

# **SMT‑based verifcation of program changes through summary repair**

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# **Abstract**

This article provides an innovative approach for verifcation by model checking of programs that undergo continuous changes. To tackle the problem of repeating the entire model checking for each new version of the program, our approach verifes programs incrementally. It reuses computational history of the previous program version, namely function summaries. In particular, the summaries are over-approximations of the bounded program behaviors. Whenever reusing of summaries is not possible straight away, our algorithm repairs the summaries to maximize the chance of reusability of them for subsequent runs. We base our approach on satisfability modulo theories (SMT) to take full advantage of lightweight modeling approach and at the same time the ability to provide concise function summarization. Our approach leverages pre-computed function summaries in SMT to localize the checks of changed functions. Furthermore, to exploit the trade-of between precision and performance, our approach relies on the use of an SMT solver, not only for underlying reasoning, but also for program modeling and the adjustment of its precision. On the benchmark suite of primarily Linux device drivers versions, we demonstrate that our algorithm achieves an order of magnitude speedup compared to prior approaches.

**Keywords** Symbolic model checking · Incremental verifcation · SMT solving · Craig interpolation · Program changes

# **1 Introduction**

Modern software is developed by multitudes of developers from all over the world. As a result, the software undergoes frequent minor changes, e.g., bug fixes, introduction of new features, optimizations, refactoring, and so on. The problem of updating software is that this might break existing features—bugs might get introduced. The

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confidence of correctness can be increased by rigorous verification before a new revision of a piece of software is checked in. Model checking techniques [[1,](#page-29-0) [2](#page-29-1)] verify program fully automatically and exhaustively. However, most software verification approaches are not designed to support sequences of program versions, and they force each changed program to be verified from scratch which often makes re-verification computationally impractical.

This work addresses the problem of efficient analysis of a program after it undergoes changes. As a viable solution to this problem, incremental verifcation is a promising approach that aims to *reuse* the invested eforts between verifcation runs. We base our approach on symbolic verifcation techniques, and in particular bounded model checking (BMC)  $\lceil 3 \rceil$ . The previous work  $\lceil 4 \rceil$  has shown that the idea of extracting and reusing *function summaries* across program changes is useful in BMC when using the so called bit-blasting approach, with the direct use of a SAT solver. However, due to the known complexity of bit-precise encoding it sufers from scalability issues. Using SAT-based function summaries also creates summaries that can be signifcantly larger than the original formulas, and that are not human-readable, impeding their reuse and maintainability.

We present an incremental BMC approach that is based on Satisfability Modulo Theories (SMT) [[5](#page-29-4)]. Since SMT formulas provide a more natural and lightweight representation than purely propositional logic  $[6]$  $[6]$  $[6]$ , it allows for a more succinct summary representation in frst-order logic. We tackle scalability issues that arise due to verifcation of industrial-size program versions by (i) modeling the program with fragments of quantifer-free frst-order logic, in particular in Linear Real Arithmetic (LRA) and Equality with Uninterpreted Functions (EUF) which allows to leverage success of nowadays SMT solvers, and (ii) reusing and maintaining *SMT-based function summaries* across the closely-related programs.

Function summaries are obtained from Craig Interpolants [[7](#page-29-6)] which play the role of conserved function specifcations. For a code that recurs in several versions, function summaries have been proven successful as a means to capture the properties relevant for BMC in a form that avoids duplicate verifcation [[4](#page-29-3), [8](#page-29-7)[–13](#page-29-8)]. Function summaries constructed in SMT are more succinct and thus their easier reuse. In this article, we exploit an SMT-based family of summaries that condenses the relevant information from a previous verifcation run to localize and speed up the checks of new program versions. In this approach, the problem of determining whether a newly changed program still meets a safety property reduces to the problem of validating the family of summaries for the new program. As a result, in practice the verifcation stays often very local, resulting in signifcant run time improvements.

Our proposed *incremental BMC* solution aims to maintain and repair over-approximating summaries of all the program functions. Overall, our solution proceeds as follows. First, a program together with safety properties is modeled in SMT, and if properties hold, function summaries are computed. Then once a change arrives, it frst determines whether the old summaries for functions are still valid after the change. This validation phase is local to the change and tends to be computationally inexpensive since it considers only the changed function bodies, their old summaries, and possibly the summaries of the predecessors of the changed functions. If this local validation phase succeeds, the new program version is also safe. If local validation fails, the approach attempts to widen the scope of the search while still maintaining some locality, by propagating the validation check to the callers of the modifed functions. After each successful validation, any invalidated summaries become a candidate to be repaired and are made available for checking the next program version.

The key idea behind the *summary repair* approach is to circumvent the deletion of invalidated function summaries and instead attempt to adapt them to the changed

functions. Our solution repairs an over-approximating summary of the program function which is coarse enough to enable rapid check but strong enough to cover more changes in an incremental checking scenario. The repair is done via two strategies: the frst *weakens* the invalid summary formulas by removing the broken parts, and the second *strengthens* the weakened summary by recomputing the corresponding interpolant and adding missing parts. We keep the conjunction of newly repaired summaries for subsequent uses. The refnement procedure accompanies our incremental summary validation algorithm for dealing with spurious behaviors that might be introduced due to imprecise over-approximative summaries.

We implemented our SMT-based incremental verifcation algorithm with the new concept of summary repair in the UpProver tool. We advocate the necessity to offer various encoding options to the user. Therefore, in addition to the provided SMT-level light-weight modeling and the corresponding SMT-level summarizations supported by our incremental verifier, the tool allows adjusting the precision and efficiency with different levels of encodings. For this purpose UpProver enables the LRA and EUF theories (and in the future, more). This was not possible in the previous-generation tools based on bit blasting (e.g., in its predecessor eVolCheck [[4](#page-29-3), [12\]](#page-29-9)), and this distinguishes UpProver from them. Furthermore, our approach not only allows the reuse of summaries obtained from SMTbased interpolation, but also provides an innovative capability of repairing them automatically and using them in the subsequent verifcation runs.

*Improvement over previous own work.* The present work is an extension of [[14](#page-29-10)] published in a conference. We build upon and extend  $[14]$  $[14]$  in a number of ways: (i) we provide a unifed and richer description of the SMT-based verifcation framework for program revisions; (ii) we propose a new algorithm to repair previously computed summaries on-the-fy and to use them in the subsequent verifcation runs; (iii) we give an algorithm for constructing formulas in the presence of various substitution scenarios and refning the over-approximation that are not accurate enough; (iv) we provide a proof of correctness of the algorithm and then discusses how it can be instantiated to a concrete theory of SMT with diferent interpolation procedures; (v) a thorough evaluation of the proposed algorithm is carried out on industrial verifcation problems created from Linux kernel device drivers with several revisions.

*Structure of the paper.* Sect. [2](#page-2-0) provides the necessary background used in the theoretical development of the work. Section [3](#page-7-0) uses a concrete example of a two-version model checking problem to demonstrate how the approach can incrementally verify the second program by reusing pre-computed summaries of the frst program. Section [4](#page-9-0) frst presents the core algorithm for incremental verifcation of program revisions, then introduces the algorithm for summary repair, and lastly describes an improvement for efficiently building formulas and refning them on-the-fy. Section [5](#page-13-0) frst discusses the correctness of our proposed algorithm and then instantiates the algorithm with respect to a concrete theory. Section [6](#page-16-0) gives an overview of the architecture and implementation of our tool, UpProver. Section [7](#page-17-0) describes the experimental results and evaluation of our tool. We discuss related work in Sect. [8](#page-26-0) and fnally conclude in Sect. [9](#page-28-0).

# <span id="page-2-0"></span>**2 Background**

We first review symbolic program modeling and interpolation-based function summarization. Then we describe how to compute function summaries and how to apply them while encoding a program.

We work in the domain of Satisfiability Modulo Theories (SMT) [[5\]](#page-29-4) where satisfiability of formulas is determined with respect to some background theory  $\mathcal T$ . In this work, we restrict our interest to quantifier-free formulas. Since our goal is an efficient analysis of a program after a change, to leverage the success of nowadays SMT solvers we concentrate on two light-weight theories of SMT for program encodings: (i) the theory of equality with uninterpreted functions (EUF), and (ii) the theory of linear real arithmetic (LRA). However, our approach is generic and is not bound to specific theories.

Bounded Model Checking (BMC) [\[3\]](#page-29-2) is one of the widely-used bug-catching techniques that trades of completeness of the state-space exploration for fnding as many counterexamples as allowed by the predetermined time- and resource constraints. SMT-based BMC has been successfully applied in verifying standalone programs [[11](#page-29-11), [15](#page-29-12), [16\]](#page-29-13). Our SMT-based BMC technique aims at verifying diferent program versions and operates on a loop-free program created from an original by unrolling all loops and recursive calls up to a given number of unwinding steps.

We write loop-free programs as tuples  $P = (F, f_{\text{main}})$  where *F* represents the finite set of unique function calls, i.e., function invocation with a unique combination of a program location, a call stack, and a target function.  $f_{main} \in F$  denotes the call of the entry point of the program. Interchangeably *F* also corresponds to the set of functions in the call tree of the unrolled program. We use relations *child*  $\subseteq$  *F*  $\times$  *F* and *subtree*  $\subseteq$  *F*  $\times$  *F*, where *child* relates each function *f* to all the functions invoked by *f*, and *subtree* is a refexive transitive closure of *child*. Since each node has at most one parent, we write *parent* $(n_2)$  to refer to  $n_1$ if  $child(n_1, n_2)$  holds.

