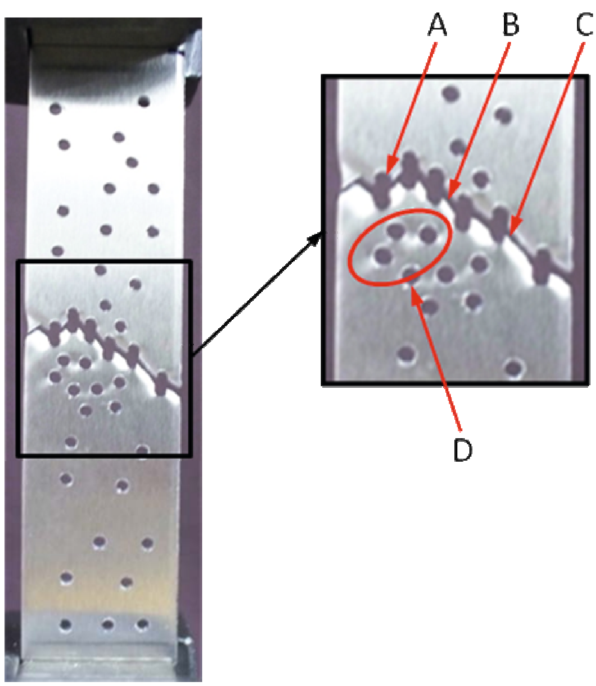

Figure 4: Crack propagation in the specimen: A. Crack initially started at the hole closest to the edge and closest to the cluster B. Shift in the direction of the crack to propagate to next closest hole C. All pores, no matter the positioning, cracked at the highest stress concentration point D. Kinks formed by stress high concentration points under loading

To quantify the effect of pore distribution on the fracture toughness of a specimen, we analyze the correlation between ultimate strength and minimum separation between the pores in the specimen. The results are presented in Figure 5. Using statistical data analysis it was shown there is a very strong positive correlation of 83% between the minimum pore separation and fracture stress. The relationship between the entities is relatively linear with an equation of

$$\sigma_u = 38.175\delta + 167.2 \tag{3.4}$$

with σ_u measured in MPa and δ - in mm.

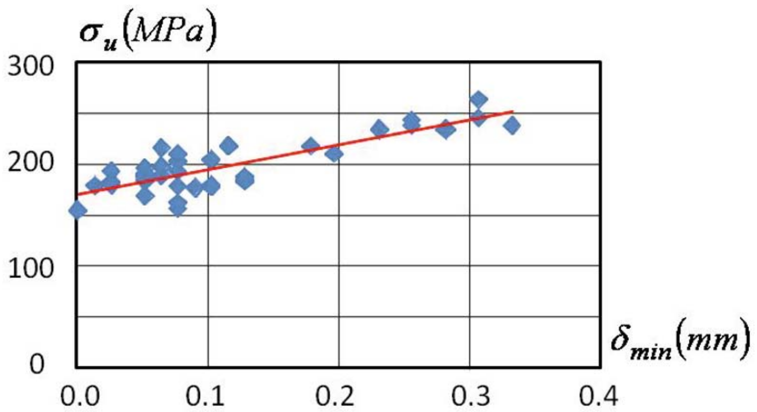

Figure 5. Dependence of the ultimate strength on minimum separation between the pores

4. Conclusions

We addressed the problem of the effect of pore distribution on mechanical properties of a porous material on the example of uniaxial tension of two dimensional Aluminum 6061-T6 specimens. It is shown that the Young's modulus is not affected by existence of distinguishable clusters and at the level of porosity up to 0.2, it can be calculated with good accuracy using differential scheme. In, contrast, fracture toughness of a porous material is very sensitive to the mutual positions of the pores. This dependence can be approximated by expression (3.4). Since the probability of the minimum separation is larger if the pores form a cluster, the fracture toughness of a specimen containing pore cluster is usually smaller than those of a specimen with randomly distributed pores. Fracture toughness also depends on cluster shape and orientation with respect to the direction of loading. As a side-result, we observed that plastic yield of a porous and virgin material starts at approximately same level of strain.

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