Foundations of Physics (2024) 54:5
https://doi.org/10.1007/s10701-023-00743-5

BRIEF REPORT

A Critical Analysis of 'Relative Facts Do Not Exist: Relational Quantum Mechanics Is Incompatible with Quantum Mechanics' by Jay Lawrence, Marcin Markiewicz and Marek Źukowski

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Received: 30 October 2023 / Accepted: 16 November 2023 / Published online: 11 December 2023

Abstract
We discuss a recent work by J. Lawrence et al. [arxiv.org/abs/2208.11793] criticizing relational quantum mechanics (RQM) and based on a famous nonlocality theorem Going back to Greenberger Horne and Zeilinger (GHZ). Here, we show that the claims presented in this recent work are unjustified and we debunk the analysis.

Keywords Relational quantum mechanics · Greenberger Horne Zeilinger nonlocality · Wigner friends paradox

1 Introduction

The Relational Quantum Mechanics (RQM) is an alternative interpretation of quantum mechanics that was proposed originally by Rovelli [1–4]. RQM can be seen as a logical completion and generalization of the Copenhagen (orthodox) interpretation but where the arbitrariness of Heisenberg's quantum 'shifty-split' or 'cut', which is separating observed and observing subsystems, is taken more seriously. Unlike, the Copenhagen interpretation the cut is not confined to the macroscopic domain and the roles of observed and observing systems are relative and can be inverted. RQM is therefore a more symmetric and general approach.

Moreover, recently RQM has been criticized and assessed by various authors. The aim of the present comment (see also the analysis by Cavalcanti et al. [5]) is to give a short reply to the recent Lawrence et al. article [6] that concerns RQM and the role of quantum contextuality (for previous claims see [7, 8] and see the replies by

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Di Biagio and Rovelli [9] and Drezet [10, 11]). Lawrence et al. have recently replied to the present analysis in Ref. [12], and to the independent analysis [5] in Ref. [13].

2 The starting point

Following the recent analysis [10, 11], I remind that in RQM the main issue concerns the interpretation of the full wavefunction $|\Psi_{SO}\rangle$ involving observer (O) and observed system (S). In RQM the fundamental object relatively to (O) is not $|\Psi_{SO}\rangle$ but the reduced density matrix

$$\hat{\rho}_{S|O}^{(red.)} = \mathrm{Tr}_O[\hat{\rho}_{SO}] = \mathrm{Tr}_O[|\Psi_{SO}\rangle\langle\Psi_{SO}|]. \tag{1}$$

As it is well known $\hat{\rho}_{S|O}^{(red.)}$ is independent of the basis chosen to represent the degrees of freedom for (O). In Ref. [10] I showed that it solves the dilemma discussed in Refs. [7, 8] concerning the 'preferred basis problem'. Here I show that the same features debunk the claims of Ref. [6] concerning non-contextuality.

Ref. [6] starts with a GHZ state [14] for a system S of three spins $m = 1, 2, 3$:

$$\begin{aligned}|GHZ\rangle_S = \frac{1}{\sqrt{2}}[&|+1^{(1)}, +1^{(1)}, +1^{(1)}\rangle_{S_1,S_2,S_3}\\ &+|-1^{(1)}, -1^{(1)}, -1^{(1)}\rangle_{S_1,S_2,S_3}]\end{aligned} \tag{2}$$

where $|p^{(1)}\rangle_{S_m} \equiv |\mathrm{sign}(p)z\rangle_{S_m}$ (with $p = \pm 1$) are spin eigenstates along the z direction. We also have in different spin bases:

$$\begin{aligned}|GHZ\rangle_S = \frac{1}{2}[&|+1^{(2)}, +1^{(3)}, -1^{(3)}\rangle_{S_1,S_2,S_3}\\ &+|+1^{(2)}, -1^{(3)}, +1^{(3)}\rangle_{S_1,S_2,S_3}\\ &+|-1^{(2)}, +1^{(3)}, +1^{(3)}\rangle_{S_1,S_2,S_3}\\ &+|-1^{(2)}, -1^{(3)}, -1^{(3)}\rangle_{S_1,S_2,S_3}]\end{aligned} \tag{3}$$