Classical BMC of software encodes an unwound program to a BMC formula [[17\]](#page-29-14). First, the unrolled program is encoded into the static single assignment form (SSA), from which a BMC formula is constructed. However the resulting formula is monolithic and all the function calls are inlined, thus it does not allow modular verifcation. Let  $\beta_f$  be the BMC encoding of the body of a function *f*, i.e., the logical formula obtained from the SSA form of the body of the function *f*. Note that  $\beta_f$  does not include inlining of called functions. A *partitioned BMC* (PBMC) formula [\[8](#page-29-7)] is constructed recursively as

<span id="page-3-0"></span>
$$
\text{InlineFormula}(f) \triangleq \beta_f \wedge \bigwedge_{h \in F: child(f,h)} \text{InlineFormula}(h) \tag{1}
$$

For each  $f \in F$ , the formula is built by conjoining the partition  $\beta_f$  and a separate partition for all nested calls. The PBMC formula  $\texttt{InlineFormula}(f_{main})$  conjoined with the negation of a safety property *errorfmain* is called a *safety query*. Typically *errorfmain* represents disjunction of the negations of each of the assertion in the program. A program is *safe* if the safety query is unsatisfable.

#### **2.1.1 Interpolation‑based function summarization**

Function summary is an over-approximation of the function behavior defned as a relation over its input and output variables. There are diferent ways to obtain a summary formula, and in this work we use the approach introduced in [\[8\]](#page-29-7) for extracting summary formulas using Craig interpolation. Craig interpolation [[7\]](#page-29-6) is widely used as a means of abstraction in symbolic model checking [\[18,](#page-29-15) [19](#page-29-16)]. Interpolants can be computed from a proof of unsatisfiability of the formula  $A \wedge B$ .

<span id="page-4-2"></span>**Defnition 1** (binary interpolation) Given an unsatisfable formula Φ partitioned into two disjoint formulas *A* and *B*, we call the pair (*A* ∣ *B*) a *binary interpolation instance*. An *interpolation algorithm Itp* is a procedure that maps an interpolation instance to a formula  $I = Itp(A | B)$  such that (i)  $A \Rightarrow I$ , (ii)  $I \wedge B \Rightarrow \bot$ , and (iii) *I* is defined over symbols appearing both in *A* and *B*.

Binary interpolation can be generalized so that the partitions of an unsatisfable formula form a tree structure. We in particular concentrate on *tree interpolants*, generalizations of binary interpolants, obtained from a single proof that guarantees the tree interpolation property as defned in the following:

<span id="page-4-1"></span>**Definition 2** (tree interpolation property.<sup>1</sup>) Let  $X_1 \wedge ... \wedge X_n \wedge Y \wedge Z$  be an unsatisfiable formula in first-order logic. Let  $I_{X_1}, \ldots, I_{X_n}$  and  $I_{X_1 \ldots X_n Y}$  be interpolants for interpolation instances  $(X_1 | X_2 \wedge ... \wedge X_n \wedge Y \wedge Z)$ , …,  $(X_n | X_1 \wedge ... \wedge X_{n-1} \wedge Y \wedge Z)$ , and  $(X_1 \wedge \ldots \wedge X_n \wedge Y \mid Z)$ , respectively. The tuple  $(I_{X_1}, \ldots, I_{X_n}, Y, I_{X_1 \ldots X_n Y})$  has the *tree interpolation property* iff  $I_{X_1} \wedge \ldots \wedge I_{X_n} \wedge Y \Rightarrow I_{X_1 \ldots X_n}$ *y*.

Note that an interpolation procedure that guarantees tree interpolation property according to Defnition [2](#page-4-1) also computes a tree interpolant for a tree interpolation problem as defined in [[21](#page-29-17)]. The properties of tree interpolant follow from Definition [2](#page-4-1) when for each node in the tree interpolation problem, the following partitioning of the formula is considered: the subtrees of the current node's children are the partitions  $X_1, \ldots, X_n$ , the label of the current node is the partition *Y*, and the rest of the formula forms the partition *Z*.

In the following, we show how interpolation can be used to extract over-approximation of the function behaviors after a successful verifcation run. By exploiting the proof of unsatisfability for the safety query we can extract *function summaries* [\[8](#page-29-7)] for each function call *f*.

<span id="page-4-0"></span><sup>1</sup> For example in [\[20](#page-29-18)] this is called the *weak tree-interpolation property.*

Algorithm 1: Function summarization in SMT-based BMC (Bootstrapping) [8]

**Input:** Program  $P = (F, f_{main})$  with function calls F and main function  $f_{main}$ ; theory  $\mathcal{T}$ . **Output:** Verification result: {SAFE, UNSAFE}, summary mappings  $\sigma_{\mathcal{T}}$  in a particular theory  $\mathcal{T}$ **Data:** PBMC formula  $\phi_{f_{main}}$ , resolution refutation:  $\pi$ 1  $\phi_{f_{main}}$  InlineFormula $(f_{main}) \wedge \neg \textit{error}_{f_{main}} // \textit{create}$  inlined formula 2  $\langle result, \pi, \epsilon \rangle \leftarrow \text{CheckSAT}(\phi_{f_{main}})$ ; // run SMT-solver a if  $result = SAT$  then 4 | return  $UNSAFE$ , ExtractCex $(\epsilon)$ ; 5 foreach  $f \in F$  do  $\sigma_{\mathcal{T}}(f) \leftarrow \texttt{GetInterpolant}(f, \pi);$  // extract summaries  $\bf{6}$ 7 return SAFE,  $\sigma_{\tau}$ ;

Algorithm 1 describes the method for constructing function summaries in BMC. At line 1 once the safety query in a certain theory  $T$  is created, it is sent to an SMT solver. If the result is unsatisfiable, i.e., the program is safe, the method  $GetInterpolant$  (line  $6$ ) computes an interpolant for each  $f \in F$  from the proof of unsatisfiability. Function summaries for a function *f* are constructed as interpolants as follows: The PBMC formula  $\Phi$  is divided into two parts  $\Phi_f^{subtree} \wedge \Phi_f^{rest}$ . First,  $\Phi_f^{subtree}$  corresponds to the partitions representing the function call *f* and its nested function calls:

<span id="page-5-1"></span><span id="page-5-0"></span>
$$
\Phi_f^{subtree} \triangleq \bigwedge_{h \in F:subtree(f,h)} \beta_h \tag{2}
$$

Second,  $\Phi_f^{rest}$  corresponds to the rest of the program including the negation of safety properties:

$$
\Phi_f^{rest} \triangleq \neg error_{f_{main}} \land \bigwedge_{h \in F : \neg subtree(f,h)} \beta_h \tag{3}
$$

Then for each *f*, formula [\(2](#page-5-0)) is considered as *A*-part and formula ([3\)](#page-5-1) as *B*-part. The GetInterpolant method generates an interpolant  $I_f$  for the interpolation instance (*A* | *B*), which acts as a summary for the function *f*. We map functions to their summaries encoded in the theory T with  $\sigma_T : F \to S$  such that  $\sigma_T(f) = I_f$ .

It is important to note that although the bootstrapping phase can take time, especially if the program being bootstrapped is large, it is a critical part of the approach as it can save significant resources in subsequent runs. If a program has only one function and no function calls, no function summary will be generated. This means that the algorithm will function similarly to a classical non-incremental BMC, and the efficiency of our algorithm may be limited in this scenario.

#### **2.2 Applying summaries in formula construction**

Construction of the formulas by inlining as in Eq. [\(1\)](#page-3-0) results in a monolithic formula where the whole function bodies are included. We describe the construction of PBMC formula in the presence of function summaries that can be substantially smaller compared to the inlining of the entire function bodies since functions summaries tend to be more compact.

Consider function *f* as a root of a subtree of a program. Suppose in the subtree of *f* there is a function *h* and its summary was already computed. Then the summary of *h* can be substituted for body of *h* while building the PBMC formula of *f*. This way, the PBMC formula  $\phi_f$  corresponding to the encoding of subtree of *f* can be considerably more succinct compared to the inlining of the entire function bodies in subtree of *f*.

Algorithm 2 creates a PBMC formula for a subtree rooted at *f*, i.e.,  $\phi_f$ . The algorithm initially accepts a *substitution scheme* for the function representations and uses it while constructing formula  $\phi_f$ .

**Defnition 3** A substitution scheme for function calls is a function *Sb* ∶ *F* → {*inline*,*sum*, *nondet*} determines how each function call should be handled.

The level of approximation for each function  $f \in F$  is determined as one of the following three cases: (i) *inline* when the entire *f* is required to be processed, (ii) *sum* when a precomputed summary substitutes *f*, and (iii) *nondet* when *f* is treated as a nondeterministic function. Since nondet abstracts away the function, it is equivalent to using a summary formula *true*.



We define three substitution schemes:  $Sb_{\text{inline}}$ :  $F \rightarrow \{ \text{inline} \}$  inlines all the function bodies.  $Sb_{eager}^N$ :  $F \to \{sum, intline\}$  inlines functions with invalid summaries accumulated in set *N* and otherwise employs summaries:

$$
Sb_{eager}^{N}(g) = \begin{cases} \text{inline, if } g \in N\\ \text{sum, otherwise} \end{cases}
$$
 (4)

Note that summary mapping  $\sigma_{\tau}$  is total and all functions initially have summaries. Finally,  $Sb<sub>lazy</sub><sup>N</sup>$ : *F* → {*sum*, *nondet*} treats functions with invalid summaries as nondeterministic calls and the rest as *sum*, as follows:

$$
Sb_{lazy}^{N}(g) = \begin{cases} \text{nondet}, & \text{if } g \in N \\ \text{sum}, & \text{otherwise} \end{cases} \tag{5}
$$

This results in a smaller initial PBMC formula and leaves the identifcation of the critical function calls to the refnement loop.

