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″The Unavoidable Interaction Between the Object and the Measuring Instruments″: Reality, Probability, and Nonlocality in Quantum Physics

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Abstract

This article aims to contribute to the ongoing task of clarifying the relationships between reality, probability, and nonlocality in quantum physics. It is in part stimulated by Khrennikov's argument, in several communications, for "eliminating the issue of quantum nonlocality" from the analysis of quantum entanglement. I argue, however, that the question may not be that of eliminating but instead that of further illuminating this issue, a task that can be pursued by relating quantum nonlocality to other key features of quantum phenomena. I suggest that the following features of quantum phenomena and quantum mechanics, distinguishing them from classical phenomena and classical physics—(1) the irreducible role of measuring instruments in defining quantum phenomena, (2) discreteness, (3) complementarity, (4) entanglement, (5) quantum nonlocality, and (6) the irreducibly probabilistic nature of quantum predictions—are all interconnected, so that it is difficult to give an unconditional priority to any one of them. To argue this case, I shall consider quantum phenomena and quantum mechanics from a nonrealist or, in terms adopted here, "reality-without-realism" (RWR) perspective. This perspective extends Bohr's view, grounded in his analysis of the irreducible role of measuring instruments in the constitution of quantum phenomena.

Keywords Complementarity · Entanglement · Quantum causality · Quantum indefinitiveness · Quantum nonlocality · Reality without realism

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1 Introduction

This article aims to contribute to the ongoing task of clarifying the relationships between reality, probability, and nonlocality in quantum theory, specifically quantum mechanics, QM.[1] My argument is based on the concept of "reality without realism," RWR, introduced by this author previously (e.g., [1–7]), and an interpretation of quantum phenomena and QM defined by this concept, which allows for a range of interpretations. The concept of reality without realism and RWR-type interpretations only assume the concept of *reality*, defined as that which is assumed to exist, while placing the *character* of this existence beyond representation or knowledge, or even conception. By contrast, realism is defined here by assuming the possibility of representing or at least forming a conception of the character of the reality considered. In RWR-type interpretations, the concept of reality without realism only applies to the ultimate reality, idealized in terms of quantum *objects*, responsible for quantum *phenomena*, observed in measuring instruments, which allow for a realist treatment (by means of classical physics). Thus, the reality considered in these interpretations is stratified into that of the RWR-type and that of the realist type.

In their famous 1935 paper, "Can the Quantum-Mechanical Description of Physical Reality be Considered Complete?" A. Einstein, B. Podolsky, and N. Rosen (EPR), proposed the concept of reality based on the following criterion: "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity" [8, p. 138] . EPR define a theory as complete if "every element of the physical reality must have a counterpart in the physical theory," which would thus represent every such element and, as result, predict it with probability equal to unity [8, p. 138]. By using the thought experiment, now known as the EPR experiment, EPR argued that QM is either incomplete or nonlocal (by allowing an instantaneous action at a distance) and thus injected the question of nonlocality into by then a decade-long debate concerning QM. EPR's argument was challenged by Bohr, who argued that EPR's "criterion of reality ... contains ... an essential ambiguity when it is applies the actual problem with which we are here concerned" [9, p. 697]. During the last half a century, in the wake of Bell's theorem and related findings, the main focus of the debate concerning reality, probability, and nonlocality in quantum theory shifted to the question of nonlocality of quantum phenomena or QM, rather than that of completeness of QM, although, as will be seen, this assessment may depend on one's concept of completeness.

This article is in part stimulated by the argument for "eliminating" the considerations of nonlocality from the analysis of quantum entanglement, advanced by Khrennikov [10–15].[2] Khrennikov in particular proposed to differentiate classical and

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[1] My argument will be restricted to the standard QM. Other theories of quantum phenomena, such as Bohmian mechanics, will only be mentioned in passing. I shall also put aside the complexities involved in using such terms as "theory," "model," or "mathematical model," considered from the RWR perspective in [1, pp. 6–10]

[2] Khrennikov, in [13, 14] in part responds to the argumentation of this author in [16, 17]. The present

quantum entanglement not by their respective locality and nonlocality, but instead by the inherent discreteness of quantum phenomena vs. the inherent continuity of classical phenomena [11, 14, 15]. It would be more accurate to speak of the continuity of the processes underlying and connecting classical phenomena, given that some classical phenomena are observationally discrete. By the same token, the question arises whether this type of continuous connectivity, is also possible to assume in considering quantum phenomena. This has been one of the main foundational questions of quantum theory from its inception on. This question is answered in the negative in RWR-type interpretations, because they preclude any claims concerning the ultimate nature of the reality responsible for quantum *phenomena*, the reality commonly, including in this article, idealized in terms of quantum *objects*.

The discreteness of quantum phenomena has rarely been addressed in recent foundational discussions. The subject was much more prominent at earlier stages of quantum theory, beginning with M. Planck's discovery of the discrete nature of radiation in certain circumstances and, especially, following Bohr's 1913 atomic theory [18], and then the discovery of QM in 1925 and Bohr's interpretation of quantum phenomena and QM in terms of complementarity. This interpretation was introduced in 1927 in the so-called Como lecture [19, v. 1, pp. 52–91], grounded in what Bohr called "the quantum postulate, which attributes to any atomic process an essential discontinuity, or rather individuality, completely foreign to the classical theory and symbolized by Planck's quantum of action [h]" [19, v. 1, p. 53]. In fact, both discontinuity and individuality are essential, because each quantum phenomenon is both individual in itself and is discontinuous with any other. Bohr was never satisfied with his Como argument and revised his views, in particular by removing certain elements of realism and causality (classically defined), in part under the impact of his exchange with Einstein in October 1927 [19, v. 1, pp. 41–47].[3]

Eventually, around 1937, Bohr introduced his concept of "phenomenon," in which, along with complementarity, he grounded the ultimate (RWR-type) version of his interpretation, with only a few minor changes added subsequently.[4] A phenomenon is defined by what is observed, in fact *what has already been observed and registered*, in each experiment, as the result of the interaction between the quantum object considered and the measuring instrument used [19, v. 2, p. 64]. As J. A. Wheeler stated: "No ... phenomenon [in Bohr's sense] is a phenomenon until it is a registered phenomenon" [21, p. 192]. The concept sharpens the point that quantum

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Footnote 2 (continued)
article is a revised version of [17]. While still responding to Khrennikov, it offers a more independent argument and new concepts, in particular that of quantum indefinitiveness.

[3] Thus, he changed his view even before the lecture, given in September 1927, was published in April 1928 [20]. Bohr, notably, dated the published version 1927 when it was reprinted in his book, *Atomic Theory and the Description of Nature* [20], now the first volume of [19].

[4] I have considered different versions of Bohr's interpretation in [22]. It is worth noting that there is no *single* Copenhagen interpretation, as even Bohr changed his views a few times. It is more fitting to speak, as Heisenberg did, of "the Copenhagen spirit of the quantum theory" [23, p. iv]. This spirit designates a spectrum of interpretations that share some, but not all, of their features.

discreteness or individuality are those of quantum *phenomena*, observed in measuring instruments, rather than of quantum *objects* [19, v. 2, pp. 32–33].

RWR-type interpretations place quantum objects or the stratum of reality this concept idealizes beyond any representation, for example, either discrete or continuous, or even beyond conception, which I shall call the strong RWR view. By contrast, in accordance with Bohr's concept of phenomenon, in the case of quantum phenomena, a representation or, again, the corresponding form of idealization is possible and indeed necessary. Thus, as noted from the outset, RWR-type interpretations assume two idealizations of reality. The first is that of the ultimate nature of reality considered, placed beyond representation or conception and idealized in terms of quantum *objects*. The second is that defined by what is observed in measuring instruments, which allows for a representation, indeed in terms of classical physics, and thus realism, and is idealized in terms of quantum *phenomena*. It is still an idealization because, this representation or even this observation is a product of our thought, and, as I. Kant argued, it may not correspond to what actually obtains in nature. Bohr spoke of "the idealization of observation" already in the Como-lecture, even before he adopted the RWR view [19, v. 1, p. 55]. In RWR-type interpretations, quantum phenomena can only be related to each other, by means of QM or other theory, in terms of probabilistic or statistical predictions or correlations. No other predictions are possible on experimental grounds, as things stand now, because the repetition of identically prepared quantum experiments in general leads to different outcomes.

Although these predictions and correlations may pertain to spatially separated quantum events, quantum phenomena and QM may be argued to be "local" insofar as they do not entail any instantaneous transmission of physical influences between such events, "a spooky action at a distance" [*spukhafte Fernwirkung*], famously invoked by Einstein [24, p. 155]. Such an action would bring QM in conflict with relativity. In his reply to EPR, Bohr, by contrast, argued for the compatibility of quantum phenomena with "all exigencies of relativity theory" and thus for its locality in this sense [9, p. 701n]. I shall term this concept (the only one considered by Einstein himself) "Einstein-nonlocality," as against "quantum nonlocality," the term introduced, along with several definitions of it, in the wake of Bell's theorem. I shall now offer one such definition, by taking advantage of the fact that one can argue for "spooky *predictions* at a distance," without assuming a spooky *action* at a distance (e.g., [3, pp. 128–130], [6, pp. 138–139], [22, 25, pp. 269–271, 315]). These predictions are "spooky" insofar as there is, at least in RWR-type interpretations, no concept to be formed of how these correlations or quantum phenomena, in the first place, come about or why these predictions are possible. At the same time, these correlations need not entail a spooky *action* at a distance or Einstein-nonlocality, including in the EPR case, where they are possible with probability one. I define "quantum nonlocality" as the existence of such correlations and the possibility of predicting them. Indeed, as I shall argue, all quantum predictions are predictions at a distance, without implying an action at a distance. Quantum nonlocality is sometimes defined differently, for example, in terms of violations of Bell's or related inequalities.[5] Such definitions, however, leave room for their physical interpretation, and quantum nonlocality as just defined provides such an interpretation, among other possible interpretations, some of which interpret quantum nonlocality as Einstein-nonlocality.

Einstein eventually admitted that Einstein-nonlocality could be avoided if one assumed that QM is only a statistical theory that does not provide a representation of the behavior of individual objects considered. He was, however, not satisfied with this alternative, because it was in conflict with his conviction that a fundamental physical theory should always do so. For one thing, why QM was able to make its statistical predictions remained unexplained, which, for Einstein, made QM made more akin to magic that a proper theory—"Jacob's pillow" and not "the real thing" (e.g., [24, pp. 155, 205], [26, p. 81]).

