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THE STATUS OF THE WAVE FUNCTION IN DYNAMICAL COLLAPSE MODELS

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The idea that in dynamical wave function collapse models the wave function is superfluous is investigated. Evidence is presented for the conjecture that, in a model of a field theory on a 1+1 lightcone lattice, knowing the field configuration on the lattice back to some time in the past, allows the wave function or quantum state at the present moment to be calculated, to arbitrary accuracy so long as enough of the past field configuration is known.

Key words: wave function collapse, lightcone field theory.

1. INTRODUCTION

The question of the status of the state vector, Psi, in standard text-book quantum mechanics has been a controversial issue since Bohr's and Einstein's time. Is the state vector a complete description of the physical state of a system, or is it incomplete and needing of completion with extra information, or does it represent a state of (someone or something's) knowledge of the system? The class of models known variously as "dynamical collapse models" and "spontaneous localisation models" are observer independent alternatives to standard quantum theory. We can ask: what happens to this question about the status of the state vector? Is it immediately resolved by the formalism of the collapse models or is there still a question to answer?

The general structure of all dynamical collapse models is similar: there is a state vector, Psi, which undergoes a stochastic evolution in

Hilbert space and there is a "classical" (c-number) entity – let's call it "q-bar" following Diósi [1] – with a stochastic evolution in spacetime. The stochastic dynamics for the two entities – Psi and q-bar – are coupled together. The stochastic dynamics in Hilbert space depends on which q-bar is realised in such a way as to tend to drive Psi into an eigenstate of an operator (q-hat) that corresponds to q-bar: this is the eponymous "collapse" in these models. And the probability distribution for the realised values of q-bar depends on Psi.

The choice of q-bar varies from model to model. In the original GRW model [2] and a proposed relativistic version [3], q-bar is a sequence of discrete "collapse centres" or spacetime events, in Diósi's model for single particle quantum mechanics [1] q-bar is a particle position (see, however, footnote 1), in Continuous Spontaneous Localisation (CSL) models [4, 5] q-bar is a scalar field. In all cases the c-number entity q-bar is defined on spacetime and is therefore covariant in essence.

The Bell ontology [6] for the GRW model states that the collapse centres are the beables or real variables. The analogous ontology for collapse models in general is that the history of q-bar – whatever it happens to be in the model – is real. Work by Diósi shows that any prediction about results of macroscopic experiments and observations that can be made using the expectation value of operator q-hat in state Psi, can also be made, For All Practical Purposes (FAPP), using only knowledge about q-bar, suitably regularised and coarse grained. Indeed, in non-relativistic theories q-bar is equal to this expectation value plus white noise with zero mean[1] [1, 7]. Put another way, suppose one has run one's computer simulations of the collapse model up to a time well to the future of anything one is interested in and in the computer memory is a history for Psi traced out in Hilbert space and a (regularised) history for q-bar traced out in spacetime. If the computer has a memory failure and loses all information about Psi, the information about q-bar would be enough, when suitably coarse grained, to make all the macroscopic predictions that could be made from Psi.

An example is the lattice field theory [8] that is the subject of this paper. In [9] it was argued that a coarse graining of q-bar – in this case q-bar is a $\{0, 1\}$-valued field on the lattice – displays the same structure, FAPP, as the coarse grained expectation value of the field operator in the quantum state Psi.

In taking this point of view, that q-bar is real, we are forced to address the question of the status of Psi. Diósi takes the view that

[1]This raises the objection that the q-bar history is not really properly defined at all as it contains a white noise term. One could fall back on the argument that spacetime is widely expected to be fundamentally discrete and this discreteness would provide a physical cutoff for the frequency of the white noise. Or turn the argument around and say that if the Bell ontology for collapse models is desirable, this suggests the necessity of fundamental discreteness.

both Psi and q-bar are real [10]. In this paper we will investigate the possibility, raised explicitly by Kent [11] for the GRW model, that Psi doesn't exist at all – that it is at most a convenience and conveys no information that is not carried by the history of q-bar itself.

One can argue that there is already a way partially to demote the quantum state in collapse models from its status as a really existing thing to that of a "dynamical law". This view can be taken in formalisms in which spacetime histories of the system are primary (including for example consistent histories [12–15], Sorkin's quantum measure theory [16, 17] and Bohmian mechanics, see *e.g.* [18]). If we consider a collapse model to be a stochastic law for the q-bar histories then the quantum state Psi can be formally relegated to the initial surface from which it need never evolve. The initial state gives us the dynamical law for the future q-bar histories in the form of the probability distribution on them and is not itself real. However, we can, if we know the q-bar history up to some spacelike surface, define an "effective" quantum state on that spacelike surface which tells us how to calculate the probability distribution on q-bar events to the future of the surface conditional on the known past history. The "evolution" of this effective quantum state from surface to surface (which is *precisely* the stochastic process in Hilbert space mentioned above) is akin to a "Bayesian" updating – on the actualisation of stochastic events – of the rule which gives the future probability distribution and is not the evolution of something physical. On this view, the quantum state is something we invent in order to render the dynamics Markovian.

It would be desirable to go further than this. The initial state on the initial surface hangs around like the smile of the Cheshire Cat – rather insubstantial but still persistently *there*. Moreover, in the quest to make a relativistic collapse model, the need to begin with a state defined on an initial surface breaks Lorentz invariance. In this paper we elaborate on the conjecture made in [8, 11], that in collapse models even the initial state can be eliminated as a necessary part of the theory (and the only information that remains from the state is a classical distribution over superselection sectors). We suggest that it can be replaced by an "initial period of q-bar history". Knowing this initial period of history would allow the calculation, FAPP, of an effective quantum state which could be used to make predictions from then on.