where we used $|p^{(2)}\rangle_{S_m} = \frac{1}{\sqrt{2}}[|+1^{(1)}\rangle_{S_m} \pm |-1^{(3)}\rangle_{S_m}] \equiv |\mathrm{sign}(p)x\rangle_{S_m}$, and $|p^{(3)}\rangle_{S_m} = \frac{1}{\sqrt{2}}[|+1^{(1)}\rangle_{S_m} \pm i|-1^{(3)}\rangle_{S_m}] \equiv |\mathrm{sign}(p)y\rangle_{S_m}$. This implies

$$\sigma_{xS_1}\sigma_{yS_2}\sigma_{yS_3}|GHZ\rangle_S = -|GHZ\rangle_S \tag{4}$$

and

$$p_{S_1}^{(2)}q_{S_2}^{(3)}r_{S_3}^{(3)} = -1. \tag{5}$$

Similar expressions are obtained by circular permutations:

$$p_{S_1}^{(3)}q_{S_2}^{(2)}r_{S_3}^{(3)} = -1, \tag{6}$$

and

$$p^{(3)}_{S_1} q^{(3)}_{S_2} r^{(2)}_{S_3} = -1. \tag{7}$$

We can also write:

$$\begin{aligned} |GHZ\rangle_S = \frac{1}{2}[| + 1^{(2)}, +1^{(2)}, +1^{(2)}\rangle_{S_1,S_2,S_3} \\ +| + 1^{(2)}, -1^{(2)}, -1^{(2)}\rangle_{S_1,S_2,S_3} \\ +| - 1^{(2)}, +1^{(2)}, -1^{(2)}\rangle_{S_1,S_2,S_3} \\ +| - 1^{(2)}, -1^{(2)}, +1^{(2)}\rangle_{S_1,S_2,S_3}] \end{aligned} \tag{8}$$

implying

$$\sigma_{xS_1}\sigma_{xS_2}\sigma_{xS_3}|GHZ\rangle_S = +|GHZ\rangle_S \tag{9}$$

and thus

$$p^{(2)}_{S_1} q^{(2)}_{S_2} r^{(2)}_{S_3} = +1. \tag{10}$$

It is well known that quantum mechanics is highly contextual. If the results of spin measurements were non-contextual Eqs. 5, 6, 7 and 10 could be true together and this, clearly, is not possible: multiplying Eqs. 5, 6, 7 contradicts Eq. 10. This incompatibility also leads to a well known proof of quantum nonlocality without inequality [14].

3 Debunking a paradox

In Ref. [6] the authors use the previous results. First, they consider a single observer Alice (A) (composed of three qubits A_1, A_2, A_3) who measures the spins of the GHZ system S in the y bases. The GHZ states of Eq. 2 reads in the y bases:

$$|GHZ\rangle_S = \sum_{p,q,r} C^{333}_{pqr} |p^{(3)}, q^{(3)}, r^{(3)}\rangle_{S_1,S_2,S_3} \tag{11}$$

where the amplitudes C^{333}_{pqr} are non vanishing for each of the eight combinations p, q, r and where $|C^{333}_{pqr}|^2 = \frac{1}{8}$. After entanglement with Alice's qubits we have

$$|GHZ\rangle_{SA} = \sum_{p,q,r} C^{333}_{pqr} |p^{(3)}\rangle_{SA_1} |q^{(3)}\rangle_{SA_2} |r^{(3)}\rangle_{SA_3} \tag{12}$$

with $|k^{(3)}\rangle_{SA_m} := |k^{(3)}\rangle_{S_m} |k^{(3)}\rangle_{A_m}$ and $|k^{(3)}\rangle_{A_m}$ is the state of the qubit A_m, $m = 1, 2, 3$ (for $k = \pm 1$).

