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Interpretations of Quantum Theory in the Light of Modern Cosmology

**Mario Castagnino[1] · Sebastian Fortin[2] ·
Roberto Laura[3] · Daniel Sudarsky[4]**

Abstract The difficult issues related to the interpretation of quantum mechanics and, in particular, the "measurement problem" are revisited using as motivation the process of generation of structure from quantum fluctuations in inflationary cosmology. The unessential mathematical complexity of the particular problem is bypassed, facilitating the discussion of the conceptual issues, by considering, within the paradigm set up by the cosmological problem, another problem where symmetry serves as a focal point: a simplified version of Mott's problem.

1 Introduction

It is a remarkable fact that the debate about the interpretation of quantum mechanics continues more than 80 years after the establishment of that theory in its modern form. This is due in part to the fact that the theory is extremely successful, and that the

✉ Roberto Laura
rlaura@fceia.unr.edu.ar

[1] Instituto de Astronomía y Física del Espacio (CONICET-UBA) and Instituto de Física Rosario (CONICET-UNR), Rosario, Argentina

[2] CONICET, Departamento de Física FCEN (Universidad de Buenos Aires), Buenos Aires, Argentina

[3] Instituto de Física Rosario (CONICET-UNR) and Facultad de Ciencias Exactas, Ingeniería y Agrimensura (UNR), Pellegrini 250, 2000 Rosario, Argentina

[4] Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, A. Postal 70-543, Mexico, DF 04510, Mexico

multiple interpretations seem to lead to exactly the same predictions[1] when applied to all the situations we have faced until now. In other words, when faced with any laboratory situation, one can rely on any of the interpretations, as they all lead, in practice, to exactly the same answers and predictions regarding the observations.

We will see that the situation changes dramatically when confronted with the challenges posed by modern cosmology. We will argue that, in that case, none of the existing interpretations are sufficient to deal successfully with the problems at hand.

We must face serious problems even before one gets into full quantum cosmology, where contact with observation is more elusive than in the case we will be focussing on. In fact, once one tries to incorporate gravitation into the quantum treatment, and quite independently of the technical issues that must be confronted, this situation entails yet another set of very serious conceptual problems, such as the disappearance of time from the theory [1–3], and many others [4].

The issue we want to consider here is one that arises already when considering the inflationary regime that, according to the current understanding, is an essential aspect of the history of our universe. We note that this situation is one where the technical difficulties associated with a full quantum theory of gravitation are essentially absent and simple perturbative treatments seems to be sufficient. We will see however that despite the relative simplicity of the situation, a serious question must be confronted.

Let us start by recalling here that the inflationary modification or adjustment to our cosmological theories arose when attempting to deal with certain "naturalness problems" of the standard Hot Big Bang Model: Namely, the Horizon problem, The Flatness problem and the Primordial relics problem [5]. The inflationary solution is obtained when one assumes that the "standard cosmological era" is preceded by an era of almost exponential expansion, which erases all inhomogeneities, dissolves all defects, and, in general, drives all quantum fields to their vacuum state. Although inflation was introduced to deal with those naturalness problems in the standard Big Bang Cosmological theory, its major success is its purported ability to predict the shape of the spectrum of primordial fluctuations that are supposed to seed all the structure in our universe, and whose earliest manifestations we see imprinted in the cosmic microwave background (CMB).

The problem we want to focus on, is exactly how does our theory account for the manner in which those first seeds of structure actually emerge from the quantum fluctuations of the inflaton field.[2] We will see that, although the problem is, in a sense, connected with the measurement problem in Quantum Theory, the particular manner in which it occurs in the inflationary context is such that issues which otherwise one might consider as having "only philosophical relevance", become acute to the point that a major shift in our thinking is required.

The core of the problem can be summarized in the following question: "How is it that from an initial situation which is supposed to be described, both at the quantum

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[1] We are ignoring the fact that certain interpretations are problematic.The point however is that to the extent that they are applied in a particular manner in concrete situations they do not offer predictions that differ from the text book version of Quantum Theory.

[2] The favored version of the theory actually deals with a composite variable representing the quantum aspects of the inflaton field and a certain component of the space-time metric [6].

and classical levels, by conditions[3] that are perfectly homogeneous and isotropic, a universe with space-time dependent density perturbations emerges, through processes that involve only dynamics which does not break the initial symmetry?

The issue has been confronted by several researchers in the field of Inflationary Cosmology, and it is worth mentioning that the majority of colleagues working on that subject do not seem to think that there is a problem, or are convinced that the problem has been solved by some clever arguments. It is noteworthy however, that these arguments tend to differ, in general, from one inflationary cosmologist to another [7–17]. Other cosmologists do acknowledge that there seems to be something unclear at this point [18], and the work of [19] might be considered as an early inquiry on the subject. Moreover, a couple of recent books on the subject acknowledge that there is a problem (see [20] and [6]).

The issue has been mostly ignored also by the community working in Foundational issues in Quantum Theory. They are probably justified in thinking that the complexity of the cosmological situation, involving as it does, not only general relativity but also quantum field theory in curved space-time, is not a particularly convenient one to consider in dealing with fundamental and conceptual questions. We believe, however, that the issue we have just described, actually offers an opportunity to focus sharply on the problems that normally concern our colleagues in that field, and that important lessons can be extracted by considering the issues in some detail. This manuscript is devoted precisely towards that goal.

Our strategy here will be to find a simpler "analogous" situation where the relevant issues appear just as in the cosmological setting, but where we have removed the complications that usually hide the fundamental aspects we want to focus on.

The paper will be organized as follows: In Sect. 2, we will review the essential aspects and details of the cosmological problem as it is treated in the works on inflationary cosmology. In Sect. 3, we will discuss briefly a problem that is often presented as analogous to the one we are confronting, the problem of breaking of rotational symmetry in the observations of nuclear decay in bubble chambers (often called Mott's problem [21]), a problem that is usually considered as solved in that work by Sir N. F. Mott, and, we will examine to which degree the analogy holds and fails to hold, and to what degree the problem has been truly solved. In addition, we will present an even simpler version of Mott's problem (which we call the Mini-Mott problem), that will allow us to write all expressions in full detail, and thus, to focus more clearly on the issue we must confront. In Sect. 4, we will then analyze the manner in which the problem would have been addressed by a scientist adhering to each one of the existing interpretational schemes for quantum theory. We end with a brief discussion of our findings, and a conclusion.

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[3] We refer here to the stage corresponding to several e-folds after the start of inflation, when the background corresponds to an inflating, flat, Robertson Walker space-time, and the "quantum fluctuations" are described by the Bunch-Davies vacuum, or some similarly highly symmetric state. This characterization is thought to be accurate up to exponentially small corrections in the number of e-folds, a detail that we will ignore as is customary in all inflationary analyses.

2 The Problem

In presenting the basic aspects of the problem here, we will be ignoring alternative views associated with certain interpretations of quantum theory, that will be discussed in more detail in the rest of the manuscript. This is done for clarity of exposition only, as is it not our aim to avoid the discussion of relevant postures.

For most of its existence, cosmology has been discussed in a classical language, as it is, in fact done in many other situations, such as the study of trajectories of space probes, while everybody knows that our world is quantum mechanical.[4] We, physicists, believe that the classical description of any system is nothing but an approximation to the truly fundamental quantum description, and, therefore, when we consider say, the classical description of the trajectory of a satellite in space, we view it as indicating that the wave function of its constituting atoms (or even that of its more elementary constituents) is a sharply peaked wave packet, where the uncertainties in the position and velocities are negligible compared with the precision of the description we are making.[5] In those situations, the classical description does not enter into a fundamental contradiction with the characteristics of our satellite trajectory. However, it would be very unsettling, for instance, if we were forced to consider at the same time, the classical elliptical trajectory of the satellite around Earth, while on the other hand we were forced to admit that, at the more fundamental level, the satellite was described by a spherically symmetric wave function. We know this is not the case,[6] and that the precise quantum description of the situation would indeed correspond to a suitable superposition of energy and momentum eigenfunctions leading to a wave packet corresponding to a sharply localized object. Of course, the precise way to do this faces, at this time, technically insurmountable problems; however, the principle is clear. In fact, we also must recognize that, in the case of the satellite, one is dealing with an open system, and its interaction with a clearly identifiable environment,—and the ensuing decoherence,—is likely to play an important role in making compatible the quantum and classical descriptions [22–26]. At this point we should note that, despite the widespread beliefs to the contrary, decoherence can not be claimed to truly solve the measurement problem[7] [27]).

In the cosmological setting, however, when we want to connect our classical descriptions of the cosmological late times, with say a quantum description of the early cosmological eras, we should seek, in a similar manner, to address the corresponding issues. That is, when considering the classical description we must regard it as nothing but the shorthand for the essential characteristics (i.e., the values corresponding to peaks of the wave functions) of a full quantum mechanical description.