In Sec. 2 we briefly describe a collapse model for a field theory on a 1+1 null lattice that we will use as a testing ground for our conjecture. In this model, q-bar is a field configuration of 0's and 1's on the lattice. In Sec. 3 we state the conjecture and in Sec. 4 we describe the simulations. The results reported in Sec. 5 suggest that if the field configuration is known to a certain depth in time $T_{converge}$, the state vector can be deduced FAPP from that configuration. Thus the evolution of the field alone would be approximately Markovian on time scales larger than $T_{converge}$. Section 6 contains a summary and

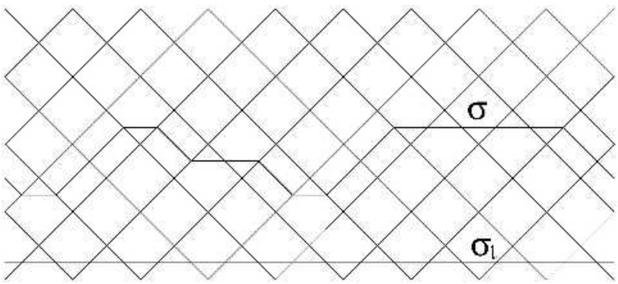

Fig. 1. The light cone lattice. σ_t is a constant time surface and σ is a generic spacelike surface.

discussion.

2. CAUSAL COLLAPSE MODEL ON A LIGHTCONE LATTICE

We briefly review the spontaneous collapse model [8] that we will use to investigate the conjecture. We follow the presentation of [9] and refer to that paper for further details. The model is a modification of a unitary QFT on a 1+1 null lattice, making it into a collapse model by introducing local "hits" driving the state into field eigenstates. The spacetime lattice is N vertices wide and periodic in space, extends to the infinite future, and the links between the lattice points are left or right going null rays. A spacelike surface σ is specified by a sequence of N leftgoing links and N rightgoing links cut by the surface; examples of spatial surfaces are shown in Fig. 1. We assume an initial spacelike surface σ_0.

An assignment of labels to the vertices to the future of σ_0, $v_1, v_2, \ldots$, is called "natural" if $i < j$ whenever v_i is to the causal past of v_j. A natural labelling is equivalent to a linear extension of the (partial) causal order of the vertices. A natural labelling, $v_1, v_2, \ldots$ is also equivalent to a sequence of spatial surfaces, $\sigma_1, \sigma_2, \ldots$ where the surface σ_k is defined such that between it and σ_0, lie exactly the vertices $v_1, \ldots v_k$. One can think of the natural labelling as giving an "evolution" rule for the spacelike surfaces: as each vertex event v_k occurs, the surface creeps forward by one "elementary motion" across that vertex. For any natural labelling and any k, the finite set of vertices $\{v_1, v_2, \ldots v_k\}$ is a *stem*, a finite set that contains its own causal past.

The local field variables α live on the links. These field variables take only two values $\{0, 1\}$, so that on each link there is a qubit Hilbert space spanned by these two states. We denote by $\{\alpha_{R_k}, \alpha_{L_k}\}$ (α_{v_k} for short) the values of the field variables on the two outgoing links (to the right R and to the left L) from vertex v_k. (Note, in paper 1 we used the hatted symbol $\hat{\alpha}$ to denote the actual value of the field variable but here we will use the unhatted α.) One can, colloquially, consider the field values 0 and 1 to represent the absence or presence (resp.) of "bare particles" on the lattice.

A quantum state $|\psi_n\rangle$ on surface σ_n is an element of the 2^{2N} dimensional Hilbert space H_{σ_n} which is a tensor product of the 2N 2-dimensional Hilbert spaces on each link cut by σ_n. The basis vectors (the "preferred basis") of this Hilbert space are labelled by the possible field configurations on σ_n, namely the $2N$-element bit strings $\{0, 1\}^{2N}$. We will often refer to the number of 1's in the bit string labelling an eigenstate as the number of particles in that state. The Hilbert space is the direct sum of $2N + 1$ sectors each of fixed particle number. We identify the Hilbert spaces on different surfaces in the obvious way using the field basis. At each vertex v_k, there is a local evolution law which is given by a 4-dimensional unitary "R-matrix" $U(v_k)$ (4-dimensional because it evolves from the two ingoing links the two outgoing links). For this paper we choose these R-matrices to be uniform across the lattice. (One can simulate external interventions by fiddling with the R-matrices.)

In the standard text-book unitary theory, one postulates the existence of an external measuring agent and then this formalism can be used to predict the results of sequences of measurements of the field. One way to do this is to identify projectors: $P(\alpha_{v_k})$ projects onto the subspace of the Hilbert space spanned by the basis vectors in which the field values at vertex v_k are α_{v_k} (recall there are two links outgoing from v_k and so α_{v_k} is really two values).

Then the joint probability distribution for the agent to *measure* a particular field configuration $\{\alpha_{v_1}, \alpha_{v_2}, \ldots \alpha_{v_n}\}$ on the lattice between the hypersurfaces σ_0 and σ_n is

$$\begin{aligned} &\mathbf{P}^{\text{standard QM}}(\alpha_{v_1}, \alpha_{v_2}, \ldots \alpha_{v_n}) \\ &= ||P(\alpha_{v_n})U(v_n) \ldots P(\alpha_{v_1})U(v_1)|\psi_0\rangle||^2 \,. \end{aligned} \tag{1}$$

This probability rule is independent of the linear ordering of the vertices and depends only on their causal order. The rule evades the potential danger of violating relativistic causality described in [19] in two ways: the causal structure of the vertices of the lattice is a partial order from the start (no transitive completion is required) and the field variables being measured are completely local quantities.