Importantly, for Alice this state shows no correlation. This is clear in RQM where the reduced density matrix reads $\hat{\rho}^{(red.)}_{S|A} = \mathrm{Tr}_A[\hat{\rho}_{SA}]$:

$$\hat{\rho}^{(red.)}_{S|A} = \frac{1}{8}\sum_{p,q,r} |p^{(3)}\rangle_{S_1S_1}\langle p^{(3)}| \otimes |q^{(3)}\rangle_{S_2S_2}\langle q^{(3)}| \otimes |r^{(3)}\rangle_{S_3S_3}\langle r^{(3)}| \tag{13}$$

Due to decoherence i.e., entanglement with the environement (Alice) we have lost coherence and correlations between spins. In particular we have

$$\begin{aligned} \mathrm{Tr}_S[\sigma_{xS_1}\sigma_{xS_2}\sigma_{xS_3}\hat{\rho}^{(red.)}_{S|A}] = 0, \\ \mathrm{Tr}_S[\sigma_{xS_1}\sigma_{yS_2}\sigma_{yS_3}\hat{\rho}^{(red.)}_{S|A}] = 0, \\ \mathrm{Tr}_S[\sigma_{yS_1}\sigma_{xS_2}\sigma_{yS_3}\hat{\rho}^{(red.)}_{S|A}] = 0, \\ \mathrm{Tr}_S[\sigma_{yS_1}\sigma_{yS_2}\sigma_{xS_3}\hat{\rho}^{(red.)}_{S|A}] = 0, \end{aligned} \tag{14}$$

which contrast with

$$\begin{aligned} \mathrm{Tr}_S[\sigma_{xS_1}\sigma_{xS_2}\sigma_{xS_3}\hat{\rho}^{(red.)}_{S}] = +1, \\ \mathrm{Tr}_S[\sigma_{xS_1}\sigma_{yS_2}\sigma_{yS_3}\hat{\rho}^{(red.)}_{S}] = -1, \\ \mathrm{Tr}_S[\sigma_{yS_1}\sigma_{xS_2}\sigma_{yS_3}\hat{\rho}^{(red.)}_{S}] = -1, \\ \mathrm{Tr}_S[\sigma_{yS_1}\sigma_{yS_2}\sigma_{xS_3}\hat{\rho}^{(red.)}_{S}] = -1, \end{aligned} \tag{15}$$

In RQM Eq. 15 actually describes the correlations available to A before the interaction occurred, i.e., when the full state of SA is still factorized. It represents a catalog of knowledge or potentiality in the sense of Heisenberg. The actualization of measurements in RQM is a debatable issue and we will not consider this problem here (see e.g., [10]).

In the next step the authors of Ref. [6] consider a second observer Bob (B) (also composed of 3 qubits B_1, B_2, B_3) who measures the spins of the entangled GHZ system SA in the x bases of the joint system. In analogy with Eq. 8 we write after entanglement with Bob qubits:

$$|GHZ\rangle_{SAB} = \frac{1}{2}\sum_{p,q,r} |p^{(2)}\rangle_{SAB_1}|q^{(2)}\rangle_{SAB_2}|r^{(2)}\rangle_{SAB_3} \tag{16}$$

with $|k^{(2)}\rangle_{SAB_m} := |k^{(2)}\rangle_{SA_m}|k^{(2)}\rangle_{B_m}$ and $|k^{(2)}\rangle_{B_m}$ is the state of the qubit B_m, $m = 1, 2, 3$ (for $k = \pm 1$) and where we introduced the entangled 'x' states for the SA system: $|k^{(2)}\rangle_{SA_m} = \frac{1}{\sqrt{2}}[| + 1^{(3)}\rangle_{SA_m} \pm i| - 1^{(3)}\rangle_{SA_m}] \equiv |\mathrm{sign}(p)x\rangle_{SA_m}$. Crucially, the numbers $p, q, r = \pm 1$ in Eq. 16 must obey the GHZ-constraint:

$$p^{(2)}_{SAB_1} q^{(2)}_{SAB_2} r^{(2)}_{SAB_3} = +1. \tag{17}$$

Once more, in RQM we need to consider the reduced density matrix $\hat{\rho}^{(red.)}_{SA|B} = \mathrm{Tr}_B[\hat{\rho}_{SAB}]$:

$$\hat{\rho}^{(red.)}_{SA|B} = \frac{1}{4}\sum_{p,q,r} |p^{(2)}\rangle_{SA_1SA_1}\langle p^{(2)}| \otimes |q^{(2)}\rangle_{SA_2SA_2}\langle q^{(2)}| \otimes |r^{(2)}\rangle_{SA_3SA_3}\langle r^{(2)}| \tag{18}$$