#### <span id="page-7-0"></span>**3 Motivating example**

In this section, we demonstrate summary reuse and summary weakening in our incremental verifcation approach. Consider two programs in Fig. [1](#page-8-0) as the base and changed program. We call them  $P_1$  $P_1$  (Fig. 1a) and  $P_2$  (Fig. [1b](#page-8-0)). Both versions consist of several functions out of which one function differs, namely max, highlighted with red. The function nondet represents a nondeterministic choice (e.g. user input) which is assumed to be in a certain range. The two assert statements capture the property of the program that should always hold after an execution of the program. Program  $P_1$  can be encoded as a LRA formula together with the negation of assertions.

Our approach first performs a bootstrapping verification for  $P_1$  preferably encoding and solving with a light theory of SMT like LRA. After successful verification of  $P_1$  using Algorithm 1, summaries of functions are created with respect to all properties, as shown in Fig. [1d](#page-8-0). These function summaries represent the relation between the inputs and outputs of each function, and are expressed using a formula that includes the return value of the function, denoted by the variable *ret* in Fig. [1](#page-8-0)d. Note that tool implements Algorithm 1 for computing summaries automatically. Summaries are stored for future usage. When it comes to verifying  $P_2$ , in order to have an efficient verification procedure, instead of performing full-verification again it is desirable to reuse the summaries of  $P_1$ . We process the changed functions of  $P_2$  and investigate if the previous summaries are good enough to over-approximate the changes.

*Summary weakening.* Let us denote  $\sigma_{\gamma}$ (max) as summary of max and  $\phi'_{\text{max}}$  as LRA encoding of function max in *P*<sub>2</sub>. Here the summary check  $\phi'_{\text{max}} \Rightarrow \sigma_{\text{T}}(\text{max})$  does not succeed. Since  $\sigma_{\tau}$ (max) is conjunctive, we can weaken the summary by dropping some conjuncts to increase the chance of being valid for the changed max. As shown in Fig. [1](#page-8-0)d  $\sigma<sub>7</sub>(max)$  is in conjunctive form:

$$
\sigma_{\mathcal{I}}(\max) := (ret \ge 2b) \land (ret \ge 2a). \tag{6}
$$

A possible weakened formula is  $\sigma^w_T(\max) := (ret \geq 2b)$ . By dropping a conjunct, the resulting formula is still a summary, but it is coarser than the previous one. Since the implication  $\phi'_{\text{max}} \Rightarrow \sigma^w_{\text{T}}(\text{max})$  is valid, it indicates that the weakened summary is coarse enough to capture the changed function  $\phi'_{\text{max}}$ . However, the validation check has to propagate towards

int D;  $//global$ int D: main  $\overline{1}$  $\overline{2}$  $int$  nondet () {  $int$  nondet() {  $\overline{3}$  $\overline{A}$ int r: int r: get assume  $(r>0 \& r<5);$  $assume(r>0 \& r<5);$  $\overline{5}$  $\mathcal{L}$  $return r;$  $return r;$  $\overline{\tau}$ fur  $\mathcal{R}$ int  $add(int x)$  $\overline{9}$ int  $add(int x)$ (max add)  $10$ return  $x+10$ ; return  $x+10$ ;  $\mathcal{V}$ ι  $11$ 12 int max $(int a, int b)$ { int max $(int a, int b)$ { 13  $(c)$  Call tree if(a >= b) return  $2^*a$ ; // if(a >= b) return  $2^*a$ ; 14  $1.5$  $return 2*b;$  $return 2*b;$  $\mathcal{E}$ 16 17 int fun(int  $a$ , int  $b$ ){ int fun(int  $a$ , int  $b$ ) { 18 Summary formula  $19$ int out =  $max(a, b)$ ; int out =  $max(a, b)$ ;  $D = add(a);$  $D = add(a);$  $20$  $assert (D - a >= 1);$  $\text{assert}(\hat{D} - a \geq 1);$  $_{21}$  $(ret \geq 2b) \wedge$  $return out + D;$  $return out + D;$  $\sigma_{\mathcal{T}}(\max)$  $22$  $(ret \geq 2a)$  $\mathcal{E}$ 23  $^{24}$  $int$  get  $(int a, int b)$  {  $int$  get  $(int a, int b)$  { 25  $\sigma_{\mathcal{T}}(\texttt{add})$  $ret \geq x + 10$  $26$  $int c = fun(a, b);$  $int c = fun(a, b);$ if  $(a)=c$  &&  $b>=c$ ) if  $(a)=c$  &  $b>=c$ ) 27  $\sigma_{\mathcal{T}}(\texttt{fun})$   $(\textit{ret} \geq 10 + a + 2b)$ return  $2 * a$ ; return  $2 * a$ ; 28  $return 2 * c;$ return  $2 * c;$ 29  $\mathcal{E}$ j  $\wedge (ret \geq 10 + 3a)$ 30 31 void main() {  $void$  main(){ 32  $(a > -5) \Rightarrow$  $int a = nondet();$ int  $a = n$  ondet (); 33 int  $b = n$ ondet(); int  $b = n$ ondet(); 34  $\sigma_{\mathcal{T}}(\texttt{get})$  $((ret > 2a) \wedge$  $D = nondet()$ ; 35  $D = \text{nondet}(x)$  $int w = get(a, b);$ int  $w = get(a, b)$ ; 36  $(ret \geq 2a + 4b + 20))$ 37  $assert(w == a+b);$  $assert(w == a+b);$  $\mathcal{F}$ 38 (d) Function summaries (a) Base program (b) Changed program of base program

<span id="page-8-0"></span>**Fig. 1** Two versions of a C program with call tree and function summaries

the caller, i.e., fun to make sure if the coarse summary is a valid over-approximation in the subtree rooted at fun.

*Summary reuse.* During the validation of fun's summary, i.e.,  $\sigma_{\tau}(\text{fun})$ , if there are summaries available in its subtree they are used. Since function add is not changed, its summary  $\sigma_{\tau}$ (add) is reused straightforwardly as well as  $\sigma_T^w$ (max). However, the summary check for function fun fails because of the change in function max, i.e.,  $\sigma_{\gamma}(\text{add}) \wedge \sigma_{\gamma}^{w}(\text{max}) \wedge \phi_{fun} \Rightarrow \sigma_{\gamma}(\text{fun})$  or (*D*≥*a* + 10)∧(*out*≥2*b*)∧(*ret* = *out* + *D*)⇒(*ret*≥10 + *a* + 2*b*)∧(*ret*≥10 + 3*a*) does not succeed. The summary of fun should be weakened to ( $ret \ge 10 + a + 2b$ ), and the summary check should be performed for function get, where the check succeeds. Thus the check does not continue further to the root of the program. This means that the changed program is safe too.

Since there are no further changed functions unprocessed, the incremental checking terminates, and we simply could conclude that  $P<sub>2</sub>$  is safe too. The weakened summaries  $(\sigma^w_\tau(\max))$  and  $\sigma^w_\tau(\text{fun})$  are stored instead of the original summaries and will be reused when a new program version arrives.

# <span id="page-9-0"></span>**4 Incremental verifcation of program changes**

This section presents our solution to the problem of efficient verification of a program after a change. The problem of incremental verifcation is stated as follows: given the frst program  $P_1$  with the safety properties included in the code, a summary mapping (certificate of correctness)  $\sigma_1$  of  $P_1$ , and the changed program  $P_2$ , adapt  $\sigma_1$  to become a summary mapping of  $\sigma_2$  for  $P_2$ , or show that  $P_2$  does not admit a correctness certificate (i.e., has a counterexample).

Note that we present the incremental verifcation algorithm instantiated in the context of BMC and SMT. However, the algorithm is more general and can be applied in other approaches relying on over-approximative function summaries.

# **4.1 Overview of the algorithm**

Algorithm 3: UPPROVER: Summary validation and repair in SMT **Input:** Program  $P = (F, f_{main})$  with function calls F; the set of changed functions  $\Delta \subseteq F$ ; total summary mapping  $\sigma_{\mathcal{T}}: F \to S$  that maps functions to summaries; theory  $\mathcal{T}$ ; **Output:**  $\langle\textit{Safe}, \sigma_{\tau}\rangle$  or  $\langle\textit{Unsafe}, \textit{CE}\rangle$ **Data:**  $N \subseteq F$  set of functions with invalidated summaries  $1 \quad WL \leftarrow \Delta;$  $2 N \leftarrow \varnothing$ ; 3 while  $WL \neq \emptyset$  do pick  $f \in WL$ , s.t.  $\forall h \in WL : \neg subtree(f, h);$  $\overline{\mathbf{4}}$  $WL \leftarrow WL \setminus \{f\};$ 5 if  $f \notin N$  then // if has summary? ß  $\phi_f \leftarrow \texttt{CreateFormula}(P, \sigma_{\mathcal{T}}, \mathcal{T}, f, Sb_{inline});$  // create formula  $\overline{7}$  $\langle result, \pi, \epsilon \rangle \leftarrow \text{CheckSAT}(\phi_f \wedge \neg \sigma_T(f));$ // run SMT solver 8 if result =  $SAT$  then 9 if  $f \neq f_{main}$  then 10  $\mid WL \leftarrow WL \cup \{parent(f)\};$  $11$ else  $12$ | return  $\langle \textit{Unsafe}, \texttt{ExtractCex}(\epsilon) \rangle;$ 13  $\mid \langle \sigma_{\mathcal{T}}(f), N \rangle \leftarrow$  WeakenSummary $(P, \sigma_{\mathcal{T}}, f, N);$ 14 else //  $res = UNSAT$ 15 for all  $h \in F$  s.t. subtree(f, h) do 16  $\sigma_{\mathcal{T}}(h) \leftarrow \sigma_{\mathcal{T}}(h) \wedge \texttt{GetInterpolant}(\pi,h);$  $17$  $N \leftarrow N \setminus \{h\};$ 18 19  $return \ \langle Safe, \sigma_{\mathcal{T}} \rangle$ ;// new version safe, return updated summary

In this section, we frst describe the main points of Algorithm 3. Then we describe the important subroutines of the algorithm; (i) an improvement of the algorithm with the summary weakening in Sec. [4.2,](#page-11-0) and (ii) summary refinement in Sec. [4.3](#page-12-0).