Inspired by the GRW model with Bell's ontology, this unitary quantum field theory requiring external agents can be turned into an observer independent theory for a closed system by replacing the projection operators for measurements in (1) by positive operators (for "unsharp measurements") and adopting the resulting formula as the probability that the corresponding field configuration *occurs*. More precisely, we define on each link (i.e., on each 2-dimensional Hilbert space associated with a link) the two operators J_0 and J_1 where

$$J_0 = \frac{1}{\sqrt{1+X^2}} \begin{pmatrix} 1 & 0 \\ 0 & X \end{pmatrix}, \quad J_1 = \frac{1}{\sqrt{1+X^2}} \begin{pmatrix} X & 0 \\ 0 & 1 \end{pmatrix}, \tag{2}$$

with $0 \leq X \leq 1$. Note that $J_0^2 + J_1^2 = 1$ and this is a positive operator valued measure. Then we define the jump operator $J(\alpha_{v_k})$ on the 4-dimensional Hilbert space on the outgoing links from v_k as the tensor product of the two relevant 2-dimensional jump operators, *e.g.*, when $\alpha_{v_k} = (0,0)$, $J(\alpha_{v_k}) = J_0 \otimes J_0$. We promote $J(\alpha_{v_k})$ to an operator on the Hilbert space of any spatial surface containing those two links by taking the tensor product with the identity operators on all the other components of the full Hilbert space. The probability of the field configuration $\{\alpha_{v_1}, \ldots \alpha_{v_n}\}$ is given by

$$\mathbf{P}(\alpha_{v_1}, \ldots \alpha_{v_n}) = \| J(\alpha_{v_n}) U(v_n) \ldots J(\alpha_{v_1}) U(v_1) |\psi_0\rangle \|^2 . \tag{3}$$

From this we can see the importance of the fact the jump operators form a positive operator valued measure, which ensures consistency:

$$\mathbf{P}(\alpha_{v_1}, \ldots \alpha_{v_{n-1}}) = \sum_{\alpha_{v_n}} \mathbf{P}(\alpha_{v_1}, \ldots \alpha_{v_n} \qquad 4)$$

$$= \sum_{\alpha_{v_n}} \langle \psi_0 | \ldots J(\alpha_{v_n}) J(\alpha_{v_n}) \ldots |\psi_0\rangle \tag{5}$$

$$= \| J(\alpha_{v_{n-1}}) U(v_{n-1}) \ldots J(\alpha_{v_1}) U(v_1) |\psi_0\rangle \|^2 . \tag{6}$$

Again, (3) depends only on the (partial) causal order of the vertices because any other choice of natural labelling of the same vertices gives the same result. These probabilities of the field configurations on all stems are enough, via the standard methods of measure theory, to define a unique probability measure on the sample space of all field configurations on the semi-infinite lattice.

We stress that whereas Eq. (1) is the probability for *measuring* a particular field configuration in standard unitary quantum theory, Eq. (3) is interpreted, in the Bell ontology, as the probability for the field to *be* in that configuration. The full content of the theory is the probability distribution (3) on possible field configurations, dependent on an initial state $|\psi_0\rangle$.

The *state* on the hypersurface σ_n that is reached after the elementary motions over vertices $v_1, \ldots v_n$ and the field values

$\{\alpha_{v_1}, \ldots \alpha_{v_n}\}$ have been realised is the normalised state

$$|\psi_n\rangle = \frac{J(\alpha_{v_n})U(v_n)\ldots J(\alpha_{v_1})U(v_1)|\psi_0\rangle}{\|J(\alpha_{v_n})U(v_n)\ldots J(\alpha_{v_1})U(v_1)|\psi_0\rangle\|}\,. \tag{7}$$

Thus, the probability for state (7) on hypersurface σ_n is (3). In order to make predictions about the field on the lattice to the future of σ_n – conditional on the past values $\{\alpha_{v_1}, \ldots \alpha_{v_n}\}$ – it is sufficient to know $|\psi_n\rangle$. Indeed, the conditional probability of $\{\alpha_{v_{n+1}}, \ldots \alpha_{v_{n+m}}\}$ is given by

$$\mathbf{P}(\alpha_{v_{n+1}}, \ldots \alpha_{v_{n+m}}) = \|J(\alpha_{v_{n+m}})U(v_{n+m} \ldots J(\alpha_{v_{n+1}})U(v_{n+1})|\psi_n\rangle\|^2\,. \tag{8}$$

The state vector provides these conditional probabilities, and is therefore a convenient way of keeping the probability distribution up to date, given past events.

3. THE STATUS OF THE WAVE FUNCTION: "AN EXECUTIVE SUMMARY"?

We are interested in investigating the possibility of doing away with the quantum state entirely as a fundamental concept in collapse models and will be using the lattice model described above as a test case. As mentioned in the introduction, we can relegate the quantum state to a state, $|\psi_0\rangle$, on an initial surface σ_0 from where it acts as a "dynamical law", specifying the probability distribution on field configurations to the future of σ_0. Can we weaken even this status?

We make the conjecture that, if the field configuration is known between σ_0 and σ_n, then even if the state on σ_0 is not known, the state on σ_n is calculable up to a correction that goes to zero as $n \to \infty$. This would mean that although the evolved state on σ_n *a priori* depends on both the initial state on σ_0 and on the field values that have actually occurred in between, its dependence on $|\psi_0\rangle$ dies away as time goes on until all we need to know to make predictions, FAPP, is the field configuration back to a certain depth in time. We would then have an interpretation not only assigning reality to a field configuration in spacetime but further demoting the wave function by denying it a role as a *necessary* entity to the theory: the state $|\psi_n\rangle$ can be deduced FAPP from the field configuration to the past of σ_n to some depth in time (or exactly if the whole infinite past history is known) and becomes an "executive summary" of the past reality containing no independent information.

More precisely, let $|\psi_0^1\rangle$ and $|\psi_0^2\rangle$ be two states on σ_0. Then they give, according to (3), two probability distributions, $\mathbf{P}_1$ and $\mathbf{P}_2$ on field configurations to the future of σ_0. Choose any linear ordering of the vertices to the future of σ_0, $v_1, v_2, \ldots$. Adopt the notation $\alpha(n)$ for a field configuration between σ_0 and σ_n, and $|\psi_n^a, \alpha(n)\rangle$ for the state on σ_n that arises from $|\psi_0^a\rangle$ on σ_0 after $\alpha(n)$ has happened ($a = 1, 2$).