with again $pqr = +1$. This defines the information available to B in RQM and this density matrix shows partial coherence since we have

$$\begin{aligned} \mathrm{Tr}_{SA}[\sigma_{xSA_1}\sigma_{xSA_2}\sigma_{xSA_3}\hat{\rho}^{(red.)}_{SA|B}] &= +1,\\ \mathrm{Tr}_{SA}[\sigma_{xSA_1}\sigma_{ySA_2}\sigma_{ySA_3}\hat{\rho}^{(red.)}_{SA|B}] &= 0,\\ \mathrm{Tr}_{SA}[\sigma_{ySA_1}\sigma_{xSA_2}\sigma_{ySA_3}\hat{\rho}^{(red.)}_{SA|B}] &= 0,\\ \mathrm{Tr}_{SA}[\sigma_{ySA_1}\sigma_{ySA_2}\sigma_{xSA_3}\hat{\rho}^{(red.)}_{SA|B}] &= 0. \end{aligned} \tag{19}$$

The first line of Eq. 19 is of course reminiscent of Eq. 17 and shows that there is a preferred pointer basis defined by the specific measurement protocol. This is associated with a specific interaction Hamiltonian $H_{SA,B}$ leading to the state given by Eq. 16.

However, the authors of Ref. [6] didn't correctly analyze the structure of RQM and the meaning of relative facts. They claim that we can find relations between facts or information available to Bob and facts or information available to Alice. This we show below is actually a misunderstanding. More precisely, they consider that Bob only measures one of the three qubits belonging to SA. In the following we consider the particular case $m = 1$ and therefore only the system SA_1 will interact with B_1. This requires to let the two other qubits of Bob B_2 and B_3 in their respective ground states. As the authors show we get the new state

$$|GHZ'\rangle_{SAB} = \frac{1}{2}\sum_{p,q,r} |p^{(2)}\rangle_{SAB_1}|q^{(3)}\rangle_{SA_2} \times|in\rangle_{B_2}|r^{(3)}\rangle_{SA_3}|in\rangle_{B_2} \tag{20}$$

with now the constraint:

$$p^{(2)}_{SAB_1}q^{(3)}_{SA_2}r^{(3)}_{SA_3} = -1. \tag{21}$$

Of course, we could develop two similar procedures acting only on the qubit B_2 or alternatively the qubit B_3. We will obtain two different states $|GHZ''\rangle_{SAB}$ and $|GHZ'''\rangle_{SAB}$ leading to the relations

$$p^{(3)}_{SA_1}q^{(2)}_{SAB_2}r^{(3)}_{SA_3} = -1, \tag{22}$$

and

$$p^{(3)}_{SA_1}q^{(3)}_{SA_2}r^{(2)}_{SAB_3} = -1. \tag{23}$$

With these mathematical properties the deduction of Ref. [6] goes as follows:

(i) It is visible from Eq. 21 (and similarly for Eqs. 22, 23 by cyclic permutation) that the number $p^{(2)}_{SAB_1}$ characterizes the entangled system SAB_1 whereas $q^{(3)}_{SA_2}$ and $r^{(3)}_{SA_3}$ characterize SA_2 and SA_3. Therefore it is tempting to call $p^{(2)}_{SAB_1}$ a relative fact for Bob and $q^{(3)}_{SA_2}$ and $r^{(3)}_{SA_3}$ relative facts for Alice. This idea, which is central for their paper, is clearly summarized by the analysis surrounding their Eq. 17 in Ref. [6]. They call $\mathcal{B}_m$ the number $k^{(2)}_{SAB_m}$ defined in our Eqs. 17, 21–23 and similarly they call $\mathcal{A}_m$ the number $k^{(3)}_{SA_m}$. Assuming this we go to the next step of their 'no-go' deduction.