Our approach for incremental verification considers two versions of the program,  $P_1$  and  $P_2$ , and the function summaries of  $P_1$ . If  $P_1$  or its function summaries are not available (e.g., at the initial stage), a bootstrapping run (Algorithm 1) is required to verify the whole program  $P<sub>2</sub>$  to generate the summaries, which are then maintained during the subsequent verifcation runs. We assume that the set of safety properties remains the same throughout the incremental verifcation.

As input, the pseudocode takes the new program  $(P_2)$ , the set of functions that have been identified as changed  $\Delta$ , the theory T, and the summaries as a total mapping  $\sigma<sub>T</sub>$  from functions  $F$  to the set of all summaries  $S$ . Initially in case no summary exists for  $f$  (e.g., newly introduced in  $P_2$ ) its summary is initialized as *false*. By initializing the summary of *f* as *false*, we are being explicit about the fact that we do not yet have any summary. As output, it reports either *Unsafe* with a concrete counterexample *CE*, or *Safe* with a possibly updated total mapping representing the summaries.

The algorithm maintains a worklist *WL* of function calls that need to be checked against the pre-computed summaries. Initially, *WL* is populated by a set of functions with code changes, namely Δ (line 1). Then the algorithm repeatedly chooses *f* from *WL* so that no function in the subtree of *f* exists in *WL* (line 4). Then it removes a function *f* from *WL* and attempts to check the validity of the corresponding summary in the new version. Note that this bottom-up traversal of the call tree ensures that summaries in the subtree of *f* have been already checked (shown either valid or invalid). The algorithm also maintains the set *N* to store the set of functions with invalid summaries and aims at repairing all of the summaries that were identifed as invalid.

The if-condition at line 6 checks whether the summary of *f* is not invalid (i.e., has a summary). If so, CreateFormula constructs the formula  $\phi_f$  that encodes the subtree of *f*. Note that here the substitution scenario *inline* is used where all function calls in the subtree of  $f$  are naively inlined. Later in Sect.  $4.3$  we introduce a more efficient way for constructing the formula.

The validation check of pre-computed summaries occurs in 8. The validity of implication  $\phi_f \Rightarrow \sigma_{\tau}(f)$  is equivalent to the unsatisfiability of the negation of the formula,  $\phi_f \wedge \neg \sigma_T(f)$ . This local formula is sent to an SMT solver for deciding its satisfiability. Performing the local check determines whether the summary is still a valid over-approximation of the new function's behavior. If the *result* is *UNSAT*, the validation is successful and the summary covers the changed function. Here, the algorithm obtains a proof of unsatisfability  $\pi$  which is used to compute new summaries to update the invalid or missing summaries (line 17). This is called *strengthening* procedure in our approach. If *result* is *SAT*, the validation of the current summary fails for the changed function (line 9). In this case, either the check is propagated to the function caller towards the root of the call tree (line 11) or a real error is identifed (line 13). In the latter case, since the validation fails for the root *f<sub>m</sub>ain* of the call tree, the algorithm extracts and reports a concrete counterexample from the result of the SMT query (line 13).

Note that if a function *f* is introduced in  $P_2$ , the caller of *f* is marked as changed by our difference-checker. In such a scenario, since summary of *f* is trivially *false*, the validation check immediately fails, *f* gets added to *N*, and the algorithm continues to check the caller. A successful validation of some ancestor of *f* with inlined *f* generates a summary for *f* (line 17).

Whenever a summary is identified as invalid (line 9), the sub-routine WeakenSummary (line 14) is called to make the summary coarser which is explained in the next section.

#### <span id="page-11-0"></span>**4.2 Summary weakening**

Algorithm 4: WeakenSummary  $(P, \sigma_{\mathcal{T}}, \mathcal{T}, f, N)$ **Input:** Program  $P = (F, f_{main})$  with function calls F; summary mapping  $\sigma_{\mathcal{T}} : F \to S$ ; Theory T; function f whose summary is being weakened, set of functions  $N$  with invalid summary **Output:** Updated summary formula of  $f \sigma_T(f)$ , updated invalid set N 1 For a formula  $\sigma_{\mathcal{T}}(f) = \mathbb{S}_1 \wedge ... \wedge \mathbb{S}_n$ , consider a set  $Cands = {\mathbb{S}_1, ..., \mathbb{S}_n}$ ;  $\mathbf{2} \ \ \phi_f \leftarrow \texttt{CreateFormula}(P, \sigma_\mathcal{T}, \mathcal{T}, f, Sb^N_{easer});$ // Create formula with substitution scheme (4) s while  $Cands \neq \emptyset$  do  $\big|$   $\langle result, \text{--}, \epsilon \rangle \leftarrow \text{CheckSAT}(\phi_f \wedge \text{--} \bigwedge_{cnj \in \text{Cands}} cnj);$  // validity  $\overline{\mathbf{4}}$ if  $result = UNSAT$  then break;  $\overline{5}$ for cand  $\in$  Cands do  $\ddot{6}$  $\overline{7}$ | if  $\epsilon \models \neg \text{cand then}$  $\left| \begin{array}{l} \lfloor\mathit{Cands} \leftarrow \mathit{Cands} \setminus \{\mathit{cand}\}; \end{array} \right|$  $\mathbf{8}$ **9**  $\sigma_{\mathcal{T}}(f) \leftarrow \bigwedge_{cnj \in \text{Cands}} cnj;$  // store valid summary conjuncts 10 if  $\sigma_T(f) = true$  then  $N \leftarrow N \cup \{f\};$ 11 return  $\sigma_{\mathcal{T}}(f)$ , N;

The previous algorithms of incremental verifcation in [[4](#page-29-3), [14](#page-29-10)] check summaries one-by-one and whenever the validation fails, the invalidated summaries are removed straight away, thus the chance to reuse summaries becomes low. In this section instead of removing the invalidated summaries right away, our proposed algorithm attempts to compute coarser over-approximation (i.e. weaker summaries) by dropping some conjuncts, thus maximizing their usability. The key insight behind the algorithm is identifying which parts of the summary break the validity of implication, removing them from the set, and repeating the validation check.

The number of top-level conjuncts in a summary formula is a measure of generalizability of the interpolant. In some applications (see, e.g.,  $[22, 23]$  $[22, 23]$  $[22, 23]$  $[22, 23]$ ) it is useful to further abstract an overapproximation. The idea was inspired by Houdini algorithm [\[24\]](#page-29-21). Weakening a summary formula is performed by dropping the conjuncts that break the validity of the summary. Note that Houdini is only meaningful for the summaries with top-level structure in conjunctive form.

In Algorithm 3 once the summary turned out to be invalid (line 9) the sub-routine WeakenSummary (i.e., Algorithm 4) is called to weaken the summaries. Algorithm 4 shows a simple implementation of an iterative check-and-refute cycle that iterates until the validation check of the subset of summary conjuncts succeeds. Initially, summary conjuncts are stored in the set *Cands* and as the algorithm proceeds, the conjuncts are removed if proven to be invalid.

In line 2 the PBMC formula  $\phi_f$  for the function *f* is constructed. In this phase  $Sb_{eager}^N$ is used as the substitution scheme, i.e., whenever a function summary is available in the subtree of *f*, the summary is used as a substitute for the function body. Then in line 4 the containment of the resulting formula  $\phi_f$  in the summary candidate is checked by the SMT solver. Once the solver generates a counterexample  $\epsilon$ , it is used to prune the summary conjuncts that break the containment check. This iterates until the solver returns *UNSAT*. In

the end, the remaining subset of summary candidates would form a new valid summary for *f*. In case no conjuncts are left, the function summary is assigned to the weakest possible summary, namely *true* (line 10) and it is added to the set *N* that contains functions whose summaries were not valid anymore in  $P_2$ .

Once the weakened summary is obtained from Algorithm 4, in Algorithm 3 the check always propagates to the caller (as the parent is already marked there) to make sure the new weakened summary is suitable in the subtree rooted at the caller. It can be the case that the weakened summary does not capture the whole relevant functionality of the changed function and needs more conjuncts (strengthening) that can be obtained during the validation check of the caller and interpolation in line 17 of Algorithm 3. Note that in this case even though the check propagates to the caller, it would be still benefcial in the sense that the actual encoding of the function body is substituted with the weakened summary, thus the overall check will be less expensive and still maintains some locality.

#### <span id="page-12-0"></span>**4.3 Summary refnement**



In Algorithm 3 line 7 creates the PBMC formula  $\phi_f$  in a way that all the nested functions in the subtree rooted at *f* are inlined. In other words, the initial substitution scheme was set to  $Sb<sub>inline</sub><sup>N</sup>$ . To further speed up the incremental check while constructing the formula  $\phi_f$ , precomputed summaries can be used to abstract away the function calls in its subtree.

