



Causal Processes in C^* -Algebraic Setting

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Abstract

In this paper, we attempt to explicate Salmon's idea of a causal process, as defined in terms of the mark method, in the context of C^* -dynamical systems. We prove two propositions, one establishing mark manifestation infinitely many times along a given interval of the process, and, a second one, which establishes continuous manifestation of mark with the exception of a countable number of isolated points. Furthermore, we discuss how these results can be implemented in the context of the Haag–Araki theories of relativistic quantum fields on Minkowski spacetime.

Keywords Causation · Process theories · C^* -dynamical systems · Local quantum physics

1 Introduction

Process theories of causation emerged in the first half of the twentieth century after the advent of the theory of relativity. Reichenbach's concept of a real sequence, as distinguished from an unreal one in terms of the method of mark [1], is considered the first attempt to define *causal processes* as distinguished from *pseudoprocesses*, while Russell's theory of causal lines [2] is a way to talk about processes in terms of regularities. The real boost, however, to this approach has been given by the work of Salmon [3, 4] and Dowe [5, 6] towards the end of the century.

Process theories try to explain the cause and effect relation in terms of mechanisms of generation of causal influence, *causal interactions*, and of mechanisms of propagation of this influence, *causal processes*. Both mechanisms are considered spacetime entities, admitting a geometric representation: causal interactions are localized, they occur at a place at a time, and they are represented, ideally, by points of spacetime, while causal processes are locally extended entities represented

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geometrically by spacetime curves. Along these curves the causal influence generated at a spacetime point is propagated.

Various attempts have been made to describe these causal mechanisms in terms of physical theories and to consider their existence contingent upon the truth of these theories. Salmon's early theory, which dates back to 1984, is based on the idea of a mark introduced into a process by a single local interaction, at a spacetime point. The generation of a mark is considered the prototype of a causal interaction. The ability of a causal process to transmit a mark at each point of its spacetime curve, without further assistance, indicates its ability to propagate causal influence and distinguishes it from a pseudoprocess. Pseudoprocesses, on the other hand, although they exhibit some kind of uniformity along their geometric representation, they lack the ability of mark transmission. The motion of a ball in space illustrates the notion of a causal process while the hit of a bat imparting in the ball momentum illustrates a causal interaction. On the other hand, the motion of a shadow or of a light spot qualifies as a pseudoprocess since any change produced locally at these objects is not propagated, without further assistance, throughout their history. The connection between the theory of relativity and this account rests on the impossibility of a mark to be transmitted at spacelike distance and the consideration of spacelike curves as possible geometric representations of pseudoprocesses. Hence, any uniformity manifesting itself in a sequence of events (process) that *can be* represented geometrically by a spacelike curve does not qualify as a causal process. In 1997, Salmon, reconsidering the early version of his theory, has admitted that it provides a *geometric* account of causation rather than a *physical* one, since no specification of the nature of causal influence in terms of physical magnitudes has been given. The recognition of this limitation led Salmon to the abandonment of the mark method and the specification of the nature of causal influence, first, in terms of relativistic invariant quantities, such as the rest mass and the electric charge, and, later, in terms of conserved quantities (energy, 3-momentum, etc.), under the influence of Dowe's Conserved Quantity Theory. According to the later versions of Salmon's theory, a causal interaction is a local exchange of an invariant or conserved quantity, while a causal process has the ability to transmit continuously these quantities from one spacetime point to another in the absence of any further assistance.

What remained unaltered in the development of Salmon's thought was his commitment to the idea of spacetime continuity in the propagation of causal influence by a causal process: once produced, causal influence is made manifest, either in the form of a mark or in that of a constant amount of an invariant and/or conserved quantity, at each spacetime point of the continuous curve. This idea, Salmon claims, provides a solution to Hume's celebrated problem of causation. It yields the connection between cause and effect that Hume was unable to trace at the empirical level. Hume attempted to explain the connection by interpolating further intermediate causes and effects between the distant events, yielding, thus, the image of a causal chain connecting the distant cause with its effect. Nevertheless, as Salmon has pointed out, this view just multiplies the instances of the problem instead of solving it, since, now, the connection between each pair of successive links in the causal chain needs to be accounted for. On the other hand, were to abandon the discrete picture of distant links in a chain in favor of a continuous connection, the problem

would be solved, since in a continuum, points are densely distributed and it is meaningless to talk about the successor of a point. In this way, Salmon developed the idea of a continuous causal connection between cause and effect. On the basis of his causal notions, the causal relata are distant causal interactions connected by a causal process which manifests continuously, i.e. at each spacetime point of the curve that represents it geometrically, either the same mark, when the process is marked at the original interaction, or a constant amount of an invariant and/or conserved quantity produced at the original interaction, in the absence of any further interaction.

One of the desiderata of a causal theory, according to Salmon, is to be time independent so that it is consistent with a causal theory of time, i.e. a definition of the time arrow in terms of causal asymmetry. Thus, in defining a causal process no distinction between past and future is taken for granted and causal influence can be equally propagated in both directions along a spacetime curve. Similarly, causal interactions that generate causal influence are taken, in principle, to be time symmetric and reversible. Causal asymmetry is not determined in terms of each singular cause–effect relationship. Causal relations as explicated in terms of causal processes and interactions inherit their asymmetry from structural properties of the web of causal relations. In particular, Salmon suggested that conjunctive forks—common cause structures consisting of two independent causal processes that emanate from a spacetime point that does not represent geometrically a causal interaction—cannot be open to the past. This condition determines the direction of propagation of causal influence in the entire web and derivatively the direction of each singular causal relation [3, p. 176].

Dowe's Conserved Quantity Theory, of 1992, is somewhat different from Salmon's process theory of causation. Firstly, while a causal process for Salmon, transmits a fixed, non-zero amount of a conserved quantity, in the absence of interactions, Dowe allows variations of the amount of conserved quantity manifested along a causal process which needs not to be non-zero. These variations are either due to causal interactions or they might be explained in terms of distant action. Thus, although Salmon's account is firmly committed to action by contact, Dowe's view leaves room for action at a distance as well. Secondly, Dowe defines a causal process in terms of objects having identity through time, while Salmon does not presuppose any account of identity of an object since it defines causal processes in terms of continuous manifestations of properties in the absence of external assistance. Thirdly, the continuity requirement is dropped by Dowe, since he claimed that an object defining a causal process in terms of its possession of a conserved quantity may not have continuous existence; admitting, thus, "gappy or discontinuous" causal processes [6, pp.118–119]. This is a very important difference between the two views, given the great philosophical importance that Salmon attributes to spacetime continuity. Fourthly, the explication of the concepts of cause and effect in terms of causal processes and interactions is different for Salmon and Dowe. But we need not to further any more about the differences between these two accounts since in this paper we focus on the early account of Salmon's theory.

In the following, we attempt to define causal processes in terms of the mark method, in C^* -algebraic framework. Firstly, we define a process to be a C^* -dynamical system and we identify an instance of a process in terms of a normal state and

an observable (self-adjoint operator) of the algebra. Secondly, we consider a state transformation, defined by a non-selective operation on the algebra, as an explication of a marking interaction of a process. Although all definitions in this paper do not distinguish between classical and quantum processes, the results we obtained are restricted by the use of a particular non-selective operation, suitable only for quantum processes, the non-selective Lüders measurement. Thus, we use a well-known lemma, Lem. 8, to provide a necessary and sufficient condition for the manifestation of the mark in terms of commutation relations between the projection employed for the marking and the observable (projection) associated with the instance of the process into which the mark has been introduced (see Cor. 9). Thirdly, a causal process is defined in terms of the ability of a process to transmit a mark continuously over an interval of values of the parameter of the one-parameter group of the dynamical system. A second definition explicates a weaker notion of continuity of mark transmission, which demands continuous manifestation of the mark with the exception of a countable number of isolated points. In this way, we intend to capture the idea of a ‘gappy’ process with respect to mark transmission. Then, we prove two propositions: the first states that for a marking operation defined in terms of a non-selective Lüders measurement of a given projection, a mark introduced into a process at one stage, and being manifested at another stage, will appear also in infinite number of intermediate stages of the process, Prop. 11. The second states that if we, further, impose a condition of analyticity for the dynamics of the system on the projection measured, then ‘gappy’ processes, in the sense explained, exist, Prop. 13. In both definitions of mark transmission and in the propositions we prove, the transmission of a mark does not presuppose any time direction, i.e. assuming that the marking interaction takes place at $t = 0$, the mark may be transmitted at stages of the process corresponding to $t > 0$ or to $t < 0$. Although this fact may seem bizarre, since one expects that the mark appears after and not before the marking interaction, it complies with Salmon’s desideratum that causal processes and interactions are independent of time direction.

