Foundations of Physics (2022) 52:64
https://doi.org/10.1007/s10701-022-00580-y

How Different Interpretations of Quantum Mechanics can Enrich Each Other: The Case of the Relational Quantum Mechanics and the Modal-Hamiltonian Interpretation

Olimpia Lombardi[1] · **Juan Sebastián Ardenghi**[2]

Received: 26 February 2022 / Accepted: 13 May 2022 / Published online: 7 June 2022

Abstract

In the literature on the interpretation of quantum mechanics, not many works attempt to adopt a proactive perspective aimed at seeing how different interpretations can enrich each other through a productive dialogue. In particular, few proposals have been devised to show that different approaches can be clarified by comparing them, and can even complement each other, improving or leading to a more fertile overall approach. The purpose of this paper is framed within this perspective of complementation and mutual enrichment. In particular, the Relational Quantum Mechanics (RQM) and the Modal-Hamiltonian Interpretation (MHI) are compared, highlighting their differences and points of contact. The final purpose is to show that, in spite of their disagreements, they are not contradictory but, on the contrary, they can be made compatible from an overarching perspective and can even complement each other in a fruitful way.

Keywords Relational Quantum Mechanics · Modal-Hamiltonian Interpretation · Quantum observables · Quantum state · Relational time

1 Introduction

The physical and metaphysical novelties introduced by quantum mechanics are so radical that the foundations of the theory have been under uninterrupted discussion for more than a century. It is therefore not surprising that all interpretations are forced to introduce elements that challenge some traditional scientific and/or philosophical assumptions. There is no way out, quantum mechanics is

✉ Olimpia Lombardi
olimpiafilo@gmail.com

1 University of Buenos Aires and CONICET, Buenos Aires, Argentina

2 National University of Bahia Blanca and CONICET, Bahía Blanca, Argentina

so peculiar that some price must be paid. Given this situation, in general, discussions about the interpretation of the theory usually confront different researchers, each one defending a particular point of view and emphasizing the difficulties or limitations of the others. The most neutral and balanced presentations try to compare different theoretical and/or interpretive approaches, making their pros and cons explicit. However, not many works attempt to adopt a more proactive perspective, aimed at seeing how different interpretations can enrich each other through a productive dialogue. In particular, few proposals have been devised to show that different approaches can be clarified by comparing them, and can even complement each other, improving or leading to a more fertile overall approach.

The purpose of this paper is framed within this perspective of complementation and mutual enrichment. In particular, the Relational Quantum Mechanics (RQM) and the Modal-Hamiltonian Interpretation (MHI) will be compared, highlighting their differences and points of contact. The final purpose will be to show that, in spite of their disagreements, they are not contradictory but, on the contrary, they can be made compatible from an overarching perspective and can even complement each other in a fruitful way. With this purpose, the paper is organized as follows. In Sects. 2 and 3, RQM and MHI will be briefly introduced, respectively. In Sect. 4, it will be shown that, despite the difference between the two interpretations, there are several agreements that open the way to a fertile complementation. Section 5 is devoted precisely to develop the different aspects in which RQM and MHI can complement each other. The article closes with some final remarks in Sect. 6.

2 Relational Quantum Mechanics

Proposed in 1996 by Carlo Rovelli [1], RQM has sparked great interest in the community of the philosophy and the foundations of quantum mechanics. Since its presentation (although not its essence) has slightly mutated over the years, we will mainly rely on the most recent formulations. As there are multiple works in relation to RQM, both by its author and by various commentators due to the interest it has aroused, the present description will be only a sketch and will refer to the relevant works.

Some generic assumptions constitute the broad framework of RQM.

- The standard formalism of quantum mechanics is retained: "RQM does not suggest changing the formalism of quantum theory—as alternative formulations of the theory do—but rather modifies the conceptual schemes with which we can interpret the formalism, and consequently, our metaphysics." ([2], p. 22).
- Quantum mechanics is "complete": it applies to any physical system, there is not "classical cut". In this sense, "RQM is a sort of «democratised» Copenhagen." ([2], p. 13).

- The interpretation is realist in the sense that, by contrast to instrumentalist or subjective standpoints, it intends to describe how the world is: "RQM can make sense of a fully quantum world" ([2], p. 2).
- There are not observers as privileged systems: "Any system, irrespectively of its size, complexity or else, can play the role of the textbook's quantum mechanical observer." ([2], p. 6).
- Measurements are ordinary physical interactions: "any interaction counts as a measurement, to the extent one system affects the other and this influence depends on a variable of the first system." ([2], pp. 6–7).
- Since "RQM is essentially a refinement of the textbook «Copenhagen» interpretation" ([2], p. 1), it adopts the following principles: Schrödinger evolution, projection postulate, and Born rule.

In addition to these generic assumptions, RQM relies on particular theses that endow it with its interpretive specificity:

(a) "*Variables take value only at interactions*" ([2], p. 1, italics in the original). Since an event is the acquisition of a value by a variable or observable, "[t]he world is […] an evolving network of sparse relative *events*" ([2], p. 1, italics in the original). In other words, events are always discrete and actual. This is a manifestation of quantum discreteness, which "is not an accessory aspect of quantum theory: it is its most characteristic feature (and it gives the theory its name). […] The history of a quantum particle, for instance, is neither a continuous line in spacetime (as in classical mechanics), nor a continuous wave function on spacetime. Rather, with respect to any other system it is a discrete set of interactions, each localized in spacetime." ([2], p. 12).

(b) "[T]he values they [the variables] take are only *relative* to the (other) system affected by the interaction." ([2], p. 1, italics in the original). As a consequence, a quantum mechanical description cannot be taken as an "absolute" description of reality [1]. Nevertheless, this does not involve any kind of subjectivity: "Here «relative» is in the same sense in which velocity is a property of a system *relative* to another system in classical mechanics." ([2], p. 1, italics in the original).

(c) The wave function—and, in general, the quantum state—is not realistically interpreted: it is "a theoretical tool to facilitate the computation of the probabilities of future events on the basis of certain given knowledge" ([2], p. 1), similar to the Hamilton–Jacobi functional in classical mechanics. Since what we measure are values of variables, in principle the quantum state is dispensable: "Quantum mechanics can be formulated without reference to the quantum state, as a theory of probabilities for sequences of events." ([2], p. 8).

The paradigmatic application of the RQM is the so-called "third person problem" [1], which is considered one of the cornerstones of RQM and the main argument for the need of the relativization of events in one system to another system. In a certain sense, the third person problem is a "depersonalized" version of the thought experiment known as "*the Wigner friend*" (see [3]). Let us consider a system S at t_1 in a state $\alpha_1|a_1\rangle + \alpha_2|a_2\rangle$, which is a superposition regarding the observable A, with eigenstates $|a_1\rangle$ and $|a_2\rangle$ corresponding to the eigenvalues a_1 and a_2. If at t_2 another system O measures S, then the observable A acquires a definite value, say, a_1. However, if a system P describes the composite system $S+O$, the state of the composite system at t_2 will be $\alpha_1|a_1\rangle|b_1\rangle + \alpha_2|a_2\rangle|b_2\rangle$, where $|b_1\rangle$ and $|b_2\rangle$ are eigenstates of an observable B of O corresponding to the eigenvalues b_1 and b_2; as a consequence, according to this description, the observable A does not have a definite value since P only describes but has not interacted with $S+O$. This situation leads to a contradiction—A has a definite value and A does not have a definite value—if the assignment of a value to an observable is conceived as absolute. But there is no contradiction in the framework of RQM, since *the event of A acquiring a definite value is defined relative to O and not relative to P*. In other words, "If the variable A of a system S takes a value in the interaction with a second system S', the value it takes is only *relative to* S'. The actualisation of an event is always relative to a system." ([2], p. 4, italics in the original). In turn, if the composite system $S+O$ is measured by P through the interaction between P and $S+O$, then both observables A and B acquire values relative to P, say, a_1 and b_1.