Conjecture: There exists a complex phase λ such that

$$\| |\psi_n^2, \alpha(n)\rangle - \lambda |\psi_n^1, \alpha(n)\rangle \| \to 0 \text{ as } n \to \infty \tag{9}$$

for all $\alpha(n)$ except those which almost surely do not occur according to both $\mathbf{P}_1$ and $\mathbf{P}_2$.

Conjecture (density matrix form):

$$\begin{aligned} &\| \sum_{\alpha(n)} \mathbf{P}_1(\alpha(n)) |\psi_n^1, \alpha(n)\rangle\langle\psi_n^1, \alpha(n)| \\ &- \sum_{\alpha(n)} \mathbf{P}_1(\alpha(n)) |\psi_n^2, \alpha(n)\rangle\langle\psi_n^2, \alpha(n)| \| \to 0 \text{ as } n \to \infty, \end{aligned} \tag{10}$$

where $\|\cdot\|$ is the operator norm, and similarly with 1 and 2 interchanged.

Note that we already know that the conjectures cannot be true strictly as stated because of the possible existence of "superselection sectors" in the Hilbert space. For example, the jump operators J preserve particle number and if the R-matrices do so also (this is the case we will study in detail in the next section) then a state in the k-particle sector can never approach a state in the l-particle sector if $k \neq l$. If the R-matrices preserved only particle number mod-2 (by allowing pair creation and annihilation of particles) then there would be two superselection sectors (even and odd particle number). It should be noted that even if there is a conserved quantity – particle number, say – this quantity is conserved in the state vector but not in the realised field configuration. We expect however that the "conservation law" will be reflected in the probability measure in the sense that a suitable property of the coarse grained field configuration will be predicted with probability close to one.

When there are superselection sectors, an initial quantum state corresponds to a classical probability distribution over the sectors and a quantum state in each sector, in the familiar way. Without loss of generality therefore, we will assume in what follows that we are restricted to a single superselection sector and the conjectures apply to each superselection sector individually becoming, effectively: two states in the same superselection sector tend to each other up to a phase for all histories except for a set of histories which has measure zero in the probability measure of both states.

4. THE SIMULATIONS

We sought evidence for the conjecture in the following way. We chose a unitary R-matrix, uniform across the lattice, of the form

$$R = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & i\sin\theta & \cos\theta & 0 \\ 0 & \cos\theta & i\sin\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \tag{11}$$

This gives a particle number preserving dynamics for the state, since the hit operators J also preserve particle number.

We chose σ_0 to be a constant time surface and we chose two initial states, $|\psi_0^1\rangle$ and $|\psi_0^2\rangle$ (in the same superselection sector, which here meant the same particle number sector). We generated, at random according to the probability distribution $\mathbf{P}_1$ or $\mathbf{P}_2$ field configurations to the future of σ_0. For each of these field configurations, $\alpha(M)$ (where M was large enough for the calculation in hand) we calculated the two states $|\psi_n^a, \alpha(n)\rangle$, $a = 1, 2$ on the surface σ_n which is the nth surface in a sequence of surfaces chosen according to the stochastic rule "choose the next elementary motion at random with uniform probability from those possible." This is not a covariant rule – it is equivalent to a probability distribution on linear extensions of the partial order on the whole future lattice but it does not give each equal weight – and moreover a covariant, Markovian rule does exist [20] but we made the choice for ease of calculation. We will comment on what significance this has for our results below.

We would like to show that the two states $|\psi_n^a, \alpha(n)\rangle$, $a = 1, 2$ become close, up to a phase, as n gets large and more precisely we would like to know how the difference behaves with n. One can argue that it is not the states themselves that should be compared but the probability distributions for the field variables that they produce. Indeed, in an interpretation in which only the field is real, it is only this probability distribution and not the state itself which has physical import.

In principle the entities that should be compared are the two probability distributions, for states 1 and 2, over field configurations to the future of any surface σ, given the values of $\alpha(M)$ lying to the past of σ. This is calculationally impractical and we used two simplifying strategies. First, we sampled the space of all surfaces by choosing a sequence $\sigma_1, \sigma_2, \ldots$ according to the rule described above. This rule does not sample uniformly in the space of surfaces, as mentioned above, and an improvement of our scheme would be to determine and then implement the covariant rule which does. Second, we compared the probabilities, not for the whole future field configuration but only for the value 1 on each of the two outgoing links from v_n for each n.

An important point is that, as discussed in [9], the interesting physical regime for these models is when the parameter X is close to one, alternatively when $\epsilon \equiv 1 - X$ is close to zero. This means that the hits are very gentle and superpositions of microscopically different states will last for a long time. In this case, however, the conditional probability of a 1 on each link becomes very close to 1/2, indeed it is equal to $1/2 + O(\epsilon)$. (Here we clearly see the white noise term in the field configuration that is to be expected from Diosi's work.) So for small epsilon the probabilities will be close, whether or not the states are coming close to each other. Indeed, let the link in question be denoted l and suppose, at some stage in the dynamics, l is one of the outgoing links from the vertex that has just been evolved over. Let the state on the current spacelike surface through l be denoted schematically by

$$|\Psi\rangle = a|0\rangle + b|1\rangle, \tag{12}$$

where $|0\rangle$ ($|1\rangle$) is short hand for the normalised superposition of all the terms in the state in which the value of the field on l is 0 (1). The probability that the field will be 1 on l (conditional on the past evolution to that stage) is

$$\frac{|a|^2X^2 + |b|^2}{1+X^2} = \frac{X^2}{1+X^2} - |b|^2\frac{1-X^2}{1+X^2}\ . \tag{13}$$

So, for the difference between the probability for a 1 on link l in state 1 and in state 2 we will obtain

$$(|b_1|^2 - |b_2|^2)(1-X^2)/(1+X^2). \tag{14}$$

When ϵ is small, this becomes

$$(|b_1|^2 - |b_2|^2)(\epsilon + \mathcal{O}(\epsilon^2))\ . \tag{15}$$

From this we see that the appropriate quantity to calculate for each link is $|b_1|^2 - |b_2|^2$: that gives a measure of the difference of the probability distributions that affects the coarse grained, renormalised field configuration (see [9]) and indeed it is a measure of the difference between the states themselves.