At this point we need to clarify our position regarding these ‘gappy’, with respect to mark transmission, processes, i.e., those processes which, if marked, exhibit discontinuities in the mark manifestation, at particular isolated points along the process. The picture we form for this sort of a process is that, if marked, it consists of denumerable parts of continuous transmission of the mark which are delimited by points of mark disappearance (exempting the upper end of the interval within which mark transmission occurs). What, then, one would think of this sort of processes? Would one still be willing to consider such processes, causal? And, what about the miraculous disappearance and reappearance of causal influence along a process? Certainly, Salmon would resist heavily to disregard such brute facts of non-manifestation of mark and, still, consider the process, causal. It would be as if he accepted a restoration of the image of a causal chain connecting the distant cause with its effect, with the links of the chain being extended over an interval of the process, manifesting continuous transmission of mark. Thus, were to accept such processes as causal, he would retreat from a very important contribution of his causal theory, namely, to deal with the problem of the Humean ‘connexion’ between the cause and the effect in terms of the continuity in the transmission of causal influence. On the other hand,

as we have already explained before, it seems that Dowe does not have a great problem to accept the causal nature of such processes, despite his denial of the mark method account of causal processes.

Far from adhering to Dowe’s view, our opinion is that causal processes should propagate *continuously* causal influence. And the desideratum of this paper was to explore the possibility of a process that has the ability to transmit a mark, to propagate causal influence, continuously. However, given the restrictions induced by the definitions we have put forward, the formal results we have obtained, in the C*-algebraic framework, were the best we could do: we preserved some intuition of continuity, without being able to exclude completely gaps in mark transmission along the process. So, we suggest the reader to consider our account a preliminary, possibly insufficient, step in establishing the possibility of causal processes in the quantum domain. To be sure to leave open the challenge for the more intuitively plausible continuous *causal processes*, we dub these ‘gappy’ processes, *causal-up-to-a-countable-set-of-isolated-points (CSIP-causal)*.

However, if discussion about causal processes ended here, one would be entitled to criticize this approach as an attempt to strip process theories of causation off their mettle, their local character, the conception of causal processes and interactions as entities that have a life in space and time. Thus, in the third section of this paper we attempt to embed the abstract mathematical approach of the second section, in the context of local quantum physics—in particular, in that of a Haag–Araki theory of relativistic quantum fields on Minkowski spacetime—where one can meaningfully be talking about spacetime entities. In this context, we will be able to consider marking operations locally defined in terms of local projections and commutation relations implied by causality–locality axiom of the theory. Prop. 11 is valid for local projections and it establishes the manifestation of the mark in infinitely many regions that result from the timelike translation of a given bounded spacetime region. However, as it is shown in “Appendix”, analytic projections do not belong to local algebras in a Haag–Araki theory and they cannot be taken to determine local marking operations. Thus, in order to obtain, a similar result about *CSIP-causal* processes, in terms of Prop. 13, we should abandon local marking operations, and talk about almost local marking operations and approximate manifestations of the mark.

2 Causal Processes and C*-Dynamical Systems

Consider a C*-dynamical system $\langle \mathcal{A}, \mathbb{R}, \tau \rangle$, defined in terms of a von Neumann algebra $\mathcal{A} \subseteq \mathcal{B}(\mathcal{H})$, where \mathcal{H} is a complex separable Hilbert space, the group $\langle \mathbb{R}, + \rangle$ of real numbers with the operation of addition, and a strongly continuous homomorphism of \mathbb{R} into the group of automorphisms of \mathcal{A} , $\tau : \mathbb{R} \rightarrow \text{Aut}(\mathcal{A}) : t \mapsto \tau_t$, induced by a strongly continuous unitary representation U of $\langle \mathbb{R}, + \rangle$ on \mathcal{H} ,

$$\forall X \in \mathcal{A}, \quad \tau_t(X) = U_t X U_t^{-1}, \quad \tau_0(X) = X. \tag{2.1}$$

Definition 1 A *process* is a C*-dynamical system $\langle \mathcal{A}, \mathbb{R}, \tau \rangle$.

The classical or quantum nature of the process depends on whether the algebra \mathcal{A} is or is not commutative.

For Salmon, processes are particular local entities identified both in terms of their geometric specifications in spacetime and the empirical manifestation of their uniformity. Yet, in Def. 1 there is nothing suggestive of a process' particular nature. We suggest a non-geometric account of a process which makes no reference to spacetime and locality, so the particular nature cannot be couched in terms of geometric specifications. In addition, although the uniformity of a process is reflected on the action of the group of isometries $t \mapsto \tau_t$ on the algebra that determines the dynamical evolution of the physical observables, a process, according to Def. 1, is not associated with any set of measurements of any observable along the process, as the specification of a process in terms of the empirical manifestation of its uniformity would seem to require. To consider the empirical manifestation of the uniformity along a process, we need to specify a particular normal state ω of the algebra \mathcal{A} and a particular observable $Q \in \mathcal{A}$. Then, the mapping $t \mapsto \omega(\tau_t(Q))$ describes the exhibited uniformity of the process. Parameter t indexes the stages of the process—be they spacetime points along a curve, in Salmon's geometric view—and $\omega(\tau_t(Q))$ is the corresponding expectation values of the observable $\tau_t(Q)$, resulting from measurements performed at each stage of the process. In what follows, we will consider the observable Q to be a projection and $\omega(\tau_t(Q))$ to denote the evolution of the probability of measuring Q along that process. The restriction to normal states of the algebra allows for such a probabilistic interpretation of $\omega(\tau_t(Q))$ and it is also supported by physical reasons.¹ In accordance with the above considerations we may define the concept of an instance of a process:

Definition 2 An *instance* of a process $\langle \mathcal{A}, \mathbb{R}, \tau \rangle$ is the quintuple $\langle \mathcal{A}, \mathbb{R}, \tau; \omega, Q \rangle$, where, $\langle \mathcal{A}, \mathbb{R}, \tau \rangle$ is a process, ω a normal state of the algebra \mathcal{A} and $Q = Q^* \in \mathcal{A}$, an observable of the algebra.

As noted, the causal character of a process is connected with its capacity to transmit a mark. The marking of a process is defined in terms of a *non-selective operation*, a state transformation

$$\omega \mapsto \omega_T : \omega_T(X) = \omega(T(X)), \quad X \in \mathcal{A}$$

induced by a linear, positive, completely positive and unital map $T : \mathcal{A} \rightarrow \mathcal{A}$.

Namely, T satisfies the following properties:

¹ Gleason's theorem and its consequences for a probabilistic interpretation of a quantum theory and the preparability of a state by means of physically realizable local operations in an open bounded region of Minkowski spacetime are some reasons for restricting physically admissible states to normal states. See [7].

$$\begin{aligned} \forall X, Y \in \mathcal{A} \quad \text{and} \quad \lambda, \mu \in \mathbb{C}, T(\lambda X + \mu Y) &= \lambda T(X) + \mu T(Y), \\ T(X^* X) &\geq 0, \\ T(I) &= I, \end{aligned}$$

and the map

$$id \otimes T : \mathbb{C}^{n \times n} \otimes \mathcal{A} \rightarrow \mathbb{C}^{n \times n} \otimes \mathcal{A}$$

is positive for every $n \in \mathbb{N}$.