A possible criticism consists in arguing that nothing seems to prevent that the observable A acquires the value a_1 relative to O and the value a_2 relative to P. The first point to stress is that this is not a *logical* contradiction, because *the event of A acquiring a definite value relative to O and the event of A acquiring a definite value relative to P are different events.* Nevertheless, this disagreement would be *physically* unacceptable. But quantum mechanics, when applied to this kind of situations, shows that the probability of such a disagreement is zero (for a similar criticism, see [4], and a response in [5]; see also [6]).

Several aspects of RQM will not be discussed here because, although important, they are not relevant in the context of the purpose of the present work. For example, the role of the concept of information in RQM and the attempt of reconstructing quantum mechanics on the basis of few postulates will be not mentioned here. Moreover, the interpretation has sparked varied and intense debates (besides the above mentioned criticism and the response to it, see [7, 8] and responses in [9]), but they are beyond the limits of the present paper.

3 Modal-Hamiltonian Interpretation

The modal interpretations of quantum mechanics find their roots in the works of Bas van Fraassen [10, 11]. Since the 1980's a series of authors began to develop different versions of the modal themes, although agreeing in some general points (see [12]):

- The interpretations are based on the standard formalism of quantum mechanics.
- Quantum mechanics is "universal": it applies to any physical system, both to microscopic and to macroscopic systems.

- The interpretations are realist, in the sense that they intend to describe quantum systems and their properties.
- Quantum measurements are ordinary physical interactions and there is no collapse.
- The quantum state contains information about the probabilities of the possible physical properties of the system.

As well known, the Kochen-Specker theorem [13] proves the impossibility of ascribing precise values to all the observables of a quantum system simultaneously, while preserving the functional relations between commuting observables. Therefore, all modal interpretations are committed to supply a rule that picks out, from the set of all the observables of a quantum system, the subset of definite-valued properties that constitutes the preferred context.

The specificity of MHI [14] is that it endows the Hamiltonian of the quantum system with a determining role, both in the definition of systems and subsystems and in the selection of the preferred context. On this basis, it relies on the following postulates:

(SP) Systems postulate: A quantum system S is represented by a pair $(\mathcal{O}, H)$ such that (i) $\mathcal{O}$ is a space of self-adjoint operators on a Hilbert space $\mathcal{H}$, representing the observables of the system, (ii) $H \in \mathcal{O}$ is the time-independent Hamiltonian of the system S, and (iii) if $\rho_0 \in \mathcal{O}'$ (where $\mathcal{O}'$ is the dual space of $\mathcal{O}$) is the initial state of S, it evolves according to the Schrödinger equation in its von Neumann version.

(CSP) Composite systems postulate: A quantum system represented by $S: (\mathcal{O}, H)$ is *composite* when it can be partitioned into two quantum systems $S^1: (\mathcal{O}^1, H^1)$ and $S^2: (\mathcal{O}^2, H^2)$ such that (i) $\mathcal{O} = \mathcal{O}^1 \otimes \mathcal{O}^2$, and (ii) $H = H^1 \otimes I^2 + I^1 \otimes H^2$, (where I^1 and I^2 are the identity operators in the corresponding tensor product spaces). In this case, we say that S^1 and S^2 are *subsystems* of the composite system, $S = S^1 + S^2$. If the system is not composite, it is *elemental*.

(AR) Actualization rule: Given an elemental quantum system represented by $S: (\mathcal{O}, H)$ the actual-valued observables of S are the Hamiltonian H and all the observables commuting with H and having, at least, the same symmetries as H.

The central idea of the MHI's actualization rule is that the Hamiltonian of the system defines actualization. Any observable that does not commute with and/or does not have the symmetries of the Hamiltonian cannot acquire a definite actual value, since this actualization would break the symmetry of the system in an arbitrary way. This rule has been applied to many physical situations—e.g. the free particle with spin, the harmonic oscillator, the free hydrogen atom, the Zeeman effect, the fine structure, and the Born–Oppenheimer approximation—resulting in descriptions consistent with empirical evidence (see [14], Sect. 5). The interpretation has also proved to be adequate in other situations, such as the problem of optical isomerism in chemistry [15], and the emergence of a relational quantum event-time [16].

MHI can be formulated under a Galilean-invariant form, in terms of the Casimir operators of the Galilean group [17, 18]. Moreover, the MHI's actualization rule can be transferred to the relativistic domain by changing the symmetry group accordingly: the definite-valued observables are those represented by the Casimir operators of the Poincaré group, that is, the mass operator and the squared spin operator [19, 20]. This agrees with the usual assumption in quantum field theory that elemental particles always have definite values of mass and spin, and those values are precisely what define the different kinds of elemental particles of the theory. From an ontological viewpoint, the MHI proposes an ontology of properties, lacking the ontological category of individual [21, 22], but does not prevent the emergence of particles under particular circumstances [23]. This ontological view supplies an adequate answer to the problem of the entanglement of indistinguishable systems [24].

From the MHI perspective, a single measurement (for the difference between single measurement, frequency measurement and state measurement, see [14], Sect. 6) is a three-stage process:

- *First Stage* (I): The system $S:\ (\mathcal{O}^S, H^S)$ to be measured and the measuring apparatus $M:\ (\mathcal{O}^M, H^M)$ do not interact. Therefore, they are elemental subsystems of the composite system $U^{(I)}:\ (\mathcal{O}^{(I)}, H^{(I)})$, where $\mathcal{O}^{(I)} = \mathcal{O}^S \otimes \mathcal{O}^M$ and $H^{(I)} = H^S \otimes I^M + I^S \otimes H^M$. The system S is in a state $\sum_{i=1}^n c_i |a_i\rangle$, where the $|a_i\rangle$ are the eigenstates of an observable A of S, and the apparatus M is in a ready-to-measure state $|p_0\rangle$, eigenstate of the pointer P of M. Thus, the state of $U^{(I)}$ in this stage is
$$\left|\Psi^{(I)}\right\rangle = \left(\sum_{i=1}^n c_i |a_i\rangle\right) \otimes |p_0\rangle.$$
- *Second Stage* (II): S and M interact through an interaction Hamiltonian H^{int} that introduces a correlation between the eigenstates $|a_i\rangle$ of A and the eigenstates $|p_i\rangle$ of P. Therefore, the whole system becomes the elemental system $U^{(II)}:\ (\mathcal{O}^{(II)}, H^{(II)})$, where $\mathcal{O}^{(II)} = \mathcal{O}^{(I)}$ and $H^{(II)} = H^S \otimes I^M + I^S \otimes H^M + H^{\text{int}}$, and whose state is
$$\left|\Psi^{(II)}\right\rangle = \sum_{i=1}^n c_i |a_i\rangle \otimes |p_i\rangle.$$
- *Third Stage* (III): The interaction ends and the whole system is again composite, $U^{(III)} = U^{(I)}$: S and M become elemental systems as in the first stage. The state in this stage is $\left|\Psi^{(III)}\right\rangle = \left|\Psi^{(II)}\right\rangle$.