In this section we present an algorithm for creating PBMC formulas in a more efficient way, then we present our solution for refning the abstract summaries on demand. The pseudocode in this section can substitute lines 7 and 8 in Algorithm 3.

Algorithm 5 consists of two key points: (i) while constructing the PBMC formula  $\phi_f$ , whenever summaries are available in the subtree of *f*, they substitute the actual body of the function calls in the subtree, (ii) while checking the validity of summaries, the infeasible behaviors that are detected during analysis of abstract summaries are refned by an iterative refnement procedure.

The initial over-approximation in Algorithm 5 is set to  $Sb_{\text{lazy}}^N$  where it sets the precision for function calls in the *lazy* style. Then the PBMC formula  $\phi_f$  is created based on Algorithm 2 with  $Sb_{lazy}^N$ . The resulting  $\phi_f$  is expected to be substantially smaller compared to the encoding whose substitution scheme was initially set to *Sb<sub>inline</sub>*.

Then the resulting  $\phi_f$  is checked for the validity whether its pre-computed summary contains the formula, i.e.,  $\phi_f \Rightarrow \sigma_T(f)$  (line 3). If the resulting formula is satisfiable, it can be either a real or a spurious violation since over-approximative function summaries were used to substitute some of the nested function calls. This can be discovered by analyzing the presence of summaries along an error trace, determined by a satisfying assignment  $\epsilon$ returned by a solver and by dependency analysis.

Based on the satisfying assignment the algorithm identifes the set of the summaries used along the counter-example and stores them in *RefCandid* (line 6). The algorithm applies dependency analysis that restricts *RefCandid* set to those possibly afecting the validity. Then every over-approximations (summary or nondet) in the *RefCandid* set is marked as *inline* in the next iteration (line 11). If the set is empty, the check fails and the summary is shown invalid. This refnement loop repeats until the validity of the summary is determined.

## <span id="page-13-0"></span>**5 Correctness of the algorithm**

This section discusses the correctness of the SMT-based incremental verifcation algorithm, i.e., given *k* unrolling steps, the algorithm always terminates with the correct answer with respect to *k*. Notice that in this article, program safety is considered with respect to the pre-determined unwinding bound *k*. In the remainder of this section, we assume the same *k* for both old and new programs. In case the user increases the bound for a specifc loop, the corresponding function needs to be validated as if changed.

#### **5.1 Correctness of the algorithm**

The correctness of Algorithm 3 is stated in the following theorem:

**Theorem 1** *Assume the interpolation algorithm for* T *guarantees tree interpolation property*. *When the Algorithm*  3 *returns safe*, *then the entire program is safe*, *i*.*e*.,  $error_{f_{main}} \wedge \Phi_{f_{main}}^{subtree} \Rightarrow \bot.$ 

*Proof* Let  $f_m$ *ain* be the entry function,  $\sigma_{\tau}(f)$  be the summaries, *f* range over the function calls satisfying *subtree*( $f_m$ *ain*,  $f$ ), and  $c_1$ , ...,  $c_n$  be the function calls in the (possibly empty) set of functions called by *f*. We frst show that the properties

<span id="page-14-1"></span><span id="page-14-0"></span>
$$
error_{f_{main}} \land \sigma_T(f_{m}ain) \Rightarrow \bot, \text{ and} \tag{7}
$$

$$
\sigma_T(c_1) \wedge \ldots \wedge \sigma_T(c_n) \wedge \beta_f \Rightarrow \sigma_T(f) \tag{8}
$$

are strong enough to prove that the entire program is safe, and then show that they hold in the algorithm both after successful bootstrapping and after a successful incremental verifcation run on a set of changes  $\Delta$ .

*Safety from properties* ([7](#page-14-0)) *and* ([8](#page-14-1)). After rewriting property ([7\)](#page-14-0) into  $\sigma_T(f_main) \Rightarrow (error_{f_{main}} \Rightarrow \bot)$ , logical transitivity and iterative application of property ([8\)](#page-14-1) to substitute all interpolants on the right hand side of property ([8](#page-14-1)) yields the inlined formula in the claim  $error_{f_{main}} \wedge \Phi_{f_{main}}^{subtree} \Rightarrow \bot$ .

*Bootstrapping phase.* We show that property [\(7](#page-14-0)) holds over the program call tree annotated by computed interpolants whenever bootstrapping verifcation in Algorithm 1 terminates. Recall that the summaries are generated only when the program is safe with respect to the property, i.e.,  $error_{f_{main}} \wedge \Phi_{f_{main}}^{subtree} \Rightarrow \bot$ . Therefore, by definition of interpolation in Definition [1](#page-4-2) (property (ii)),  $error_{f_{main}} \wedge I_{f_{main}}$  is unsatisfiable, i.e., property [\(7\)](#page-14-0) holds. Property [\(8\)](#page-14-1) follows from our assumption that the interpolation algorithm guarantees the tree interpolation property. This can be seen by choosing the following partitions in Defni-tion [2:](#page-4-1)  $X_i \equiv \Phi_{c_i}^{subtree}$  for  $i \in 1 \dots n, Y \equiv \beta_f$ , and  $Z \equiv \Phi_f^{rest}$ .

*Incremental phase.* Assume that properties [\(7](#page-14-0)) and ([8\)](#page-14-1) hold before the changes in  $\Delta$ are introduced. We show that if Algorithm 3 running on a set of changes  $\Delta$  successfully returns *Safe*, both properties are maintained, from which the claim in the theorem follows. If Algorithm 3 successfully terminates, then each function call  $c$  obtains an updated summary  $\sigma_{\tau}(c)$  (line 17) when some of its predecessor f passed the summary validity check (line 15). Otherwise, the check propagates towards the root of the call tree and eventually may lead to an *UNSAFE* result. Thus, it suffices to show that the recomputed or repaired interpolants satisfy property [\(8](#page-14-1)). For this purpose, we again rely on the assumption that the interpolation algorithm guarantees the tree interpolation property. When constructing the formula of a function, Algorithm 2 uses all valid summaries in the subtree of the function. This is sound as we know from property (i) of Definition [1](#page-4-2) that  $\sigma_T(c_i) \Rightarrow I_{X_i}$  where  $I_{X_i}$  is an interpolant obtained from the proof of unsatisfability corresponding to the successful validity check of the predecessor of *ci* .

In case some change in  $\Delta$  is not contained in pre-computed summary  $\sigma_T(f)$  but the algorithm introduced a weakened summary  $\sigma^w_{\mathcal{T}}(f)$  that contains the change, the algorithm still propagates to the caller function (line 11) to determine whether property ([8](#page-14-1)) holds. In case  $\sigma_T^w(f)$  is not precise enough in the subtree of the caller, the algorithm proceeds with the refnement of the weak summaries. Once the refned check succeeds, the proof of unsatisfiability is used (through interpolation) to strengthen  $\sigma^w_{\mathcal{T}}(f)$  (line 17). Technically, the weakened summary and the recomputed summary are conjoined to form a new summary  $\sigma_T^{inp}(f) \wedge \sigma_T^w(f)$ . Again relying on the assumption that the interpolation algorithm guarantees the tree interpolation property, the recomputed and repaired interpolants satisfy property  $(8)$  $(8)$ .

#### **5.2 Interpolation algorithms in a concrete theory**

We instantiate our generic SMT-based incremental verifcation approach to certain theories of SMT. In particular, we focus on interpolation algorithms that are the low-level primitives in our approach and discuss the requirements they shall fulfll. The theories of our interest are Linear Real Arithmetic (LRA) and Equality Logic and Uninterpreted Functions (EUF). Having diferent theories and interpolation algorithms is of great practical interest since the choice of a good interpolation algorithm may well determine whether an application terminates quickly or diverges.

In the theory of linear arithmetic over the reals, LRA, there are several efficient proofbased interpolation algorithms proposed in the literature so that the resulting interpolants can difer in ways that have practical importance in their use in incremental verifcation. For the theory of LRA, many SMT solvers produce interpolants using application of the Farkas lemma [[25](#page-29-22)]. The most widely used approach computes weighted sum defned by Farkas coefficients of all inequalities appearing in A part of  $(A | B)$  [[26](#page-29-23)]. The interpolant computed in this way is always a *single* inequality. We call this approach Farkas interpolation procedure and denote it as *Itp<sup>F</sup>*.

Recently [\[27\]](#page-29-24) introduced a new algorithm called decomposing Farkas interpolation procedure which is able to compute interpolants in linear arithmetic in the form of a *conjunction of inequalities*. The algorithm is an extension of  $Itp^F$ ; it uses techniques from linear algebra to identify and separate independent components from the interpolant structure. We denote the decomposing interpolation procedure as  $Itp^D$ . Intuitively,  $Itp^D$ works as follows: Instead of using the whole weighted sum of *A*, it tries to *decompose* the vector of weights (Farkas coefficients) into several vectors. This effectively decomposes the single sum into several sub-sums. If each of the sub-sum still eliminates all *A*-local variables, the resulting inequalities can be *conjoined* together to yield a valid interpolant.

Since  $Itp<sup>D</sup>$  is able to produce interpolants in the form of a conjunction of inequalities, it provides the opportunity to make more efective use of summaries in our incremental verifcation algorithm. Hence, Algorithm 4 can beneft from computing coarser overapproximation (i.e. weaker interpolants) by dropping conjuncts. In the later sections, we will experimentally verify the usefulness of the decomposition scheme by comparing two LRA interpolation algorithms.