Definition 3 A *mark* introduced into an instance $\langle \mathcal{A}, \mathbb{R}, \tau; \omega, Q \rangle$ of a process $\langle \mathcal{A}, \mathbb{R}, \tau \rangle$ is a change of the normal state ω of \mathcal{A} , $\omega \mapsto \omega_T$, defined by a non-selective operation, in terms of a linear, positive, completely positive and unital map $T : \mathcal{A} \rightarrow \mathcal{A}$, that is manifested as a change in the expectation value of the observable $\tau_{t_1}(Q)$, $\omega(\tau_{t_1}(Q)) \neq \omega_T(\tau_{t_1}(Q))$, for at least one $0 \neq t_1 \in \mathbb{R}$.

At this point, let us try to get a clearer understanding of the aforementioned definitions. Notice, firstly, that we consider two instances of the process, the unmarked instance, $\langle \mathcal{A}, \mathbb{R}, \tau; \omega, Q \rangle$, and the marked one, $\langle \mathcal{A}, \mathbb{R}, \tau; \omega_T, Q \rangle$, which provide different manifestations of the uniformity along the process. The relationship between these two instances can be understood in two complementary ways: (a) temporally, and (b) counterfactually. In the first interpretation, the two instances $\langle \mathcal{A}, \mathbb{R}, \tau; \omega, Q \rangle$ and $\langle \mathcal{A}, \mathbb{R}, \tau; \omega_T, Q \rangle$ describe different time parts of a single entity occupying subsequent time intervals, before and after the marking interaction. However, the result of putting together the two instances is not a new instance of the process. Each part of the composite entity is described independently in terms of a parameter t that indexes the stages of the process which extends over all real numbers, $t \in \mathbb{R}$ and is not restricted to any semi-closed interval of \mathbb{R} . This provision is essential in order to provide a time reversible account of the marking of a process. There is nothing paradoxical in this picture, since a plane wave that passing through a narrow slit on a barrier produces a spherical wave cannot be described by a single solution of the wave equation although both the spherical and the plane waves are solutions of the equation for every $t \in \mathbb{R}$. And the time-reversed picture of this physical situation represents a spherical wave with shrinking wavefronts converging to the slit for $t < 0$, being turned into a plane wave after having passed through the slit for $t > 0$. In the second interpretation, we say that $\langle \mathcal{A}, \mathbb{R}, \tau; \omega, Q \rangle$ would have been the instance of the process and no composite entity would have existed, *had the marking not occurred* while if the time arrow is reversed then $\langle \mathcal{A}, \mathbb{R}, \tau; \omega_T, Q \rangle$ would have been, respectively, the instance of the process had the marking not occurred. The similarity to the wave propagation still holds good for the second interpretation, since we may claim that had the plane wave not met the pierced barrier it would have continued to be a plane wave for every $t \in \mathbb{R}$. Salmon has been aware that the definition of a causal process in terms of the marking method involves the truth of counterfactual statements of this sort, and he deemed this a major problem to overcome only within the formulation of the invariant/conserved quantity theory [3, p. 148, 8].

Secondly, the marking is a physical interaction between at least two processes one of which is the process we aim to mark while the other may be, for example, a physical apparatus. In the case of the wave propagation through a single slit, the pierced barrier plays the role of the second process; more generally, however, we may consider any scattering system as producing a mark. In the latter case, the unmarked instance of the process, $\langle \mathcal{A}, \mathbb{R}, \tau; \omega, Q \rangle$, is the incoming process, before the scattering, and it is usually taken to occur asymptotically, in the infinite past. Respectively, the marked process $\langle \mathcal{A}, \mathbb{R}, \tau; \omega_T, Q \rangle$ is the outgoing process which occurs asymptotically in the infinite future. Disregarding the initial and the final states of the other processes (the scatterer or the physical apparatus) engaged in the physical interaction, as well as what happens in all finite time instants where the interaction occurs, we may describe the marking in terms of a mapping $\omega \mapsto \omega_T$ in the state space that is induced by a mapping T in the C^* -algebra \mathcal{A} . The mathematical details for the derivation of the operation induced by T from the assumption that a unitary “scattering” operator describes the interaction of a quantum system with an apparatus can be found in [9].

Thirdly, although the notion of operation has been developed in the context of quantum theory, Def. 3 does not exclude classical processes. In information theory, such classical operations are known as classical channels and they describe manipulation of classical information. They are defined in terms of linear, positive and unital maps from a commutative C^* -algebra to another—since complete positivity is equivalent to positivity for commutative algebras (see [10, Prop. 3.9, p. 199])—and they induce state transformations that are determined in terms of classical transition probability distributions in the state space.²

Although the definitions we provide cover both classical and quantum processes, in what follows, we are going to focus on the exploration of quantum processes. In particular, the normal state of \mathcal{A} we referred to in defining the instance of a process is described in terms of a density operator $W \in \mathcal{A}$ i.e. a positive operator in $\mathcal{A} \subseteq \mathcal{B}(\mathcal{H})$ with $tr(W) = 1$. Namely,

$$\omega(X) = tr(WX) \quad \text{for every } X \in \mathcal{A}.$$

This assumption implies restrictions about the type of the von Neumann algebra employed: \mathcal{A} can be a type I or II_1 but not a type II_∞ or III von Neumann algebra. This mathematical fact excludes the possibility of defining a process in terms of local algebras in many Haag–Araki theories on Minkowski spacetime since it has been proved that they are type III von Neumann algebras. Yet, we may define a

² For a finite state space $\mathcal{X} = \{x_1, \dots, x_n\}$, the algebra of observables is $C(\mathcal{X}) = \{f : f : \mathcal{X} \rightarrow \mathbb{C}\}$ and

$$\text{let } T : C(\mathcal{X}) \rightarrow C(\mathcal{X}) \text{ be a classical operation, such that } g = T(f) : \begin{bmatrix} g(x_1) \\ \vdots \\ g(x_n) \end{bmatrix} = \begin{bmatrix} T_{x_1 x_1} & \cdots & T_{x_1 x_n} \\ \vdots & \ddots & \vdots \\ T_{x_n x_1} & \cdots & T_{x_n x_n} \end{bmatrix} \begin{bmatrix} f(x_1) \\ \vdots \\ f(x_n) \end{bmatrix} \text{ for every } f \in C(\mathcal{X}).$$

process using the global algebra of the system (see below) which in many interesting cases is type I .³

Next, we define the marking interaction in terms of a non-selective operation described by the Lüders rule for a projection $P \in \mathcal{A}$.⁴ Hence, a mark is introduced by the state transformation

$$\omega \mapsto \omega_p,$$

where $\omega_p(X) = \text{tr}(W_p X)$ for every $X \in \mathcal{A}$ and $W_p \in \mathcal{A}$,

$$W_p = PWP + (1 - P)W(1 - P), \tag{2.2}$$

and it changes the expectation value of the observable $\tau_{t_1}(Q)$, for some $t_1 \in \mathbb{R}$.

$$\omega(\tau_{t_1}(Q)) = \text{tr}(W\tau_{t_1}(Q)) \neq \text{tr}(W_p\tau_{t_1}(Q)) = \omega_p(\tau_{t_1}(Q)).$$

As it will become clear later (see Lem. 8), this way of restricting the definition of the marking interaction delimits us to consider only quantum processes, since in classical processes represented by commutative von Neumann algebras the mark will never become manifest at any observable.

Fourthly, the choice of a non-selective operation for the description of the marking interaction can be justified by reference to Clifton and Halvorson’s distinction between physical and conceptual operations on an ensemble of physical systems [15]. They claimed, roughly, that given an ensemble of physical systems and a device that operates on the systems of the ensemble, if the ensemble in the final state includes all systems from the original ensemble without ignoring anyone, then the transformation of the ensemble is non-selective. If, on the other hand, in the final ensemble we keep only those systems that responded in a certain way to the operation of the device, selecting, thus, a desired outcome, then we do not consider only the physical interaction that takes place, but we perform, additionally, a conceptual operation to result in the final ensemble. Hence, our favoring non-selective operations for marking interactions reflects the choice of considering marking in terms of a physical interaction.

A typical example⁵ of a non-selective operation is provided by the Stern–Gerlach experiment in which a beam of electrons travels through an inhomogeneous magnetic field, it divides in two and it is deflected in two opposite directions, depending

³ That for several Haag–Araki theories of physical significance the local algebras $\mathcal{R}(O)$ of finite regions of Minkowski spacetime are type III_1 factors, see [11, Thm. 1.3.12, pp. 35 and 254]), and that the global algebra \mathcal{R} of a system may be a type I_∞ von Neumann factor if the system does not have superselection sectors, see [11, p. 24]).