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[4] There are, apparently, some people who disagree with this view, but we will not consider their thinking any further here.

[5] It even seems possible to construct wave packets with high n in an hydrogen atom that resemble to some degree the situation above.

[6] There are apparently philosophical views inspired in Kantian ontology where this statement could be questioned.

[7] In fact in order to do that one would need not only to define the privileged basis but also to add a postulate about actualization.

The universe that we inhabit today is certainly very well described at the classical level by an in-homogeneous and anisotropic classical state, and thus we must consider that such description, is, in accordance with the previous paragraph, nothing but a concise and imperfect characterization of an equally in-homogeneous and anisotropic quantum state, where the wave functions are peaked at those values of the variables corresponding to those indicated by the classical description. This would, in principle, involve no essential differences from the case of the classical and quantum description of our satellite, except for the lack of a clearly identifiable environment, given that we take the universe to include, by definition, all the degrees of freedom of our theory. However, there is nothing that indicates that, even without the identification of an environment, we should not be able to make, in principle, such quantum semi-classical description through the use of the sharply peaked wave functions and taking into account all the interactions in the analysis of its dynamics. The situation changes dramatically, however, if we want to seriously consider a theory, in which the early quantum state of the universe was particularly simple in a very special and precise way. This is the case in the inflationary paradigm, and in particular as it refers to the predictions about the spectrum of perturbations that, in that paradigm, are believed to arise from the uncertainties or fluctuations characterizing the quantum state of the inflaton, and which, according to these ideas, constitute the seeds of cosmic structure of our universe today.

Let us remind the reader of the basic mechanism by which inflation is meant to deal with the "naturalness problems" of standard Big Bang Cosmology discussed in the introduction. The essential idea is that if the Universe undergoes an early epoch of accelerated (almost exponential) expansion (lasting at least some 80 e-folds), it would come out of this period as an essentially flat and homogeneous space-time with an extreme dilution of all relics and, indeed, of all particle species. The states of all fields would thus be extremely well described by suitable vacua. The deviations from this state will be exponentially small (with the exponent characterized by the number of e-folds). What is required to achieve this, is something that behaves early on as a cosmological constant, but that is later "turned off" as a result of its own dynamics, returning the universe to the standard Big Bang cosmological evolutionary path. This is generically thought to be the result of a scalar field with a potential of certain specific characteristics called the "inflaton field". The remarkable fact is that this scheme also results in the perdition of a spectrum of primordial quantum uncertainties of the inflaton field that matches the form of the famous Harrison-Z'eldovich [28,29] spectrum of primordial perturbations and which has been observed in the multiple analysis of the extraordinary data on the CMB sky collected in the various recent experiments [30–37].

This is the basis of the claim that inflation "accounts for the seeds of the cosmic structure". They "emerge from the quantum vacuum", continue to evolve after inflation has ended, and after leaving their mark on the CMB, result in the emergence of the structure of our universe. That structure which at late times is characterized by galaxy clusters, galaxies, stars, planets and, later on, is tied to the development of the conditions permitting our own existence.

The issue we must face is: Can any of the interpretations of quantum theory be consistently used to justify the standard inflationary scenario, by which a simple state

that is supposed to characterize the early state of the universe in terms of the vacuum state of all fields,[8] and the flat FRW space-time, and which corresponds to a situation that is completely homogeneous and isotropic, would lead to the anisotropic and inhomogeneous universe in which we live.[9]

This article will be devoted, to a large extent, to deal with the conceptual issues above, and will not include the developments that are possible when adding new elements to deal with the shortcomings we encountered in the present context. We refer the readers interested on those matters to previous works [38–45].

Here, we want to consider the most popular interpretations of quantum theory, and analyze their usefulness in dealing with those problems in cosmology. On the other hand, we will be focussing for the most part of the article, and for simplicity of the discussion on a much simpler example, where all calculations can be made explicitly, however,in order to ensure that the lessons from one case can be used in the other, we will be forcing ourselves to avoid, in the corresponding treatment, the use of any element that would be absent in the cosmological situation which motivates our study.

3 Mott's Problem

How does a quantum system lose a symmetry present in the initial state if the interactions do not break it? Historically,it seems, the first time that this issue was faced within the newly formulated quantum theory concerned the decay of an excited atom or nucleus, from a spherical symmetric state, to an unexcited nucleus or atom and an emitted particle usually taken—and in fact observed—to be escaping along a particular direction, which is clearly not a spherically symmetric state of affairs. The issue is whether or not this can be fully accounted for within quantum theory.

The problem was considered in [21] in early days of quantum theory, and its treatment is thought, by many colleagues, to have clarified the issue completely. However, let us look at it a new: The setting considered consists of a nucleus located at the origin of spatial cartesian coordinates ($\vec{X} = \vec{0}$) in an excited (unstable) state $\left|\Psi^+\right\rangle$ which is spherically symmetric, and ready to decay into an unexcited nucleus $\left|\Psi^0\right\rangle$, plus an α particle in state $\left|\Xi_\alpha\right\rangle$, which is also spherically symmetric. The setting includes also two hydrogen atoms with their nuclei fixed at positions $\vec{a}_1$ and $\vec{a}_2$, and their corresponding electrons, in the corresponding ground states. The issue that is discussed is the degree to which the nuclei should be aligned with the origin (i.e $\vec{a}_2 = c\vec{a}_1$ with c real) if both atoms are to be excited by the outgoing α particle.

The analysis indicates that the probability of both atoms getting excited is significant only when there is a large degree of alignment, thus explaining the fact that the α particle traces straight paths in a bubble chamber.

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[8] Except, of course the zero modo of the inflaton.

[9] This point is sometimes characterized as the "transition from the quantum regime to the classical regime", but we find this a bit misleading: most people would agree that there are no classical or quantum regimes. The fundamental description ought to be always a quantum description. However, there exist regimes in which certain quantities can be described to a sufficient accuracy by their classical counterparts represented by the corresponding expectation values. All this depends, of course, on the physical state, the underlying dynamics, the quantity of interest, and the context **which one is considering**.

Thus, one might think that one has an example in which an initial state possessing spherical symmetry $\left|\Psi^+\right\rangle$ evolves into a final state lacking such symmetry, despite the assumption that the hamiltonian (governing the decay $\left|\Psi^+\right\rangle \to \left|\Psi^0\right\rangle\left|\Xi_\alpha\right\rangle$ and the α particle evolution) is symmetric under rotations. Thus the problem would seemed to have disappeared and the contradictory conclusions seemed to have vanished without trace. This seems quite remarkable indeed.

However, a closer look reveals the fallacy: As indicated, the setting includes the two unexcited atoms, which, through the localizations of their nuclei, break the rotational symmetry. Indeed, the discussion is based, not on the Hamiltonian for the evolution of the free α particle, but rather on the Hamiltonian for the joint evolution (including the interaction) of the α particle and the two electrons corresponding to the two localized hydrogen atoms. In fact, the projection postulate associated with a measurement is also coming into play in the analysis of [21] when computing probabilities by projecting on the subspace corresponding to the two atoms being excited. It is clear that if we were to replace these atoms by some hypothetical detectors whose quantum description corresponded to spherically symmetric wave functions , each one with support, say, on a thin spherical shell with radius r_i, a similar calculation would not lead to straight lines, but rather it would lead us to expect a spherical pattern of excitations. We would simply find that there was a certain probability for the detectors corresponding to the shells i^{th} & j^{th} being excited, and the symmetry would not have been compromised.

3.1 An Even Simpler Problem: Mini-Mott

In order to deal with the mainly conceptual issues that confront us here, we can make use of an even simpler problem where the symmetry in question is the discrete spatial inversion in 1+1 dimensions. The problem consists of a free non-relativistic particle of mass M moving on a line and interacting with suitable detectors located at two fixed points.

Consider a particle and two detectors with levels $|-\rangle$ (un-excited) $|+\rangle$ (excited) located at $x = x_0$ and $x = -x_0$. Initially the detectors are unexcited and the particle's wave function $\varphi(x, 0) = \langle x|\varphi_0\rangle$ is a simple gaussian centered at $x = 0$.

The Hamiltonian is:

$$\hat{H}_P = \frac{1}{2M}\hat{p}^2$$

for the free particle part. The free hamiltonian for the detector located at $x = +x_0$ is

$$\hat{H}_1 = \varepsilon|+\rangle_1\langle+|_1 - \varepsilon|-\rangle_1\langle-|_1 \qquad (1)$$

being $+\varepsilon$ $(-\varepsilon)$ the energy of the detector in the exited (unexcited) state $|+\rangle_1$ $(|-\rangle_1)$.