As for the theory of EUF, we use the EUF Interpolation algorithm in [\[28\]](#page-29-25) that relies on a congruence [[26](#page-29-23)] graph data structure constructed while solving an EUF problem. EUF interpolation algorithm combines propositional and EUF interpolation which is useful for a model checking setting and some of the conjuncts in EUF interpolants are coming from the propositional structure.

For the propositional part, we use the well-known Pudlák's interpolation algorithm [[29](#page-30-0)], which is more suitable for function summaries than McMillan's, since it constructs weaker interpolants and can capture more changes in incremental verifcation. Note that internally Craig interpolation in propositional logic and theory are combined.

Tree interpolants are used in our incremental algorithm for determining the satisfability of a frst-order logic formula that reuses summaries generated after the unsatisfability of a slightly diferent formula is determined. Among the widely used family of interpolation algorithms for LRA, EUF, and PROP we rely on the ones that can guarantee the tree interpolation property, and thus are suitable for the application in incremental verifcation of program versions.

The following two lemmas state which LRA interpolation algorithms can guarantee tree interpolation property. The proofs can be found in [[20](#page-29-18)] where we discussed under which conditions LRA interpolation procedures guarantee tree interpolation property.

**Lemma 1** LRA *interpolants computed by Farkas interpolation algorithm have tree interpolation property*.

**Lemma 2** LRA *interpolants computed by decomposing Farkas interpolation algorithm with gradual decomposition defned in* [\[20\]](#page-29-18) *have tree interpolation property*.

The following lemma considers the tree interpolation property of interpolants generated from the same resolution proof in EUF, proven in [[30](#page-30-1)].

**Lemma 3** EUF *interpolation algorithm* [[28](#page-29-25)] *guarantees tree interpolation property*.

In a purely propositional setting, we rely on the results from [\[12,](#page-29-9) [31\]](#page-30-2) which proves the correctness of the propositional tree interpolation algorithms, stated in the following lemma.

**Lemma 4** *Propositional interpolation algorithm introduced by Pudlák* [\[29\]](#page-30-0) *guarantees tree interpolation property*.

The proposed solution relies on the above lemmas for the correctness of the incremental verifcation algorithm in the context of bounded model checking. However, the correctness of the algorithm is not solely restricted to the preservation of the tree interpolation property *by construction*. Instead the correctness of the algorithm can be preserved by checking for the tree interpolation property *on-the-fy*. This means that the algorithm can still produce a correct output even if the tree interpolation property is passed by performing real-time checks in verifcation run.

It is worth mentioning that our incremental algorithm with theory  $\mathcal T$  gives the same result as the verification from scratch used in the bootstrapping with the theory  $T$ . Although automatically identifying a proper level of encoding is non-trivial (and not a subject of this paper), our approach at least allows various encoding options to the user. In case the algorithm is instantiated with less-precise theory (e.g., EUF), if a bug is reported, it might be due to the abstract theory usage, and it is recommended to repeat the verifcation with a more precise theory (and accordingly, more precise summaries).

## <span id="page-16-0"></span>**6 Tool architecture and implementation**

This section describes the implementation of our algorithms in the UpProver tool which is a bounded model checker written in C++. UpProver concentrates on incremental verifcation of program versions written in C. After each successful verifcation run, it maintains a database of function summaries to store its outputs, which become available as inputs for verifcation of each subsequent program version. For bootstrapping verifcation UpProver uses HiFrog standalone bounded model checker [\[11\]](#page-29-11). For satisfability checks and interpolation UpProver uses SMT solver OpenSMT. For pre-processing as a front-end, UpProver uses the framework from Cprover  $5.11<sup>2</sup>$  $5.11<sup>2</sup>$  $5.11<sup>2</sup>$  to symbolic encoding of C by transforming C program to a monolithic unrolled BMC representation that we use as a basis for producing the fnal partitioned logical formula.

The architecture of UpProver tool is depicted in Fig. [2.](#page-18-0) UpProver implements proposed algorithms by maintaining three levels of precision—LRA, EUF, and purely propositional logic (PROP)—to check the validity of pre-computed summaries. The rest of this section describes UpProver's key components in more detail.

*Diference analyzer*: UpProver performs source code diferencing at the level of SSA forms for both the old and the new program to identify a set of functions with code changes. It annotates the lines of code changed between  $P_1$  and  $P_2$ . This defines the scope of summary validations. The user may choose an inexpensive syntax-level diference or a more expensive and precise semantic-level diference that compares programs after some normalization and translation to an intermediate representation [\[4\]](#page-29-3). The functions that have been identified as changed are stored in set  $\Delta$  in Algorithm 3.

*Call tree traversal*: The call tree traversal guides the check of pre-computed summaries for the modifed functions in bottom-up order. It exploits the SMT solver to perform summary validation. When necessary it performs an upwards refnement to identify parent functions to be rechecked using SMT solver or performs summary refnement to refne the imprecise summaries in the subtree.

*Summary repair:* Summaries of  $P_1$  (of the selected level of precision) are taken as input and used in the incremental summary validation when necessary. The tool iteratively checks if the summaries are valid for  $P_2$  and repairs them on demand, possibly by iterative weakening and then strengthening using interpolation over the refned summaries.

*SMT solving and interpolation engine*: For checking BMC queries and computing interpolants, UpProver interacts with the SMT solver OpenSMT [\[32\]](#page-30-3). The solver produces a quantifer-free frst-order interpolant as a combination of interpolants from resolution refutations [[33](#page-30-4)], proofs obtained from a run of a congruence closure algorithm in EUF [[28](#page-29-25)], Farkas coefficients obtained from the Simplex algorithm in LRA [\[27\]](#page-29-24), and Decomposed Farkas interpolation in LRA [[27](#page-29-24)].

*Summary storage*: UpProver takes summaries  $\sigma_1$  of  $P_1$  as input and outputs summaries  $\sigma_2$  of  $P_2$ . The user defines the precision of  $\sigma_1$ , and it uniquely determines the precision of  $\sigma_2$ . In the best-case scenario, the tool validates  $\sigma_1$  and copies it to  $\sigma_2$ . When some of the summaries require repair, the tool produces new interpolants from the successful validity checks of the parent functions and stores them as the corresponding summaries in  $\sigma_2$  (while all other summaries are again copied from  $\sigma_1$ ). No summaries are generated when the tool returns *Unsafe*.

### <span id="page-17-0"></span>**7 Experimental evaluation**

To evaluate our algorithm, we aim to answer the following research questions:

- **RQ 1** Is the use of SMT instead of SAT in incremental verification efficient for realworld programs?
- **RQ 2** Is reusing function summaries beneficial in incremental verification?

<span id="page-17-1"></span><sup>2</sup> <http://www.cprover.org/>



<span id="page-18-0"></span>**Fig. 2** Overview of the UpProver architecture. UpProver operates at one particular level of precision at each run

**RQ 3** How does our approach compare with other incremental verifers?

**Benchmarks and setup:**[3](#page-18-1) We chose 2670 revision pairs of Linux kernel device drivers from [[34](#page-30-5)]. The benchmarks were chosen so that they are parsable by Cprover 5.11, and contain at least one safety property (code assertion). The crafted benchmarks mainly stress-test our algorithm and have changes such as function addition/deletion, signature change, semantic/syntactic change in function-bodies, etc. In addition, we included 240 tricky hand-crafted smaller programs. The crafted benchmarks mainly stress-test our implementation and include function additions/deletions, signature changes, semantic/syntactic changes in function-bodies, etc. On average, the benchmarks have 16'000 LOC, the longest ones reaching almost 71'000 LOC. For each run, we set a memory limit of 10 GB and a CPU time limit of 900 s. The experiments were run on a CentOS 7.5 x86\_64 system with two Intel Xeon E5-2650 CPUs, clocked at 2.30 GHz, and 20 (2 x 10) cores. UpProver is available as open-source software. Technical information about the setup of the tool can be found at [http://verify.inf.usi.ch/upprover.](http://verify.inf.usi.ch/upprover)

## **7.1 Demonstrating usefulness of diferent theories**

To answer RQ 1, we compare the run time of incremental verifcation using diferent encodings. Each point in Fig. [3](#page-20-0) corresponds to an incremental verifcation run of a single benchmark (changed program  $P_2$ ). Figure [3](#page-20-0)a and b compare the EUF/LRA-based encod-ings in UpProver against the PROP-based encoding in UpProver<sup>[4](#page-18-2)</sup>. Almost universally, whenever run time exceeds one second, it is an order of magnitude faster to verify with LRA and EUF than with PROP. In addition, a large number of benchmarks on the top horizontal lines suggests that it is possible to solve many more instances with LRA/EUF -based encoding than with PROP-based encoding. However, the loss of precision is seen on

<span id="page-18-1"></span><sup>&</sup>lt;sup>3</sup> The experimentation data including benchmarks, evaluation results, and the source code repositories are available online at<http://verify.inf.usi.ch/upprover>

<span id="page-18-2"></span><sup>&</sup>lt;sup>[4](#page-21-0)</sup> The PROP-based summary reuse in UpProver depicted in Fig. 4c uses the same algorithm from its predecessor eVolCheck but has been signifcantly optimized compared to its earlier version. Thus PROP-based UpProver can be seen as representative of eVolCheck.

the benchmarks on the vertical line labeled potentially spurious (PS), indicating if the verifcation result using LRA/EUF is unsafe, the result might be spurious because of abstraction. Since UpProver only operates at one particular level of precision at each run, once the tool reports *Unsafe* in EUF/LRA it is recommended to confrm it by a stronger theory encoding.<sup>3</sup>

The results for benchmarks show the trade-off between the precision and run time of incremental verifcation. In fact the theories are complementary. This can be contrasted to the plot in Fig. [3](#page-20-0)c where we extracted benchmarks that have successful bootstrapping phase in both LRA and EUF and ended up with 2516 versions out of which 50% are strictly faster in EUF and 30% are strictly faster in LRA. The time overhead observed in LRA compared to EUF is due to the more expensive decision procedure.