⁴ For a brief and informative presentation of the Lüders rule, see [12]. The original article in German is [13] and an English translation is [14].

⁵ More examples are provided by quantum channels in quantum information theory. For instance, the amplitude-damping channel provides a simple model for the spontaneous emission of a photon in the transition of a two-level atom from its excited to its ground state. It is defined in terms of the map $T : \mathcal{B}(\mathbb{C}^2) \rightarrow \mathcal{B}(\mathbb{C}^2) : X \mapsto M_0^* X M_0 + M_1^* X M_1$ where $M_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{bmatrix}$, $M_1 = \begin{bmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{bmatrix}$ and p is the probability of decay.

on the orientation of the spin of the electrons. Electrons with “spin-up” are deflected toward the north pole while those with “spin-down” are deflected toward the south pole. This physical operation is non-selective since its outcome contains both branches resulting from splitting the original beam and it is described, mathematically, in terms of a map T that satisfies the property $T(I) = I$. If, on the other hand, we decided to focus on one branch of the divided beam—say the one consisting of electrons with “spin-up”—the physical operation would be followed by a conceptual operation and it would be *selective*; in this case, the map T would satisfy the property $T(I) < I$. Moreover, assume that the electrons in the original beam have already been prepared with their spin aligned in the positive direction of the z -axis of some Cartesian coordinate system (“spin-up” in \hat{z}) and we operate on the beam using a Stern–Gerlach apparatus that measures the spin along the x -axis of the same coordinate system. If we didn’t operate with the Stern–Gerlach apparatus the expectation value of the observable “spin in direction \hat{n} at an angle θ from the z axis on the xz -plane” would be proportional to $\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}$, $0 \leq \theta \leq \frac{\pi}{2}$. The same observable, after the operation with the Stern–Gerlach apparatus, has an expectation value equal to 0. This difference in the expectation values of an observable is required by Def. 3 to say that the instance of a process, defined in terms of that observable, is marked.

Finally, to return to Salmon’s views, to mark a process requires a local interaction that takes place at a spacetime region. This stipulation is not made here unless \mathcal{A} is considered the global algebra of a local quantum system defined in terms of a net of von Neumann algebras indexed by spacetime regions of a background spacetime. In this case, the state transformation described by the Lüders rule, (2.2), in terms of a local projection, corresponds to a non-selective measurement which takes place in a specified spacetime region; hence, marking a process becomes the outcome of a local interaction. Nevertheless, as mentioned earlier, these definitions do not intend to include geometric considerations and to regard causal processes as spacetime entities; locality issues will be postponed for the next section.

Next, we attempt to explicate the concept of mark transmission by providing two alternative formulations which make different demands regarding the continuity of the manifestation of the mark along a process. Without loss of generality we assume that the mark introduced into an instance of a process is manifested for a real number $t_1 > 0$ and we examine the continuity of its transmission over the interval $(0, t_1]$.⁶

Definition 4 Let $\langle \mathcal{A}, \mathbb{R}, \tau; \omega, Q \rangle$ be an instance of a process $\langle \mathcal{A}, \mathbb{R}, \tau \rangle$ and assume that a mark defined in terms of the Lüders rule for a projection $P \in \mathcal{A}$ is introduced into $\langle \mathcal{A}, \mathbb{R}, \tau; \omega, Q \rangle$ and is manifested for a $t_1 > 0$, i.e. $\omega(\tau_{t_1}(Q)) \neq \omega_P(\tau_{t_1}(Q))$. We say that the mark is *continuously transmitted* over an interval $(0, t_1]$ by that process, if

⁶ If we do not make this assumption and we consider any real number $t_1 \neq 0$, the interval over which the mark is transmitted is $(a, b) \cup \{t_1\}$, where $a = \min\{0, t_1\}$ and $b = \max\{0, t_1\}$. Thus, in Defs. 4 and 5 there is no hidden preference about the direction of transmission of the mark along a process, in conformity with Salmon’s desideratum of providing definitions of the fundamental causal notions that are not committed to a preferred time direction.

and only if it is manifested as a change in the expectation value of every observable $Q_t = \tau_t(Q)$, for $t \in (0, t_1]$.

Hence,

$$\omega(Q_t) = \text{tr}(WQ_t) \neq \text{tr}(W_P Q_t) = \omega_P(Q_t),$$

for every $t \in (0, t_1]$.

Definition 5 Let $\langle \mathcal{A}, \mathbb{R}, \tau; \omega, Q \rangle$ be an instance of a process $\langle \mathcal{A}, \mathbb{R}, \tau \rangle$ and assume that a mark defined in terms of the Lüders rule for a projection $P \in \mathcal{A}$ is introduced into $\langle \mathcal{A}, \mathbb{R}, \tau; \omega, Q \rangle$ and is manifested for a $t_1 > 0$, i.e. $\omega(\tau_{t_1}(Q)) \neq \omega_P(\tau_{t_1}(Q))$. We say that the mark is *continuously transmitted up to a countable set of isolated points* over an interval $(0, t_1]$ if and only if it is manifested as a change in the expectation value of every observable $Q_t = \tau_t(Q)$, for $t \in (0, t_1]$, with the exception of a countable set of values of the parameter t which have to form a discrete metric space with the usual metric of the real numbers.

Hence,

$$\omega(Q_t) = \text{tr}(WQ_t) \neq \text{tr}(W_P Q_t) = \omega_P(Q_t),$$

for every $t \in (0, t_1]$ with the exception of a countable set of values of the parameter which form a discrete metric space with the usual metric of the real numbers.

According to Def. 5, continuous mark transmission up to a countable set of isolated points allows denumerable gaps, either finite or countably infinite, in the manifestation of a mark over an interval of a process. It is demanded, however, the real values of the parameter in which the mark disappears to form a discrete space in the usual topology of the real numbers, i.e. to be isolated. Thus, for an instance $\langle \mathcal{A}, \mathbb{R}, \tau; \omega, Q \rangle$ of a process and a mark defined by a projection P , if

$$S = \left\{ t : t \in (0, t_1] \text{ and } \omega(Q_t) = \text{tr}(WQ_t) = \text{tr}(W_P Q_t) = \omega_P(Q_t), \right\},$$

is the subset of $(0, t_1]$ in which the mark disappears, Def. 5 requires that for every $t_0 \in S$, there is a neighborhood $\mathcal{N}(t_0, r) = \{t : |t - t_0| < r\}$ such that $\mathcal{N}(t_0, r) \cap S = \{t_0\}$. Put more simply, if a mark is continuously transmitted up to a countable set of isolated points over an interval $(0, t_1]$, the time instants in which the mark disappears are not densely distributed over $(0, t_1]$, i.e. they cannot be represented by the rational numbers in $(0, t_1]$.

Nevertheless, for Salmon, mark transmission over a continuous segment of a spacetime curve representing geometrically a causal process is continuous, i.e. the mark appears at each spacetime point of that segment. This conception is expressed in the following definition, given that t is taken to be a curve parameter that indexes spacetime points along that segment:

Definition 6 A process $\langle \mathcal{A}, \mathbb{R}, \tau \rangle$ is *causal* if and only if there exists an instance $\langle \mathcal{A}, \mathbb{R}, \tau; \omega, Q \rangle$ of the process and a mark, defined in terms of the Lüders rule for a

projection P , which can be transmitted continuously over a semi-closed interval of the real numbers.

A weaker definition of a causal process would allow ‘gappy’ causal processes without eliminating completely the continuity.

Definition 7 A process $\langle \mathcal{A}, \mathbb{R}, \tau \rangle$ is *CSIP-causal* or *causal-up-to-a-countable-set-of-isolated-points* if and only if there exists an instance $\langle \mathcal{A}, \mathbb{R}, \tau; \omega, Q \rangle$ of the process and a mark, defined in terms of the Lüders rule for a projection P , which can be continuously transmitted up to a countable set of isolated points over a semi-closed interval of the real numbers.