Thus, we calculated for each vertex v_n (recall the sequence $v_1, v_2, \ldots$ is equivalent to a linear extension and is the one chosen at random by our evolution rule described above) and for each outgoing link, l, from v_n, the quantity $|b_1|^2 - |b_2|^2$ which we denote by $B(l)$. Of course this is a very approximate measure of the difference between the two states and to overcome this, we took the sum of this quantity over every link in a block, a certain number m of lattice time steps long and the width of the whole spatial lattice:

$$B_m(t) \equiv \sum_l B(l), \tag{16}$$

where the sum is over all links with lattice time coordinate from t through $t+m-1$ (the lattice time step is 1). Our convergence criterion was $B_m(t) < \delta$ and we define the convergence time T_c to be the smallest time such that $B_m(t) \leq \delta$, $\forall t > T_c$.

With the help of numerical simulations on 8, 9 and 10 vertex lattices we studied the dependence of T_c on ϵ, on particle number, on θ and on different types of initial state within fixed particle number sectors. In total about 600 simulations were run. We also studied the convergence of states for field configurations not generated according to the probability distributions from either state, for example the field configuration (a) of all 1's, (b) of all 0's and (c) randomly generated with uniform probability distribution of 1/2 for a 1 on each link. We failed to find convergence only in the cases (a) and (b) mentioned above when the field configuration was all 1's or all 0's which is consistent with the conjecture because they almost surely do not occur in $\mathbf{P}_1$ and in $\mathbf{P}_2$. Convergence occurred but was slower for the field configurations of type (c) than for those generated by (and therefore likely in) the probability distibutions of states 1 or 2.

In our simulations we were limited as to lattice size by the exponential growth of the problem in vertex number, and it is at present unclear whether the limited size of the lattice has important implications for our results, in particular the question whether the periodic boundary conditions of the lattice stimulate convergence remains open.

5. RESULTS

To begin by giving a flavour of the kind of simulations run, the convergence of two initial states is illustrated in Fig. 2. Each cell corresponds to a single link of the lattice (so there are twice as many cells across the lattice width as vertices) and the darkness of the cell is (positively) proportional to $|b|^2$ (see equation (12)). In plot (a) we show the evolution of state 1, which begins as an eigenstate with 4 particles on the left hand side of the lattice in the leftmost panel, time proceeds up the page and then the lattice continues at the bottom of the next panel and so on. Plot (b) shows the evolution of state 2, which begins as a state with 4 particles on the right hand side of the lattice. The parameters for the evolution are $X = 0.65$ and $\theta = 0.26\pi$. Comparing (a) and (b) it can be seen that the plots are indistinguishable from halfway up the first panel in each. The plots after this time are somewhat superfluous but we show them to emphasize that we checked that the convergence persists long after our convergence criterion is reached.

Figure 3 shows the quantity $B_{10}(t)$ defined in equation (16) plotted against lattice time t, for an 8 vertex run with $X = 0.95$, $\theta = 0.1\pi$ and initial states which are two different one-particle eigenstates. It shows a pleasingly sharp falloff to zero.

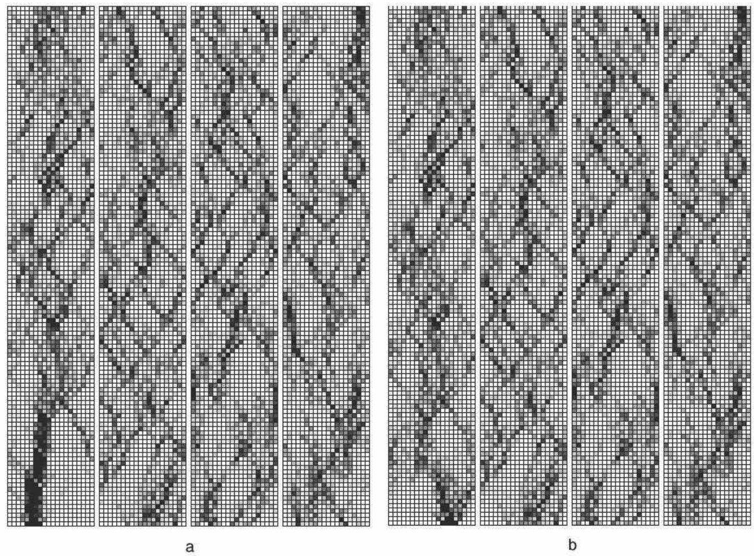

Fig. 2. (a) and (b) are plots of the evolution of four-particle eigenstates $|\psi_1\rangle$ and $|\psi_2\rangle$ respectively, in a field configuration generated according to the probability distribution of $|\psi_1\rangle$, with $X = 0.65$ and $\theta = 0.26\pi$.

Figure 4 is a plot of $\log(T_c)$ against $\log(\epsilon)$ for many 8 vertex runs of varying ϵ, where we chose B_{10} as our measure of difference and $\delta = 10^{-4}$ to define the convergence time. The other parameters were $\theta = 0.1\pi$ and $|\psi_1\rangle$ and $|\psi_2\rangle$ are two fixed one-particle eigenstates. The field configuration is chosen according to the probability distribution from $|\psi_1\rangle$. The plot is consistent with a dependence of

$$T_c \propto 1/\epsilon^2 \tag{17}$$

As a consequence of this, in a continuum limit in which the lattice spacing $a \to 0$ and $\epsilon = \mathcal{O}(\sqrt{a})$, the "physical" convergence time, aT_c would tend to some finite non-zero value.