I would argue, then, that the question is not that of "*eliminating* the issue of quantum nonlocality," but instead that of *illuminating* this issue, even though the ultimate nature of quantum nonlocality may remain beyond illumination, beyond any picture or concept our thought can form. That, however, does not mean that the *issue* cannot be further illuminated. One could, I argue, be helped in this task by relating quantum nonlocality to other key features of quantum physics.[6] I do not of course imply that this article will accomplish this task. Instead, I hope to contribute to the ongoing collective endeavor of doing so.

I shall argue that the following key defining features of quantum phenomena and QM, possibly distinguishing them from classical phenomena and classical physics (there are quite a few of them!): (1) the irreducible role of measuring instruments in defining quantum phenomena, (2) discreteness, (3) complementarity, (4) entanglement, (5) quantum nonlocality, and (6) the irreducibly probabilistic or statistical nature of quantum predictions, which pertains to our quantum theories rather than quantum phenomena—are all interconnected so that it is difficult to give an unconditional priority to any one of them. I am not saying that it is in principle impossible to distinguish quantum and classical phenomena or quantum and classical theory by a single feature, as has been suggested in the case of QM, although not quantum phenomena, by recent (reconstruction) projects of deriving QM for discrete variables.[7] In the present context, it is tempting to argue, following Bohr, that, if there were such a single feature, it would be the irreducible role of measuring instruments in defining quantum phenomena. One might, however, prefer to err on the side of caution. Besides, these features may still not be exhaustive in defining quantum phenomena vs. classical ones. For one thing, there is the role of Planck's constant, h. Quantum phenomena were initially defined by the fact that, in considering them, h, must be taken into account, which is still the case. While, however, the role of

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[5] The literature on the subject is extensive, and my limits here only allow me to mention a very small portion of it.

[6] It is worth noting that Khrennikov, too, brings into consideration complementarity and the role of Planck's constant, h [15].

[7] Two such cases are "the continuity axiom" of L. Hardy's derivation [27] and "the purification postulate" of that of D'Ariano et al. [28].

h is irreducible in quantum phenomena, their specificity as quantum appears to be defined by a broader set of features, such as those under discussion, some of which are not connected to h, at least not expressly. Some of these features are also exhibited by classical phenomena or found in "toy" models different from those of QM.[8] Nevertheless, measuring any quantum phenomenon known thus far involves h.

I would like, in closing this introduction, to emphasize that most of my claims in this article only concern *interpretations* of quantum phenomena and QM, those of the RWR type amidst others, some of which are realist. While I, unavoidably, make claims concerning quantum phenomena, observed in measuring instruments, I make no claims concerning how nature ultimately works. Such claims would, in any event, be precluded by the RWR view, because it places the ultimate workings of nature beyond representation or even conception, at least as things stand now.

2 Measurement and Reality in Quantum Physics

2.1 Quantum Measurement, Reality Without Realism, and Quantum Indefinitiveness

The concept of reality without realism is grounded in more general concepts of reality and existence, assumed here to be primitive concepts and not given analytical definitions. These concepts are, however, in accord with most, even if not all (which would be impossible), available concepts of reality and existence in realism and nonrealism alike. By "reality" I refer to that which is *assumed* to exist, without making any claims, defining realist theories, concerning the *character* of this existence. The absence of such claims allows one to place this character beyond representation or knowledge, or even conception. I understand existence as a capacity to have effects on the world. The very assumption that something, including the world, is real is made on the basis of such effects. Following L. Wittgenstein, I understand "the world" as "everything that is the case" [31, p. 1].[9] To ascertain observable effects of reality entails a representation of them, but not necessarily of how they come about. This implies that a given theory might use different types of idealizations of reality, some allowing for a representation or at least conceptions and others not. As noted in the Introduction, Bohr's and the present interpretation use both types. The behavior of the macroworld and specifically of the observable parts of measuring instruments, defining quantum phenomena, is idealized as representable. By contrast, the reality ultimately responsible for these phenomena is idealized by quantum objects and their behavior as that which cannot be represented or even conceived of. Even the latter, strong, RWR view, is, however, still a product of thought, which still makes it a human idealization. But then, so is any other concept of reality.

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[8] See, for example, [29, 30]. I have discussed this subject in detail in [1, pp. 33–34].

[9] While in physics the primary reality considered is that of matter, a reality, including a reality without realism, can be mental, for example, in mathematics [32, pp. 203–210].

Realist or ontological thinking is defined by the corresponding theories, commonly representational in character.[10] Such theories aim to represent the reality they consider by mathematized models suitably idealizing this reality. All modern, post-Galilean, physical theories are such mathematized idealizations, as is QM, in this case, an idealization that, in the RWR view, does not involve a representation or even conception of the ultimate nature of the reality at stake. It is also possible to assume that the reality considered has an independent architecture of some sort, while admitting that it is not possible to either adequately represent this architecture or even to form a well-defined concept of it, either at a given point or perhaps ever. In the first eventuality, a theory that is merely predictive could be accepted for lack of a realist alternative, usually with a hope that a future theory will do better by being a properly representational theory. Einstein adopted this attitude toward QM. Even in the second eventuality, however, this architecture is usually conceived on the ontological model of classical physics (which need not mean that the physics governing this architecture is assumed to be classical). What, then, grounds realism most fundamentally is the assumption that the ultimate constitution of reality possesses properties and the relationships among them, or, as in structural realism [33], at least a structure of some kind, that may be either (a) known to the degree allowing it to be ideally represented by a theory or (b) unknown or even unknowable, but still conceivable, usually with a hope that it will be eventually so represented.[11]

Physical theories prior to quantum theory have been realist theories. Thus, classical mechanics (used in dealing with individual objects and small systems, apart from chaotic ones), classical statistical mechanics (used in dealing with large classical systems), or chaos theory (used in dealing with classical systems that exhibit a highly nonlinear behavior) are realist. While classical statistical mechanics does not represent the overall behavior of the systems considered because their great mechanical complexity prevents such a representation, it assumes that the individual constituents of these systems are represented by classical mechanics. In chaos theory, one assumes a mathematical representation of the behavior of chaotic systems. ("Quantum chaos" is different, because it is a quantum theory.) The status of these theories as realist could be questioned, on Kantian lines, even in classical mechanics, where the representational idealizations used are more in accord with our phenomenal experience, which, however, does not mean that these idealizations correspond to how things, as things-in-themselves, are in nature [37]. Our phenomenal experience can only serve us partially in the case of relativity. This is because, while one can give the relativistic behavior of photons a concept and represent it mathematically, which makes relativity a realist and classical causal (in fact, deterministic) theory, we have no means of visualizing this behavior, or the behavior represented

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[10] Although terms "realist" and "ontological" sometimes designate more diverging concepts, they are close in their meaning and will be used interchangeably here.

[11] One could in principle see the assumption of the existence or reality of something to which a theory can relate without representing it as a form of realism. This use of the term is found in advocating interpretations of QM that are nonrealist in the present sense (e.g., [34–36]), although none of these authors entertains the strong RWR view. In any event, I would argue that the present definition is more in accord with most understandings of realism in physics and philosophy.

by Einstein's velocity-addition formula for collinear motion $c = \frac{v+u}{1+vu/c^2}$. In all these theories, however, we can observe the phenomena considered without disturbing them appreciably. As a result, we can identify these phenomena with the objects in nature in their independent behavior for all practical purposes.

The representation of individual quantum behavior became partial in the so-called old quantum theory, in particular Bohr's atomic theory, introduced in 1913 [18]. The theory only provided representations, in terms of orbits, for the so-called stationary states of electrons in atoms (in which electrons had constant energy), but not for the discrete transitions, "quantum jumps," between stationary states. This radical concept was not only incompatible with classical mechanics and electrodynamics alike, but also with classical causality. As Bohr said later: "The very idea of stationary states is incompatible with any directive for the choice between such transitions and leaves room only for the notion of the relativity probabilities of the individual transition processes" [19, v. 2, p. 35]. The concept became central for Heisenberg, who built on it by abandoning the orbital (or any other) representation of stationary states, which led him to his discovery of QM [38]. In his 1925 assessment of QM, by then developed into a full-fledged matrix mechanics by Born and Jordan [39], Bohr said:

> In contrast to ordinary mechanics, *the new quantum mechanics does not deal with a space–time description of the motion of atomic particles*. It operates with manifolds of quantities [matrices] which replace the harmonic oscillating components of the motion and symbolize the possibilities of transitions between stationary states ... . These quantities satisfy certain relations which take the place of the mechanical equations of motion and the quantization rules [of the old quantum theory]. [19, v. 1, p. 48; emphasis added]

As was Heisenberg's thinking at the time, this assessment was based in an RWR-type view. By contrast, as indicated above, the 1927 Como version of Bohr's interpretation, attempted to restore realism and classical causality to QM. This attempt was abandoned by Bohr, following his discussion with Einstein in October of 1927, which initiated Bohr's path toward an RWR-type interpretation (e.g., [22, pp. 41–70], [25, pp. 179–238]).[12] According to Heisenberg himself (back in 1925): "What I really like in this scheme [QM] is that one can really reduce *all interactions* between atoms and the external world ... to transition probabilities" (Heisenberg, Letter to Kronig, 5 June 1925; cited in [40, v. 2, p. 242]). By speaking of the "*interactions* between atoms and the external world," this statement suggests that QM was about predicting effects of these interactions, observed in the measuring instruments. This view was adopted by Bohr, eventually leading him to his ultimate, strong RWR-type, interpretation.

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[12] Both Dirac [41] and von Neumann [42], followed Bohr's Como argument or, in any event, adopted the same type of view, allowing for realism and classical causality in considering the independent behavior of quantum objects, with probabilities only introduced by measurement (see [25, pp. 197–214], [7, p. 1279].