(ii) The four relations Eqs. 17, 21–23. are clearly incompatible. If we multiply Eqs. 21 by 22 and 23 we obtain

$$\begin{aligned} & p^{(2)}_{SAB_1} q^{(2)}_{SAB_2} r^{(2)}_{SAB_3} (p^{(3)}_{SA_1} q^{(3)}_{SA_2} r^{(3)}_{SA_3})^2 \\ & = p^{(2)}_{SAB_1} q^{(2)}_{SAB_2} r^{(2)}_{SAB_3} = -1, \end{aligned} \tag{24}$$

which clearly contradicts Eq. 17. Now as they emphasize it clearly: '*Note that, most importantly for the sequel, the three [unitary] transformations* [*acting on* $m = 1, 2, 3$] *mutually commute, and thus their order of application is immaterial.*'. In other words: Since the three operations leading to $|GHZ'\rangle_{SAB}$, $|GHZ''\rangle_{SAB}$, $|GHZ''\rangle_{SAB}$ and thus Eqs. 21–23 are acting 'locally' only on one of the sub systems SA_m their meaning should be non contextual and absolute. This following [6] justifies why we should apriori compare these states with $|GHZ\rangle_{SAB}$ and Eq. 17. This noncontextual reading is what they believe is contained in RQM.

If we accept this reasoning then RQM contradicts quantum mechanics. Relative facts for Alice and Bob are contradictory.

However, this deduction is false and we now debunk the contradiction. First, consider point (i): It is clear that $p^{(2)}_{SAB_1}$ must be a relative fact for Bob. However, there is no reason to consider that $q^{(3)}_{SA_2}$ and $r^{(3)}_{SA_3}$ should be relative facts for Alice. Actually, the quantum state Eq. 20 contains quantum numbers p, q, r but in RQM the fundamental description for Bob is the reduced density matrix $\hat{\rho}'^{(red.)}_{SA|B} = \mathrm{Tr}_B[\hat{\rho}'_{SAB}]$ obtained by using $\hat{\rho}'_{SAB} = |GHZ'\rangle_{SAB\,SAB}\langle|GHZ'|$. We have:

$$\begin{aligned} \hat{\rho}'^{(red.)}_{SA|B} = & \frac{1}{2}|+1^{(2)}\rangle_{SA_1\,SA_1}\langle+1^{(2)}| \\ & \otimes|\Phi\rangle_{SA_2,SA_3\,SA_2,SA_3}\langle\Phi| \\ & +\frac{1}{2}|-1^{(2)}\rangle_{SA_1\,SA_1}\langle-1^{(2)}| \\ & \otimes|\Psi\rangle_{SA_2,SA_3\,SA_2,SA_3}\langle\Psi| \end{aligned} \tag{25}$$

where

$$|\Phi\rangle_{SA_2,SA_3} = \frac{1}{\sqrt{2}}[|+1^{(3)}\rangle_{SA_2}|-1^{(2)}\rangle_{SA_3} + |-1^{(3)}\rangle_{SA_2}|+1^{(2)}\rangle_{SA_3}],$$
$$|\Psi\rangle_{SA_2,SA_3} = \frac{1}{\sqrt{2}}[|+1^{(3)}\rangle_{SA_2}|+1^{(2)}\rangle_{SA_3} + |-1^{(3)}\rangle_{SA_2}|-1^{(2)}\rangle_{SA_3}]. \tag{26}$$

are two Bell states. The fact that EPR-like Bell states are present show that there is a coherence preserved in the description of the system SA by Bob. In particular we deduce

$$\begin{aligned}
\mathrm{Tr}_{SA}[\sigma_{xSA_1}\sigma_{xSA_2}\sigma_{xSA_3}\hat{\rho}'^{(red.)}_{SA|B}] &= +1,\\
\mathrm{Tr}_{SA}[\sigma_{xSA_1}\sigma_{ySA_2}\sigma_{ySA_3}\hat{\rho}'^{(red.)}_{SA|B}] &= -1,\\
\mathrm{Tr}_{SA}[\sigma_{ySA_1}\sigma_{xSA_2}\sigma_{ySA_3}\hat{\rho}'^{(red.)}_{SA|B}] &= 0,\\
\mathrm{Tr}_{SA}[\sigma_{ySA_1}\sigma_{ySA_2}\sigma_{xSA_3}\hat{\rho}'^{(red.)}_{SA|B}] &= 0.
\end{aligned} \tag{27}$$