The free Hamiltonian for the detector located at $x = -x_0$ is

$$\hat{H}_2 = \varepsilon|+\rangle_2\langle+|_2 - \varepsilon|-\rangle_2\langle-|_2 \qquad (2)$$

The Hamiltonian corresponding to the interaction of the particle and the detector located at $x = +x_0$ is

$$\hat{H}_{1P} = \lambda g(\hat{x} - x_0 \hat{I}_p) \otimes (|+\rangle_1 \langle -|_1 + |-\rangle_1 \langle +|_1) \tag{3}$$

where $\hat{x}$ is the position operator of the particle, $\hat{I}_p$ the identity in the Hilbert space of the particle $\left(\hat{I}_p = \int dx \, |x\rangle \langle x| \right)$, and $g(y)$ is a function with support in a small interval centered at $y = 0$ [46]. Analogously, the Hamiltonian for the interaction of the particle and the detector located at $x = -x_0$ is

$$\hat{H}_{2P} = \lambda g(\hat{x} + x_0 \hat{I}_p) \otimes (|+\rangle_2 \langle -|_2 + |-\rangle_2 \langle +|_2)$$

The total Hamiltonian for the system composed by the particle and the two detectors is

$$\hat{H} = \hat{H}_P \otimes \hat{I}_1 \otimes \hat{I}_2 + \hat{I}_P \otimes \hat{H}_1 \otimes \hat{I}_2 + \hat{I}_P \otimes \hat{I}_1 \otimes \hat{H}_2 +$$
$$+ \hat{H}_{1P} \otimes \hat{I}_2 + \hat{H}_{2P} \otimes \hat{I}_1 \tag{4}$$

where $\hat{I}_1 \equiv |+\rangle_1 \langle +|_1 + |-\rangle_1 \langle -|_1$ and $\hat{I}_2 \equiv |+\rangle_2 \langle +|_2 + |-\rangle_2 \langle -|_2$.

The Schrödinger equation can be solved explicitly with the initial condition

$$|\Psi(t = 0)\rangle = |\varphi_0\rangle \otimes |-\rangle_1 \otimes |-\rangle_2 \tag{5}$$

Recall that the initial wave function for the particle is symmetric, so $< -x|\varphi_0 >=< \varphi_0|x >$. Thus, at time t we have:

$$|\Psi(t)\rangle = e^{-\frac{i}{\hbar}\hat{H}t} |\Psi(t = 0)\rangle$$
$$= |\varphi_{+-}(t)\rangle \otimes |+-\rangle + |\varphi_{-+}(t)\rangle \otimes |-+\rangle + |\varphi_{--}(t)\rangle \otimes |--\rangle$$
$$+ |\varphi_{++}(t)\rangle \otimes |++\rangle \tag{6}$$

where we have used $|+-\rangle \equiv |+\rangle_1 \otimes |-\rangle_2, |-+\rangle \equiv |-\rangle_1 \otimes |+\rangle_2, |--\rangle \equiv |-\rangle_1 \otimes |-\rangle_2$ and $|++\rangle \equiv |+\rangle_1 \otimes |+\rangle_2$. The last two terms represent the failure to detect (no detector is ever perfect), and double detection (involving something like a bounce, and corresponding to a small effect of order λ^2).

One might think that the first two terms (the relevant ones for our discussion) already show what one wants: we end up with the two alternatives $|+-\rangle$ or $|-+\rangle$ breaking the symmetry, and we say that we just do not know which one of the alternatives is selected by nature, or is actualized. However, we will see that the situation is not that simple, because the pair $(|+-\rangle, |-+\rangle)$ does not represent the only way to characterize these alternatives.

3.2 The Symmetry of the Problem

An inversion operator $\hat{\mathbb{P}}$ can be defined in such a way that it changes x by $-x$ in the wave function of the particle, and simultaneously it interchange the states of the detectors, i.e.

$$\hat{\mathbb{P}} |\varphi\rangle \otimes |\eta\rangle_1 \otimes |\chi\rangle_2 \equiv \left(\hat{P} |\varphi\rangle\right) \otimes |\chi\rangle_1 \otimes |\eta\rangle_2$$

where $\langle x| \hat{P} |\varphi\rangle \equiv \langle -x|\varphi\rangle$.

It is easy to prove that $\hat{\mathbb{P}}$ is a symmetry of the Hamiltonian of Eq. (4), and that the vector of Eq. (5), representing the initial state of the composed system, is an eigenstate of $\hat{\mathbb{P}}$, i.e.

$$\left[\hat{H}, \hat{\mathbb{P}}\right] = 0, \hat{\mathbb{P}} |\Psi(t = 0)\rangle = (+1) |\Psi(t = 0)\rangle$$

and therefore the value of $\hat{\mathbb{P}}$ is preserved by the time evolution $(\hat{\mathbb{P}} |\Psi(t)\rangle = (+1) |\Psi(t)\rangle)$. From this eigenvalue equation and the definition of the operator $\hat{\mathbb{P}}$ we obtain

$$\hat{P} |\varphi_{--}(t)\rangle = |\varphi_{--}(t)\rangle$$
$$\hat{P} |\varphi_{++}(t)\rangle = |\varphi_{++}(t)\rangle$$
$$\hat{P} |\varphi_{-+}(t)\rangle = |\varphi_{+-}(t)\rangle$$
$$\hat{P} |\varphi_{+-}(t)\rangle = |\varphi_{-+}(t)\rangle \tag{7}$$

The probabilities for the different possibilities of the two instruments pointer states are

$$\Pr(+-) = \langle\Psi(t)| \left\{\hat{I}_P \otimes |+-\rangle \langle+-|\right\} |\Psi(t)\rangle = \langle\varphi_{+-}(t)|\varphi_{+-}(t)\rangle$$
$$\Pr(-+) = \langle\Psi(t)| \left\{\hat{I}_P \otimes |-+\rangle \langle-+|\right\} |\Psi(t)\rangle = \langle\varphi_{-+}(t)|\varphi_{-+}(t)\rangle$$
$$\Pr(++) = \langle\Psi(t)| \left\{\hat{I}_P \otimes |++\rangle \langle++|\right\} |\Psi(t)\rangle = \langle\varphi_{++}(t)|\varphi_{++}(t)\rangle$$
$$\Pr(--) = \langle\Psi(t)| \left\{\hat{I}_P \otimes |--\rangle \langle--|\right\} |\Psi(t)\rangle = \langle\varphi_{--}(t)|\varphi_{--}(t)\rangle$$

Taking into account Eq. (7), we obtain

$$\Pr(-+) = \langle\varphi_{-+}(t)|\varphi_{-+}(t)\rangle = \left\langle\hat{P}\varphi_{+-}(t)|\hat{P}\varphi_{+-}(t)\right\rangle$$
$$= \left\langle\varphi_{+-}(t)|\hat{P}^2\varphi_{+-}(t)\right\rangle = \langle\varphi_{+-}(t)|\varphi_{+-}(t)\rangle = \Pr(+-)$$

As it was expected from the symmetry of the Hamiltonian and the initial condition, the probability $\Pr(+-)$ to have only the measurement instrument at $x = x_0$ excited is equal to the probability $\Pr(-+)$ to have excited only the instrument at $x = -x_0$.

3.3 Alternative Choice of Basis States

Everything should be fine if one adopts an interpretation such as Bohr's, in which the measurement instruments are classical objects, external to the quantum theory.

However, when the detectors are treated as quantum objects themselves, things become more problematic: we will see that one seems to be forced, not only to identify the quantum variables that are considered as detectors and subject these to slightly different rules of treatment, but also one would need to specify exactly how they are used. In other words, it seems one should specify a-priori which variables are the appropriate ones we must use in describing the situation. In the particular case we are dealing with here, this issue can be easily illustrated.

In the previous subsections, we have used the vectors $|+-\rangle, |-+\rangle, |--\rangle$ and $|++\rangle$ as a basis for the pointer states of the two instruments.

But we might choose to work with the basis given by the following four vectors

$$|S\rangle \equiv \frac{1}{\sqrt{2}} (|+-\rangle + |-+\rangle)$$

$$|A\rangle \equiv \frac{1}{\sqrt{2}} (|+-\rangle - |-+\rangle)$$

$$|D\rangle \equiv |--\rangle$$

$$|U\rangle \equiv |++\rangle \tag{8}$$

This basis seems to be particularly convenient when discussing symmetry related aspects of the problem. They can be used to expand the time dependent state vector $|\Psi(t)\rangle = e^{-\frac{i}{\hbar}\hat{H}t} |\Psi(t=0)\rangle$ already obtained in Eq. (6)

$$|\Psi(t)\rangle = |\varphi_S(t)\rangle \otimes |S\rangle + |\varphi_A(t)\rangle \otimes |A\rangle + |\varphi_{--}(t)\rangle \otimes |D\rangle + |\varphi_{++}(t)\rangle \otimes |U\rangle \tag{9}$$

where $|\varphi_S(t)\rangle \equiv \frac{1}{\sqrt{2}} \{|\varphi_{+-}(t)\rangle + |\varphi_{-+}(t)\rangle\}$ and $|\varphi_A(t)\rangle \equiv \frac{1}{\sqrt{2}} \{|\varphi_{+-}(t)\rangle - |\varphi_{-+}(t)\rangle\}$. The last equation clearly exhibits the preservation of the initial symmetry.