This interpretation was first presented in Bohr's 1937 article, "Complementarity and Causality." It was grounded in the feature that defined the difference between classical and quantum phenomena in all of Bohr's interpretations: the irreducible role of the interactions between quantum objects and measuring instruments in the constitution of quantum phenomena. Bohr does not use the language of reality without realism, but his understanding of quantum measurement clearly amounts to the RWR view:

> The renunciation of the ideal of causality in atomic physics which has been forced on us is founded logically only on our not being any longer in a position to speak of the autonomous behavior of a physical object, due to the unavoidable interaction between the object and the measuring instruments which in principle cannot be taken into account, if these instruments according to their purpose shall allow the unambiguous use of the concepts necessary for the description of experience. In the last resort an artificial word like "complementarity" which does not belong to our daily concepts serves only briefly to remind us of the epistemological situation here encountered. [43, p. 87]

I shall discuss complementarity, which does more than this, in Sect. 3, and shall only note here that it is complementarity that enables this unambiguous use by making some of these concepts complementary: mutually exclusive and yet equally necessary for a comprehensive account of quantum phenomena. The concept of causality that grounds this ideal of causality is defined by the claim that the state, X, of a physical system is determined, in accordance with a law, at all future moments of time once it is determined at a given moment of time, state A, and A is determined in accordance with the same law by any of the system's previous states. This assumption, thus, implies a concept of reality, which defines this law, and makes this concept of causality ontological. This concept has a long history, beginning with the pre-Socratics, and it has governed classical physics from its inception on. I shall term this concept "classical causality." As discussed in Sect. 3, it is possible to introduce alternative, probabilistic, concepts of causality, applicable in QM, including in RWR-type interpretations, and relate them to complementarity, which Bohr saw as a "generalization of causality" [19, v. 2, p. 41].

Although the concept of classical causality is in accord with the history of the idea of causality, one might question calling this concept "causality," because it need not imply that A is a cause of X, in accord, say, with Kant's definition of causality [37, p. 305]. The fact that the physical state of a falling body at point t_1 determines, by Newton's law of gravity, the state of this body at any other point t_2 does not mean that t_1 is the cause of t_2. One might say that gravity, encoded in Newton's law, is the cause of this determination, although this claim involves further complexities. While keeping these qualifications in mind, I shall retain the designation classical causality for this concept, although some use "determinism" instead.[13] I prefer to define "determinism" as an epistemological category

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[13] These qualifications in part explain the history of questioning of the idea of causality in fundamental physics, while allowing for the type of view of classical physics or relativity termed here classically causal, beginning with Russell's 1913 essay [44]. See [45] for a reconsideration of Russell's argument from a contemporary perspective, allied with structural realism [33]. As will be seen, in the case of quantum causality, one could speak of events as causes.

referring to the possibility of predicting the outcomes of classical causal processes ideally exactly. In classical mechanics, when dealing with individual objects or small systems (apart from chaotic ones), both notions in effect coincide. On the other hand, classical statistical mechanics or chaos theory are classically causal but not deterministic in view of the complexity of the systems considered, which limit us to probabilistic or statistical predictions concerning their behavior.

In quantum physics, deterministic predictions are no longer possible even in considering individual quantum objects, however elementary. This is because the repetition of identically prepared experiments in general leads to different outcomes, and unlike in classical physics, this difference cannot be diminished beyond the limit defined by Planck's constant, h, by improving the capacity of our measuring instruments. This impossibility is manifested in the uncertainty relations, which would remain valid even if we had perfect instruments and which pertain to quantum data, rather than to any particular theory. Hence, the probabilistic or statistical character of quantum predictions must also be maintained by interpretations of QM or alternative theories of quantum phenomena that are classical causal. Such interpretations and theories are also, and in the first place, realist because classical causality implies a law governing it and thus a representation of the reality considered (in these cases, defined by the behavior of quantum objects) in terms of this law.

By contrast, as Bohr says above, RWR-type interpretations are not classically causal because of the absence of realism in considering the behavior of quantum objects or the reality thus idealized. Given, however, that it is possible to argue for interpretations of QM or alternative theories of quantum phenomena that are realist and possibly classical causal, Bohr's claim should be seen as representing an RWR-type interpretation, adopted by Bohr by this point. This interpretation fulfilled his imperative in his 1935 reply to EPR, still alongside the same appeal to "a final renunciation of the classical ideal of causality," that quantum phenomena required "a radical revision of our attitude toward the problem of physical reality" [9, p. 697]. A revision of an *attitude* toward the problem of physical reality is not the same as a revision of a given *concept* of reality, on which I shall comment in closing this article. Bohr, however, clearly undertook such a revision, by adopting the RWR view, as confirmed by the passage under discussion and other statements, such as: "In quantum mechanics, we are not dealing with an arbitrary renunciation of a more detailed analysis of atomic phenomena [an analysis reaching quantum objects], but with a recognition that such an analysis is *in principle* excluded" [19, v. 2, p. 62].

Bohr's position represents the strong form of the RWR view, placing the ultimate nature of reality beyond conception, even if not the strongest possible one. For, if, as Bohr says, we are "not being any longer in a position to speak of the autonomous behavior of a physical object, due to the unavoidable interaction between the object and the measuring instrument," this behavior, or the reality so idealized, must be also beyond conception. If we had such a conception, we would be able to say something about it. It is true that there is a difference between some conception of this reality and a rigorous conception that enables us to provide a proper representation of it by means of a theory. Bohr, however, claims that we are not in a position to speak of the autonomous behavior of quantum objects and hence this reality at all. Hence, we cannot have a conception of this reality either.

The question then becomes that of whether our inability to do so only (A) characterizes the quantum-mechanical situation as things stand now, or (B) places this reality beyond the reach of our thought altogether. While Bohr, thus, at least assumes (A) and while there are intimations that he entertained (B), he never stated so, which leaves whether he assumed (B) or only assumed (A) to an interpretation. Logically, once (A) is assumed, then (B) is possible. There does not appear to be any experimental data compelling one to prefer either. Both views are in effect equivalent as far as physics is concerned. They are, however, different philosophically in defining how far our mind can, in principle, reach in investigating nature.[14]

The qualification "as things stand now" applies, however, to (B) as well, even though it might appear otherwise. It applies because a return to realism is possible. This return may take place either because quantum theory, as currently constituted, is replaced by an alternative realist theory, or because (B), or (A), becomes obsolete for those who hold it with quantum theory in its present form. It is possible, however, that the RWR view, either of (A) or (B) type, will remain viable in grounding interpretations of QM or QFT. It is also conceivable that a physical theory would emerge, perhaps the one bringing gravity and other forces of nature into a harmony or even unify them, that will require a view that is neither realist nor that of RWR-type, difficult as it may be to imagine such a view now.

I shall now introduce the quantum indefinitiveness postulate, which is a consequence the strong RWR view of either type, (A) or (B). The postulate dictates *the impossibility of making definitive statements of any kind, including mathematical ones, concerning the relationship between any two individual quantum phenomena or events, indeed to definitively ascertain the existence of any such relationship*. The postulate allows for definitive statements concerning single *individual* events, statements related to measurements which, and only which, define them. It also allows statements concerning the relationships between multiple events, in this case statistical in nature, such as events exhibiting quantum correlations. It is crucial that the postulate concerns events *that have already happened*, rather than possible future events, in which case one can make probabilistic statements concerning them.[15]

Heisenberg's statement, discussed below, suggests the quantum indefinitiveness postulate: "There is no description of what *happens* to the system between the initial observation and the next measurement" [49, pp. 47, 145; emphasis added]. The same would apply to the word "happen" or "system," or any word we use, whatever concept it may designate, including reality. Bohr is reported to have said: "We must

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[14] There is yet another alternative, that of simply disregarding such questions, captured by N. D. Mermin's often cited maxim "shut up and calculate," an attitude not adopted by Mermin himself, who said on the same occassion: "But I will not shut up" [46, p. 24].

[15] The concept of quantum *indefinitiveness* is different from A. Shimony's realist concept of "objective *indefiniteness*," which sounds similar [47]. Shimony's concept implies a statement concerning a relation of between individual quantum events, a relation established by QM. The concept of quantum indefinitiveness is independent of QM. Shimony's concept is noteworthy as revealing subtler dimensions of realism in quantum theory. See [48] for an instructive discussion.

never forget that 'reality' too is a human word" [50, p. 234]. Unlike the quantum indefitiveness postulate or the corresponding interpretations, such as that of Bohr or the present one, Heisenberg's statement in principle allows for a mathematical representation of what "happens" between quantum events. Heisenberg adopted this view at the time of this statement, although not at the time of his discovery of QM.

2.2 Quantum Phenomena and Quantum Objects as Idealizations

The nature of the idealization of the ultimate constitution of physical reality as quantum objects in RWR-type interpretation is very different from that used in classical mechanics, say, in terms of dimensionless massive points mathematically idealizing material objects. Elementary particles are often seen as dimensionless, point-like entities. If they had volume, charged particles would be torn apart by the electromagnetic force within them. They cannot, however, be considered as idealized point particles of classical mechanics, and when they are understood in terms of quantum fields, this concept, too, is very different from that of classical or relativistic fields.[16] The reason is that, while what is observable in measuring instruments is always uniquely and indeed classically defined, what can be considered as the object under investigation or what is considered as a measuring instrument (beyond its observable stratum) is not uniquely defined. According to Bohr:

> This necessity of discriminating in each experimental arrangement between those parts of the physical system considered which are to be treated as measuring instruments and those which constitute the objects under investigation may indeed be said to form a *principal distinction between classical and quantum-mechanical description of physical phenomena*. It is true that the place within each measuring procedure where this discrimination is made is in both cases largely a matter of convenience. While, however, in classical physics the distinction between object and measuring agencies does not entail any difference in the character of the description of the phenomena concerned, its fundamental importance in quantum theory ... has its root in the indispensable use of classical concepts in the interpretation of all proper measurements, even though the classical theories do not suffice in accounting for the new types of regularities with which we are concerned in atomic physics. In accordance with this situation there can be no question of any unambiguous interpretation of the symbols of quantum mechanics other than that embodied in the well-known rules which allow to predict the results to be obtained by a given experimental arrangement described in a totally classical way, and which have found their general expression in the transformation theorems, already referred to [9, pp. 697–697n.]. [9, p. 701]

This statement may suggest that, while observable parts of measuring instruments are described by classical physics, the independent behavior of quantum objects is

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[16] For discussion of the concept of quantum field from the RWR perspective, see [5].

represented by the quantum-mechanical formalism. While this type of view was adopted by Bohr in the Como lecture (1927) and then others, such as Dirac [41] and von Neumann [42], it was not Bohr's view after he revised his Como argument. Bohr does not say that the independent behavior of the objects under investigation is described by quantum-mechanical formalism, the "symbols" of which are assumed here, as elsewhere in Bohr, to have only a probabilistically or statistically predictive role. His statement only says that quantum objects cannot be treated classically.