The measurement problem consists in explaining why the pointer P of the apparatus M acquires a definite value in Stage III. According to the MHI's actualization rule, P is a definite-valued observable because P commutes with H^M and does not break its symmetries. These features are supported by plausible physical reasons: (i) for the reading of the pointer to be possible, the eigenvectors $|p_i\rangle$ of P have to be stationary, that is, $\left[P, H^M\right] = 0$, and (ii) since P is a "collective", highly degenerate observable (see [25, 26]), in general it has more degeneracies than H^M, which is general is non-degenerate and, as a consequence, has no symmetries.

This MHI account of quantum measurement problem supplies an answer to the problem both in its ideal and its non-ideal versions, overcoming some well known-criticisms to the original modal interpretations (see [27–29]). In particular, it was successfully applied to the Stern-Gerlach experiment taking into account the possibility of infinite "tails" [30]. In the non-ideal case, it gives a criterion to distinguish between reliable and non-reliable measurements ([14], Sect. 6), a criterion that can be generalized when expressed in informational terms [31]. Moreover, this MHI view of measurement can also account for the correlations observed in consecutive measurements [32].

4 Agreements Despite the Differences

The brief presentations of the previous sections clearly highlight the differences between RQM and MHI. The first evident difference is that, whereas according to RQM the properties of a system acquire definite values only relative to another system, MHI endows a closed system with certain definite-valued non-relational properties. As will be argued below, this fact is not an obstacle to the compatibility between the two interpretations, but rather an opportunity for complementarity.

The second aspect that distinguishes the two interpretations is that RQM admits collapse, whereas MHI is a non-collapse interpretation. However, this is not a deep discrepancy to the extent that in neither of the two cases the quantum state is conceived as a physical field. In particular, according to RQM, collapse is an epistemic fact, not a physical phenomenon: "Unitary evolution requires the system to be isolated, which is exactly what ceases to be true during the measurement, because of the interaction with the observer. If we include the observer into the system, then the evolution is still unitary, but we are now dealing with the description of a different observer." ([1], p. 1672). More recently it has been insisted that RQM explicitly departs from "physical collapse" theories and that it does not admit "mysterious collapse and jumps" ([2], p. 8).

Despite these differences, RQM and MHI agree in several substantial aspects. First, it is worth noting that MHI accepts most of the generic assumptions of RQM: adoption of the standard formalism, no classical cut, realism, no need of undefined concept of observer, measurements as ordinary physical interactions. These points constitute a relevant common ground on which the two interpretations can engage in a fertile dialogue.

Another point of contact between the two interpretations is the fact that in both cases they start with the observables of the system. This is explicit in MHI, which adopts the algebraic approach as a departing point (see [14], p. 384; for the ontological relevance of this formal choice, see also [22–24]). The priority of physical variables representing properties is also a leitmotiv of RQM: "we do describe the world in terms of «properties» that the systems have and values assumed by various quantities, not in terms of states in the Hilbert space." ([1], p. 1646). This means that, even if not so explicit, the algebraic approach is also presupposed in this case, according to which "[e]ach physical system can be characterized by a set (in fact, an algebra) of physical variables $A_1, ..., A_n$" ([5], p. 1). As van Fraassen states, the algebra of observables is a stable observer-independent feature, an "absolute" in RQM ([33], p. 391).

Precisely due to the priority of the observables/variables, according to the two interpretations the quantum state does not represent a physical field of stuff or properties. The state in quantum mechanics is a device to compute probabilities for the values of the observables/variables. As Federico Laudisa and Rovelli ([2], p. 7) clearly stress: "The interpretation of the wave function in the context of RQM is akin the interpretation of the Hamilton–Jacobi functional in classical mechanics: a theoretical tool to facilitate the computation of the probabilities of future events on the basis of certain given knowledge." An analogous idea is expressed in the framework of MHI: "To the extent that states are defined as expectation-value functionals on the algebra of observables, their «nature» is exhausted in fulfilling the task of computing the expectation values of the observables of the algebra." ([24], p. 237).

The probabilities encoded in the state are applied to the possible values of the observables/variables of a quantum system; they measure a certain tendency or potentiality of each value to become actual. Both in RQM and in MHI, actualization is a spontaneous phenomenon that cannot be explained or predicted by the theory. This feature is not conceived as a shortcoming of quantum mechanics, but as the manifestation of the indetermination of nature. As Laudisa [3] points out, RQM gives no deeper justification or underlying dynamical representation of the actualization of quantum events at interactions: "Quantum mechanics gives probabilities for quantum events to happen, not a story representing how they happen." ([2], p. 19). MHI is also clear about this point: "among the possible facts belonging to that set [the set of possible facts where actualization occurs], one and only one becomes actual. But, as a consequence of its intrinsic probabilistic nature, quantum mechanics does not determine which one of those possible facts is the actual one." ([14], p. 430). In other words, according to both RQM and MHI, the actual acquisition of a definite value by an observable/variable is an event: events are actual, objective, and irreducibly indeterministic phenomena, which occur at the interactions.

In summary, although RQM and MHI are clearly different interpretations, they share many conceptual assumptions that frame their views about quantum mechanics. It is this conceptual framework of shared assumptions that provides the ground on which the differences between them can be conceived as complementary rather than as rival interpretive aspects.

5 RQM and MHI as Complementary Interpretations

5.1 Closed and Open Systems

In RQM, interactions play a leading role: variables only acquire definite values when systems interact. But, what about closed systems? RQM says nothing beyond that they unitarily evolve according to the Schrödinger equation: it gives no clue about closed systems' properties. This point has been noted by Claudio Calosi and Cristian Mariani when they ask: "What about noninteracting quantum systems? It follows that they do not have definite value properties. Does this mean they have no properties at all?" ([34], p. 161).

According to Mauro Dorato [35], RQM does not need to deny the existence of isolated quantum system. In his attempt to assign properties also to closed systems, Dorato offers a dispositional interpretation of RQM according to which closed systems only have dispositions that actualize in interactions: "both non-interacting quantum systems S and observers O have no intrinsic properties, except *dispositional* ones. In other words, such systems S have intrinsic dispositions to correlate with other systems/observers O, which manifest themselves as the possession of definite properties q relative to those Os" ([35], p. 239, italics in the original).