Thus the question we must face is the following: Why would it be incorrect to describe everything: the full Hilbert space, the evolution, including the interaction of detectors with the particle using this last choice of basis. Is there anything in the theory that would indicate which one is the correct basis to talk about the problem?. Why is it that it seems less natural to use the second rather than the first choice of basis? By the way, we note that each one of the four elements of the full Hilbert space, appearing in the above expression, are by themselves eigenstates of $\hat{P}$ with eigenvalue $+1$.

3.4 Decoherence

One might object to the above discussion pointing out that only one dynamical variable was considered for each measurement instrument, and it was the variable associated with the pointer position, allowed to have only two possibilities (excited and unexcited). This is clearly a highly idealized representation. A real measurement instrument is a macroscopic object, composed by an enormous amount of atoms. A more realistic description is to consider the states of the instrument represented by vectors of a Hilbert space which is the tensor product of a vector space associated with the pointer vari-

able and another vector space corresponding to an enormous number of microscopic variables of the instrument, playing the role of what we might call the environment.

Following the standard arguments of the theory of decoherence [47,48], the interaction pointer-microscopic variables for the instrument located at $x = +x_0$ may be described by the transformations

$$|-\rangle_1 |\varepsilon_-\rangle_1 \longrightarrow |-\rangle_1 |\varepsilon_-\rangle_1 \quad |+\rangle_1 |\varepsilon_-\rangle_1 \longrightarrow |+\rangle_1 |\varepsilon_+\rangle_1 \quad \langle\varepsilon_-|\varepsilon_+\rangle_1 \cong 0 \qquad (10)$$

In this very rapid process, the two possible pointer states $|-\rangle_1$ and $|+\rangle_1$ become correlated to the approximately orthogonal environment states $|\varepsilon_-\rangle_1$ and $|\varepsilon_+\rangle_1$.

Analogously, the interaction pointer-microscopic variables for the instrument located at $x = -x_0$ gives

$$|-\rangle_2 |\varepsilon_-\rangle_2 \longrightarrow |-\rangle_2 |\varepsilon_-\rangle_2 \quad |+\rangle_2 |\varepsilon_-\rangle_2 \longrightarrow |+\rangle_1 |\varepsilon_+\rangle_1 \quad \langle\varepsilon_-|\varepsilon_+\rangle_2 \cong 0 \qquad (11)$$

The interaction particle-instruments described in the previous subsections produce the time dependent state described by Eq. (6). If, following the standard approach, this interaction is followed by the interactions pointer-microscopic variables for both measurement instruments, we obtain

$$|\Psi(t)\rangle = |\varphi_{+-}(t)\rangle |+-\rangle |\varepsilon_+, \varepsilon_-\rangle + |\varphi_{-+}(t)\rangle |-+\rangle |\varepsilon_-, \varepsilon_+\rangle$$
$$+ |\varphi_{--}(t)\rangle |--\rangle |\varepsilon_-, \varepsilon_-\rangle + |\varphi_{++}(t)\rangle |++\rangle |\varepsilon_+, \varepsilon_+\rangle \qquad (12)$$

where we used the notations $|\varepsilon_\pm, \varepsilon_\pm\rangle \equiv |\varepsilon_\pm\rangle_1 \otimes |\varepsilon_\pm\rangle_2$ and omitted all tensor product symbols $\otimes$ to produce a more compact expression.

Any observable involving only the pointer variables should have the form

$$\hat{O} = \hat{I}_P \otimes \hat{O}_{pointers} \otimes \hat{I}_{E_1} \otimes \hat{I}_{E_2} \qquad (13)$$

where $\hat{I}_P$ is the identity operator for the particle, and $\hat{I}_{E_1}$ ($\hat{I}_{E_2}$) is the identity operator for the environment of the instrument located at $+x_0$ ($-x_0$).

The mean value of the pointer operator (13) in the decohered state (12) is

$$\langle\Psi(t)| \hat{O} |\Psi(t)\rangle = \langle\varphi_{+-}(t)|\varphi_{+-}(t)\rangle \langle+-| \hat{O}_{pointers} |+-\rangle$$
$$+ \langle\varphi_{-+}(t)|\varphi_{-+}(t)\rangle \langle-+| \hat{O}_{pointers} |-+\rangle$$
$$+ \langle\varphi_{--}(t)|\varphi_{--}(t)\rangle \langle--| \hat{O}_{pointers} |--\rangle$$
$$+ \langle\varphi_{++}(t)|\varphi_{++}(t)\rangle \langle++| \hat{O}_{pointers} |++\rangle \qquad (14)$$

We can now define an effective statistical operator in the space of the pointer variables

$$\hat{\rho}_{pointers} \equiv \langle\varphi_{+-}(t)|\varphi_{+-}(t)\rangle |+-\rangle \langle+-| + \langle\varphi_{-+}(t)|\varphi_{-+}(t)\rangle |-+\rangle \langle-+|$$
$$+ \langle\varphi_{--}(t)|\varphi_{--}(t)\rangle |--\rangle \langle--| + \langle\varphi_{++}(t)|\varphi_{++}(t)\rangle |++\rangle \langle++| \quad (15)$$

This effective statistical operator can be used to compute the mean value of Eq. (14) in the space of the pointer variables

$$\langle \Psi(t) | \hat{O} | \Psi(t) \rangle = Tr \left(\hat{\rho}_{pointers} \hat{O}_{pointers} \right) \tag{16}$$

The decoherence process has produced an effective state which is diagonal in the basis $\{|+-\rangle, |-+\rangle, |--\rangle, |++\rangle\}$ for the pointer states. However, in the present case we could not argue that this is "the" basis privileged by the decoherence process. In Sect. 3.2, we proved that $\langle \varphi_{+-}(t)|\varphi_{+-}(t)\rangle = \langle \varphi_{-+}(t)|\varphi_{-+}(t)\rangle$, as a consequence of the symmetry of the problem. Therefore the effective statistical operator $\hat{\rho}_{pointers}$ is also diagonal in any basis including two orthogonal linear combinations of $|+-\rangle$ and $|-+\rangle$, together with the vectors $|--\rangle$ and $|++\rangle$. One of these basis is the one defined by the vectors $\{|S\rangle, |A\rangle, |D\rangle, |U\rangle\}$ of Eqs. (8). Therefore, the decoherence process on the symmetric problem we have considered is not useful to privilege a basis of "physical states", unless the two environments were initially in different states. (See appendix for a theorem establishing the generality of this problem).

3.5 Predictability Sieve Criterion

In the previous section, we studied the model of interest adding an environment, according to the Zurek's recipe. We concluded that the introduction of this environment is not enough to determine the actualization basis of the pointer. The situation is the same as in Zurek [47], where the environment itself does not select the privileged basis. According to Zurek, the determination of a privileged basis can be carried out considering an additional criterion. This criterion is called "predictability sieve criterion" and establishes that the privileged basis is given by the dominant term in the Hamiltonian of the system. In the typical case, the system-environment interaction Hamiltonian is dominant, hence the privileged basis will be the eigenvector basis of the Hamiltonian of interaction H_{int}. For example, in the model of Zurek

$$H_{int} = \frac{1}{2} (|\Uparrow\rangle \langle\Uparrow| - |\Downarrow\rangle \langle\Downarrow|) \otimes \sum_{i=1}^{N} g_i (|\uparrow_i\rangle \langle\uparrow_i| - |\downarrow_i\rangle \langle\downarrow_i|)$$

where $\{|\Uparrow\rangle, \langle\Uparrow|\}$ are the eigenvectors of S_Z for the system and $\{|\uparrow_i\rangle, \langle\uparrow_i|\}$ are the eigenvectors of S_{Z_i} for the environment, the privileged basis are spin states with spin in $\hat{z}$ direction and the pointer indicates the spin in $\hat{z}$.