This situation is sometimes referred to as the arbitrariness of the "cut" or, because the term [*Schnitt*] was favored by Heisenberg and von Neumann, the "Heisenberg-von-Neumann cut." As Bohr noted, however, while "it is true that the place within each measuring procedure where this discrimination [between the object and the instrument] is made is ... largely a matter of convenience," it is true only largely, but not completely. This is because "in each experimental arrangement and measuring procedure we have only a free choice of this place within a region where the quantum-mechanical description of the process concerned is effectively equivalent with the classical description" [9, p. 701]. In other words, the ultimate constitution of the reality responsible for quantum phenomena observed in measuring instruments is always, in any possible experiment, on the other side of the cut. So are those quantum strata of the measuring instruments through which the latter interact with this reality. It is the reality that is always on the other side of the cut that quantum objects and their behavior idealize.[17] What is a quantum object can be different in each case, including possibly something that, if considered by itself, would be classical, as in the case of Carbon 60 fullerene molecules, which were observed as both classical and quantum objects [51]. But a quantum object is always on the other side of the cut, and what is responsible for its quantum behavior is defined by the microscopic RWR-type reality that is never on the measurement side of the cut.[18] Quantum objects can never be extracted from the phenomena enveloping them, the impossibility that defines the wholeness of phenomena in Bohr [19, v. 2 pp. 72–73].

Bohr's view of the indispensability of classical physical concepts is often misunderstood, in part by disregarding the quantum aspects of measurement. Bohr does insist on "the indispensable use of classical concepts in the interpretation of all proper measurements, even though the classical theories do not suffice in accounting for the new types of regularities with which we are concerned in atomic physics" [9, p. 701]. The instruments, however, also have quantum strata, through which they interact with quantum objects. This interaction, which, as discussed below, is a form of entanglement, is quantum and thus cannot be observed. It is "irreversibly amplified" to the macroscopic classical level, such as a spot left on a silver screen (e.g., [19, v. 2, p. 73]).[19] Bohr, it is true, does not speak in terms of the quantum stratum

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[17] The concept "quantum object" could be defined otherwise, for example, on more realist lines, as, in part via Shimony's concept of objective indefiniteness, mentioned above, in [48].

[18] A somewhat similar argument concerning the stratified character of the reality defining quantum phenomena, if without adopting the RWR view, was proposed in [52].

[19] The nature of this "amplification" is part of the problem of the transition from the quantum to the classical, which and related subjects, such as "decoherence, " are beyond my scope here.

of the apparatus, but the role of this stratum is a consequence of what he is saying about the interaction between the object and the instrument. How could an instrument interact with a quantum object otherwise?

2.3 Classical Causality and RWR Probability

The RWR view makes the absence of classical causality automatic, because assuming this nature to be classically causal would imply at least a partial conception or even representation of this reality as concerns the law that governs it. This, as explained earlier, does not mean that interpretations of QM or alternative theories of quantum phenomena that are realist and classical causal are impossible. But they can only concern what underlies quantum phenomena, because one cannot, regardless of which theory one uses, track individual quantum objects, in the way one can individual classical objects, by separating the behavior of quantum objects from their interactions with measuring instruments. One can only deal with the effects of these interactions under the constraints of the uncertainty relations, which, too, are independent of any theory. This leaves no room for determinism, but only, in Schrödinger's apt language for "expectation-catalogs" of outcomes of future experiments [53, p. 154]. Hence, while in classical physics or relativity, where all systems are classically causal, some of them can be handled deterministically and others must be handled probabilistically or statistically, in quantum physics all systems considered can only be handled probabilistically or statistically. Nor do they need to be assumed to behave classically causally, and they are not in RWR-type interpretations. The implies a very different nature of "the recourse to probability laws," which may be designated as "RWR-probability." According to Bohr:

> [I]t is most important to realize that the recourse to probability laws [in quantum physics] is essentially different in aim from the familiar application of statistical considerations as practical means of accounting for the properties of mechanical systems of great structural complexity. In fact, in quantum physics we are presented not with intricacies of this kind, but with the inability of the classical frame of concepts to comprise the peculiar feature[s] of the elementary [quantum] processes. [19, v. 2, p. 34]

This should, again, be seen as expressing the RWR-type interpretation adopted by Bohr here, because, as just discussed, some (realist) interpretations of QM, even on Copenhagen lines, or alternative theories, such as Bohmian mechanics, assume classically causal views of the behavior of quantum objects, with probability or statistics brought in by measurement.[20] "The classical frame of concepts" may appear to

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[20] The difference between the statistical and probabilistic (such as Bayesian) views of QM would require a separate treatment. Khrennikov, in the works cited here, adoptes a statistical view, on more realist lines, as does, on RWR lines, the present author [4, 7]. Quantum Bayesianism, QBism, offers an RWR-type Bayesian approach [35]. RWR-type statistical interpretations of QM are uncommon. A compelling example of a statistical interpretation that may be interpreted along RWR lines, even if it is not by the authors themselves, is offered in [54]. Their position appears to allow for this interpretation because they argue that one should only interpret outcomes of pointer indications, and leave the richer quantum structure, which has many ways of expressing the same identities, without interpretation. In RWR-type interpretations, this structure would only be seen as that enabling statistical predictions, without representing

refer to the concepts of classical physics, and this frame does include these concepts. However, by this time (in 1949), Bohr adopts the strong RWR view, which gives the phrase a broader meaning: all concepts that we can form are classical. The question is only whether they could one day become applicable in the ultimate nature of reality at stake in quantum theory. As discussed below, purely mathematical concepts are a possible exception, although not in Bohr's or the present view.

2.4 Bohr's Concept of Phenomenon, Quantum Indefinitiveness, and the Completeness of Quantum Mechanics

By the time he reaches his ultimate, RWR-type, interpretation, Bohr defines quantum phenomena strictly in terms of effects, observed in measuring instruments as a result of their interaction with quantum objects. As he said:

> I advocated the application of the word phenomenon exclusively to refer to the *observations* obtained under specified circumstances, including an account of the whole experimental arrangement. In such terminology, the observational problem is free of any special intricacy since, in actual experiments, all observations are expressed by unambiguous statements referring, for instance, to the registration of the point at which an electron arrives at a photographic plate. Moreover, speaking in such a way is just suited to emphasize that the appropriate physical interpretation of the symbolic quantum-mechanical formalism amounts only to predictions, of determinate or statistical character, pertaining to individual phenomena appearing under conditions defined by classical physical concepts [describing the relevant observable parts of measuring instruments]. [19, v. 2, p. 64]

As "*the observations [already] obtained under specified circumstances*," phenomena refer to events that have already occurred, and not to future events that one can predict. Speaking, phenomenologically, of observations, rather than, ontologically, of observed situations, explains Bohr's choice of the term "phenomenon." This "idealization of observation" [19, v. 1, p. 55], is the same as in classical physics and allows one to identify phenomena with objects (instruments), because our observations do not disturb them, in contrast to the way instruments disturb quantum objects or the reality they idealized by interacting with them. By constrast, in the RWR-type interpretation adopted by Bohr at this point, this reality is no longer available to a representation or even conception, at least as things stand now.

Footnote 20 (continued)

the ultimate reality responsible for the outcomes of quantum experiments and hence pointer indications. Finally, there are also arguments (which are, it follows, realist) for classical causality in the case of discrete events, arguments advocated, for example, by L. Smolin, following R. Sorkin [55, pp. 257–261]. These arguments, in my view, pose problems, beginning with that of establishing discontinuous physical mechanisms by means of which classical causality can be established for such events, although one could have mathematics for predictions concerning them. It is not discreteness but classical causality that is the main difficulty.

The epistemological cost of the RWR view is not easily absorbed by most physicists and philosophers, and to some, beginning with Einstein, is unacceptable. This is not surprising because the features of quantum phenomena that are manifested in many famous experiments and that led to RWR-views defy many assumptions commonly considered as basic. These assumptions, arising due to the neurological constitution of our brain, have served us for as long as human life has existed, and within certain limits are unavoidable, although, as noted, while respected by classical physics, they were already challenged by relativity. QM have made this challenge much greater. Thus, it is natural and even humanly unavoidable to assume that *something happens* between observations. However, in the RWR-type view, the expression "something happened" is ultimately inapplicable to the independent behavior of quantum objects, or the reality they idealize. According to Heisenberg:

> There is no description of what *happens* to the system between the initial observation and the next measurement. ... The demand to "describe what happens" in the quantum-theoretical process between two successive observations is a contradiction in adjecto, since the word "describe" [or "represent"] refers to the use of classical concepts, while these concepts cannot be applied in the space between the observations; they can only be applied at the points of observation. [49, pp. 47, 145; emphasis added]

As noted, this is in effect an expression of the quantum indefinitiveness postulate. The same would apply to the word "happen" or "system," or any word we use, whatever concept it may designate. While quantum physics cannot avoid using ordinary language, especially in describing experiments, it imposes severe limits on this use. Our language and concepts cannot apply to the reality idealized by quantum objects and behavior. Heisenberg noted these limits in the same book: "But the problems of language are really serious. We wish to speak in some way about the structure of the atoms and not only about 'facts'—the latter being, for instance, the black spots on a photographic plate or the water droplets in a cloud chamber. *But we cannot speak about the atoms in ordinary language*" [49, pp. 178–179; emphasis added]. Nor is it possible in terms of ordinary concepts, from which ordinary language is indissociable, or even in terms of physical concepts, assuming that they can be free from ordinary concepts. In the strong RWR view, expressions like "quantum objects interact with *each other*," which refers to something between observations, can be used provisionally, because there is no term or concept, such as "interaction" or "relation," or "taking place," applicable to what "takes place." "Reality," in this case is a term without a concept associated to it, akin to a mathematical symbol.

As indicated, Heisenberg's claim allows for the mathematical representation of what happens between the experiments. Mathematics, as Heisenberg said on an earlier occasion, is "fortunately" free from the limitations of ordinary language and concepts, fortunately because one could use this freedom in physics, as Heisenberg did in creating QM [23, p. 11] . At the time, Heisenberg, adopting the RWR view, used this freedom to construct a theory, QM, designed to predict the probabilities of events observed in measuring instruments, without representing the behavior of quantum objects. By contrast, in his later writings he assumed the possibility of a mathematical representation of the ultimate nature of reality [49, pp.