However, it is still worth asking why there are no non-dispositional properties that can be assigned to the system. In principle, the assumption that a system has both intrinsic *and* dispositional properties does not seem unreasonable, except to a pandispositionalist. But physics is not friendly to pandispositionalism since, in fact, properties that are not state-dependent, such as mass, charge, and spin, are usually considered intrinsic properties of quantum systems. This is the point at which MHI has something to say, not only about dispositions, but also about the intrinsic properties of closed systems. According to MHI, although most of the properties of a system may be conceived as dispositional, the Casimir operators of the Galilean group represent intrinsic properties that always have actual values: mass, squared spin, and the Hamiltonian—which agrees with the internal energy if the reference frame is located at the system's center of mass.

On the other hand, MHI only applies to non-interacting systems. Not only it describes the intrinsic properties in the non-interacting case: also the propensities to actualization of the non-intrinsic properties are assigned to the closed system. Nevertheless, MHI has nothing to say during interactions. For instance, in Stage II of the measurement process as introduced in Sect. 3, it only describes the composite system $S + M$, but is silent about the properties of its open parts S and M: MHI cannot tell us what properties each of the systems has with respect to the other. This is exactly what RQM can provide with its relational perspective.

The above remarks show that, on the basis of their general agreements, the two interpretive stances can be made complementary if applied at different levels: the MHI gives the perspective of the closed system, whereas RQM describes the objective and relative values acquired by the properties of the subsystems in their incessant interactions. In other words, the assignment of properties to the whole system according to MHI and the assignment of relations to the different parts of that system according to RQM can complement each other (we will come back to this point in the next subsection).

This strategy makes it possible to address a question arising in certain measurements, such as the Stern-Gerlach experiment, in which the system S to be measured and the measuring apparatus M interact during a period t_1 to t_2, and at time t_2 they stop interacting. What happens after t_2, when the interaction ceases? The phenomenon whereby two systems S and M stop interacting after they have done so, is it an event? It is certainly not an interaction, but it is clear that something happens at t_2 when the composite system $S + M$ becomes two closed systems S and M as before the interaction. Conceiving events exclusively in terms of interactions gives us no clue as to how to treat this case. If, by contrast, it is admitted, in the spirit of MHI, that events happen when closed systems are constituted as such, there are two events in the above case: at t_1 the event of the constitution of the composite system $S + M$ at the beginning of the interaction, and at t_2 the event of the constitution of the systems S and M at the end of the interaction. In this experimental situation:

- From t_1 to t_2, subsystems S and M have only relational properties with respect to each other, as stated by RQM.
- For $t > t_2$, systems S and M, now closed, acquire non-relational properties according to the actualization rule of MHI.

This is an example of how the two interpretations can fruitfully collaborate to account for a specific physical situation.

5.2 Properties or/and Relations

In the previous subsection it was said that the assignment of properties to the whole system according to MHI and the assignment of relations to the different parts of that system according to RQM can complement each other. Of course, this requires that both assignments be compatible. This issue will be addressed in this subsection.

Let us begin by recalling the logical-ontological difference between monadic properties, usually called 'properties', and dyadic properties, usually called 'relations'. Both cases belong to the logical-ontological category of "property", but they are distinguished by their arity n: $n = 1$ for monadic properties, $n = 2$ for dyadic properties. The application of a monadic property to one individual or a dyadic property to two individuals leads to a fact, which is logically expressed by a proposition that can be true or false. The beginning of the occurrence of a fact is an event. For example:

- "Having the value a of the observable A" is a monadic property, a so-called 'property', let us call it P_a, that applies to a system S leading to the fact $P_a(S)$.
- "Having the value a of the observable A relative to" is a dyadic property, a so-called 'relation', let us call it R_a, that applies to two systems S_1 and S_2, leading to the fact $R_a(S_1, S_2)$.

With this terminology, the following can be said:

- According to MHI, in a closed system S, once the privileged context is fixed by the actualization rule, the observables belonging to it indeterministically acquire definite values. This means that the properties involved in actualization are always monadic, so the corresponding facts have the form $P_a(S)$.
- According to RQM, when two systems S_1 and S_2 interact, the observables of each of them indeterministically acquire definite values relative to the other system. This means that the properties involved in actualization are always dyadic, so the corresponding facts have the form $R_a(S_1, S_2)$.

The question is, then, whether the two assignments are compatible. Let us consider the measurement process as described in Sect. 3, in particular, Stage II, in which the system S and the apparatus M are in interaction leading to the whole system $S + M$. According to MHI, in this stage the Hamiltonian $H^{(II)}$ of $S + M$ has a definite value, but the observable A of S does not acquire a definite value. This means that, for the value a_k—and for any other value a of the observable A—, the fact that S has the value a_k of the observable A does not hold: $\neg P_{a_k}(S)$. According to RQM, in turn, S has a definite value, say a_k, of the observable A relative to M: $R_{a_k}(S, M)$. Of course, the two assignments are different. But the facts $\neg P_{a_k}(S)$ and $R_{a_k}(S, M)$ are not and cannot be incompatible because they involve different properties: the monadic property P_{a_k} and the dyadic property R_{a_k}, respectively.

If the assignments of monadic and dyadic properties introduced by MHI and RQM, respectively, are compatible, why should only one type of property be retained while discarding the other? This issue will be discussed in the light of a structuralist perspective.

Although Rovelli does not take a very explicit position in this regard, he admits to sympathize with an ontological perspective that dispenses with the notion of substance: "My sympathy for a natural philosophical home for relational QM is an anti-foundationalist perspective where we give up the notion of primary substance-carrying attributes." ([36], p. 10). Hence, the structuralist reading of RQM offered by Laura Candiotto [37] seems natural. According to Candiotto, the best philosophical framework for RQM is Ontic Structural Realism [38–40], which postulates the fundamental character of relations: objects emerge as relational "nodes" [41]. But in this structure, all properties are dyadic, there are only relations: "there are no intrinsic properties that can be assigned to systems independently of their interactions" ([2], p. 22). In this ontological picture, monadic properties—in particular, state-independent and observer-independent properties such as mass, charge, and spin—find no comfortable place (see [42]).

For its part, MHI explicitly adopts an ontology devoid of substances and of the category of object in general [14, 21–23]. From this perspective, a quantum system is a bundle of properties, the natural ontological correlate of an algebra of observables. The properties involved in the quantum ontology are physical properties, directly or indirectly defined in terms of the symmetry group of the theory, that is, the Galilean group (see [14], Subsection 3.2). But whereas the aim of the traditional bundle theory is to construct objects out of properties, this bundle view completely dispenses with the ontological category of object: bundles of properties do not behave as individual objects at all since they belong to a different ontological category. As a consequence, "[n]either does Leibniz's Principle of the Identity of Indiscernibles nor the Kantian category of quantity apply to them." ([24], p. 238). As noticed by Steven French [43], this view of non-individual bundles finds a natural resonance with Ontic Structural Realism, which was mainly motivated by the ontological challenges of quantum mechanics [39]. In this ontological view, however, dyadic properties are not taken into account. This leaves unexplained the relationships that are established between systems, not only while they interact, but also when they stop interacting, because entanglement does not disappear when interaction ceases.

But as noted above, the monadic and dyadic property assignments introduced by MHI and RQM, respectively, are compatible. Therefore, nothing prevents combining both types of properties to constitute a structural ontology, since nothing prevents a structure from incorporating properties of either arity. It would then be an ontology with a single fundamental ontological category, that of property, but in which the type of properties is not restricted to a specific arity. According to this view, quantum systems are not mere bundles of properties, nor just nodes of relations, but are the nodes where both properties and relations converge. Despite including monadic properties, this ontological picture still retains the structural spirit of Ontic Structural Realism.