# **7.2 Demonstrating the efect of summary reuse**

To answer RQ 2 we demonstrate how summary reuse and summary repair beneft incremental verifcation. To this end we frst compare the performance of the tool with and without summary reuse, and then we show how summary repair results in more summaries while the overhead of producing more summaries is negligible.

#### **7.2.1 Incremental BMC vs monolithic BMC**

The purpose of this section is to compare verifcation time of *reusing* the summary against not reusing it. As opposed to summary-based incremental checking in UpProver that maintains and reuses over-approximating summaries of the functions across program versions, a standalone BMC tool, e.g., HiFrog  $[11]$  $[11]$  $[11]$  and cbmc  $[16]$ , creates a monolithic BMC formula and solves it as a standalone run without reusing information from previous runs of other versions. In this section, we compare UpProver with HiFrog as a representative sample of non-incremental BMC tool. This choice is made because both tools use the same infrastructure from Cprover v5.11 to transform C program to obtain a basic unrolled BMC representation that UpProver uses as a basis for producing the fnal logical formula. Since UpProver and HiFrog share the same parser and the same SMT solver OpenSMT the comparison is not afected by unrelated implementation diferences.

The plots in Fig. [4](#page-21-0) compare UpProver against HiFrog (non-incremental) with three encodings EUF, LRA, and PROP. Each point in each plot corresponds to verifcation run of a changed program, with the running time of UpProver when *reusing* the summary of the frst version on *x*-axis, and the running time of HiFrog without reuse on *y*-axis. The plots demonstrate the performance gains of incremental verifcation with reusing function summaries against not reusing it. A large amount of points on the upper triangle lets us conclude that UpProver is an order of magnitude faster than the corresponding non-incremental verifcation for most benchmarks.

Table [1](#page-22-0) provides more details on each encoding that would clarify the scatter plots further. We use acronyms  $P_1$  and  $P_2$  for two versions of a program. The row *Safe* indicates the number of programs reported safe by each encoding. In total out of 2910

<span id="page-19-0"></span>The work in [[9\]](#page-29-26) demonstrated the relation among some theories of SMT from the perspective of overapproximation.



<span id="page-20-0"></span>**Fig. 3** Demonstrating the impact of theory encoding by comparing timings of LRA/EUF encodings in UpProver vs. PROP encoding. The inner lines **TO** and **MO** refer to the time and memory limit. The outer lines **PS** refer to the results that are potentially spurious due to the use of abstract theory

benchmarks, UpProver with LRA verified safe the largest amount of  $P_2$ , i.e., 92% and with EUF and PROP verifed 85% and 40% respectively. The row *Unsafe* indicates the number of programs reported unsafe by each encoding. The unsafe results might be spurious when theory encodings were used (indicated by an asterisk). The row TO shows that while UpProver with PROP times out in 53% of the benchmarks, for LRA and EUF this happens for less than 1%. The row MO shows that with PROP encoding, UpProver exceeds the memory limit in 105 benchmarks of  $P_1$  and  $P_2$ , for LRA and EUF this does not occur.

The row *uniquely verified* programs indicates how many  $P_2$  can be incrementally verifed safe in each encoding exclusively. The count of the uniquely verifed using LRA is comparable to other encodings where 41 instances are not solved by any other encoding. Even though, LRA solves the most safe program versions, there are several benchmarks that can be uniquely verifed by EUF (11 instances) and by PROP (4 instances). These distinctly verifed programs in each encoding can be included in a portfolio.

The row *no* − *sum* represents the cases where there are no possibility to perform incremental verifcation because no function summaries were produced in the bootstrapping phase. This can happen when the bootstrapping verification of  $P_1$  results in *Unsafe*, TO, or MO. The PROP encoding results in the highest rate of *no* − *sum*, i.e., 57% (1661) which is



<span id="page-21-0"></span>**Fig. 4** Incremental verifcation time of UpProver versus non-incremental verifcation time of HiFrog on (a) EUF, (b) LRA, and (c) PROP encoding

the summation of Time Out, Memory Out, and Unsafe results of bootstrapping of *P*1. This asserts that using the rigid approach of bit-blasting for majority of our real-world benchmarks obstructs the incremental verifcation. On the contrary, LRA and EUF encodings have a relatively small rate of *no* − *sum*.

It is worth noting that we compared the results of UpProver with the expected results of SVCOMP, since most benchmark names indicate the expected result. Out of 2910 pairs of benchmarks, 2794 pairs had both versions classifed as safe. However, for the remaining 116 benchmarks, no expected result was provided and they were marked as unknown, thus we are unable to obtain precise numbers for *Unsafe* benchmarks. For *Safe* benchmarks, we never encountered any disagreement with the expected results. This indicates a high degree of accuracy for UpProver in verifying safe benchmarks with theories. However, for EUF/LRA, *Unsafe* results might be either real error or false alarms. Due to these unknown benchmarks, we are unable to report the exact number of false alarms for *Unsafe* benchmarks in theory and we mark them with asterisk.

The overall fndings from our experiments show evidence for the following key points: precision and performance-gain present a trade-of. UpProver with EUF and LRA have a better performance compared to the bit precise PROP encoding and are crucial for scalability. At the same time, there is a small number of benchmarks that require PROP. Despite

<span id="page-22-0"></span>

the fact that bit-blasted models are more expensive to check than the EUF and LRA models, we fnd it surprising that the light-weight encodings succeed so often. In practice, the encodings complement each other, and the results imply an approach where the user gradually tries diferent precisions until one is found that suits the programs at hand.

#### **7.2.2 Number of repaired summaries**

In this section, we measure the number of repaired summaries that are generated by two out of the box LRA interpolation algorithms. Recall the two phases of the repair in our algorithm: once an existing summary is marked invalid for a changed function *f*, Algorithm 4 frst *weakens* the summary by removing broken conjuncts of the summary. In case the weakened summary is not strong enough, Algorithm 3 (line 17) *strengthens* the weakened summary by recomputing interpolants for *f* and conjoining with the weakened summaries. We shortlisted 43 pairs of C programs whose summaries was repaired at least once during incremental verifcation.

Figure [5](#page-23-0) depicts the count of two types of repair in LRA. We use acronyms  $W$  for weakening, *Itp<sup>D</sup>* for decomposing Farkas interpolation algorithm, and *Itp<sup>F</sup>* for Farkas interpolation algorithm. We ran Algorithm 1 with LRA encoding over the shortlisted programs and generated 3837 LRA summaries in total, out of which 1043 summaries were strengthened by  $Itp^D$  interpolation and 49 summaries were weakened by W. The remaining summaries were either used without any repair, or unused at all due to their corresponding functions did not have change, thus no summary validation performed. Similarly, 1007 and 29 of summaries were repaired by  $Itp^F$  and by W respectively.

We can also view the result of this experiment from a diferent perspective. It can be seen as a way to test how good the interpolants are and how benefcial is the summary weakening. In the summary validation phase in UpProver, the more general interpolants are, the more likely they contain changes of the functions in the new version. Experimenting with *Itp<sup>D</sup>* and *Itp<sup>F</sup>* algorithms for producing LRA interpolants, shows that almost always they were as good as they could obtain more summaries with  $W$  technique. Observing that 49 summaries out of 3837 could be weakened further, implies that pre-computed interpolants are already as weak as possible but strong enough to be safe.

Now that we have an estimate of the number of function summaries repaired by  $W$ , the question becomes how much overhead time this approach adds up to the overall process of UpProver.

<span id="page-23-0"></span>

#### **7.2.3 Overhead of summary repair**

In the following, we study the overhead of weakening  $(\mathcal{W})$  process in the verification time. Figure [6](#page-24-0) compares the runtime of UpProver when reusing summaries generated by *Itp<sup>D</sup>* with and without  $W^6$  $W^6$ . In 60% of the benchmarks *Itp<sup>D</sup>* with W outperform or equal to *Itp<sup>D</sup>*. In 32% strictly *Itp<sup>D</sup>* with W faster than *Itp<sup>D</sup>*, whereas in 41% *Itp<sup>D</sup>* is strictly faster than *Itp<sup>D</sup>* with  $W$ . This reveals that in most of the cases not only weakening of summaries did not introduce considerable overhead, but also sometimes outperform the cases without  $W$ . For instance, the cases that are above 200 s, 29 benchmarks with  $Itp<sup>D</sup>$  and W strictly outperform  $Itp^D$ , suggesting that the  $Itp^D$  is not undesirable.

Overall, the results imply that summaries repaired by  $W$  and strengthened by  $Itp^D$  are benefcial in a sense that leads to more summaries in the end, and even shows speed-up in some benchmarks compared to disabling  $W$ , thus did not affect the overall performance.

In Fig. [7](#page-24-1) we compared the runtime of UpProver in theory of EUF with and without  $W$ . Concretely, at the area of around 350 to 800 s we observe 6 points above diagonal line confirms out performance of  $W$ , whereas 3 points below diagonal shows that pure EUF without  $W$  performs better.