145, 167–186].[21] Heisenberg conceived of this representation in terms of symmetry groups and defined "elementary particles" accordingly, without considering them as "particles" in any physical sense. A particle is elementary if the corresponding representation of the symmetry group defining it is irreducible [57]. Bohr, by contrast, rejected the possibility of a mathematical representation of quantum objects and behavior, or the reality they idealize, along with a physical one, at least in his ultimate, RWR-type, interpretation. It is true that Bohr often speaks of this reality as being beyond our phenomenal intuition, also involving visualization, sometimes used, including by Bohr, to translate the German word for intuition, *Anschaulichkeit* (e.g., [19, v. 1 p. 51, 98–100, 108; v. 2, p. 59]). It is clear, however, that, apart from the Como lecture, Bohr saw the ultimate nature of this reality as being beyond any representation or even conception, including a mathematical one, at least as things stand now.

In RWR-type interpretations, then, QM is incomplete if one adopts Einstein's view of completeness, because it offers no representation of the ultimate constituents of nature and their individual behavior. I shall call such theories "Einstein-complete," in parallel with the Einstein-local. Einstein wanted fundamental theories to be both (local realism). Ideally, he also wanted them to be classically causal and even deterministic. QM may, however, be seen, as it was by Bohr, as complete in a different sense,"Bohr-complete:" it is as complete a theory of quantum phenomena as nature allows a theory to be. There has been no change in this regard. QM remains the standard theory of nonrelativistic quantum phenomena, as, in relativistic quantum regimes, does QFT, which allows for RWR-type interpretations [5].

2.5 What Does a Quantum Measurement Measure and What Does a Quantum Theory Predict?

As Bohr eventually came to realize, in any quantum experiment, the quantum object under investigation and the measuring instrument become entangled as a result of their interaction with each other. The interaction between the object and the measuring instruments (the quantum stratum of the instruments) leading to this entanglement is not the measurement: this interaction occurs *before* the measurement takes place. Once performed, the measurement, say, that of the momentum, disentangles the object and the instrument, with the observed outcome "irreversibly amplified" to the level of *the classically observed stratum of the apparatus* [19, v. 2, p. 73]. This outcome pertains to the quantum stratum of the apparatus *after* this interaction, rather than to the object. It is, as Schrödinger nicely explains, this disentangling that enables one to predict the probability that the momentum measurement at a given future moment in time will be within a certain range [53, pp. 162–163]. Alternatively, if the initial measurement, which could be equally set up *after* this interaction, was that of the position, one could predict the probability that the position measurement at a given future moment in time will locate the trace of the

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[21] See [56], for an assessment of Heisenberg's overall later views of QM, including in [49].

interaction between the object and the instrument within a certain area. Quantum phenomena are never entangled. Only quantum objects and ψ-functions are. But quantum objects are never observed, and ψ-functions never represent either quantum objects apart from measurement in RWR-type interpretations or measurements themselves, even if one adopts a realist view of ψ-functions as representing what happens between measurements. According to Bohr:

> After a preliminary measurement of the momentum of the diaphragm, we are in principle offered the choice, when an electron or photon has passed through the slit, either to repeat the momentum measurement or to control the position of the diaphragm and, thus, to make predictions pertaining to alternative subsequent observations. It may also be added that it obviously makes no difference, as regards observable effects obtainable by a definitive experimental arrangement, whether our plans of considering or handling the instruments are fixed beforehand or whether we prefer to postpone the completion of our planning until a later moment when the particle is already on its way from one instrument to another. [19, v. 2, p. 57][22]

If, then, one always makes a measurement after the object has left the location of the measurement, what does one measure? One measures the state of the measuring instrument, either its momentum or its position, given that both, again, can never be measured in the same arrangement, as reflected by the uncertainty relations.[23] More accurately, either measurement relates the state of the quantum stratum of the instrument, which interacted with the object in the past (however recently, but always in the past!), amplifying this state to the classical level of the observation, to which and only to which such concepts as momentum or position, can rigorously apply in the RWR view. One might assume that, because of the exchange of momenta between the object and the instrument, the momentum of the object will correspond to the difference between two momentum measurements of the instrument before and after the interaction with the object. Physically, however, one never measures that momentum, given that the object has already left the location of the instrument and that one could have performed instead the position measurement after it did. In any event, one can ascertain, *regardless of an interpretation*: (a) that one can perform either measurement concerning the state of the quantum stratum of the instruments, with the outcome amplified to the classical level of the observable part of this instrument; and correlatively (b) the quantum-nonlocal nature of quantum prediction, because by changing one's decision which measurement to perform, one can make two alternative predictions concerning distant future events, to which one is not physically connected at the time of either measurement.

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[22] This observation, as Wheeler notes, anticipates the delayed choice experiment [21, pp. 182–192].

[23] This point appears to have been missed or not addressed either in commentaries on Bohr or by treatments of quantum measurement elsewhere. Subtle as it is, Schrödinger's analysis of quantum measurement in his cat-paradox paper does not consider this point [53, pp. 158–159]. Von Neumann's analysis comes close, but, while it is conceivable that von Neumann realized this point, he did not comment on it, and some of his statements suggest a realist view, which attributes the measured quantity to the object at the time of measurement [42, pp. 355–356].

Thus, using the measurement of the state of the apparatus, one can predict, at a distance, by means of the ψ-function (cum Born's rule), a possible outcome of a future measurement of either variable, without "in any way disturbing the system," just as in the EPR type experiment [8, p. 138]. It is true that there was an interaction between the object and the instrument before that measurement. But this is also the case for the two objects of the EPR pair, which have been in an interaction, entangling them. In a standard measurement, the probability of such a prediction will not be equal to one. As Bohr realized, however, with some simple additional arrangements one can reproduce the EPR case in considering the standard quantum measurement [9, pp. 699–700], [19, v. 2, p. 60], [58, pp. 101–103]. It *might seem* that, in either the standard or the EPR case, because either of the two complementary quantities could be predicted at a distance for one quantum object by an alternative measurement on another quantum object, the first object can be assigned both quantities, as, in EPR's language, "elements or reality" [8, p. 777], or alternatively that our predictions are Einstein-nonlocal, because a measurement defines the reality at a distance. This was, essentially, EPR's argument, although the possibility of predicting either quantity with probability one strengthened their case. As, however, should already be apparent and as will be discussed in detail in Sect. 4, this is not necessarily the case, because, in any *actual* experiment, only one of these quantities could be so predicted. There is no experiment that would allow one to physically realize the prediction of both quantities for the same object. Nor need one assume that our predictions are Einstein-nonlocal, because, even if one can predict the quantity in question with probability one, one can measure the complementary quantity and thus establish a different element of reality from the one predicted. Accordingly, a measurement performed on one quantum object cannot be claimed to define an element of reality pertaining to another, spatially separated, quantum object. Even in the standard quantum measurement, however, one must always consider not only with the object under investigation as in classical physics (where one can disregard the role of measuring instruments), but a composite entangled quantum system, consisting of the object and the quantum stratum of the instrument.[24] In each EPR-type experiment, at the last critical stage (when one makes the EPR prediction with probability one), one deals with four systems, two objects and two instruments.

3 Complementarity and Quantum Causality

3.1 Complementarity

Defined arguably most generally, complementarity is characterized by:

(a) a mutual exclusivity of certain phenomena, entities, or conceptions; and yet

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[24] I am indebted to D'Ariano for drawing my attention to the significance of considering composite systems in quantum theory, the point emphasized in his recent works (some of which are cited here), in contrast to other quantum-informational approaches to quantum foundations.

(b) the possibility of considering each one of them separately at any given point; and
(c) the necessity of considering all of them at different moments for a comprehensive account of the totality of phenomena that one must consider in quantum physics.

The concept was never given by Bohr a single definition of this type. However, this definition may be surmised from several of Bohr's statements, such as: "Evidence obtained under different experimental conditions cannot be comprehended within a single picture, but must be regarded as *complementary* in the sense that only the totality of the phenomena [some of which are mutually exclusive] exhaust the *possible* information about the objects" [19, v. 2, p. 40; emphasis added] . In classical mechanics, we can comprehend all the information about each object within a single picture because the interference of measurement can be neglected: this allows us to identify the phenomenon considered with the object under investigation and to establish the quantities defining this information, such as the position and the momentum of each object, in the same experiment. In quantum physics, this interference cannot be neglected, which leads to different experimental conditions for each measurement on a quantum object (assumed to be prepared in some way, by a previous event, such as an emission of this object by some device) and their complementarity, in correspondence with the uncertainty relations. This implies two incompatible pictures of what is observed, as phenomena, in measuring instruments. Hence, the *possible* information about a quantum object, the information *to be found* in measuring instruments, could only be exhausted by the mutually incompatible evidence obtainable under different experimental conditions. On the other hand, once made, either measurement, say, that of the position, will provide the complete *actual* information about the system's state, as complete as possible, at this moment in time. One could never obtain the complementary information, provided by the momentum measurement, about this object at this moment in time, because in order to do so one would need simultaneously to perform a complementarity experiment on it, which is never possible. In fact, as explained, if one performs the first, position, measurement again with the same preparation, the outcome will be different. Accordingly, when one speaks, as Bohr does here, of any *possible* information about the object, this information, unlike in classical mechanics, is always probabilistic or statistical in character.[25]

It follows that one cannot assume that two complementary measurements represent *parts* of the same *whole*, the same single reality. Each measurement establishes, by a decision, the only reality there is, and the alternative decision would establish a different reality, at both levels of idealization, quantum phenomena and quantum objects, even though in the latter case this reality is each time unknowable and even unthinkable. It may still be assumed to be each time different because each of its effects, observed as a phenomenon, is different. Rather than arbitrarily selecting one

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[25] This situation is also responsible for what is known as "contextuality," which is a statistical concept. I have considered the relationships between complementarity and contextuality from the RWR perspective in [6].

or other part of a preexisting physical reality, as in classical physics, our decisions concerning which experiment to perform establish the *single* reality which defines what *type* of quantity (although not its value, of course) can be observed or predicted and precludes the complementary alternative. Accordingly, parts (b) and (c) of the above definition are as important as part (a), and disregarding them can lead to a misunderstanding of Bohr's concept.

It may be noted that wave-particle complementarity, with which the concept of complementarity is often associated, had not played a significant role in Bohr's thinking, especially after the Como lecture. Bohr's solution to the dilemma of whether quantum objects are particles or waves was that they were neither. Instead, either "picture" refers to one of the two sets of discrete individual effects of the interactions between quantum objects and measuring instruments, particle-like, which may be individual or collective, or wave-like, which are always collective, but composed of discrete individual effects. The example of the latter are "interference" effects, composed of the large number of discrete traces of the collisions between the quantum objects and the screen in the double-slit experiment in the corresponding setup (when both slits are open and there are no means to know through which slit each object has passed). These two sets of effects may be seen as complementary.