5.3 What are Interactions?

The question of what an interaction is turns out to be central to both RQM and MHI, because in both cases the concept of interaction plays a leading role. It could be considered that in the two proposals it is a primitive concept. However, even if this is the case, it is necessary to positively identify under what circumstances an interaction can be said to have occurred.

Although Rovelli does not offer an explicit definition of interaction, he insists in that 'interaction' and 'measurement' must be taken as synonyms: "any interaction counts as a measurement, to the extent one system affects the other and this influence depends on a variable of the first system." ([2], pp. 6–7]). In turn, when he explains the "third person problem" [1], it is clear that the interaction between S and O during the interval $\Delta t = t_2 - t_1$ leads to a perfect correlation between the eigenstates $|a_i\rangle$ of the observable A of S and the eigenstates $|b_i\rangle$ of the observable B of O:

$$\overset{t_1}{\left(\sum \alpha_i |a_i\rangle\right) \otimes |b_{init}\rangle} \quad \rightarrow \quad \overset{t_2}{\sum \alpha_i |a_i\rangle \otimes |b_i\rangle}$$

It is in this situation that the relation—dyadic property—, say R_{a_k}, actualizes, leading to the fact.

$R_{a_k}(S, O)$: the system S acquires the value a_k of the observable A relative to O. Despite this characterization seems to exclude imperfect correlation, this is not the case. According to Rovelli, imperfect correlation does not mean that the interaction has not occurred: "imperfect correlation does not imply no measurement performed, but only a smaller than 1 probability that the measurement has been completed" ([1], p. 1652). In other words, an interaction is a process that, after some time, establishes a perfect correlation between observables of the interacting systems. Rovelli [44] even claims that quantum mechanics provides a probabilistic prediction of the time at which the perfect correlation is established and, with it, the corresponding relations actualize. In particular, he shows that an observable M of the system $S + O$ can be defined, with eigenvalue 1 when the correlation is perfect and 0 when it is not. The expectation value of this observable in the state $\Psi(t)$ at the intermediate time t between t_1 and t_2 gives the probability at t that the interaction has been completed [44].

Given the central role of Hamiltonians in MHI, it is not surprising that, in this interpretation, the interaction is characterized in terms of an interaction Hamiltonian that represents the energetic link between the interacting systems. More precisely, two systems S and O interact when the Hamiltonian of the composite system $S + O$ can be expressed as $H_{SO} = H_S \otimes I_O + I_S \otimes H_O + H_{SO}^{\text{int}}$, where H_S, H_O are the Hamiltonians of S and O respectively, and H_{SO}^{int} is the interaction Hamiltonian. When the composite system is formed, the monadic property P_{ω_k} actualizes—as well as the properties corresponding to observables that commute with H_{SO} and do not break their symmetries—leading to the fact.

$P_{\omega_k}(S + O)$: the system $S + O$ acquires the value ω_k of the observable H_{SO}. In this case, the correlations between observables of the interacting systems are not mentioned.

Despite the differences between the two interpretations as to how to identify interactions, the two positions are not incompatible. On the one hand, Rovelli obviously recognizes the role played by the interaction Hamiltonian in establishing the perfect correlation: "It is not difficult to construct model Hamiltonians that produce evolutions of this kind, and that can be taken as models for the physical interactions that produce a measurement." ([1], p. 1643). More precisely, "we can describe the interaction between the two systems, say, in terms of an interaction term in the Hamiltonian that depends, in particular, say, on a variable A of the system S: then A is the variable that takes value. The reason is that the interaction Hamiltonian depends on the property of S responsible in determining the effect of S on O." ([5], p. 3). On the other hand, in the context of MHI it is explicitly acknowledged that the specific form of the interaction Hamiltonian determines which observables of the interacting systems turn out to be perfectly correlated. In particular, an interaction Hamiltonian H_{SO}^{int} depending on the observable A in the following way,

$$H_{SO}^{\text{int}} = -\frac{\lambda\hbar}{\Delta t}\left(A \otimes P^B\right),$$

where λ is a constant and P^B is the observable conjugate to B, establishes the perfect correlation between the eigenstates of A and B (see [14], p. 408, following [45]).

The point that still seems to make a difference is the exact time at which the events associated with an interaction occur. According to RQM, the event corresponding to A acquiring a definite value relative to O takes place at the time when the interaction is completed, that is, at some time in the interval $\Delta t = t_2 - t_1$ during which the systems interact. According to MHI, the event corresponding to the observable H_{SO} acquiring a definite value takes place when the interaction begins and the new closed system $S+O$ arises as a composite of the two interacting systems. Nevertheless, this difference is not very relevant since it refers to different instants in the time of the unitary evolution, but it does not affect the relationships between events that lead to a relational event-time: we will come back to this point in Subsection 5.5.

5.4 What are Systems?

If the concept of interaction plays a central role in both RQM and MHI, and an interaction is a physical relationship between systems, it is necessary to make clear how systems are defined and identified in both interpretations. This is by no means a trivial issue because, as is well known, a single closed system can be decomposed in many ways into component systems. Precisely, the so-called "quantum structure studies" deal with the different ways in which a quantum system can be decomposed into subsystems (quantum structures) (see, e.g., [46–48]). Even entanglement is relative to the partition of the closed system into parts (see, e.g., [49–51]).

According to MHI, the issue is straightforward, since the only quantum systems are closed systems. From the very moment when two closed systems S_1 and S_2 interact to form a new closed system, they are no longer subsystems but parts of the new composite system. This top-down approach, which endows the closed system with ontological priority, enables the MHI to deal with the possibility of decomposing the system in many ways: all of these ways, leading to different partitions, are equally legitimate (this top-down approach has also been applied to the relativization of the phenomenon of decoherence: see, e.g., [52–56]). Nevertheless, as already pointed out, MHI says nothing about the properties of the open parts into which the whole closed system can be decomposed.

In the context of RQM, the possibility of identifying quantum systems in different ways is scarcely discussed. An exception is Matthew Brown [57], who asks if it is necessary to introduce "canonical cuts" in RQM. In some passages of his works, Rovelli also mentions that there are many ways of defining the systems that interact, although he does not discuss the question: "I assume that the world can be decomposed (possibly in a large number of ways) into a collection of systems, each of which can be equivalently considered as an *observing system* or as an *observed system*." ([1], p. 1655, italics in the original); "The basic idea is that the world can be decomposed (in many alternative manners) into «physical systems» that interact among themselves." ([5], p. 1). But once it is accepted that a closed system can be decomposed in a multiplicity of ways, a new layer of relationalism needs to be incorporated into the RQM's proposal.