Table [2](#page-25-0) gives further details on 14 representative pairs of benchmarks whose change type are substantially diferent and whose summaries had a chance of being repaired at least once. The table consists of four confgurations in LRA. Each row refers to a pair  $(P_1, P_2)$  of programs. The columns highlighted in gray color refers to enabling *summary weakening* feature in the algorithm. The columns highlighted in blue color refers to disabling *summary weakening* feature in which the summaries are repaired only by re-computation through interpolation. The column *interpolationtime* shows the time for generating all summaries after successful bootstrapping of  $P_1$  and *initialsummary* the number of function summaries in  $P_1$  which are non-trivial, i.e., are not simply *true* formula. The column *preserved* refers to the number of functions that stayed the same in  $P_2$  and  $\Delta$  to

<span id="page-23-1"></span> $6$  The results with  $Itp^F$  is similar.



<span id="page-24-1"></span><span id="page-24-0"></span>**Fig. 6** Incremental verifcation time of UpProver with LRA decomposed interpolants with and without weakening  $(W)$ 

the number of changed functions in  $P_2$ . The column *difftime* shows the time taken by difference-checker to identify changes between  $P_1$  and  $P_2$ . The column *validation check* refers to iterative validation checks of summaries for checking the containment of summaries. The columns *repaired by itp* and *repaired by* W indicate the number of newly established summaries by re-computation through interpolation and weakening respectively.

Overall, the numbers in the column *validation check* are higher when  $W$  is used since the algorithm has to iterate more to fnd a subset of conjuncts in the summary formula. Nevertheless, there are benchmarks, highlighted in bold, that show that the performance improves with the use of  $W$ . This happens because the weakened summaries can contain more changes and thus be more suitable for incremental verifcation. Interestingly, the columns *repaired by itp* and *repaired by* Wshow that whenever  $W$  is used, more summary formulas are produced. This is desirable for incremental verifcation. We see from the experiments that the increase in the number of summary formulas results both directly

<span id="page-25-0"></span>

from weakening the summaries and indirectly because each successful validation check generates new interpolants.

### **7.3 Comparison of UpProver and CPAchecker**

To answer RQ1, we compare UpProver with a widely-used tool CPAchecker which is able to perform incremental verifcation by reusing abstraction precisions. It is an orthogonal technique to ours, i.e., it is an unbounded verifer and aims at fnding loop invariants. Thus, comparing running times does not make sense since running times in UpProver crucially depend on the chosen bound.<sup>7</sup> Instead, we focus on comparing the speedups obtained with the two techniques since the change of a bound afects a speedup less.

Here we report the results only on device driver instances which both tools could handle. Out of 250 device drivers categories given in [https://www.sosy-lab.org/research/cpa](https://www.sosy-lab.org/research/cpa-reuse/predicate.html)[reuse/predicate.html](https://www.sosy-lab.org/research/cpa-reuse/predicate.html), we selected 34 categories which are suitable for UpProver.<sup>[8](#page-26-2)</sup> These categories contain in total 903 verifcation tasks.

Figure [8](#page-27-0) shows the comparison of speedup in UpProver and CPAchecker. A large amount of points on the lower triangle reveals that summary reuse in UpProver achieves superior speedup than the precision reuse in CPAchecker. The average speedup in UpProver with LRA summary-reuse is 7.3 with a standard deviation of 6 and in CPAchecker the average speedup is 2.9 with a standard deviation of 1.7. UpProver reported 4 slowdowns among 34 categories, whereas this was not the case for CPAchecker.

# <span id="page-26-0"></span>**8 Related work**

The problem of incremental verifcation is not as studied as model checking of standalone programs. There are still several techniques and tools [\[12,](#page-29-9) [34](#page-30-5)[–39\]](#page-30-6) which the central incentive behind these lines of work is the ability to reuse intermediate results that were costly computed in previous verifcation runs, thus achieving performance speedup in the verifcation of later revisions compared to verifcation of programs in isolation. Works in this area vary based on the underlying non-incremental verifcation approach used, which defines what information to be reused and how efficiently so. Various information has been proposed for reuse, including state-space graphs [[40](#page-30-7)], constraint solving results [[41](#page-30-8)], and automata-based trace abstraction [\[38\]](#page-30-9). However, these groups of techniques are orthogonal to our approach as we store and reuse the interpolation-based function summaries in the context of BMC for verifying revisions of programs. Moreover, apart from pure reusing the previous computations, our technique repairs the already generated summaries to increase the chance of reusability.

Another approach towards efficiently verifying evolving programs, which is the one we compare in this article, is based on the reuse of previously abstraction precision in predicate abstraction CPAchecker [\[34\]](#page-30-5). Apart from the inherent diference that CPAchecker is an unbounded verifer and UpProver is a bounded model checker, we difer from this in that we base our approach on proof-based computing of interpolants and repairing them on the fy, therefore in some sense are able to store more information from the previous runs.

<span id="page-26-1"></span><sup>7</sup> For instance, the average running times in CPAchecker is 285.3 s and in UpProver with LRA is 13.4 seconds for chosen bound 5. For other bounds UpProver would have diferent average running times.

<span id="page-26-2"></span><sup>&</sup>lt;sup>8</sup> The reported version of UpProver is restricted by its dependency on the Cprover 5.11 framework which impedes its frontend from processing some benchmarks.

<span id="page-27-0"></span>



Other techniques for incremental verifcation of program revisions include reusing inductive invariants in Constrained Horn Clause across programs by guessing syntactically matching variable names [\[42,](#page-30-10) [43\]](#page-30-11). However, these techniques can be applied only for programs sharing the same loop structure. In contrast, our approach is applicable for all sorts of program changes in a bounded model. However, when the changes are drastic, there would not be much summary reuse even with the summary repair.

Other techniques for verifying program versions are based on *relational verifcation* (also known as regression verifcation or equivalence checking) which are used to prove equivalence of closely related program versions. To tackle the problem of formally verifying all program revisions various techniques and tools have been proposed for the last two decades [[44](#page-30-12), [45](#page-30-13)]. Existing relational verifcation approaches leverage the similarities between two programs so that they verify the frst revision, and then prove that every pair of successive revisions is equivalent [\[46](#page-30-14)[–50\]](#page-30-15). Since checking exact equivalence is hard to fulfll and not always practical, there is a group of techniques that check for partial equivalence between pairs of procedures [\[45,](#page-30-13) [51](#page-30-16), [52](#page-30-17)] or check conditional equivalence under certain input constraints [[46](#page-30-14)]. Despite the evident success, these techniques are sound but not complete.

The work in [[53](#page-30-18)] investigates the efects of code changes on function summaries used in dynamic test generation. This approach is also known as white-box fuzzing which includes running a program while simultaneously symbolically executing the program to collect constraints on inputs. The aim of [[53](#page-30-18)] is to discover summaries that were afected by the modifcations and cannot be reused in the new program version. Since this approach relies on testing, it sufers from the problem of path explosion, i.e., all program paths are not covered. However, this work is orthogonal to our approach as we construct and repair function summaries in a symbolical way, thus our approach allows encoding of all paths of an unrolled program into a single formula.

A group of related work includes techniques using interpolation-based function summaries (such as  $[8, 54, 55]$  $[8, 54, 55]$  $[8, 54, 55]$  $[8, 54, 55]$  $[8, 54, 55]$ ) for the standalone programs. Although these do not support program versions, we believe that our incremental algorithm may be instantiated in their context similar to how we instantiated it in the context of HiFrog [[11](#page-29-11)]. The bootstrapping phase of our work is built on top of HiFrog [\[11\]](#page-29-11), an approach for extracting and reusing interpolation-based function summaries in the context of Bounded Model Checking. In later work [[9](#page-29-26)] we propose to use function summaries more efficiently by lifting function summaries into various SMT levels, thus information obtained from one level of abstraction could be reused at a diferent level of abstraction.

The work we fnd most closely related to ours is eVolCheck [\[4,](#page-29-3) [12](#page-29-9)], the predecessor of UpProver, which works only at the propositional level and uses the function summaries only in a bit-precise encoding. Consequently, despite being an incremental approach, eVolCheck is computationally expensive in many cases in practice. Whereas, our approach allows fexibility in balancing between verifcation performance and precision through both program encoding and the choice of summarization algorithms. As a result of the highlevel encodings, UpProver summaries serve as human-readable *certifcates of correctness* expressing function specifcations.

# <span id="page-28-0"></span>**9 Conclusion**

We addressed the problem of verifying a large number of programs, in particular, when they are closely related. To avoid expensive full re-verifcation of each program version and repeating a signifcant amount of work over and over again, our proposed algorithm operates incrementally by attempting to maximally reuse the results from any previous computations. The key contribution of this work lies in enabling this fexibility by SMT encoding and exploiting the SMT summarization. Having SMT encoding allows for a lot of fexibility when reusing and repairing the summaries leading to the optimization of the whole process which was not possible in the previous SAT-based approach. To achieve incrementality, our algorithm extracts and reuses SMT-based function summaries to overapproximate program functions. It also provides an innovative capability of repairing previously computed summaries by means of iterative weakening and strengthening procedures. Moreover, it offers an efficient way of building formulas and refining them on-thefy. Through extensive experimentation, we demonstrate that our approach advances the state of the art in incremental verifcation of program revisions and is signifcantly more efficient than its predecessor eVolCheck and non-incremental BMC approach.

In future, we plan to extend the tool to handle summaries from diferent theories simultaneously [[9](#page-29-26)], possibly with the feature of function summarization in theory combination.

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