The concept of complementarity is better exemplified by complementarities of spacetime coordination and the application of momentum or energy conservation laws, correlative to the uncertainty relations. Technically, the uncertainty relations, $\Delta q \Delta p \approx h$, only prohibit the simultaneous exact measurement of both variables, always possible, at least in principle, in classical physics. In Bohr's interpretation, however, one not only cannot measure both variables simultaneously but also cannot define them simultaneously. As Bohr said:

> In the phenomena concerned we are not dealing with an incomplete description characterized by the arbitrary picking out of different elements of physical reality at the cost of [sacrificing] other such elements, but with a rational discrimination between essentially different experimental arrangements and procedures which are suited either for an unambiguous use of the idea of space location, or for a legitimate application of the conservation theorem of momentum. [9, p. 699]

By the same token, the uncertainty relations are not due to the limited accuracy of measuring instruments, and, as noted, they would be valid even if we had perfect instruments. As Bohr said: "we are of course not concerned with a restriction as to the accuracy of measurement, but with a limitation of the well-defined application of space-time concepts and dynamical conservation laws, entailed by the necessary distinction between measuring instruments and atomic objects" [19, v. 3, p. 5].

3.2 Quantum Causality

It follows from the preceding analysis that, *as a quantum-theoretical concept*, complementarity acquires probabilistic or statistical aspects, rarely addressed in considering it. (Complementarity could be considered more generally and used beyond

QM, in which cases these aspects need not play a role.) Indeed, complementarity was seen by Bohr as a generalization of the idea of causality. Bohr never explained the nature of this generalization, but it can be understood by means of the concept of quantum causality, introduced by this author previously [1, 3, pp. 206–206], [59].

"Quantum causality" is *the probabilistic or (if the experiment is repeated) statistical determination* of what may happen in a future observation at time t_2 as *a result of what has happened* previously as a quantum event, such as that of the measurement defined by our decision which experiment to perform at a given moment in time t_1. I add emphases because, as explained earlier, one can at t_2, perform an alternative measurement and thus, by another decision, establish a reality different from the one predicted, even if this prediction was made with probability one. Whatever is registered as a quantum event defines a set of probabilistically or statistically predictable future events and, by complementarity, precludes certain other types of predictions. All such predictions are quantum-nonlocal, but they respect Einstein-locality. In contrast to classical causality, quantum causality defines, by our decision, what *may happen*, although, *not necessarily what will happen*. This makes the event of an experiment the cause of what may happen as its effects.

This definition of quantum causality has affinities with several recent approaches to causality in quantum information theory (e.g., [60–62], except that it brings in complementarity, rarely considered in these arguments. D'Ariano defines causality in physics in general by means of this type of concept, which is consistent with complementarity, although the latter is not considered by D'Ariano either [62]. Classical causality as defined here or determinism (the term used, essentially equivalently, by D'Ariano) is merely a special case of causality in his sense, which may allow, as in classical mechanics, for ideally exact predictions. This is an important concept, providing a very general definition of causality, applicable beyond physics. Just as the present concept of quantum causality, it is consistent with both Einstein-locality and quantum nonlocality. I adopt the term "quantum causality, " in part for historical reasons, given the previous use of the term causality.

D'Ariano's concept is expressly linked to the arrow of time. The arrow of time is, however, also inherent in quantum causality as defined here. In the present, RWR-type, view, the arrow of time, as quantum causality itself, is only manifested classically in the observable phenomena. D'Ariano, on the other hand, appears to see the arrow of time as found in the ultimate workings of reality responsible for quantum (or classical) phenomena. As indicated earlier, however, the present view does not imply that at the ultimate level of reality there is no change or multiplicity but only permanence and oneness, sometimes suggested in recent literature (e.g., [63, 64]). In the RWR view, this concept would not apply to the ultimate constitution of reality any more than those of change or motion, or any other concepts, such as space or time. In the RWR view, the equations of QM or QFT, such as Schrödinger's or Dirac's equation, are not equations of motion of quantum objects, but are mathematical structures providing (with the help of Born's or analogous rules) expectation-catalogs concerning the outcomes of future experiments, which implies the arrow of time. Accordingly, their formal time reversibility, used to argue against the arrow of time or temporality itself, has no physical significance. What can, in the RWR view, be objectively ascertained is that the ultimate nature of reality is such that all our interactions with it, on all scales, by means of experimental technology entail the arrow of time, which quantum causality reflects in the case of quantum phenomena.

It is instructive to consider here the case of predictions with "probability equal to unity." As discussed in the next section, such predictions define EPR's argument, based their criterion of reality: "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity" [8, p. 138] . One might ask first: What does it mean to predict with probability equal to unity in physics? It means that one assumes that, if one measures what is so predicted, the measurement will confirm the prediction. But does the predicted quantity correspond to an element of physical reality, unless the measurement is performed? Technically, this is not so even in classical physics or relativity. An outside interference could change the reality thus predicted. It is also difficult and even impossible to strictly obtain the predicted value by a measurement. Such predictions are idealizations. Obviously, however, these qualifications do not pose serious difficulties for using EPR's criterion.[26]

The situation is fundamentally different in the case of quantum phenomena because of the irreducible role of measuring instruments and complementarity. Suppose that one had predicted, on the basis of the position measurement at time t_1, a future value of the position of an object at time t_2 with probability equal to unity, by means of a measurement performed on a different quantum object and thus without in any way disturbing the first one, which is ideally possible in EPR-type experiments. This prediction can then be confirmed, ideally, by the corresponding position measurement at t_2. However, measuring at t_2 the value of the complementary variable, that of momentum, will make it impossible to assign the position variable to the object at t_2, even though if we had measured the position, the outcome would correspond to our prediction. There is no experiment that could allow us to make this assignment, as opposed to classical physics, where one can measure both variables simultaneously, and assign both properties to the object independently of measurements. A prediction with probability equal to unity is applicable only if it is in principle verifiable, which cannot be assured in considering quantum phenomena, in the way it could be in classical physics. In quantum physics, establishing any measurable quantity unavoidably interferes with the object, and one can always interfere differently from the way necessary for verifying a given prediction and thus preclude establishing the predicted quantity as representing an element of reality.[27]

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[26] One might also note, along Bayesian lines, that predictions with any probability are only meaningful insofar as those who made them or know of them are still alive.

[27] That a prediction with probability equal to unity is not the same as establishing the reality of what is so predicted has been stressed by quantum Bayesians (QBists), on the grounds of the subjective nature of Bayesian probability, rather than the reasoning used here (e.g., [35, 46, pp. 231–238]).

4 "Without in Any Way Disturbing the System": Reality, Locality, and Complementarity in EPR-type Experiments

As noted from the outset, during the last half a century, following the Bell and Kochen-Specker theorems, the focus of the debate concerning quantum foundations has shifted towards the questions of quantum correlations and nonlocality, although the questions of completeness of QM and realism have remained an unavoidable background. Most of the key findings and arguments involved in these debates deal with discrete variables and Bohm's version of the EPR experiment. The main reason is that the thought-experiment proposed by EPR cannot be performed in a laboratory. Bohm's version of the EPR experiment can and has been performed, confirming the existence of quantum correlations, which can be ascertained experimentally, apart from QM. Among the most famous of these findings are those of D. M. Greenberger, M. Horne, A. Zeilinger, and L. Hardy, and, from the experimental side, A. Aspect's experiment and related experimental work, such as that by A. Zeilinger and his group [65–68].[28] The meaning of these findings have been debated as well. I shall bypass these debates here.[29] I would argue, however, at stake in these findings are situations in which complementarity plays a key role, a role rarely appreciated, and which can be viewed from the RWR perspective. In order to support this argument, I shall reexamine the key features of Bohr's reply to EPR.

Bohr contested Einstein's argumentation by analyzing the irreducible roles of measuring instruments and complementarity in all quantum phenomena, which, he argued, were underappreciated by EPR. This analysis allowed him to conclude that QM "would seem to fulfill, within its scope, all rational demands for completeness," at least Bohr-completeness [9, pp. 696, 700n], [19, v. 2, p. 57] . He also argued that QM fulfills these demands without sacrificing Einstein-locality, by virtue of the compatibility of his argument with "all exigencies of relativity theory," which implies Einstein-locality [9, p. 701n]. Bohr's interpretation in his reply was somewhat different from his ultimate interpretation, which no longer allowed for an assignment of elements of reality to quantum objects (before, at the time, or after measurement) and which would make his logic easier to apply. In his reply, this assignment was assumed to be possible at the time of measurement. This argument was, however, still of the RWR-type, because it implied tht quantum objects could not be considered independently of their interaction with measuring instruments. Bohr argued that the irreducible role of measuring instruments in considering quantum phenomena, disallow the application EPR's criterion of physical reality, or at least, the unqualified way in which the criterion was used by them, making this

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[28] I only cite some key earlier experiments. There have been numerous experiments performed since, some in order to find loopholes in these and subsequent experiments.

[29] The literature dealing with these subjects is immense. Among the standard treatments are [69–73]. There are also realist and causal views of quantum entanglement and correlations, either in realist interpretations of QM, such as the many worlds interpretation, or in alternative theories, such as Bohmian mechanics or that of classical random fields [7, 74]. Superdeterminism is another realist view, which explains away the complexities discussed here by denying an independent decision of performing one or the other EPR measurements (e.g., [75, 76]).

criterion "essentially ambiguous." It is true that Bohr only argued for (along with Einstein-locality) the Bohr-completeness of quantum mechanics, rather than for its Einstein-completeness. As noted, QM is expressly not Einstein-complete in Bohr's interpretation, including that in his reply, because it does not offer a representation of the independent behavior of quantum objects. However, this is all Bohr needed to do. EPR did not contend that QM was not Einstein-complete, but rather that it was not even Bohr-complete because its predictions were not exhaustive, unless one allowed for Einstein-nonlocality. EPR's definition of completeness was realist and thus in accord with Einstein's completeness, because it required that "every element of the physical reality must have a counterpart in the physical theory," which would thus represent every such element and, as result, predict it with probability equal to unity [8, p. 138]. But their argument was that, while all elements of reality in question could be so established, QM could not predict them, which made it even Bohr-incomplete. As I noted, Einstein later acknowledged that, if QM is Bohr-complete, insofar as its statistical predictions are exhaustive, one can see it as Einstein-local [24, pp. 155, 205]. This could not satisfy him, because he saw Einstein-completeness as a requirement for a fundamental theory (e.g., [24, pp. 155, 166–170, 205], [26, p. 81] .