A plot of the convergence time (defined by B_{10} and $\delta = 10^{-4}$) against θ is given in Fig. 5 for $X = 0.95$ and fixed initial one-particle eigenstates.

The plot is difficult to interpret. It seems particularly odd when one realises that for $\theta = \pi/2$ and $\theta = 0$ (the two limits of the range of θ shown) the R-matrix is such that it does not introduce any superpositions into the states. Indeed the evolution is completely deterministic: for $\theta = \pi/2$ an initial one-particle field eigenstate remains essentially constant – just acquiring a phase of i at each lattice time step – and for $\theta = 0$ it propagates at the speed of light along the null direction it

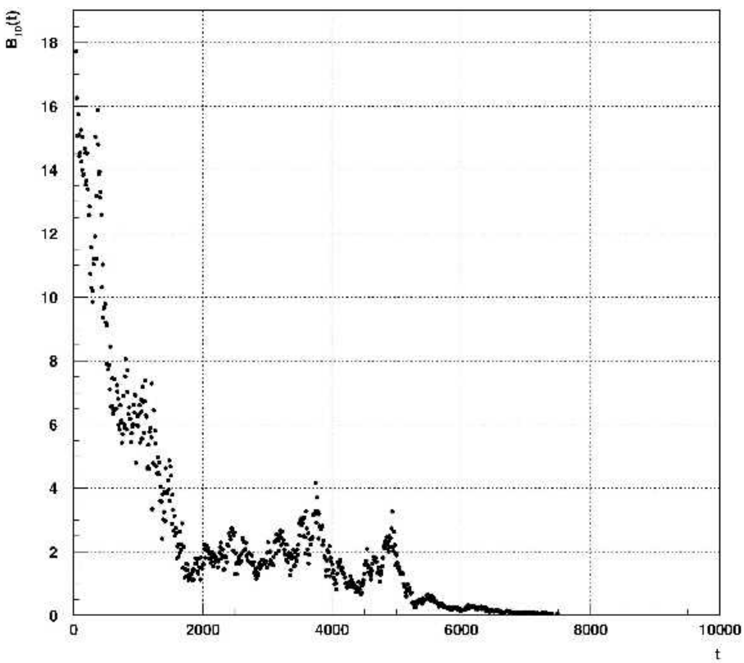

Fig. 3. Plot of $B_{10}(t)$, against lattice time t, for $X = 0.95$, $\theta = 0.1\pi$ and two different one-particle eigenstates for initial states.

starts off in. The evolution in these cases, therefore, does not "mix" the Hilbert space and states 1 and 2 can never converge.

The plot, however, suggests that for values of θ close to these limiting ones, convergence is faster than for θ in the middle of the range, and indeed the convergence time is tending to zero. We speculate that this has something to do with the competing effects of the mixing by the R-matrices and the converging effect of the hits. If we imagine starting with two states which have support over the whole of the one-particle sector of Hilbert space, the harder the hits, the faster the states will converge. In the extreme case, if $\epsilon = 1$ then the hit operators are projectors, state 1 will collapse into one of the eigenstates after one time step, the field configuration will be the one given by that eigenstate and state 2 will be forced into that state also. When ϵ and θ vary, there is a competition between the driving towards eigenstates by the hits and the mixing (introduction of superpositions) by the R-matrices. In the runs plotted here, we kept ϵ fixed so the strength of the hits does not vary but as θ tends towards the two limiting values it could be that the R-matrix evolution loses the competition. If the hits drive state 1 very

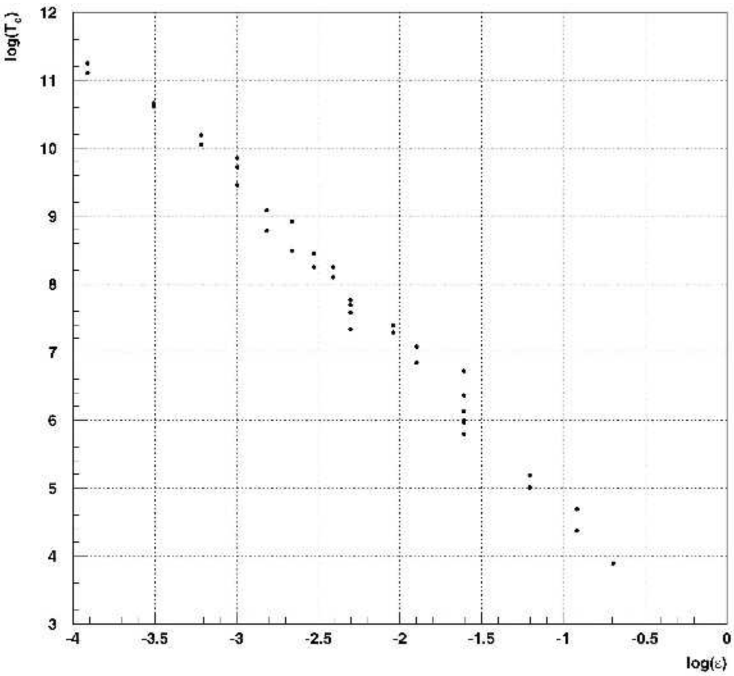

Fig. 4. Plot of $\log(T_c)$ against $\log(\epsilon)$

quickly into an eigenstate, then as long as there's been enough mixing so that there is even a tiny amplitude for that eigenstate in state 2, there will be convergence.

Figure 6 shows results from runs on an 8 vertex lattice with $X = 0.9$, $\theta = \pi/4$. In any given run the two initial states are eigenstates with the same particle number, which varies across the runs. The plot is of T_c (defined by B_8 and $\delta = 10^{-4}$) against particle number.