At this point we can not continue with generic analysis of the previous section, where we studied the influence of a generic environment. To apply the "predictability sieve criterion" it is necessary to clarify which is the environment Hamiltonian and which is the interaction Hamiltonian. Following the steps of Zurek, we can choose an interaction Hamiltonian. As an example we can specify that

$$H_{int} = (e_{++} |++\rangle \langle++| + e_{+-} |+-\rangle \langle+-| + e_{-+} |-+\rangle \langle-+| + e_{--} |--\rangle \langle--|) \otimes O_E$$

where O_E is some observable of the environment. Then, the eigenstates of H_{int} are

$$H_{int} |++\rangle |\varepsilon_{++}\rangle = e_{++}\varepsilon_{++} |++\rangle |\varepsilon_{++}\rangle$$
$$H_{int} |+-\rangle |\varepsilon_{+-}\rangle = e_{+-}\varepsilon_{+-} |+-\rangle |\varepsilon_{+-}\rangle$$

$$H_{int} \left|-+\right\rangle \left|\varepsilon_{-+}\right\rangle = e_{-+}\varepsilon_{-+} \left|-+\right\rangle \left|\varepsilon_{-+}\right\rangle$$
$$H_{int} \left|--\right\rangle \left|\varepsilon_{--}\right\rangle = e_{--}\varepsilon_{--} \left|--\right\rangle \left|\varepsilon_{--}\right\rangle$$

where $\left|\varepsilon_{\pm\pm}\right\rangle$ are the eigenvectors of O_E with eigenvalues $\varepsilon_{\pm\pm}$.

Thus, the privileged basis is the H_{int} eigenstates basis, i.e. $\{\left|+-\right\rangle, \left|-+\right\rangle, \left|++\right\rangle, \left|--\right\rangle\}$ and the pointer indicates the correct observable. Therefore, the problem seems solved because the decoherence selects the possible states of the pointer in the proper way. However, this method has two difficulties:

- The first is that it is necessary to introduce an interaction Hamiltonian specially designed to obtain the desired results. If we choose a different interaction Hamiltonian

$$H_{int} = (e_S \left|S\right\rangle \left\langle S\right| + e_A \left|A\right\rangle \left\langle A\right| + e_D \left|D\right\rangle \left\langle D\right| + e_U \left|U\right\rangle \left\langle U\right|) \otimes O_E$$

the result is that the pointer is actualized in the basis $\{\left|S\right\rangle, \left|A\right\rangle, \left|D\right\rangle, \left|U\right\rangle\}$. Therefore, we must make the choice of H_{int} carefully, i.e. the introduction of the interaction Hamiltonian is ad hoc.

- Second, in the general case the introduction of such interaction Hamiltonian breaks the symmetry of the total Hamiltonian. This is because the Hamiltonian privileges one direction. In fact, if we permute 1 and 2 in H_{int} we have

$$H_{int} \left|++\right\rangle \left|\varepsilon_{++}\right\rangle = e_{++}\varepsilon_{++} \left|++\right\rangle \left|\varepsilon_{++}\right\rangle$$
$$H_{int} \left|+-\right\rangle \left|\varepsilon_{+-}\right\rangle = e_{-+}\varepsilon_{+-} \left|+-\right\rangle \left|\varepsilon_{+-}\right\rangle \neq e_{+-}\varepsilon_{+-} \left|+-\right\rangle \left|\varepsilon_{+-}\right\rangle$$
$$H_{int} \left|-+\right\rangle \left|\varepsilon_{-+}\right\rangle = e_{+-}\varepsilon_{+-} \left|-+\right\rangle \left|\varepsilon_{-+}\right\rangle \neq e_{-+}\varepsilon_{-+} \left|-+\right\rangle \left|\varepsilon_{-+}\right\rangle$$
$$H_{int} \left|--\right\rangle \left|\varepsilon_{--}\right\rangle = e_{--}\varepsilon_{--} \left|--\right\rangle \left|\varepsilon_{--}\right\rangle$$

the only case where the symmetry is not broken is $e_{-+}\varepsilon_{-+} = e_{+-}\varepsilon_{+-}$. But if we take this case we have degeneration, thus any lineal combination of $\{\left|+-\right\rangle, \left|-+\right\rangle, \left|++\right\rangle, \left|--\right\rangle\}$ is an eigenstate of H_{int}, therefore $\{\left|S\right\rangle, \left|A\right\rangle, \left|D\right\rangle, \left|U\right\rangle\}$ is other H_{int} eigenstates basis. In the present case, the predictability sieve criterion can not select univocally a preferred basis, and, in particular, can not be used to identify what we might intuitively feel is the correct one.

The theorem we present in the appendix ensures that we will face this issue in the cosmological problem at hand. Thus we conclude that, even taking into account the predictability sieve criterion, the approach based just on decoherence is not helpful in offering a solution to our predicament.

4 Addressing the Problem in the Various Interpretational Schemes

One of the most clear evidences of the persistent state of confusion about quantum theory is the existence of a plethora of interpretations. Nothing like this happens with the other physical theories. There is no pressing/critical questions about the interpretation of Maxwell electrodynamics, or that of Einstein's theories of Relativity, either Special or General (see however [49–51]).

An exhaustive analysis of each one of those interpretations, or a detailed comparative study of their relative advantages and disadvantages is clearly outside the scope of the present manuscript. However, we will briefly survey the field in order to show that, in facing the problem that concerns us here, they all seem to come short. Before embarking in a more detailed way on that path, let us give the definitions of some concepts that will be used frequently in what follows.

Given a quantum theoretical description of a problem, we assume that one is given a Hilbert space, a Hamiltonian, and the set of observables. However, one often wants to demand that the discussion be carried out in a certain basis of the Hilbert space. Such a choice of preselected and privileged basis is called a context, and it often dictates essential aspects of the interpretation such as "collapse or actualization". How is that choice made, will, in general, depend on the particular interpretative scheme one wants to employ. Let us recall that selecting a context is equivalent to choosing an orthogonal basis of the Hilbert space, and requiring that all vectors and operators be described in such basis whenever the interpretation of the mathematics is required. In this setting, the coefficients of the corresponding expansions are then taken as yielding the corresponding probabilities. The concept associated with this is that of "actualization" or that of some alternative notion of "a possibility becoming actual". The precise meaning of the word "actual" naturally depends on the type of interpretation. In the Bohm De Broglie interpretation (which is often considered as involving hidden variables) the actualization is permanent, as it refers to the value of the hidden variable representing the "particle's position" —or, more generally, the point in configuration-space that together with the wave function represents the physical situation—corresponds to the actualized value of the position $\vec{x}$. In the other cases, the notion of actualization is associated with a change in the state of the system that, depending on the interpretation, is brought about by various causes and has different connotations.

It is well known that the logic of quantum mechanics is not a Boolean logic but a quantum logic [52–54]. Our brain knows how to reason with Boolean logic, but it is unable to use quantum logic (at least at the present time). Then, in applying the theory, we must somehow combine quantum mechanics with Boolean logic. Moreover, we can consider the problem in just a particular instant for some instantaneous type of interpretation or consider periods of time within one of the historical interpretations. There are interpretations that consider a special role for the apparatuses, often taken as classical and outside the scope of the theory, and consider that the theory only pertains to the results of the usage of these apparatuses to study a system. In some of them, the process of measurement produces "the collapse of the wave function", e.g. in the Copenhagen interpretation (see [55]). In one extreme we find the approaches where the posture is that "the system does not even exists" in the same physical sense as the apparatuses and observers. In the realistic interpretations, the measureing apparatus are missing, or not considered as essential, and they are substituted by an actualization of the wave function (e.g. in the modal interpretations, see [56]).

Here, we will discuss how the most popular interpretations deal with the simplified version of Mott's problem and with the cosmological problem that motivates our analysis.

4.1 Classical Apparatuses Interpretation

In this interpretation, there is a coexistence between the classical world and the quantum world. The context (i.e. the basis of the Hilbert space in which one analyzes the situation) is determined by the classical measuring apparatus (one assumes that these are clearly specified). This interpretation is supposed to be the interpretation "for all weathers". Accordingly, one is supposed to take the view that the only things that truly exists are those measuring apparatus, including, of course, the preparation apparatuses which are just other kind of measuring apparatuses. Those are taken to be always macroscopic, and, therefore, so the posture states, they must be treated classically with the usual boolean classical interpretation. The rest of the formalism is just a mathematical characterization of a microscopic world, but not a realistic description thereof, something that, in any event, is seen as lying outside the realm of science, and thus it is not considered as corresponding to one which could be taken as having physical reality. This seems to be the way in which many experimental physicists have learned to think, and which ensures that they never make mistakes. In this interpretation, the fundamental requirement is the existence of a clearly identified classical measuring apparatus, the description of which lies outside of the scope of the quantum theory.