It is not possible to give justice to EPR's paper and Bohr's reply here. I shall only offer a sketch of the exchange and then define "the EPR complementarity," which reflects some of the deepest aspects of Bohr's concept.[30] The crux of the EPR argument is that the EPR experiment allows for predictions with certainty concerning quantum objects without physically interfering with them by means of measurement, and thus "without in any way disturbing the system, " in accordance with their criterion of reality. An EPR prediction concerning a quantum object, S_2, of the EPR pair (S_1, S_2), is enabled by performing a measurement on another quantum object, S_1, with which, S_2, has previously been in interaction that entangled them, but from which it is spatially separated at the time of the measurement on S_1. Specifically, once S_1 and S_2, are separated, QM allows one to simultaneously assign both the distance between the two objects and the sum of their momenta, because the corresponding Hilbert-space operators commute. With these quantities in hand, by measuring either the position or the momentum of S_1, one can predict exactly either the position or the momentum for S_2 without physically interfering with it, which, EPR assumed, implies that one can simultaneously assign to S_2 both quantities as elements of reality to S_2.

"The authors [EPR]," Bohr said in his reply, "therefore want to ascribe an element of reality to each of the quantities represented by such variables. Since, moreover, it is a well-known feature of the present formalism of quantum mechanics that it is never possible, in the description of the state of a mechanical system, to attach definite values to both of two canonically conjugate variables, [EPR] consequently deem this formalism to be incomplete, and express the belief that a more satisfactory

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[30] I have considered Bohr's reply in detail previously [3, pp. 136–154], [22, pp. 107–136], [25, pp. 237–312] . The present discussion, however, modifies these treatments in several respects.

theory can be developed" [9, p. 696] .[31] It follows, then, if EPR are correct, that the formalism would not even be Bohr-complete, because it does not predict all that is possible to establish as real. The only alternative, as EPR saw it, would be the Einstein-nonlocal nature of quantum phenomena or QM [8, p. 141] .

Bohr counterargued that the situation does not allow one to dispense with the role of measuring instruments, because this role entails limitations on the types of measuring arrangements used in determining the quantities in question, even if one does so without performing a measurement on the object, S_2, concerning which predictions are made. These limitations result from "an influence *on the very conditions which define the possible types of predictions* [concerning S_2, by meareements performed on S_1]" [9, p. 700] . Disregarding this influence (which is not a physical influence on S_2!), as EPR do, gives EPR's criterion of reality its "essential ambiguity," in the case quantum phenomena. On the other hand, taking this influence and, in the first place, these conditions into account allowed Bohr to counterargue EPR's argument on both counts, (Bohr) incompleteness and (Einstein) nonlocality.

Both EPR and Bohr assume that the EPR experiment for (S_1, S_2) can be set in two alternative ways so as to predict, with probability equal to unity, either one or the other complementary measurable quantities for S_2 on the basis of measuring the corresponding quantities for S_1. Let us call this assumption "*assumption A*."

EPR infer from this assumption that both of these quantities can be assigned to S_2, even though it is impossible to do so simultaneously, in view of the uncertainty relations for the corresponding measurements on S_1. This makes QM incomplete (under EPR's criterion) because it has no mechanism for this assignment, unless one allows for Einstein-nonlocality (Einstein et al, p. 141). Let us call this inference "*inference E*" (for Einstein).

Bohr argued that, while *assumption A* is legitimate, *inference E* is unsustainable because a realization of the two situations necessary for the respective assignment of these quantities would involve two incompatible experimental arrangements and, thus, by implication, two different quantum objects, and two different EPR pairs to prepare them. There is no physical situation in which this joint assignment is possible for the same object. If one makes the EPR prediction, with probability equal to unity, for the second object, S_{12}, of a given EPR pair, (S_{11}, S_{12}), one would need a different EPR pair (S_{21}, S_{22}) to make the measurement on S_{21}, in order to make an alternative EPR prediction concerning S_{22}. I designate this inference as "*inference B*" (for Bohr).

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[31] EPR's actual argument is more elaborate. They derive a contradiction between the assumption that QM is complete and the assumption of the impossibility of attaching definite values to both variables in question, which, since this impossibility is inherent in QM, implies that QM is uncomplete. But this conclusion is essentially the same as stated by Bohr here. Even though one can predict (exactly) the two quantities considered only alternatively, EPR still contend that both quantities correspond to the elements of reality jointly pertaining to S_2, according to their criterion, which does not require simultaneity of such measurements or predictions, a requrement that would, in their their view, impliy Einstein-nonlocality [8, p. 141] . QM, however, only allows one to predict either one or the other of these two quantities. Hence, it is incomplete, because it cannot predict all that is possible to establish as real, unless it is Einstein-nonlocal.

One can diagrammatically represent the situation as follows. Let X and Y be two complementary variables, either continuous or discrete, in the Hilbert-space formalism $XY - YX \neq 0$ and x and y the corresponding physical measurable quantities ($\Delta q \Delta p \approx h$); (S_1, S_2) is the EPR pair of quantum objects; and p is the probability of predictions, via the wave functions, Ψ_1 and Ψ_2 associated with S_2, of either x or y, on the basis of two alternative measurements of either x or y performed on S_1 Then:

The EPR experiment in EPR's and Einstein's view, which considers one EPR pair:

$$\begin{array}{lll} S_1 & & S_2 \\ x_1 & \Psi_1 \,(\text{with } p = 1) \Longrightarrow & x_2 \\ y_1 & \Psi_2 \,(\text{with } p = 1) \Longrightarrow & y_2 \end{array}$$

EPR admit that x_1 and y_1 are and, because of the uncertainty relations, could only be, measured alternatively by disturbing the system S_1. But, because either x_2 or y_2 can be assigned to S_2 "*without in any way disturbing it*," S_2 can, EPR argue, be claimed to posses both x_2 or y_2, as elements of reality, at the time of either prediction.

The EPR experiment (in Bohr's view, according to which two EPR pairs are always required for two EPR predictions):

$$\begin{array}{lll} S_{11} & & S_{12} \\ x_{11} & \Psi_1 \,(\text{with } p = 1) \Longrightarrow & x_{12} \\ S_{21} & & S_{22} \\ y_{21} & \Psi_2 \,(\text{with } p = 1) \Longrightarrow & y_{22} \end{array}$$

The first diagram is, I argue, impossible to realize physically. The second can be physically realized, and it is that of a complementarity, which may, against EPR's own grain, be called "the EPR complementarity." This complementarity can be described as follows. Once one type of measurement (say, that of variable x) is performed on S_{11}, enabling the corresponding prediction on S_{12}, we irrevocably cut ourselves off from any possibility of making the alternative, complementary, measurement (that of y) on S_{11} and, thus, from the possibility of ever predicting the second variable for S_{12} [9, p. 700] . There is no way to define that variable for S_{12}, except, by a measurement and thus by disturbing S_{12}, which defeats the very purpose of EPR's argument, and doing so no longer allows one to ever verify the original prediction, thus requiring one to further qualify EPR's criterion or reality, a qualification that, as will be seen, is crucial in considering locality. By prediction, this could only be done on S_{22}, that is, by preparing another EPR pair and performing a measurement of y on S_{21}, which will, however, irrevocably prevent us from establishing x for S_{22}. It is only possible to establish both quantities for two EPR pairs, (S_{11}, S_{12}) and (S_{21}, S_{22}), and never for one. If we had predicted the second quantity, instead of the first one, for S_{12}, it would not, in general, be the same as it is for S_{22}. If we repeat the experiment for yet another pair, (S_{31}, S_{32}), so as to make predictions concerning the position of S_{32}, we can, again, make such a prediction exactly, but the outcome of the measurement on S_{32} will not in general be the same as for S_{12} or S_{22}, because the corresponding measurements on S_{12}, S_{22}, and S_{32} will be different.

Bohr does not explain the situation in terms of two different objects and EPR pairs necessary in order to make both EPR predictions. This is, however, at least an implication of his argument, given his insistence in his reply and elsewhere that "in the problem in question we are not dealing with a *single* specified experimental arrangement, but are referring to *two* different, mutually exclusive, arrangements" [19, v. 2 p. 57, 60] . In view of this mutual exclusivity, the second quantity in question cannot in principle be assigned to the same quantum object, once the first is assigned. The joint assignment is not possible even if one accepts EPR's criterion of reality, whereby such an assignment is made on the basis of a prediction, *unless we add the context of measurement to this criterion*, which in effect is what Bohr suggested.[32] The second prediction is not possible once an experiment enabling one to make the first prediction is made. The simultaneous assignment of both quanties is precluded by the uncertainty relations, as recognized by EPR. They, however, aim to show that this limitation could be circumvented by arguing that both variables could be assigned at any point, even though only one of them could be measured or predicted. Bohr counterargues that the uncertainty relations and complementarity, defined by the irreducible role of measuring instruments, disallow one ever to assign both quantities to any quantum object, even in the EPR case. Bohr concludes:

> From our point of view we now see that the wording of the above mentioned criterion of physical reality proposed by Einstein, Podolsky, and Rosen contains an ambiguity as regards the meaning of the expression "without in any way disturbing a system." Of course there is in a case like that just considered no question of a mechanical disturbance of the system under investigation [the second object of the EPR pair considered, concerning which we make the EPR prediction at a distance] during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of an *influence on the very conditions which define the possible types of predictions regarding the future behavior of the system.* Since these conditions constitute an inherent element of the description of any phenomenon to which the term "physical reality" can be properly attached, we see that the argumentation of the mentioned authors does not justify their conclusion that quantum-mechanical description is essentially incomplete. On the contrary this description, as appears from the preceding discussion, may be characterized as a rational utilization of all possibilities of unambiguous interpretation of measurements, compatible with the finite [quantum] and uncontrollable interaction between the object and the measuring instruments in the field of quantum theory. In fact, it is only the mutual exclusion of any two experimental procedures, permitting the unambiguous definition of complementary physical quantities, which provides room for new physical laws the coexistence of which might at first sight appear irreconcilable with the basic principles of science. It is just this entirely new situation as regards the description of physical phenomena that the notion of *complementarity* aims at characterizing. [9, p. 700; Bohr's emphasis]

This elaboration, especially Bohr's claim that "the essential ambiguity" of EPR's criterion (in the EPR experiment) pertains to "the meaning of the expression

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[32] This fact was important for Khrennikov's arguments concerning quantum nonlocality. It was also recently stressed, on more realist lines, by Grangier (e.g., [77]).