The plot is consistent with expected behaviour. The fixed particle number, m, sectors have dimension $(2N)!/m!(2N-m)!$ which increases as m increases to N and then decreases symmetrically as m increase further to $2N$. When the Hilbert space is larger, we expect that convergence will take longer as it takes longer for each state to mix and acquire amplitudes for all the different eigenstates. We expect the plot to be symmetric because, further, there is a duality in the models between field value 0 and field value 1.

We checked our results by taking several runs and calculating the quantity

$$C_n = 1 - \|\langle\psi_n^1, \alpha(n)|\psi_n^2, \alpha(n)\rangle\|^2 \tag{18}$$

and comparing it to the quantity $B_{10}(t)$ for the same run (recall that

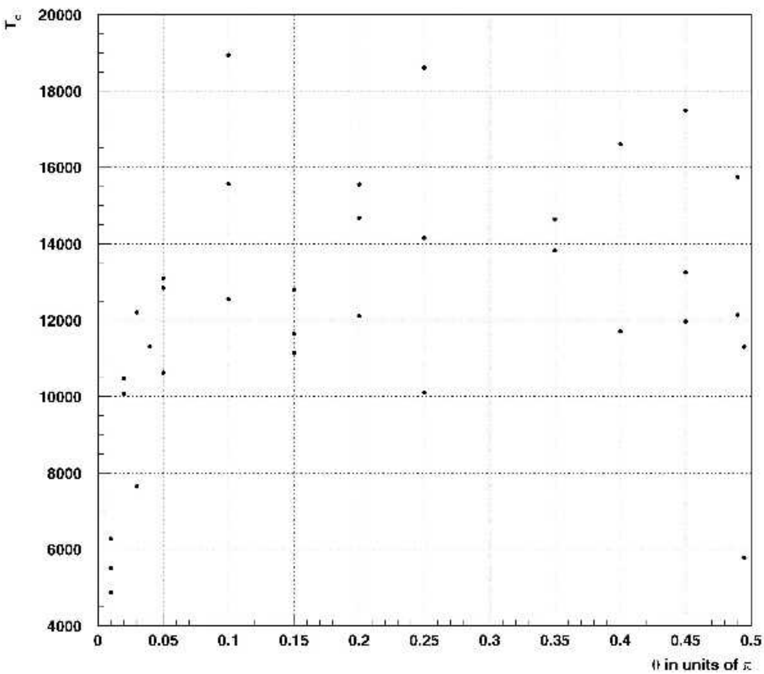

Fig. 5. Convergence time vs θ.

the way $B_{10}(t)$ is defined, there is one value for every tenth lattice time coordinate, so there are 80 times as many C_n data as B_{10} data). Figure 7 is a C_n plot of the run shown in 3. This is a more direct check of the conjecture and in the future we would want to redo our analysis using this method.

However, we present more evidence in Figs. 9, and 11 that indicates that the results will be the same. Indeed, even in the details of how the convergence occurs in each run, the behaviours of the measures $B_{10}(t)$ and C_n match each other very well. On noting that the number of elementary motions is 8 times the lattice time, it can be seen that the main features of the two types of plot are well matched in time. Figures 8, 9 show a plot of $B_{10}(t)$ and C_n data for a run with the same initial states and parameters as for the simulation whose data is shown in Figs. 3 and 7, while Figs. 10, 11 show a plot of $B_{10}(t)$ and C_n data for a run with the same states and parameters as for 8 and 9 but a different $\theta = 0.25\pi$.

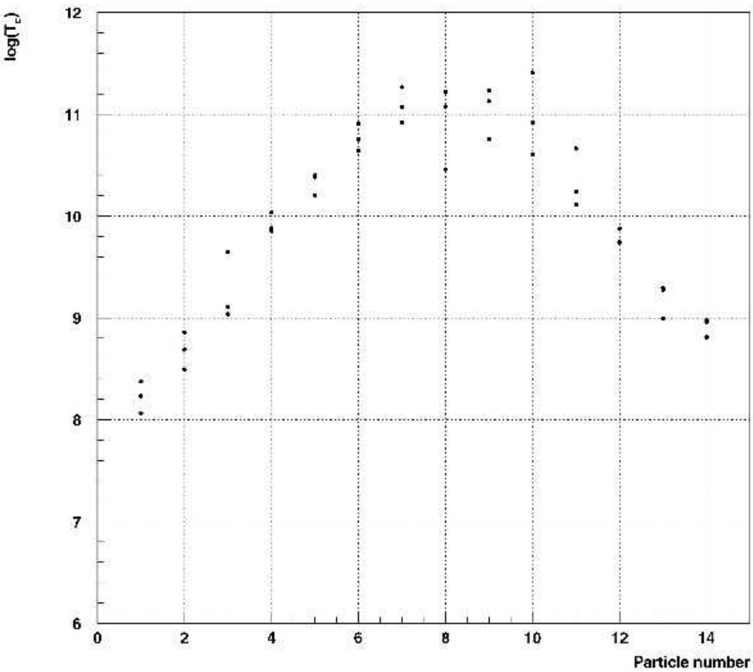

Fig. 6. Plot of $\log T_c$ against particle number.

6. DISCUSSION

We can state the import of the conjecture we have made thus: given some particular field history from $t = -\infty$ to $t = 0$ then there is a physical probability distribution on the field histories for $t > 0$ which we can express conveniently in the form (3), using a quantum state at $t = 0$ which is precisely specified by the past field history. If one could discover an algorithm for transforming the data in the field history directly into the probability distribution then one would have built a model in terms only of the field variables which makes no reference to a quantum state.

We have presented evidence for this conjecture. The falloff seen in Figs. 3 and 7-11 suggest the stronger conjecture that there is a time scale T_c such that even if we know only the field history from $t = -T_c$ to $t = 0$ then we can construct a quantum state that gives the correct predictions FAPP. We would like to study this further by investigating the dependence of T_c on the degree of convergence δ.