Here, one would have to say that the description of the detectors at the quantum level is simply inappropriate. The detectors must be regarded as macroscopic, and, thus, intrinsically classical systems, and the classical states are, therefore, those which would tie them with the first basis. I.e. they are excited or non excited (but of course, being classical they are not described in the language of state vectors or operators in any Hilbert state). The problem, of course, is that such posture violates the rules we have set up to ourselves to solve the problem only within a scheme that would be applicable to the cosmological problem that motivated our analysis in the first place. The point is that, in that situation there are simply no systems that can be envisioned as playing the role of the measuring apparatuses. Apparatuses will emerge only after complex measuring instruments would be designed and built by sapient beings. And complex instruments and beings require the existence of planets, stars and generally, inhomogeneous and anisotropic regions to live and evolve, and, thus, this view is simply unsuitable to address the cosmological problem.

4.2 Copenhagen Interpretation

This interpretation is accepted as the official one [55]. The vast majority of the books are written based on its rules and concepts. Here, measurements are viewed as forcing the quantum collapse of the state of the system into the eigenvector determined by the measurement and its outcome. The collapse is associated with measurement. According with the textbook by Cohen Tanuodji et al. [57, p. 221], the collapse postulate is the following: "If the measurement of a physical quantity A on the system of state gives the result a_i, the state of the system, immediately after the measurement is the projection onto the subspace associated with $|a_i\rangle$". This interpretation takes the view that, even if the apparatuses might be described at a quantum level, there are distinct physical processes called measurements, which are governed by very special rules:

When a measurement takes place, the state of the system undergoes a sudden jump into one of the eigenvalues of the observable being measured and the probability for such jump is given by the Bohr's rule. This interpretation, thus, involves the notion of measurement as an independent concept, or in some presentations, as lying outside the scope of theory: It is something that can not be described in terms of the other concepts of the theory: states and/or operators on a Hilbert space. This is, in a sense, the most widely used interpretation, is presented in most text books, and has been subjected to multiple criticisms (see [58] for criticisms). In this interpretation, the essential component, the existence of which is taken for granted, is an external measuring device, a quantum system, which somehow produces/ induces the collapse.

In this case, one takes the measurements as triggering the quantum collapse of the state of the system. Thus Mini-Mott problem would be solved by describing the state of the system (now the particle and detectors) using the basis which is appropriate to describe the measurement. The point is that the measurement would have to be described by something that goes beyond the mere identification of the interaction hamiltonian, because, as we have seen, we can describe it in either, the symmetric or the non-symmetric basis. In the case of Mini-Mott, we would have to say that, the detectors are somehow constructed to detect the particle either at one position or the other, and this characterization can not be made simply by writing the interaction hamiltonian. The measurement is identified as a special type of interaction that is subject to spacial rules that do not apply to all interactions. The problem again is that this kind of solution would not be applicable to the cosmological problem, as in that situation there are no measuring apparatuses, and no measurements. As before, measurements require complex beings and those require planets, stars and generally inhomogeneous and anisotropic conditions to emerge. Thus, unless one want to invoke some God-like entity predating the emergence of structure in our universe, and which can perform measurements, we must acknowledge that, there is, within this interpretation, no solution to our cosmological problem.

In short, the problem with the instrumentalist interpretations, such as the previous two, is that in the situations at hand, there are simply no instruments and no observers (recall we are dealing with the inflationary regime and the process of generation of inhomogeneities and anisotropies that would eventually evolve into galaxies and stars that can in turn be the regions where life, intelligence, and even instruments can arise). In fact, it is one of the goals of cosmology to provide an explanation of the emergence of those conditions which lead to the generation in our universes of structures such as galaxies, planets and eventually living organisms such as ourselves, capable of making observations, and building instruments. Therefore, the instrumentalist path seems to be closed to us, at least in as much as we are focussing on the cosmological problem and on the related ones such as the Mini-Mott problem.

4.3 Statistical Interpretation

In this interpretation of quantum mechanics, the quantum state is interpreted as an abstract quantity that characterizes the probability distribution for an ensemble of identically prepared systems. That is, ensembles, and not individuals systems are con-

sidered as central to the theory, i. e. idealized sets containing infinite copies of identical systems. The quantum state corresponds to a collective description of all elements of the ensemble but not of each individual element. A quantum state corresponding to a superposition of different macroscopic states,is not seen as constituting any problem within this approach. It is just taken to represent a potential set of results and not the coexistence thereof. Thus, the main element to which the theory applies is the statistical ensemble,and not the individual system. We note that the application of the formalism within this interpretation, to any specific situation, requires the identification of a context (i.e. the basis of the Hilbert space in which one analyzes and discusses the situation) In the case where the experiment under consideration involves the measurement of a property, the context is determined by that property, and thus indirectly by the measuring devices. However, when there are no measuring devices identified, the interpretation often presupposes some choice of a preferred context. Without the context, one does not know what exactly is the ensemble one wishes to discuss, and, in particular, one does not know how should, the individual elements of the ensemble, be characterized.

Within this interpretation one considers that there must be an ensemble of copies of the system and that the individual systems in the ensemble become actualized in the various possibilities. However, as we have indicated previously, this interpretation requires a selection of the privileged basis to talk about the corresponding statistics. In the absence of measuring devices and/or anything that can play the role of observers, we have no way of doing so. Exactly in the same way that we could not argue convincingly that we should choose, for instance, the first over the second basis in Mini-Mott problem.

Furthermore, let's recall that, according to the statistical interpretation, one must adopt the position that quantum theory does not describe individual systems, (in this case, the Universe), but only some statistical ensembles of similarly prepared systems. This raises an important point. In order to make statistics over an ensemble one needs to be able to talk about each individual system that makes up the ensemble. Statistical averages of quantities are defined in terms of the individual values those quantities takes on each element of the ensemble. If one takes the view that individual systems are described in classical terms, one immediately faces the problem of having, in principle, the possibility of assigning to each individual system values of quantum incompatible observables. If, alternatively, one invokes a quantum characterization of individual systems, one must face the problem of having to talk about the measurement problem or some counterpart thereof. In particular, if we want to consider the statistical characterization of each system one must face the choice of basis or context one will use to talk about them.

Thus, we are faced again with the issue: how are we to distinguish "a measurement" from other interactions? If we presuppose that there are macroscopic variables that are accessible to us as observers, as done in Ballentine's book, one is, of course, bringing the observer into the picture as the means to make the selection of the privileged basis. However, "macroscopic" and "accessible" are clearly words that have a deep anthropocentric connotation.

In the case of our Mini-Mott example, one would need, not only to identify the detectors as playing the role of measuring apparatus, but one would have to postulate

the appropriate basis to talk about the system (or at least about the detectors) and use the statistical interpretation which is deemed to be the natural one for macroscopical and accessible variables of the apparatus. The choice which seems natural to us, given our experiences, does not seem to be indicated by anything present in the theory.[10]

In fact, we must wonder why is it that the second basis might not be considered as tied with an accessible and macroscopic variable. It seems we must argue that the values "symm" and "anti" are for some reason, not accessible, but the theory does not tell us why. This suggests that there is something that escapes Quantum theory and needs to be understood at some deeper level. Moreover, even if the exact nature of the ensemble were fully determined, it seems clear that the appropriate context is not identified and unambiguously selected by the theory. The fact is that the statistical interpretation lacks a clear criterion for such context selection.

4.4 Modal Interpretations

The name of these interpretations comes from the fact that in the early versions they were related to a certain type of Modal logic proposed by van Frasen [59]. According with the Stanford Encyclopedia of Philosophy, the general features of modal interpretations are:

- The interpretation is based on the standard formalism of quantum mechanics, with one exception: the projection postulate is left out.
- The interpretation is realist, in the sense that it assumes that quantum systems possess definite properties at all instants of time.
- Quantum mechanics is taken to be fundamental: it applies both to microscopic and macroscopic systems.
- The dynamical state of the system (pure or mixed) tells us what the possible properties of the system and their corresponding probabilities are. This is achieved by a precise mathematical rule that specifies a probabilistic relationship between the dynamical state and possible value states.
- A quantum measurement is an ordinary physical interaction. There is no collapse of the dynamical state: the dynamical state always evolves unitarily according to the Schrödinger equation.

These are interpretations that do not depend on the existence of instruments or observers as differentiated objects outside the quantum theory. These interpretations replace the postulate of collapse by one of actualization. There is a privileged context in which system properties take definite values. The difference between them is that they choose different contexts.

One of the appealing features of the modal interpretations is that there is no collapse, and the evolution is always unitary. In our example this means that the symmetric initial state evolves with a symmetric Hamiltonian, and then the symmetry will be present

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[10] On the other hand, it is worth noting that the Hamiltonian of interaction between particle and detector has a explicitly local form in the first basis but not in the second. This might be used but it would have to be explicitly formulated as part of the theory. Spontaneous localization theories, and de-Broglie Bohm approaches, for instance focus on position as playing a preferential role.

in the state at all times. This is a general theorem[11] and, as has been shown explicitly in Sect. 3.2 , the state that results from the evolution never loses their symmetry. One fundamental issue that the modal interpretations would have to face, in attempting to address the problem at hand, is the following: If the context selected by the particular modal interpretation is such that includes, as an element of the preferred basis, a state that has the property of homogeneity and isotropy such as the initial state of Mini-Mott, or the Bunch Davies state of the quantum field in inflationary cosmology, why would that cease to be the case at all other (later) times. I.e. why would the symmetric states cease to be part of the preferred basis at later times. This issue seems to be an inescapable one, because, unless such change of context takes place, one could not explain the breaking of the symmetry. Additionally, some of the modal interpretations that we present have "no-go" theorems [60–62] that make them unsuitable for the general interpretation of the theory.