'without in any way disturbing a system,"' have posed difficulties for Bohr's readers. However, the elaboration and Bohr's meaning in this clause pose no special difficulties given the preceding analysis. Once one quantity in question is established (even on the basis of a prediction, in accordance with EPR's criterion of reality) for S_{12}, we cannot ever establish the second quantity involved without measuring and hence disturbing S_{12}. Only one of these quantities could be established for S_{12} without disturbing it, but once it is established, never the other *quantity without disturbing it*. We can establish such an alternative quantity without disturbing it only for a different quantum object, S_{22}, via a different EPR pair (S_{21}, S_{22}), by a measurement of a complementary type on S_{21}. These two determinations cannot be coordinated so as to assume that both quantities could be associated with the same object of the same EPR pair. The coordination of such events could only be statistical. Hence, we cannot establish both quantities for the same system "without in any way disturbing it." The only way to do so would be to perform a measurement on and thus disturb it, which, however, would erase the determination of the first quantity. Thus, the ambiguity in question indeed relates to the clause "without in any way disturbing the system," which, if one wants to apply it in the EPR situation, requires qualifications explained here but not provided by EPR. These qualifications amount to the fact that both quantities in question can never be predicted for the same quantum system, which in fact disables not only EPR's argument concerning the incompleteness (Bohr-incompleteness) of QM, but also their claim concerning Einstein-nonlocality as the only alternative.

Before I consider this issue, I reiterate that the argument just given could be transferred, with easy adjustments, to Bohm's version of the EPR experiment and spin variables (such as measuring spin in a given direction). In this case, too, there is the EPR complementarity insofar as any assignment of the complementary quantity to the same quantum objects becomes impossible, once one such quantity is assigned. An assignment of the other would require an alternative type of measurement, mutually exclusive with the first, on the first object of a given pair, and hence, another identically behaving EPR-Bohm pair, which cannot be guaranteed. Nothing other than statistical correlations, the Bell-EPR correlations, between such assignments is possible, which also allows for the Einstein-locality of correlations.

EPR noted a possible objection to their argument by requiring that "two or more physical quantities can be regarded as simultaneous elements of reality only when they can be simultaneously measured or predicted," in, they clearly imply, the same location [8, p. 41] .[33] They, however, see this as implying Einstein-nonlocality:

> One could object to this conclusion on the grounds that our criterion of reality is not sufficiently restrictive. Indeed, one would not arrive at our conclu-

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[33] As has been noted by several authors, Schrödinger arguably the first of them [53, p. 160] , one could simultaneously make the position measurement on S_1 and the momentum measurement on S_2, and thus simultaneously predict (ideally exactly) the second variable for each system, the momentum for S_1 and the position for S_2. This determination, however, is not simultaneous in the same location, and measuring the complementary variable instead of the predicted one in either location would irrevocably preclude one from verifying this prediction and would define a different reality.

> sion if one insisted that two or more physical quantities can be regarded as simultaneous elements of reality only when they can be simultaneously measured or predicted. On this point of view, since either one or the other, but not both simultaneously, of the quantities P and Q can be predicted, they are not simultaneously real. This makes the reality of P or Q depend upon the process of measurement carried out on the first system, which does not disturb the first system in any way. No reasonable definition of reality could be expected to permit this. [8, p. 141]

Einstein-nonlocality indeed follows if one assumes, as EPR do, that the measurement, say, of P, on S_1 fixes the physical state itself of S_2 by a spooky action at a distance, rather than only defines a spooky prediction at a distance concerning such as state, by fixing the conditions of this prediction. Or, as Einstein argued on later occasions, one is left with a paradoxical situation insofar as, if QM is complete, two mutually incompatible states could be assigned to the same distant quantum object, S_2, by different measurements performed on S_1. Einstein thought that Bohr accepted the alternative of Einstein-locality vs. completeness (Bohr-completeness), and retained completeness by allowing for Einstein-nonlocality. Einstein, however, misread Bohr's argument, which only allows for a spooky *prediction*, and *not action*, at a distance. Einstein presented Bohr's argument as, in his words, "translated into [Einstein's] own way of putting it" [78, p. 681]. Beginning with bypassing the role of measuring instruments and ending with seeing Einstein-nonlocality as acceptable to Bohr, Bohr's argument is lost in Einstein's "translation." This "translation" reads Einstein's own reasoning into Bohr's very different reasoning, which "ensures the compatibility between [Bohr's] argument and all exigencies of relativity theory," and thus Einstein-locality [9, p. 701n], [25, pp. 245–247] .

There is a difference between *establishing* the state of a physical object by a prediction with probability equal to unity and *possibly establishing* such a state on the basis of such a prediction. In Bohr's reply, physical states of quantum objects cannot be seen as finally determined, even when we have predicted them exactly, unless either the actual measurement is made or the possibility of *verifying* the prediction is assured insofar as such a measurement could in principle be performed so as to yield the predicted value. This last requirement in turn becomes a necessary qualification of EPR's criterion of reality in the case of quantum phenomena. This is because, as considered here, the measurement of the alternative quantity, Q, on S_2 would automatically disable any possible verification of the original prediction. It is crucial and is central to complementarity that it is always possible to perform this alternative measurement. This is one of the reasons why the assumption of the independent existence or reality of quantum objects or something in nature so idealized is important for Bohr's analysis of the EPR experiment and of the question of locality in it. This independent existence ensures the possibility of this measurement, and unless either measurement, that corresponding to the prediction or the alternative one is performed, it is meaningless to speak of the physical reality, that of P or that of Q, associated with S_2. However, once this alternative measurement is performed, the original prediction becomes meaningless as in principle unverifiable. Hence, QM could not be shown to be Einstein-nonlocal by EPR's logic, any more than it can be shown to be (Bohr) incomplete by their logic, which does not of course means that either case cannot made by a different argument.[34]

As noted earlier, Einstein eventually acknowledged that the "paradox" is eliminated if QM is only a statistical theory of ensembles and not of individual events, because, in this case, no single measurement of a given variable on S_1 or, more accurately, S_{1_n} determines with certainty the value of the corresponding variable on S_{2n} [24, p. 205] . These statistics involve correlations between spatially separated events, which has important implications for the question of locality, as Mermin shows, in considering (on lines of Bell's theorem) the data obtained in the Bohm-EPR type experiments for two spatially separated devices for spin directions, A and B:

> [It] is wrong to apply to individual runs of the experiment the principle that what happens at A does not depend on how the switch is set at B. Many people want to concluded from this that what happens at A *does* depend on how the switch is set at B, which is disquieting in view of the absence of any connections between the detectors. The conclusion can be avoided, if one renounces the Strong Baseball Principle, maintaining that indeed what happens at A does not depend on how the switch in set at B, but that this is only to be understood in its statistical sense, and most emphatically cannot be applied to individual runs of the experiment. To me this alternative conclusion is every bit as wonderful as the assertion of mysterious [spooky] action at a distance. I find it quite exquisite that, setting quantum metaphysics entirely aside, one can demonstrate directly from the data and the assumption that there are no mysterious actions at a distance, that there is no conceivable way consistently to apply the Baseball Principle [what happens at A does not depend on how the switch in set at B] to individual events. [72, p. 109]

In the present view, the impossibility of definitively claiming for the relationships between any two single events at A and B to be either independent or dependent in Einstein-nonlocal is a consequence of the quantum indefinitiveness postulate, which preclude any assertion concerning the relationships between any two individual quantum events. We cannot predict these correlations correctly on the basis of the data observed in one detector: "There is no way to infer from the data at one detector how the switch was set in other. Regardless of what is going on in detector

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[34] The argumentation just given complicates speaking, as is common, of entangled objects as forming "an indivisible whole." Bohr never does so, although his reply to EPR has been misread in this way, by confusing Bohr's use of this language for describing a phenomenon in his sense as forming an indivisible whole with the quantum object considered. Bohr's concept of phenomenon was introduced later. But, if one applies, and one can, this concept to Bohr's argument in his reply to EPR, a measurement performed on S_1 forms an indivisible whole composed of S_1 and the measuring instrument used, but does not in any way involve S_2. It only enables one to make a prediction concerning S_2 and the corresponding possible future phenomenon. A prediction is, however, not a phenomenon, only a measurement is; and a measurement on S_2 would establish its own phenomenon, with its own indivisible wholeness between S_2 and the measuring instrument.

B, the data for a great many runs at detector A is simply a random string of R's [red signals] and G's [green signals]" [72, p. 97] . We can only predict them correctly, if we know both settings. If somebody, unbeknownst to us, changed the setting in one detector, our predictions will no longer correspond to what is actually observed, and there would be no way to confirm them. Hence, there is no experimental basis to ascertain that both "elements of reality," one defined by one setting and the other by the other, could be assigned. As Mermin notes, the EPR-Bohr exchange "could be stated quite clearly" in term of his thought experiment, "a direct descendant of the rather more intricate but conceptually similar" original EPR experiment [72, pp. 90–91] .

Einstein, as I have emphasized here, never accepted that a fundamentally probabilistic or statitical theory like QM or QFT could, even as an Einstein-local theory, offer an exhaustive account of physical reality: "To believe this is logically possible without contradiction; it is so very contrary to my scientific instinct that I cannot forego the search for a more complete conception" [79, p. 375] . He required such a theory to be Einstein-complete, as well as Einstein-local. Perhaps, the question is not what we require from a fundamental theory, unless experimental evidence leads to such requirements (which was, however, not the reason for Einstein's imperative), but what a fundamental theory, either one already in place or one we need to develop, requires from us. One of the things it may require is a change of our attitude toward problems, such as that of physical reality, that we confront. I would argue, given Bohr's customarily careful way of expressing his points, that his invocation, cited earlier, of "a radical revision of our *attitude* toward the problem of physical reality" [9, p. 697; emphasis added] need not mean that one should necessarily adopt any particular concept of reality, even though Bohr did adopt an RWR-type one as against a realist one. More important is our attitude itself toward this problem or other problems, such that of quantum nonlocality: we should not be bound by previously established views, no matter how ingrained or cherished, and be ready to change our ways of thinking, *that of RWR-type included*, if physics requires it.

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