We say, once more, that although *CSIP-causal* processes save some part of the original intuition of continuity of mark transmission, they fail to provide an adequate account of Salmon’s original intuition of continuous propagation of causal influence. In the remaining section, we shall present facts about the manifestation of a mark in a process based on the above definitions.

Lemma 8 (Clifton et al. [16]) *For a von Neumann algebra \mathcal{A} and a state transformation $\omega \mapsto \omega_P$, defined by a non-selective Lüders operation for a projection $P \in \mathcal{A}$, the expectation value of an observable $Q \in \mathcal{A}$ is invariant under this state transformation if and only if $[P, Q] = 0$.*

Corollary 9 *For an instance $\langle \mathcal{A}, \mathbb{R}, \tau; \omega, Q \rangle$ of a process $\langle \mathcal{A}, \mathbb{R}, \tau \rangle$ and a mark defined in terms of the Lüders rule for a projection P , a necessary and sufficient condition for the mark that is introduced into $\langle \mathcal{A}, \mathbb{R}, \tau; \omega, Q \rangle$ to be manifested at any stage $t_1 \in \mathbb{R}$ of the process, at $\tau_{t_1}(Q)$, is the non-commutation of P and $\tau_{t_1}(Q)$,*

$$\omega(\tau_{t_1}(Q)) = \text{tr}(W\tau_{t_1}(Q)) = \text{tr}(W_P\tau_{t_1}(Q)) = \omega_P(\tau_{t_1}(Q)) \iff [P, \tau_{t_1}(Q)] = 0.$$

Lemma 10 (Borchers [17]) *Assume we have a strongly continuous one-parameter group U_t of unitary operators on a Hilbert space whose generator H has a spectrum bounded from below. Let E, F be two projections such that*

$$U_t E U_t^{-1} F = F U_t E U_t^{-1}$$

for $|t| < \varepsilon$, for some $\varepsilon > 0$. If we have $EF = 0$ then $U_t E U_t^{-1} F = 0$ for all t .

In Lem. 10 a spectrum condition is imposed on the dynamics which is commonly taken as expressing the requirement that the energy is positive. In addition, spectrum condition in the Haag–Araki theories of relativistic quantum fields is interpreted as further requiring that effects propagate at velocities less or equal to the velocity of light in the vacuum. Thus, one may suggest that a causal process is defined in terms of “a dynamical system + an adequate spectrum condition”; a suggestion expressing the idea that causality restricts the dynamics. First of all, the interpretation of spectrum condition as a causality condition is not straightforward and unobjectionable.

In a recent article, Earman and Valente conclude that “... while it is wrong to regard the spectrum condition as a straightforward causality condition, it does lie at a node of a complex web of interconnected causality concepts of AQFT” [18]. Secondly, apart from issues of interpretation of the spectrum condition, this suggestion regarding causal processes is not directly related to Salmon’s early process theory of causation, which is our source of inspiration in this paper, and unless one has provided a connection, we will not consider it here. The following proposition is proven by means of Lem. 10, hence, it requires the spectrum condition; however, as we will see, it is not sufficient to establish a causal process in Salmon’s sense. In this proposition, we assume that the projection defining the marking operation is identical with the observable which exhibits the uniformity of the process in the specified instantiation.

Proposition 11 *For an instance $\langle \mathcal{A}, \mathbb{R}, \tau; \omega, P \rangle$ of a process $\langle \mathcal{A}, \mathbb{R}, \tau \rangle$, consider a mark defined in terms of the Lüders rule for a projection $P \in \mathcal{A}$. If the mark is introduced into $\langle \mathcal{A}, \mathbb{R}, \tau; \omega, P \rangle$ and is manifested at $P_{t_1} = \tau_{t_1}(P)$, for some $t_1 > 0$, then it will be manifested at $P_t = \tau_t(P)$, for infinitely many $t \in (0, t_1]$.*

Proof Assume for *reductio* that for every $t \in (0, t_1)$ the mark disappears. Then, by Cor. 9, we infer that $[P, P_t] = 0$ for all $t \in (0, t_1)$ which can be extended to $t \in (-t_1, t_1)$.

Next, by applying Lem. 10 for the pair of orthogonal projections $P, P^\perp = I - P$, where I is the unit of \mathcal{A} , we conclude that $[P, P_t] = 0$ for all $t \in \mathbb{R}$; which is not true, by hypothesis.

Hence, there is a $t_2 \in (0, t_1)$ in which the mark appears, i.e. $t_2 \notin S$. By applying the same argument for the transmission of the mark over $(0, t_2]$, we may conclude the existence of a $t_3 \notin S$, and so on ad infinitum. □

From the proof of Prop. 11 it can be easily understood that the causal nature of the process has not been established, since to conclude the manifestation of a mark at the intermediate stages of the process, we applied the same argument countably many times; hence we established the manifestation of a mark along an interval of a process countable infinitely many times but not continuously. Moreover, the mark is not even transmitted continuously up to a countable set of isolated points, since one cannot exclude on a priori grounds that the mark appears *only* at time instants $t_n = \frac{1}{r} + \frac{1}{nr}$ for some $0 \neq r \in \mathbb{R}$ and for every $n \in \mathbb{N}$. Thus, we need stronger assumptions, as those stated in Prop. 13, to meet the conditions of Def. 5, and prove at least the existence of CSIP-causal processes. But first, let us show another useful lemma.

Lemma 12 *For an instance $\langle \mathcal{A}, \mathbb{R}, \tau; \omega, Q \rangle$ of a process $\langle \mathcal{A}, \mathbb{R}, \tau \rangle$, and a mark defined in terms of the Lüders rule for a projection P , a necessary and sufficient condition for the mark that is introduced into $\langle \mathcal{A}, \mathbb{R}, \tau; \omega, Q \rangle$ not to be manifested at stage $t_1 \in \mathbb{R}$ of the process, at $\tau_{t_1}(Q)$, is the following:*

$$\omega(\tau_{t_1}(Q)) = tr(W\tau_{t_1}(Q)) = tr(W_P\tau_{t_1}(Q)) = \omega_P(\tau_{t_1}(Q)) \iff \omega([P, [P, \tau_{t_1}(Q)])] = 0.$$

Proof (By simple calculations). Firstly, for W_P defined as in 2.2,

$$W_P \tau_{t_1}(Q) = W \tau_{t_1}(Q) + PWP \tau_{t_1}(Q) - PW \tau_{t_1}(Q) - WP \tau_{t_1}(Q) + PWP \tau_{t_1}(Q). \tag{2.3}$$

Next, given the property of the trace, $tr(AB) = tr(BA)$, by properly grouping the factors in 2.3, one may show directly,

$$tr(W_P \tau_{t_1}(Q)) = tr(W \tau_{t_1}(Q)) - tr(W[P, [P, \tau_{t_1}(Q)]]).$$

Hence, $\omega(\tau_{t_1}(Q)) = \omega_P(\tau_{t_1}(Q)) \iff \omega([P, [P, \tau_{t_1}(Q)]]) = 0. \quad \square$

As in Prop. 11, we assume that the projection defining the marking operation is identical with the observable which exhibits the uniformity of the process in the specified instantiation.

Proposition 13 *For an instance $\langle \mathcal{A}, \mathbb{R}, \tau; \omega, P \rangle$ of a process $\langle \mathcal{A}, \mathbb{R}, \tau \rangle$, consider a mark defined in terms of the Lüders rule for the projection $P \in \mathcal{A}$ which is analytic for τ .⁷ The mark is continuously transmitted up to a countable set of isolated points over an interval $(0, t_1]$, if it is manifested at $P_{t_1} = \tau_{t_1}(P)$. Hence, according to Def. 7, $\langle \mathcal{A}, \mathbb{R}, \tau \rangle$ is a CSIP-causal process.*

Proof Observe, firstly, that the set of τ -analytic elements elements is τ -invariant, i.e. if $P \in \mathcal{A}$ is analytic for τ then $P_t = \tau_t(P)$ is also analytic for τ for every $t \in \mathbb{R}$ and $P_0 = P$. In addition, the sum and the product of any two τ -analytic elements elements is analytic for τ (for a proof see lemma in “Appendix”). As a consequence the element

$$P_{-t}(P_{-t}P - PP_{-t}) - (P_{-t}P - PP_{-t})P_{-t},$$

where $P_t = \tau_t(P)$ for any $t \in \mathbb{R}$ and $P_0 = P$, is also τ -analytic.