For the purposes of this paper we chose $\delta = 10^{-4}$ to define T_c and we presented evidence that T_c is of order ϵ^{-2} as $\epsilon \to 0$. In a continuum

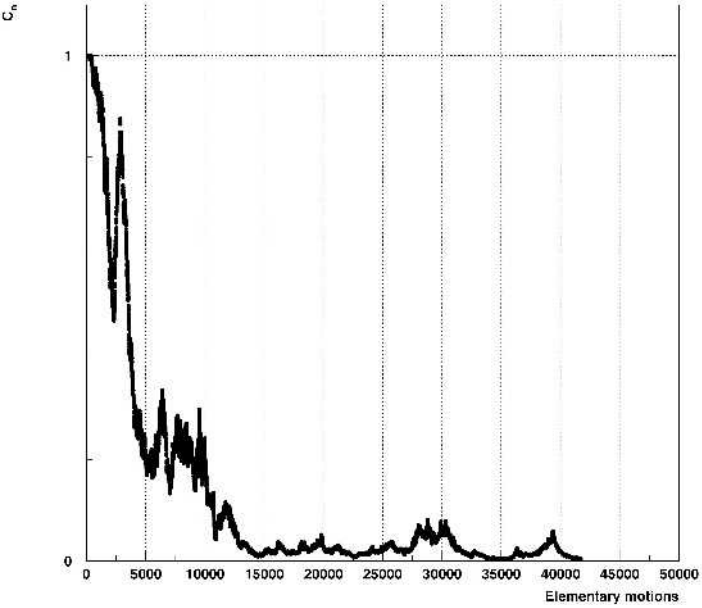

Fig. 7. Plot of C_n, against the number of elementary motions, for $X = 0.95$, $\theta = 0.1\pi$ and two different one-particle eigenstates for initial states. The data is from the same run as shown in Fig. 3

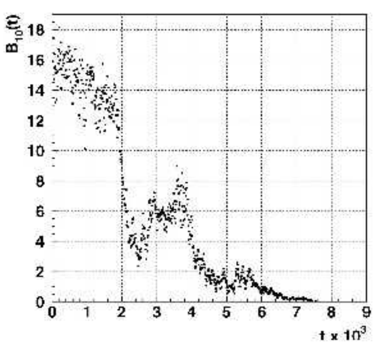

Fig. 8. Plot of $B_{10}(t)$, against lattice time t, for $X = 0.95$, $\theta = 0.1\pi$ and two different one-particle eigenstates for initial states.

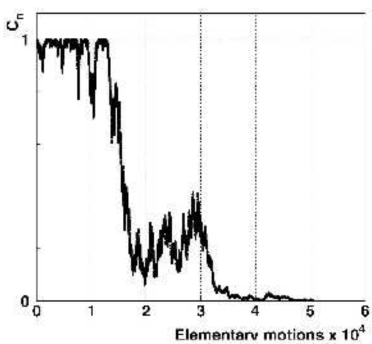

Fig. 9. Plot of C_n, against the number of elementary motions. The data is from the same run as shown in Fig. 8.

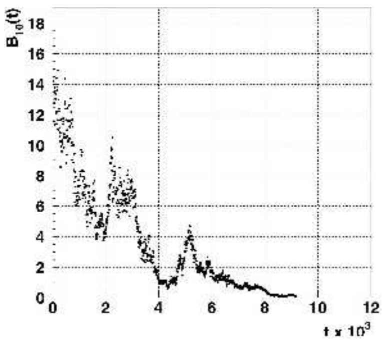

Fig. 10. Plot of $B_{10}(t)$, against lattice time t, for $X = 0.95$, $\theta = 0.25\pi$ and two different one-particle eigenstates for initial states.

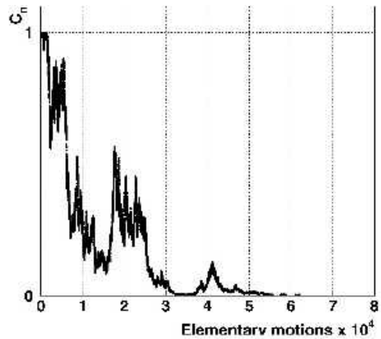

Fig. 11. Plot of C_n, against the number of elementary motions. The data is from the same run as shown in Fig. 10.

limit where the lattice spacing $a \to 0$ and $\epsilon = O(\sqrt{a})$ then the physical convergence timescale, aT_c would remain finite and the dynamics would be approximately Markovian for time scales larger than this.

It would be valuable to check all our results by redoing the simulations and calculating, instead of $B_m(t)$, C_n on each sampled surface and examining how it tends to 0 as we did for some runs described in the last section. Improvements on our methods would include calculating and implementing the covariant evolution rule for surfaces which would make our sampling of surfaces uniform. We would like to analyse quantitatively the dependence of T_c on the dimension of the particle number sector implied by the results shown in Fig. 6.

Results with different types of R-matrices as well as general initial states are still to be investigated. In particular, further evidence for the conjecture can be obtained by choosing pair-particle conserving matrices, as well as general matrices with no conservation laws.

We stress that the analysis and simulations presented in this paper are at a rather mathematical level. The question of physics has not been addressed. This would involve the settling of the issue of the competition between the R-matrices and the hits in the collapse of superpositions of eigenstates [9]. This bears on the conclusions of the current paper. The physically interesting range of parameters is when ϵ is very small and θ is also small so that "microscopic" superpositions persist for a long while but eventually collapse. In this regime, the hits are very gentle and the "mixing" of the Hilbert space by the R-matrices is slow. Investigating this regime is essential if we are to draw physically relevant conclusions about collapse models of this sort.

Finally we extend our conjecture to all collapse models. It would be interesting to study it in other cases such as the GRW model and Diósi's single particle model.

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