In fact, not only are the properties that actualize always dyadic, leading to facts of the form $R_{a_k}(S,O)$: "the system S acquires the value a_k of the observable A relative to O". In addition, actualization itself is relative to how the systems S and O are identified. For example, let us consider two systems S and O, which interact leading to a state that establishes the perfect correlation between the eigenstates of the observable A of S and those of the observable B of O. After the interaction, the two systems can be considered as subsystems of the composite system $Q = S + O$. But if the system Q can also be decomposed into two systems S' and O', $Q = S + O = S' + O'$, then, according to the Biorthogonal Decomposition Theorem, the same state establishes the perfect correlation between the eigenstates of an observable A' of S' and of an observable B' of O'. Therefore, besides the actualization of the dyadic property R_{a_k}, leading to the fact $R_{a_k}(S, O)$, the dyadic property $R_{a'_k}$ also actualizes, leading to the fact $R_{a'_k}(S', O')$: "the system S' acquires the value a'_k of the observable A' relative to O'".

As far as we know, Rovelli does not discuss explicitly this situation. Nevertheless, it is not unreasonable to suppose that he would accept the need to relativize the actualization of relational properties with respect to the definition of the interacting systems. In a proposal as strongly relational as RQM, a further layer of relationalism seems completely natural. Furthermore, Rovelli himself recognizes that the definition of the physical systems is arbitrary: "It is a setting in which two specific distinguishable physical systems are singled out, say S and P. Quantum mechanics gives descriptions of the world *conditional to this (arbitrary) choice* and describes how one system affects the other when they interact." ([5], p. 3, italics added). This "conditionality" seems to point toward that further relational dimension.

5.5 Event-Time

As explained above, Rovelli emphasizes that quantum mechanics provides a probabilistic prediction of the time at which the perfect correlation is established. More precisely, an observable M of the composite system can be defined such that its eigenvalue is 1 when the correlation is perfect and 0 when it is not. On this basis, "if we follow the Schrödinger evolution $\Psi(t)$ of the state of the coupled system from $t = 0$ to $t = T$, then at every intermediate t we can compute the probability $P(t)$ that the measurement has happened

$$P(t) = \langle \Psi(t) | M | \Psi(t) \rangle$$

For a good measurement, $P(t)$ will be a smooth function that goes monotonically from 0 to 1 in the time interval 0 to T." ([44], p. 1037). That probability is represented by a function on the time of the unitary evolution, that is, the linear and continuous time on which the state evolves according to the Schrödinger equation.

However, in other points of his works Rovelli argues that time itself is constituted by the relationships between events, in such a way that asking what happens between events makes no sense: "The question of «what happens between quantum events» is meaningless in the theory. The happening of the world is a very fine-grained but discrete swarming of quantum events, not the permanence of entities that have well-defined properties at each moment of a continuous time." ([36], p. 9). This is how Dorato seems to interpret time in RQM when he states that spacetime "is constituted by the collection of all definite quantum events, which, in their turn can be regarded the outcomes of interactions between different systems." ([58], p. 15). But if time is constituted by the events themselves, how is it possible to calculate the probability distribution of the occurrence of an event over time?

This seeming conflict in the temporal aspects of the RQM is resolved if two different notions of time are distinguished in the quantum realm (see [59, 60]):

- The *parameter-time* is the time along which the system unitarily evolves. It is represented by the variable t as it appears in the Schrödinger equation.
- The *event-time* is the time at which individual events occur. Those events are measurement results or, more generally, any acquisition of a definite value by a certain observable.

The parameter-time is the notion of time involved in the characterization of the Galilean group; it is supposed to be continuous, homogeneous, and isotropic as in the classical case. The Schrödinger equation rules how the probabilities on the possible values of all the observables of the system change along the parameter-time. The event-time, on the contrary, has no formal representation in quantum mechanics. Nevertheless, it is essential to endow the theory with physical meaning: testing the theory is only possible through the registration of specific events, such as a hit of an electron on a screen, or the absorption of a photon by an atom. In Rovelli's words: "There are two independent notions of time in ordinary quantum mechanics: the time in which the system evolves, and the "time" that orders the measurements of the observer. These two are not related and may be non-coincident." ([60], p. 130). It is interesting to note that these two notions of time, parameter-time and even-time, are deeply correlated to possibility and actuality, respectively, when conceived as irreducible modes of being [61].

Rovelli's relational program started before the formulation of RQM, with the idea of tackling the so-called "problem of time in quantum gravity", derived from the difference between the notions of time in quantum theory—a Galilean time—and in general relativity—a theory invariant under general coordinate transformations (see [62, 63]). He introduced a concrete proposal in this sense with a relational reconstruction of the parameter-time of quantum mechanics. [60, 64, 65]. The question is whether a relational reconstruction of the quantum event-time is also possible. Here, once again, the collaboration between different interpretations can be fruitful.

In a recent paper, a relational reconstruction of the event-time in the context of MHI was offered [16]. On the basis of a consecutive-measurement toy-model, an event-time of four event-instants was obtained. Although extremely simplified, this model can be extrapolated to generic situations: the discrete event-time emerges from the net of interaction relations between the systems that compose the whole universe. In the light of the leading role played by the Hamiltonian in MHI, it is not surprising that the structure of the so-reconstructed event-time is embodied in the internal structure of the Hamiltonian of the universe. That reconstruction shows that the instants of the event-time are related by a partial order, since it is possible that, given two instants, neither of them is prior to the other so that the relation "being earlier than" does not hold for them. For example, the event-time might bifurcate into two different temporal lines when two systems stop interacting and never interact again, generating event-evolutions completely disconnected, or two disconnected event-evolutions might converge into a single temporal line when two systems interact and yield a single temporal line. The fact that the resulting event-time does not have the structure of classical time, far from being a shortcoming of the relational reconstruction, “should be considered as an advantage if the aim is to obtain a truly *fundamental* quantum gravity.” ([16], p. 9, italics in the original).

Collaboration between RQM and MHI can be fruitful also on this point. As noted in previous sections, among the several aspects shared by both interpretations, the notion of event is one of them: the actual acquisition of a definite value by an observable is an event. Events are conceived as actual, objective, and irreducibly indeterministic. Therefore, it is easy to suppose that the construction of the event-time as proposed in the above-mentioned paper could be adapted to the perspective of RQM, giving rise to an event-time with the same features as that obtained in the context of MHI.

6 Final Remarks

Like most interpretations of quantum mechanics, the traditional modal interpretations were specifically designed to solve the measurement problem. In fact, they successfully achieved this goal in the case of ideal measurements. However, a series of articles in the nineties [27–30, 66] showed that those traditional approaches based on the modal views did not select the right properties for the apparatus in non-ideal measurements. As perfectly ideal measurements can never be obtained in practice, this shortcoming was considered a “silver bullet” to kill modal interpretations (Harvey Brown, cited in [67]). This explains the decline of the interest in modal interpretations since the end of the nineties.

What was not sufficiently noted at the time was that the difficulties of those original modal interpretations in dealing with non-ideal measurements were not due to their modal nature. Those shortcomings were the result of the fact that their rule of definite-value ascription made the set of definite-valued observables dependent on the instantaneous state of the system. An author who did realize this was Jeffrey Bub, whose preference for Bohmian mechanics in those days can be understood in this context. In fact, if Bohmian mechanics is conceived as a member of the modal family, whose definite-valued observables are defined by the position observable [68], it turns out to be a natural alternative given the difficulties of the original modal interpretations.