In view of this fact, there is an \mathcal{A} -valued analytic function $f : I_\lambda \rightarrow \mathcal{A}$, where I_λ is a strip in the complex plane, $I_\lambda = \{z \in \mathbb{C} : |Imz| < \lambda, \lambda > 0\}$, such that for

$$z = t \in \mathbb{R}, \quad f(t) = \tau_t(P_{-t}(P_{-t}P - PP_{-t}) - (P_{-t}P - PP_{-t})P_{-t}),$$

or

$$z = t \in \mathbb{R}, \quad f(t) = [P, [P, P_t]].$$

Hence, for every state ω of \mathcal{A} , the complex-valued function

$$F : I_\lambda \rightarrow \mathbb{C}, \quad F(z) = \omega(f(z))$$

is also analytic (see “Appendix”).

⁷ See “Appendix”

By hypothesis, a mark defined in terms of the Lüders rule for the projection P and introduced into the instance $\langle \mathcal{A}, \mathbb{R}, \tau; \omega, P \rangle$ of the process $\langle \mathcal{A}, \mathbb{R}, \tau \rangle$ is manifested at $P_{t_1} = \tau_{t_1}(P)$. This fact entails (by Lem. 12) that F is not a constant function, i.e.,

$$F(0) = \omega(f(0)) = 0 \neq \omega(f(\tau_{t_1}(P))) = F(t_1).$$

Now, since I_λ is an open connected set, by a well-known theorem of complex analysis [19, Thm. 1.2, p. 90], the set of zeros of F is at most countable and discrete, i.e. for every $z_0 \in I_\lambda$, $F(z_0) = 0$, there is a disc $D(z_0, r)$ of some radius $r > 0$ such that for no $z \in D(z_0, r)$, $z \neq z_0$, $F(z) = 0$. □

3 Causal Processes and Local Quantum Physics

To discuss the locality aspect of a causal process as defined in the previous section, one needs to be able to refer meaningfully to local observables and to local states as well as to formulate locality conditions. A friendly environment for this type of investigation, which combines the C*-algebraic setting with essential reference to spacetime concepts is local quantum physics [20]. In particular, we will consider a relativistic quantum field theory on Minkowski spacetime whose models are structures of the following type:

$$\langle \mathcal{H}, O \mapsto \mathcal{R}(O), (a, \Lambda) \mapsto U(a, \Lambda) \rangle,$$

where \mathcal{H} is a complex Hilbert space, $O \mapsto \mathcal{R}(O)$ is a net associating a von Neumann algebra $\mathcal{R}(O)$ with every open bounded region O of Minkowski spacetime, and $(a, \Lambda) \mapsto U(a, \Lambda)$ is a strongly continuous unitary representation of the (proper orthochronous) Poincaré group in \mathcal{H} . The vacuum state is supposed to be the unique Poincaré invariant state and is represented by a normalized vector $\Omega \in \mathcal{H}$. The models are supposed to satisfy the usual Haag–Araki axioms, [11, pp.8–23],⁸ so that, in particular, the global von Neumann algebra $\mathcal{R} = \left(\bigcup_{O \subset \mathbb{R}^4} \mathcal{R}(O) \right)''$, associated with the whole Minkowski spacetime is well-defined and the vacuum vector Ω is cyclic for every local algebra and separating for every local algebra over a region with non-empty causal complement—courtesy of the celebrated Reeh–Schlieder theorem (see [24] and [11, Thm. 1.3.1, p. 25]). Moreover, we assume explicitly that the global algebra \mathcal{R} is a type I von Neumann algebra.

Without loss of generality, we restrict our attention to the time evolution of a system moving along a timelike direction a . The observables measured in any

⁸ In 1964, Haag and Kastler in a well-known paper provided an axiomatic formulation of quantum theory of fields in terms of nets of abstract C*-algebras of local observables instead of quantum fields that, henceforth, is known as the Haag–Kastler formulation [21]. The same year, Araki published two papers [22] and [23], in which he used nets of von Neumann algebras of bounded operators on a Hilbert space to axiomatize quantum field theory using analogous axioms to the Haag–Kastler ones. This formulation is known as the Haag–Araki approach to quantum field theory.

open bounded region O of Minkowski spacetime are represented by self-adjoint elements of $\mathcal{R}(O)$. They are evolving with time according to the law:

$$U_t \mathcal{R}(O) U_t^{-1} = \mathcal{R}(O + ta),$$

where $O + ta = \{x \in \mathbb{R}^4 : x = y + a, y \in O\}$, $t \in \mathbb{R}$. Respectively, any element A of the global von Neumann algebra \mathcal{R} is evolving with time to the element $U_t A U_t^{-1} \in \mathcal{R}$. Hence, the process we are interested in is the C*-dynamical system $\langle \mathcal{R}, \mathbb{R}, \tau \rangle$, where the group of automorphisms τ_t , defined by 2.1, describes translations along a given timelike direction a . Furthermore, to define an instance of that process, $\langle \mathcal{R}, \mathbb{R}, \tau; \omega, Q \rangle$ we need to single out a normal state ω and a projection Q of the global algebra \mathcal{R} .

In a relativistic quantum field theory we can say more about the local features of such a process. Firstly, the geometric representation of such a process is a timelike line $\{ta : t \in \mathbb{R}\}$. Further, if we associate an open bounded spacetime region O with the process, we may consider the process to be represented geometrically by a tube defined as the translation of O along the vector a . Secondly, for every $t \in \mathbb{R}$, region $O + ta$ represents geometrically a stage of the process of which the spacelike complement is well-defined:

$$(O + ta)' = \text{int}\{x \in \mathbb{R}^4 : (x - y)^2 < 0, y \in O + ta\}$$

According to the axiom of causality–locality [11, Ax. III, p. 14],

$$\forall t \in \mathbb{R}, \forall K \subseteq \mathbb{R}^4, \text{ open and bounded, } K \subset (O + ta)' \implies [\mathcal{R}(K), \mathcal{R}(O + ta)] = 0,$$

where, two algebras commute if and only if every element of the first commutes with every element of the second algebra and vice versa.

The causality–locality axiom stipulates the independence of statistical predictions associated with the expectation values of any two observables belonging to local algebras of spacelike separated regions—a fact expressed by the commutativity of the algebras.

In terms of the local algebras $\mathcal{R}(O)$, one may understand the marking of a process as the result of a local interaction which takes place in O by demanding the projection P in 2.2 to be a local projection, $P \in \mathcal{R}(O)$. Moreover, from the axiom of causality–locality and Lem.8 it can be inferred that a mark introduced into an instance $\langle \mathcal{R}, \mathbb{R}, \tau; \omega, Q \rangle$ of a process $\langle \mathcal{R}, \mathbb{R}, \tau \rangle$ by a local interaction in O is not manifested at any local observable $Q' \in \mathcal{R}(K)$ associated with an open bounded region $K \subset O'$. Conversely, any change produced in the spacelike complement of the tube that represents geometrically the stages of a process associated with a real interval I is not manifested as a change in the expectation value of the characteristic observable $\tau_t(Q)$ for every $t \in I$. The non-manifestation of a mark at spacelike distance from the region in which it is introduced as well as the shielding of any part of a process from influences produced in spacelike distance from it, builds in this account the demand of the theory of relativity that no causal influence is propagated at velocities greater than the velocity of light in the vacuum.

To obtain a causal process one needs to establish the continuity in the manifestation of the mark along the process. By application of Prop. 11 we can infer that a mark introduced by a local non-selective Lüders operation for a projection $P \in \mathcal{R}(O)$, in a spacetime region O , into an instance $\langle \mathcal{R}, \mathbb{R}, \tau; \omega, P \rangle$ of a process $\langle \mathcal{R}, \mathbb{R}, \tau \rangle$, is manifested at infinite time instants in the interval $(0, t_1)$, in spacetime regions $O + ta$, at observables $P_t = \tau_t(P) \in \mathcal{R}(O + ta)$, given that it is manifested at $P_{t_1} = \tau_{t_1}(P) \in \mathcal{R}(O + t_1a)$ for some $t_1 \in \mathbb{R}$. Nevertheless, as we have already explained above, this result is insufficient to justify any intuition of continuity, thus, to establish the causal character of a process.