4.4.1 Atomic Modal Interpretation

This interpretation assumes that there is, in nature, a fixed set of mutually disjoint atomic quantum subsystems that constitute the building blocks of all the global quantum systems. i. e. it establishes a preferred factorization of the Hilbert space [63]. It decomposes the system (called molecular) in "atomic" blocks $\{\alpha_q\}$, each in a state $\hat{\rho}_q$, now the privileged base is $\{|iq\rangle\}$. The reduced state of each block is

$$\hat{\rho}_q = \sum_i \rho_i^{(q)} |iq\rangle \langle iq| \tag{17}$$

Thus, one can set properties on the subsystems of the total system. However, the basis is undetermined in each subsystem.

    The main problem for the Atomic modal interpretation is to justify the assumption that there is a preferred partition of the universe, and to provide some idea about what this factorization should look like. Moreover we generally do not end up with a well specified basis for the complete system. In other words, the shortcomings of this interpretation are intimately connected with some the the issues we must resolve in our quest to address the Mini Mott or the Cosmological problems : The choice of the preferred basis.

4.4.2 Biorthogonal-Decomposition Modal Interpretation

This interpretation sometimes is known as "Kochen–Dieks modal interpretation" [64–67]. The definite-valued observables are picked out by the biorthogonal (Schmidt) decomposition of the pure quantum state of the system separated into subsystems. The state is decomposed into Schmidt basis

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[11] Let $\hat{S}$ be a symmetry operator and $|\Psi(0)\rangle$ an initial symmetric state, i.e. $\hat{S}|\Psi(0)\rangle = |\Psi(0)\rangle$. Let $\hat{H}(t)$ be the system's hamiltonian, taken to be invariant under the symmetry i.e. $[\hat{H}(t), \hat{S}] = 0$. Then $\hat{S}|\Psi(t)\rangle = \hat{S}e^{i\int_0^t H(t')dt'}|\Psi(0)\rangle = e^{i\int_0^t H(t')dt'}\hat{S}|\Psi(0)\rangle = e^{i\int_0^t H(t')dt'}|\Psi(0)\rangle = |\Psi(t)\rangle$ i.e. the evolved state is also symmetric.

$$\left|\psi^{\alpha\beta}\right\rangle = \sum_j c_j \left|c_j^{\alpha}\right\rangle \otimes \left|c_j^{\beta}\right\rangle$$

And the states $\left|c_j^{\alpha}\right\rangle \left|c_j^{\beta}\right\rangle$ define the properties that take a defined value. This interpretation has the obvious difficulty that a system can be decomposed into subsystems in a variety of different ways.

However, the fact that a system can be decomposed in a variety of different ways leads to multiple alternative choices for the biorthogonal decomposition and, thus, introduces the following problem [56]: In order to apply this interpretation, we need to know in advance what is the privileged basis (or decomposition). Again, the problem with this interpretation, is the problem we want to solve. Many authors believe that those problems can be solved by appealing to quantum decoherence. But as we have seen, in the situations under consideration here, decoherence simply can't not perform the task one expects from it.

4.4.3 Perspectival Modal Interpretation

In this interpretation the properties of a physical system have a relational character and are defined with respect to another physical system that serves as a "reference system" [68]. The starting point is that the universe is in a pure state $|\psi\rangle \langle\psi|$, evolves according to the Schrödinger equation and never collapses. Using the partial trace it is possible to compute the state of a subsystem S with respect to the rest of the universe

$$\rho_U^S = Tr_{(U\backslash S)} |\psi\rangle \langle\psi|$$

the spectral resolution of ρ_U^S defines the properties that take a defined value.

As in shown in our analysis of the role of decoherence in the situations at hand, here we need to divide the whole system into relevant sub-system and environment, and, given the symmetry of the situation it is impossible to find the privileged basis (See the appendix for a theorem exhibiting the generality of the problem). In fact, if we were to admit different partitions between system and environment we could face logical contradictions [69,70].

4.4.4 Modal-Hamiltonian Interpretation

According to this interpretation, a quantum system S is represented by a pair $(\mathcal{O}, \hat{H})$ where (i) $\mathcal{O}$ is a space of all possible operators, (ii) $\hat{H} \in \mathcal{O}$ is the time-independent Hamiltonian of the system S, and (iii) the state evolves according to the Schrödinger equation [71]. Given a quantum system S, the actual-valued observables of S are the Hamiltonian $\hat{H}$, and all the observables commuting with $\hat{H}$ and having, at least, the same symmetries as $\hat{H}$. This interpretation is particularly suitable to be applied to closed systems where there is a no time-dependent Hamiltonian. But it cannot be used in the general case.

In fact, for this particular modal interpretation, one of it's axioms prevents its application to systems with truly time dependent Hamiltonians. In general, one assumes

that, if one has a time dependent Hamiltonian, it is because one has failed to consider the complete system, and that what one has been considering is just a part of a larger system with a time independent Hamiltonian. In other words, in order to apply the formalism one has to correctly identify the complete system, having a time independent Hamiltonian. While there is some progress in the attempts to apply this interpretation in quantum field theory, the proponents have never considered a curved spacetime [72]. However, we must stress that, in a general relativistic setting there is, in general, no such time independent Hamiltonian: In fact, if one includes gravity, the full Hamiltonian vanishes, and when one restricts consideration to the matter sector alone, the Hamiltonian depends on arbitrary choices of lapse and shift functions. In the cosmological context at hand, the matter Hamiltonian depends on time due to the expansion of the universe.

4.5 de Broglie-Bohm Interpretation

One of the many perspectives offering an interpretation to the results of the quantum mechanics experiments can be traced back to L. de Broglie, and was resurrected and refined by David Bohm. In the work [73,74], Bohm wrote the Schrödinger equation in a particular way: He separated the module R and phase S of the wave function obtaining a set of coupled equations governing the evolution of R and S. One of these equations is easily interpreted as a probability conservation equation upon the introduction of an appropriate ensemble of particles and an equation determining each particle's velocity in terms of the gradient of the phase S at the particle's instantaneous position. The other equation is formally identical to the Hamilton-Jacobi equation of classical mechanics, where classical potential is added to the quantum terms. This novel terms are then interpreted as a quantum contribution to the potential. This perspective indicates that the phase S can be interpreted as the generating function, which allows the calculation of the possible trajectories of the particle. Thus one obtains a deterministic quantum mechanics in which the particles have definite positions and velocities at any given time, i.e. in this approach particles do have well defined paths. In fact, this approach is sometimes considered, not just as an interpretation of quantum mechanics, but as a different theory, and under certain circumstances (see [75]) one can expect predictions that differ from standard quantum theory.

These ideas have been discussed and refined, and a more complete version can be found in the book "Quantum Theory of Motion" by Holland [76]. Within this scheme, the trajectory of a quantum particle is well defined once its initial condition is determined as in the classic case. For example, in the double-slit experiment, the quantum potential has the shape of "gutters". The gutters start at the particle gun and end at the screen. The potential is such that the density of "gutters" is higher in regions where, from the orthodox view, constructive interference is expected, and is smaller in destructive interference areas. In this experiment, for reasons of practical nature, it is impossible to determine which of the gutters will be taken by a particle which departs from the gun. However, if we observe the arrival point of the particle on the screen, it is possible to determine which gutter was taken and, consequently, its initial condition.

Bohm's theory as it exists today is not directly applicable to solve our proposed problem in a completely satisfactory way. That approach is tied intrinsically with particle quantum mechanics, as it involves, in a sense, a choice of a preferential basis, through the fact that the theory singles out the particle's position variable, as the one that is permanently actualized. In any attempt to apply this approach to the cosmological problem at hand, the first thing we would need to do is to select the corresponding special variable for the case of a field theory. One might be inclined to take the field amplitude as playing such spacial role, by arguing, for instance, that , in general, such role must be assigned to be the configuration variables. The issue would become more delicate and problematic, if what we have is a gauge field theory. In any event, it is clear that some nontrivial choices must be made. Once such choices are made, one might apply the field theoretical version of the d' Brogile-Bohm approach ,to the cosmological problem. This is in fact, a path taken in the works given in reference [77,78].