Bub showed that the shortcomings of the original modal interpretations can be overcome by making the rule of definite-value ascription independent of the system’s state and only dependent on an observable of the system. This was certainly an important step. But what was not realized at the time is that position is not the only observable that can be appealed to in order to define the state-independent rule of definite-value ascription in a modal interpretation. It is at this point that MHI enters the scene: it endows the Hamiltonian of the quantum system with the role of selecting the definite-valued observables. With this strategy, MHI not only solves the problems of the original modal interpretations, but can also be successfully applied to many physical situations. Unfortunately, this was not enough to rehabilitate modal interpretations in the eyes of many philosophers of physics.

In his early work on RQM, Rovelli considered modal interpretations to point out their limitations [1, 44]. Of course, those comments referred to the traditional modal interpretations. He later stopped mentioning them, perhaps under the influence of the "silver bullet" criticism. But at the present stage of the development of modal interpretations, it is worth revisiting them. Here we have focused on MHI, which, despite its differences from RQM, agrees with it on several relevant respects. The purpose has been to show that these agreements make possible a constructive complementation of the two views.

Both in the early paper on RQM [1] and in recent works [69], the concept of information plays a prominent role in Rovelli's proposal: quantum mechanics is conceived as a theory that only describes the information that systems have about each other, where 'information' is understood in terms of Shannon's theory [70]. Although in the case of MHI information is not a central concept, in this interpretive context the measurement process has been reconstructed as an informational situation in the sense of Shannon: on this basis, MHI provides a criterion to distinguishing between reliable and non-reliable non-ideal measurements and to quantifying reliability [31]. The appeal to Shannon's theory in both cases may open a way to explore a further point of contact between the two interpretations in a future work.

Acknowledgements This work was supported by Grant PICT-04519 of ANCyT, Argentina (O.L), and by Grants PICT-1770 of ANCyT, PIP-14-200901-00272 and PIP-114-200901-00068 of CONICET (J.S.A).

References

1. Rovelli, C.: Relational Quantum Mechanics. Int. J. Theor. Phys. **35**, 1637–1678 (1996)
2. Laudisa, F., Rovelli, C.: Relational Quantum Mechanics. In Zalta, E.N. (ed.), *The Stanford Encyclopedia of Philosophy*, (Spring 2021 Edition). https://plato.stanford.edu/entries/qm-relational/ (Page numbers are taken from the printed version) (2021). Accessed 9 June 2022
3. Laudisa, F.: Open problems in relational quantum mechanics. J. Gen. Philos. Sci. **50**, 215–230 (2019)
4. Muciño, R. Okon, E., Sudarsky, D.: Assessing Relational Quantum Mechanics. https://arXiv.org/2105.13338 (2021)
5. Rovelli, C.: A response to the Muciño-Okon-Sudarsky's assessment of Relational Quantum Mechanics. https://arXiv.org/2106.03205 (2021a)
6. Smerlak, M., Rovelli,: Relational EPR. Found. Phys. **37**, 427–445 (2007)
7. Brukner, Č.: Qubits are not observers—a no-go theorem. https://arxiv.org/:2107.03513 (2021)
8. Pienaar, J.L.: A quintet of quandaries: five no-go theorems for Relational Quantum Mechanics. https://arXiv.org/2107.00670 (2021)
9. Rovelli, C.: Relational Quantum Mechanics is about facts, not states: a reply to Pienaar and Brukner. https://arXiv.org/2110.03610 (2021b)
10. van Fraassen, B.C.: A formal approach to the philosophy of science. In: Colodny, R. (ed.) Paradigms and Paradoxes: The Philosophical Challenge of the Quantum Domain, pp. 303–366. University of Pittsburgh Press, Pittsburgh (1972)
11. van Fraassen, B.C.: The Einstein-Podolsky-Rosen paradox. Synthese **29**, 291–309 (1974)
12. Lombardi, O., Dieks, D.: Modal interpretations of quantum mechanics. In Zalta, E.N. (ed.), *The Stanford Encyclopedia of Philosophy*, (Winter 2021 Edition) https://plato.stanford.edu/entries/qm-modal/ (2021). Accessed 9 June 2022

13. Kochen, S., Specker, E.: The problem of hidden variables in quantum mechanics. J. Math. Mech. **17**, 59–87 (1967)
14. Lombardi, O., Castagnino, M.: A modal-Hamiltonian interpretation of quantum mechanics. Stud. Hist. Philos. Mod. Phys. **39**, 380–443 (2008)
15. Fortin, S., Lombardi, O., Martínez González, J.C.: A new application of the modal-Hamiltonian interpretation of quantum mechanics: the problem of optical isomerism. Stud. Hist. Philos. Mod. Phys. **62**, 123–135 (2018)
16. Fortin, S., Lombardi, O., Pasqualini, M.: Relational event-time in quantum mechanics. Found. Phys. **52**, 10 (2022)
17. Ardenghi, J.S., Castagnino, M., Lombardi, O.: Quantum mechanics: modal interpretation and Galilean transformations. Found. Phys. **39**, 1023–1045 (2009)
18. Lombardi, O., Castagnino, M., Ardenghi, J.S.: The modal-Hamiltonian interpretation and the Galilean covariance of quantum mechanics. Stud. Hist. Philos. Mod. Phys. **41**, 93–103 (2010)
19. Ardenghi, J.S., Castagnino, M., Lombardi, O.: Modal-Hamiltonian interpretation of quantum mechanics and Casimir operators: the road to quantum field theory. Int. J. Theor. Phys. **50**, 774–791 (2011)
20. Lombardi, O., Fortin, S.: The role of symmetry in the interpretation of quantum mechanics. Electron. J. Theor. Phys. **12**, 255–272 (2015)
21. da Costa, N., Lombardi, O.: Quantum mechanics: ontology without individuals. Found. Phys. **44**, 1246–1257 (2014)
22. da Costa, N., Lombardi, O., Lastiri, M.: A modal ontology of properties for quantum mechanics. Synthese **190**, 3671–3693 (2013)
23. Lombardi, O., Dieks, D.: Particles in a quantum ontology of properties. In: Bigaj, T., Wüthrich, C. (eds.) Metaphysics in Contemporary Physics, pp. 123–143. Brill-Rodopi, Leiden (2016)
24. Fortin, S., Lombardi, O.: Entanglement and indistinguishability in a quantum ontology of properties. Stud. Hist. Philos. Sci. **91**, 234–243 (2022)
25. Omnés, R.: The Interpretation of Quantum Mechanics. Princeton University Press, Princeton (1994)
26. Omnés, R.: Understanding Quantum Mechanics. Princeton University Press, Princeton (1999)
27. Albert, D., Loewer, B.: Wanted dead or alive: two attempts to solve Schrödinger's paradox. In Proceedings of the 1990 Biennial Meeting of the Philosophy of Science Association, vol. 1, pp. 277–285. Philosophy of Science Association, East Lansing (1990)
28. Albert, D., Loewer, B.: Some alleged solutions to the measurement problem. Synthese **88**, 87–98 (1991)
29. Albert, D., Loewer, B.: Non-ideal measurements. Found. Phys. Lett. **6**, 297–305 (1993)
30. Elby, A.: Why 'modal' interpretations don't solve the measurement problem. Found. Phys. Lett. **6**, 5–19 (1993)
31. Lombardi, O., Fortin, S., López, C.: Measurement, interpretation and information. Entropy **17**, 7310–7330 (2015)
32. Ardenghi, J.S., Lombardi, O., Narvaja, M.: Modal interpretations and consecutive measurements. In: Karakostas, V., Dieks, D. (eds.) EPSA 2011: Perspectives and Foundational Problems in Philosophy of Science, pp. 207–217. Springer, Berlin (2013)
33. van Fraassen, B.C.: Rovelli's world. Found. Phys. **40**, 390–417 (2010)
34. Calosi, C., Mariani, C.: Quantum relational indeterminacy. Stud. Hist. Philos. Mod. Phys. **71**, 158–169 (2020)
35. Dorato, M.: Rovelli's relational quantum mechanics, anti-monism, and quantum becoming. In: Marmodoro, A., Yates, D. (eds.) The Metaphysics of Relations, pp. 235–161. Oxford University Press, Oxford (2016)
36. Rovelli, C.: Space is blue and birds fly through it». Philos. Trans. R. Soc. A **376**, 20170312 (2018)
37. Candiotto, L.: The reality of relations. G. Metafis. **2017**, 537–551 (2017)
38. French, S., Ladyman, J.: In defence of ontic structural realism. In: Bokulich, A., Bokulich, P. (eds.) Scientific Structuralism, pp. 25–42. Springer, Dordrecht (2011)
39. Ladyman, J.: What is structural realism? Stud. Hist. Philos. Sci. **29**, 409–424 (1998)
40. Ladyman, J., Ross, D.: Every Thing Must Go: Metaphysics Naturalized. Oxford University Press, Oxford (2007)
41. French, S.: Structure as a weapon of the realist. Proc. Aristot. Soc. **106**, 170–187 (2006)
42. Oldofredi, A.: The bundle theory approach to Relational Quantum Mechanics. Found. Phys. **51**, 18 (2021)