The second line is of course reminiscent of Eq. 21 but the meaning is here different from what is claimed in Ref. [6]. Indeed, this a relation for Bob measurements not for Alice. The idea to have a mixture of information for Bob and Alice in the same Eq. 21 is thus unjustified and is not a part of RQM but instead of the reading of RQM made by the authors of Ref. [6]. The first line is also very interesting: It shows coherence associated with the $|\Phi\rangle_{SA_2,SA_3}$ and $|\Psi\rangle_{SA_2,SA_3}$ Bell states. Moreover this result can be compared with Eq. 19 for $\hat{\rho}^{(red.)}_{SA|B}$. The difference clearly stresses that the measurement procedures are more invasive that claimed in Ref. [6]. In RQM like in the Copenhagen interpretation the experimental context and the choice of the interaction Hamiltonian is key. This allows us us to answer to point ii) concerning non-contextuality. Indeed, the non contextuality supposed by the authors of Ref. [6] is not a part of the RQM. For RQM (like in the orthodox interpretation) the different contexts are not compatible and we have no right to compare Eqs. 17 and 21–23 as claimed. This would otherwise contradict the central axioms of RQM and therefore the reasoning discussed in Ref. [6] is unjustified.

We conclude this analysis by adding that RQM is a self-consistent interpretation of quantum mechanics extending the old Copenhagen interpretation. The formalism is perfectly agreeing with standard quantum mechanics and recovers the orthodox interpretation at the limit where observers are essentially macroscopic but without the problems concerning the definition of agents in Qbism. Several issues concerning interactions can still be debated but we should not focus on wrong problems. Often objections are done without carefully considering the philosophy of RQM but by adding some prejudices or preconceptions concerning the interpretation (i.e., [6–8]). I hope this note will contribute to clarify a bit this issue.

The author thanks very useful discussions with Carlo Rovelli, Eric Cavalcanti, and Marek Źukowski.

Author Contributions I am the sole author of the manuscript.

Data Availability No Data associated in the manuscript.

Declarations

Competing interest The Author declares no competing interest for this work.

References

1. Rovelli, C.: Relational quantum mechanics. Int. J. Phys. **35**, 1637 (1996)
2. Rovelli, C.: Helgoland: making sense of the quantum revolution. Riverhead Books, New York (2021)
3. Laudisa, F., Rovelli, C.: Relational quantum mechanics. In: Zalta, E.N. (ed.) The stanford encyclopedia of philosophy. Metaphysics Research Laboratory, Stanford University, Stanford (2019)
4. Di Biagio, A., Rovelli, C.: Stable facts, relative facts. Found. Phys. **51**, 30 (2021)
5. Cavalcanti, E.G., Di Biagio, A., Rovelli, C., On the consistency of relative facts (2023) arxiv.org/abs/2305.07343
6. Lawrence, J., Markiewicz, M., Źukowski, M.: Relative facts do not exist: relational quantum mechanics is incompatible with quantum mechanics (2022) arXiv:arxiv.org/abs/2208.11793
7. Brukner, Č.: Qubits are not observers-a no-go theorem (2021) arXiv: arxiv.org/abs/2107.03513v1
8. Pienaar, J.: A quintet of quandaries: five no-go theorems for relational quantum mechanics. Found. Phys. **51**, 97 (2021)
9. Di Biagio, A., Rovelli, C.: Relational quantum mechanics is about facts, not states: a reply to Pienaar and Brukner. Found. Phys. **52**, 62 (2021)
10. Drezet, A.: In defense of relational quantum mechanics: a note on ‘Qubits are not observers’ (2022). arxiv.org/abs/2202.02615
11. Drezet, A.: Can a Bohmian be a Rovellian for all practical purposes? Found. Phys. **51**, 97 (2023)
12. Lawrence, J., Markiewicz, M., Źukowski, M.: Relative facts do not exist: relational quantum mechanics is incompatible with quantum mechanics. Response to the critique by Aurélien Drezet (2022).. arxiv.org/abs/2210.09025
13. Lawrence, J., Markiewicz, M., Źukowski, M.: Relational quantum mechanics is still incompatible with quantum mechanics (2023). arxiv.org/abs/2310.18008
14. Greenberger, D., Horne, A.M., Zeilinger, A.: Going beyond Bell’s theorem. In: Kafatos, M. (ed.) Bell’s theorem, quantum theory and conceptions of the universe, pp. 69–72. Springer, Dordrecht (1989)