To make, however, stronger demands, as those required by Prop. 13, we need to abandon some essential locality conditions, since there are no analytic elements for time translations which belong to a local algebra associated with any open bounded spacetime region in a Haag–Araki theory of relativistic quantum fields that satisfies the usual axioms (see, “Appendix”). Hence, we are not allowed to assume that a mark introduced into a process is the result of a local interaction associated with a state transformation of the form of 2.2, defined in terms of an *analytic* local projection P . Moreover, given that we cannot consider the marking operation localized in a bounded spacetime region, we cannot talk meaningfully of the manifestation or not of the mark in spacelike distance from its locus of introduction. In addition, the causality–locality axiom does not apply since it is valid only for local observables.

On the other hand, since analytic elements for time translations form a norm-dense subset of \mathcal{R} , one may take the introduction of a mark into an instance $\langle \mathcal{R}, \mathbb{R}, \tau; \omega, P \rangle$ of a process $\langle \mathcal{R}, \mathbb{R}, \tau \rangle$ to be described in terms of an *analytic* projection P which differs, in the norm of \mathcal{R} , from a local projection $P' \in \mathcal{R}(O)$ less than a quantity $\delta > 0$,

$$\|P - P'\| < \delta \implies |\omega(P) - \omega(P')| < \delta,$$

for every state ω on \mathcal{R} ; where δ denotes the experimental error. Hence, although the *actual* or the *real* physical operation of marking a process takes place in a spacetime region and calls for a representation in terms of a local projection, one may *decide* to represent it *approximately* in terms of an analytic for time translations element which is practically indistinguishable from the local projection for every physically admissible (normal) state of the system. In this way, one may talk of an almost local mark introduced into an instance of a process $\langle \mathcal{R}, \mathbb{R}, \tau \rangle$ in some open bounded region O .

Next, consider the manifestation of the mark in an instance $\langle \mathcal{R}, \mathbb{R}, \tau; \omega, P \rangle$ of a process $\langle \mathcal{R}, \mathbb{R}, \tau \rangle$. Since $\tau_t, t \in \mathbb{R}$ is a group of isometries, and P , as before, is a τ -*analytic* projection which differs, in the norm of \mathcal{R} , from a local projection $P' \in \mathcal{R}(O)$ less than a quantity $\delta > 0$,

$$\|P - P'\| < \delta \implies \|\tau_t(P) - \tau_t(P')\| < \delta,$$

and the mark manifestation is required to be detected in $\tau_t(P)$, the time translation of the analytic element P , instead of the time translation of the local projection P' that would represent the mark, had it been local. Hence, we talk about an almost

local mark associated with a region O which may be approximately manifested in the translation of O , $O + ta$, along the timelike vector a .

At this point one may object that, on the one hand, we employed analytic elements in the representation of an almost local marking operation claiming that they are practically indistinguishable from local projections belonging to open bounded spacetime regions—i.e. the difference in their expectation values, in any state, is undetectable given the limits experimental error—while on the other, we seem to be satisfied with Defs. 3 and 4 which defines mark manifestation in terms of a difference in the expectation value of an observable, without taking into account whether this difference is detectable or not. To be consistent with the introduction of experimental error as a limit to the detectability of the difference in the expectation values of observables in a given state, we should demand

$$|\omega(\tau_t(P)) - \omega_P(\tau_t(P))| = |\text{tr}(W\tau_t(P)) - \text{tr}(W_P\tau_t(P))| \geq \delta, \quad (3.1)$$

where $W \in \mathcal{R}$ is a density operator and $W_P \in \mathcal{R}$ is defined as in 2.2, to claim that the mark is manifested at $\tau_t(P)$ for some $t \in \mathbb{R}$. The objection is correct and it requires further investigation to obtain results that would comply with condition 3.1. In this paper, however, we follow Defs. 3 and 4 for characterizing a causal process.

Now, by applying Prop. 13 one concludes that a process $\langle \mathcal{R}, \mathbb{R}, \tau \rangle$ represented geometrically by a tube defined as the translation of O along a timelike vector a qualifies as a *CSIP-causal process* according to Def. 7 since it is capable to transmit continuously up to a countable set of isolated points a mark defined for an analytic for time translations projection P , introduced into an instance of a process almost locally, in the sense explained, in a bounded spacetime O , over an interval $(0, t_1]$, according to Def. 5.

Valente [25] has raised serious objections against the use of analytic elements for time translations to describe operations in local quantum systems.⁹ One objection says that since analytic elements cannot be local in any standard Haag–Araki theory, they represent uninstantiated events in spacetime and they cannot be taken, in any meaningful way, to be in relation to any event or to have any local effect in spacetime. The second objection relates to the way analytic elements can be constructed by smearing local observables of a bounded region O (see, “Appendix”), providing, thus, some type of approximate localization in O (by disregarding the ‘tails’ of the smearing function). However, essentially, analytic elements are global observables and to talk about almost *local* operations is rather a euphemism. The third objection says that since analytic observables are not local they are not real observables and no actual measurement can be performed of these fictional observables. Hence, no real effect can be produced by such fictional operations.

Both the first and the third objection fit nicely for the critical discussion of Argeorgis and Stergiou’s argument [26], as intended. There, the operation is defined in terms of analytic elements and the manifestation of the mark introduced is a change

⁹ The discussion in this Valente’s paper refers to almost local operators which are constructed as analytic elements for the generators of the group of translations of Minkowski spacetime.

of the expectation value of a local observable; hence, one may reasonably object that uninstantiated events in spacetime cannot be in relation with proper events and that fictional operations cannot produce real effects. In the present discussion, however, for better or worse, the relation is established between homologous events, both uninstantiated, which lie at a fictional level. Here, we tentatively explore what would happen if local operations and local observables were represented by non-local ones which, nevertheless, bear a special relation to their local counterparts that allows us to talk about almost local marking operations and approximate manifestations.

To defend somewhat further the tentative hypothesis of using analytic elements for the description of operations, let me refer to an old paper of Steinmann [27] in which (selective) operations representing particle detection experiments are defined in terms of analytic elements. There, Steinmann proved a result that captures a basic intuition of particle causal processes. Namely, he has proven that in an one-particle state of a field theory, "...the probability that two counters are triggered by the particle is appreciable only if their separation is approximately parallel to the momentum of the particle." In addition, if a third counter is placed, then "...all three counters can be triggered only if they lie roughly in a straight line." Still, one may object to the description of a particle's motion in terms of locality considerations. Yet, we, surely, would like to save the intuitive plausibility of such particle motion as a good candidate for an 'approximate' process.

As a final comment, I would like to point out the problematic implementation of relativistic causality when discussing causal processes in terms of analytic for time translations elements in local quantum physics. As explained before, the axiom of causality–locality guarantees that when considering causal processes in terms of elements of local algebras, the mark introduced into a process is not manifested at spacelike distance from the locus of its introduction. One cannot make the same stipulation when dealing with analytic for time translation projections that are associated with some spacetime region O in the aforementioned sense, since the relevant axiom is not applicable any more. Still we may claim that the mark defined in terms of analytic projection P does not become manifest at observables which commute with P ; however, we cannot any more claim that every observable associated with an open bounded region in the spacelike complement of O , be that a local one or a practically indistinguishable (within δ) from a local observable, commutes with P , making, thus, the relevant mark disappear.

4 Summary

Process theories of causation employ three types of locality conditions in the explanation of the cause and effect relation: (a) the production of causal influence and its manifestation along a process occurs at spacetime points; (b) the propagation of causal influence is continuous in spacetime; (c) for any given spacetime point there is a set of spacetime points which are not connectable by means of causal processes, hence, these spacetime points are shielded from causal influence produced at the given point.