43. French, S.: What is this thing called structure? (Rummaging in the toolbox of metaphysics for an answer). http://philsci-archive.pitt.edu/id/eprint/16921 (2020). Accessed 9 June 2022
44. Rovelli, C.: Incerto tempore, incertisque loci: can we compute the exact time at which a quantum measurement happens? Found. Phys. **28**, 1031–1043 (1998)
45. Mittelstaedt, P.: The Interpretation of Quantum Mechanics and the Measurement Process. Cambridge University Press, Cambridge (1998)
46. Dugić, M., Jeknić-Dugić, J.: What is system: the information-theoretic arguments. Int. J. Theor. Phys. **47**, 805–813 (2008)
47. Harshman, N.L., Wickramasekara, S.: Tensor product structures, entanglement, and particle scattering. Open. Syst. Inf. Dyn. **14**, 341–351 (2007)
48. Viola, L., Barnum, H.: Entanglement and subsystems, entanglement beyond subsystems, and all that. In: Bokulich, A., Jaeger, G. (eds.) Philosophy of Quantum Information and Entanglement, pp. 16–43. Cambridge University Press, Cambridge (2010)
49. Earman, J.: Some puzzles and unresolved issues about quantum entanglement. Erkenntnis **80**, 303–337 (2015)
50. Harshman, N.L., Ranade, K.S.: Observables can be tailored to change the entanglement of any pure state. Phys. Rev. A **84**, 012303 (2011)
51. Terra Cunha, M.O., Dunningham, J.A., Vedral, V.: Entanglement in single-particle systems. Proc. R. Soc. A **463**, 2277–2286 (2007)
52. Castagnino, M., Fortin, S., Lombardi, O.: Is the decoherence of a system the result of its interaction with the environment? Mod. Phys. Lett. A **25**, 1431–1439 (2010)
53. Castagnino, M., Laura, R., Lombardi, O.: A general conceptual framework for decoherence in closed and open systems. Philos. Sci. **74**, 968–980 (2007)
54. Fortin, S., Lombardi, O.: A top-down view of the classical limit of quantum mechanics. In: Kastner, R.E., Jeknić-Dugić, J., Jaroszkiewicz, G. (eds.) Quantum Structural Studies: Classical Emergence from the Quantum Level, pp. 435–468. World Scientific, Singapore (2016)
55. Fortin, S., Lombardi, O.: A closed-system approach to decoherence. In: Lombardi, O., Fortin, S., López, C., Holik, F. (eds.) Quantum Worlds. Perspectives on the Ontology of Quantum Mechanics, pp. 345–359. Cambridge University Press, Cambridge (2019)
56. Fortin, S., Lombardi, O., Castagnino, M.: Decoherence: a closed-system approach. Braz. J. Phys. **44**, 138–153 (2014)
57. Brown, M.J.: Relational quantum mechanics and the determinacy problem. Br. J. Philos. Sci. **60**, 679–695 (2009)
58. Dorato, M.: Rovelli's relational quantum mechanics, anti-monism and quantum becoming. http://philsci-archive.pitt.edu/9964/ (2013). Accessed 9 June 2022
59. Busch, P.: The time-energy uncertainty relation. In: Muga, J., Mayato, R.S., Egusquiza, I. (eds.) Time in Quantum Mechanics. Lecture Notes in Physics, vol. 734, pp. 73–105. Springer, Berlin (2008)
60. Rovelli, C.: Is there incompatibility between the ways time is treated in general relativity and in standard quantum mechanics? In: Ashtekar, A., Stachel, J. (eds.) Conceptual Problems of Quantum Gravity, pp. 126–136. Birkhauser, New York (1991)
61. Lombardi, O., Fortin, S., Pasqualini, M.: Possibility and time in quantum mechanics. Entropy **24**, 249 (2022)
62. Isham, C.J.: Canonical quantum gravity and the problem of time. In: Ibort, L.A., Rodríguez, M.A. (eds.) Integrable Systems, Quantum Groups, and Quantum Field Theories, NATO ASI Series (Series C: Mathematical and Physical Sciences), vol. 409, pp. 157–287. Springer, Dordrecht (1993)
63. Kuchař, K.: The problem of time in canonical quantization. In: Ashtekar, A., Stachel, J. (eds.) Conceptual Problems of Quantum Gravity, pp. 141–171. Birkhäuser, Boston (1991)
64. Rovelli, C.: Quantum mechanics without time: a model. Phys. Rev. D **42**, 2638–2646 (1990)
65. Rovelli, C.: Forget time. Essay written for the *FQXi contest on the Nature of Time* (2008)
66. Ruetsche, L.: Measurement error and the Albert-Loewer problem. Found. Phys. Lett. **8**, 327–344 (1995)
67. Bacciagaluppi, G., Hemmo, M.: Modal interpretations, decoherence and measurements. Stud. Hist. Philos. Mod. Phys. **27**, 239–277 (1996)
68. Bub, J.: Interpreting the Quantum World. Cambridge University Press, Cambridge (1997)
69. Adlam, E., Rovelli, C.: Information is physical: cross-perspective links in Relational Quantum Mechanics. https://arXiv.org/2203.13342. (2022)
70. Shannon, C.: The mathematical theory of communication. Bell Syst. Tech. J. **27**, 379–423 (1948)