In this paper, we attempted to define causal processes in the absence of any reference to spacetime. We referred to C^* -dynamical systems their states and state transformations described in terms of projections in a von Neumann algebra realizing marking processes (Defs. 1, 2, 3, 4, 6 and 7). Well-known conditions for the shielding of certain observables characterizing quantum processes from the influence of marking operations, couched in terms of the invariance of the expectation value of these observables under the marking state transformations, were formulated (Lem. 8). The originality of our contribution rests, mainly, on the attempt to establish some satisfactory conception of continuity in the propagation of the mark along such a process. Proper continuity in mark transmission, required for a causal process could not be established. Nonetheless, we have shown that a mark is manifested infinitely many times in an interval of a process (Prop. 11), and that, on certain conditions, the mark is manifested continuously up to a countable set of isolated point (Prop. 13), establishing, thus, the existence of CSIP-causal processes. Thus, in this abstract algebraic setting we expressed conditions for process theories which if successfully associated with spacetime properties and relations they would yield a local account of causal processes.

To obtain a local account of causal processes, we examined a Haag–Araki theory of relativistic quantum fields on Minkowski spacetime. In this context, local observables belong to von Neumann algebras associated with open bounded spacetime regions and a local marking operation can be defined in terms of a local projection. In addition, we explained how one can perceive an almost local marking operation, as a representative of a local marking operation, and an approximate manifestation of this mark associated with an analytic projection for time translations. Analytic elements do not belong to the algebra of any open bounded region but they can be practically indistinguishable from local elements. In this way, the first of the three aforementioned locality conditions for causal processes was treated. The axiom of causality–locality satisfied by the Haag–Araki theories provides a necessary and sufficient condition for the non-manifestation of a local mark at spacetime distance from the region in which it is generated. A similar condition for marking operations in terms of analytic elements could not be established and I am not sure whether it can be assumed independently. On the other hand, continuity has been established, in the rather weak form of CSIP mark transmission by Prop. 11 in terms of almost local marking operations, while if our attention were to be restricted to local marking operations, the same goal would not be attained. Although we could not establish the existence of causal processes in Salmon’s sense, yet, we did try to explore, formally, different approaches which emphasize on aspects of the philosopher’s conception within some limitations.

Appendix: On Analytic Elements for the One-Parameter Group of Time Translations in Local Quantum Physics

Given a $\sigma(X;F)$ -continuous group of isometries of a complex Banach space X , an element $A \in X$ is said to be *analytic for τ* (τ -analytic) if there exist a strip

$$I_\lambda = \{z \in \mathbb{C} : |Imz| < \lambda\}$$

in \mathbb{C} , and a function $f_A : I_\lambda \rightarrow X$ such that

- (i) $f_A(t) = \tau_t(A)$ for $t \in \mathbb{R}$,
- (ii) $z \mapsto \eta(f_A(z))$ is analytic for all $\eta \in F$ and $z \in I_\lambda$.

On these conditions, $f_A(z) = \tau_z(A)$, $z \in I_\lambda$ is an X -valued analytic function; if $\lambda = \infty$ then f_A is entirely analytic [28, Def. 2.5.20, p. 99]. Moreover, condition (ii) is equivalent to the existence of the limit

$$\lim_{h \rightarrow 0} \frac{f_A(z+h) - f_A(z)}{h}$$

in norm in X for $z \in I_\lambda$ as h tends to zero in \mathbb{C} , [28, Prop. 2.5.21, p. 99].

The groups of isometries we consider in this paper are $\sigma(\mathcal{A}, \mathcal{A}^*)$ -continuous i.e. for all $A \in \mathcal{A}$, $t \mapsto \eta(\tau_t(A))$ is continuous for every $\eta \in \mathcal{A}^*$, which amounts to the requirement that $t \mapsto \tau_t(A)$ be continuous in norm for every $A \in \mathcal{A}$ where \mathcal{A} is a von Neumann algebra.

Lemma *The sum and the product of two analytic elements for a group of automorphisms τ in a von Neumann algebra \mathcal{A} is a τ -analytic element.*

Proof Consider two τ -analytic elements $A, B \in \mathcal{A}$ and let $f_A(z) = \tau_z(A)$, $z \in I_{\lambda_1}$, $f_B(z) = \tau_z(B)$, $z \in I_{\lambda_2}$ for $I_{\lambda_i} = \{z \in \mathbb{C} : |Imz| < \lambda_i\}$, $i = 1, 2$ be the two corresponding \mathcal{A} -valued functions that satisfy conditions (i) and (ii). To show that the product AB is τ -analytic we should prove that the \mathcal{A} -valued function $f_{AB} : I_\lambda \rightarrow \mathcal{A}$, $f_{AB}(z) = f_A(z)f_B(z)$ which extends the function $f_{AB}(t) = f_A(t)f_B(t) = \tau_t(A)\tau_t(B) = \tau_t(AB)$ for $t \in \mathbb{R}$, satisfies the condition of analyticity:

$$\lim_{h \rightarrow 0} \frac{f_{AB}(z+h) - f_{AB}(z)}{h}$$

in $I_\lambda = \{z \in \mathbb{C} : |Imz| < \lambda\}$, $\lambda = \min\{\lambda_1, \lambda_2\}$. But

$$\begin{aligned} \frac{f_{AB}(z+h) - f_{AB}(z)}{h} &= \frac{f_A(z+h)f_B(z+h) - f_A(z)f_B(z)}{h} \\ &= \frac{f_A(z+h) - f_A(z)}{h} f_B(z+h) + f_A(z) \frac{f_B(z+h) - f_B(z)}{h}. \end{aligned}$$

Since f_A and f_B satisfy the analyticity condition, which implies also the continuity of f_A and f_B , each factor in the last expression in the above equality has a well-defined limit for $h \rightarrow 0$ and the analyticity condition for f_{AB} is satisfied, for every $z \in I_\lambda$. Hence, AB is a τ -analytic element of \mathcal{A} .

The case of the sum of two τ -analytic elements can be proven along the same line of reasoning by a simple direct calculation. □

To obtain a family of analytic elements for the group of time translations one may begin with any element localized in some open bounded region O of Minkowski spacetime, $A \in \mathcal{R}(O)$, and smear its time translation over \mathbb{R} with a Gaussian function depending on a parameter $n \in \mathbb{N}$. Namely,

$$A_n = \sqrt{\frac{n}{\pi}} \int_{-\infty}^{+\infty} e^{-nt^2} \tau_t(A) dt, \quad n \in \mathbb{N}.$$

Each A_n is an entire analytic element for τ and the family $\{A_n\}_{n \in \mathbb{N}}$ converges to A in the $\sigma(X;F)$ topology. Moreover, in [28, Cor. 2.5.23, p. 101] it is shown that the set of entire analytic elements form $\sigma(\mathcal{R};\mathcal{R}^*)$ -continuous group of isometries form a norm-dense subset of \mathcal{R} .

Fact (Fredenhagen, 2019)¹⁰ In a Haag–Araki theory of relativistic quantum fields on Minkowski spacetime, no non-trivial analytic elements for time translations can be localized in open bounded regions of Minkowski spacetime.

Proof Let A be analytic for time translations element which is also localized in a bounded region O . Let B be localized in the spacelike complement of an ε -neighbourhood of O , for some $\varepsilon > 0$. Then the commutator,

$$[\tau_t(A), B]$$

vanishes in an open interval of t . For $|t| < \varepsilon$ we have

$$\langle B^* \Omega, \tau_t(A) \Omega \rangle = \langle \tau_t(A^*) \Omega, B \Omega \rangle,$$

where $\Omega \in \mathcal{H}$ is the vacuum vector.

By the spectrum condition, which demands the generator of the time translations to be positive, the term on the left hand side defines a function that is analytic in the upper halfplane while the term on the right hand side, an analytic function in the lower halfplane. By assumption, they are also analytic in a strip around the real axis. Since these analytic functions coincide in an open interval of the real axis, we obtain an entire bounded analytic function which, therefore, is constant.

By the Reeh–Schlieder Theorem, these observables B generate from the vacuum Ω a dense set of vectors in the Hilbert space \mathcal{H} . Thus, $A\Omega$ is invariant under time evolution and, due to the uniqueness of the vacuum,

$$A\Omega = \langle \Omega, A\Omega \rangle \Omega.$$

Thereby, we get the result that an analytic local observable is a multiple of the identity of the algebra. \square

¹⁰ Private communication.

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