The second point we should make is that in this scheme, the initial conditions involve not only the initial wave function of the system, but also the initial value of the configuration variables (i.e the particle's position in the case of non-relativistic quantum mechanics). In that sense, in considering the question of symmetry of the initial conditions, one must consider both aspects. In fact, given any system, the key of the explanation of its behavior is to be found in the initial condition for the configuration variables.

For instance if we study our problem from the Bohmian perspective, it is clear that the symmetry of the arrangement, and the initial wave function will induce a symmetric quantum potential. However, the initial condition for an individual particle should, according to the spirit of the approach, be chosen in a random fashion from an appropriate "equilibrium distribution". Generically, such initial condition will NOT be symmetric. This would seem to account for the fact that, in practice, we observe that the particle (e. g. in the mini-Mott example) is detected only by one of the detectors. In this sense, the path of the particle was determined by the initial condition, and was predetermined from the beginning. This feature, thus exhibits the fact that the initial condition was not symmetrical. I. e. in this theory, the asymmetry that we intend to deduce must be taken as introduced from the start. Bohm offered a replied to a critique related to this point which can be found in [79]. There he argued that, in any given experiment, the particle interaction with the environment would push the particle to one side or the other. This argument is valid, of course, in any realistic experimental situation, but we can not apply it to the cosmological case because, by definition, the universe as a whole, has no environment. Moreover, as we have seen already, the introduction of an environment, which is subject to a quantum mechanical treatment, and which is assumed to share the symmetry properties of the problem, gives rise to the same problems which appeared in the discussion of the decoherence perspective. Thus, in a strict dBB approach to the problem, the origin of asymmetry would be found in the initial conditions of the "hidden variable" of the theory, (i.e. the position, or in general something like the configuration variable) just as in the decoherence proposal it must be associated with some asymmetry in the state of the environment. Our argument is supported by the fact that all the work in cosmology which was done under this perspective introduces the asymmetry in the initial condition of the universe

[77,78]. For this reason we must consider that the d'Broglie- Bohm approach can not be said to offer a satisfactory explanation of the emergence of asymmetry: Simply speaking, the asymmetry is there from the start.

4.6 Many Worlds Interpretation

The fundamental idea of this interpretation is that, in addition to the world that we can see, there are many parallel worlds that make up the totality of what exists [80]. At every time a quantum experiment involving different potential outcomes with non-zero probability is performed, all outcomes are actually obtained, each in a different world, even if we are aware only of the world with the outcome we have seen. In fact, quantum experiments take place everywhere and very often, not just in physics laboratories: even the irregular blinking of an old fluorescent bulb is a quantum experiment [80]. In Everett words:

"*We thus arrive at the following picture: Throughout all of a sequence of observation processes there is only one physical system representing the observer, yet there is not a single and unique state of the observer (i.e. a collapse state)(which follows from the representation of the interaction-system). Nevertheless, there is a representation in terms of superposition, each element of which contains a definite observer state and a corresponding system state (i.e. the system«s ordinary state). Thus with each succeeding observation (or interaction) the observer state 'branches' into a number of different states. Each branch represents a different outcome of the measurement and the corresponding eigenstate of the object-system state. All branches exist simultaneously in the superposition after any given sequence of observations. Thus if we represent the free evolution of the system as bifurcating paths at actualization points from which the multiple alternatives emerge, to bifurcate again and again offering a multiplicity of worlds, we end up with the multi-temporal object to which this interpretation refers. By the way each one of the vertices or bifurcations points must be associated with a corresponding context that determines the bifurcation basis at that event. It is amazing how much can be said about such bizarre picture of reality*." See [81, p. 459].

In this interpretation, one of the problems one must face is that it is not specified on what basis the bifurcation occurs, or exactly when, and under which conditions it takes place. Thus, again, it becomes necessary to specify, among other things, a preferred basis, or context. According to some authors, this basis can be obtained from a decoherence type of analysis [82]. As the many worlds approach requires a privileged basis which characterizes the branchings, it is clear that in the case of the Mini-Mott problem we would have to determine if the branchings are to be associated, for instance, with the first or second decomposition of the state, i.e the expression given in Eq. (6), or rather the one in Eq. (9). There seems to be simply nothing at all, intrinsic to the setting, to help us determine how should this selection must be made. It is only if we invoke something like an observer with a conscious brain which for its own intrinsic reasons is entangled in a particular simple (diagonal) way, that one might be able to argue that the description of the detectors of our Mini Mott situation should be made in the first rather than the second basis.

Once more, in the cosmological problem we do not have any way to make such selection in the absence of conscious observers. Moreover, as we have seen, given the symmetry of the situation, decoherence does not help us at all in this case.

4.7 Interpretations Based on Histories

Interpretations based on histories consider a formalism suitable to give descriptions of quantum systems involving properties at different times. We know two versions:

a. *Consistent Histories* [83–86] The theory of consistent histories is a framework to consider, using a quantum language, the properties of a quantum system at different times. It deals with a series of times $\{t_i\}_{i \in I}$ and the specification at each t_i of a decomposition of the Hilbert space of the system into suitably chosen subspaces. The choice of one such subspace at each t_i is known as a "coarse grained history". The resulting set of coarse grained histories is called a realm if the quantum mechanical amplitude for interference between two coarse grained histories in the set vanish.

Under such circumstances, the scheme assigns probabilities to each coarse grained history in a manner that would correspond to Born's rule. The point, however, is that only when the consistency condition is satisfied for the set of histories, can it be regarded as proving a consistent characterization of the system's development. This is the origin of the name "interpretation of consistent histories".

More specifically, the scheme is based on the consideration, given a quantum state of the system represented by a density matrix $\hat{\rho}$ at time t_0, of families of histories characterized by a set of projection operators $\{\hat{P}_n(t_n)\}$, each of which is associated with the system possessing a value of certain physical property in a given range at a given time. A family F of such projectors is called self consistent, if the resulting histories do not interfere among themselves. Then, the scheme assigns probabilities to each individual " coarse grained history" within the family according to the rule:

$$P = Tr(\hat{P}_n(t_n)U(t_n, t_{n-1})\hat{P}_{n-1}U(t_{n-1}, t_{n-2})\ldots \hat{P}_2 U(t_2, t_1)\hat{P}_1 U(t_1, t_0)\hat{\rho} U(t_n, t_0)^{\dagger})$$
(18)

where the U«s stand for the standard unitary evolution operators connecting two times. The fact, however, is that, in general, there exists a multiplicity of possible choices of the realm, and the scheme does not indicate, at fundamental level, which one is to be used in each circumstance.

b. *Generalized contexts formalism* [87] It is also a formalism suitable to give descriptions of quantum systems involving properties at different times. It is, in a sense, a refinement of the previous interpretation.

Again, the interpretation deals with a time series $\{t_i\}_{i \in I}$ and the specification at each t_i of a characterization of the system by a suitably chosen set of properties. Each of the possible selections of properties of a quantum system is called a "generalized context" or "description".

The quantum properties of a generalized context should satisfy two types of compatibility conditions:

(1) For properties at the same time, the corresponding projection operators should be generated by a projective decomposition of the identity operator, i.e. by a collection of mutually orthogonal projectors adding up to the identity operator.

(2) For properties at different times, the corresponding projectors should commute when translated to a common time.

This formalism was successfully tested to give suitable descriptions of a measurement process [88], the double slit experiment with and without measurement instruments [89], and the quantum decay process [90].

This formalism corresponds to a modification of the consistent histories approach, having the advantage that some problematic histories that can arise in the consistent theory are eliminated. Moreover, in contrast with the former, here, the compatibility conditions for properties at different times are state independent, and therefore the allowed generalized contexts giving descriptions of a quantum system are also state independent. This is an interesting feature, because in the usual axiomatic theories of quantum mechanics the state is considered as a functional on the space of observables, and it enters into the theory in a somehow subordinate position.

As we indicated, the main problem with this type of approach is that, although the scheme seems satisfactory, one has selected a particular decoherent family, and there exists, in principle, an infinitude of such families, which are, however, mutually inconsistent. This is addressed in this approach by the so called "single family rule" which indicates one should never consider more than one family, at a time. One might wonder, in fact, where does such rule come from?. In [91] it is described as rather *ad hoc*, but lets not focus on that issue here. The issue we shall be concerned with, is the following: The need to single out one particular family or realm providing the alternatives for the particular history that becomes actual. The fact that one assigns probabilities within a family, strongly suggests that the interpretation must be that one of the histories in that family is actualized in our world. Otherwise, one must wonder what these probabilities refer to (i.e. the probabilities assigned are probabilities of what? (see however [92]). Recall that, in the context of the present problem we do not adopt the position that these are probabilities of observing a certain value of a physical quantity when that quantity is measured, because, as already discussed, we do not want to bring concepts like measurement or observation into the discussion. In other words, there is, in principle, no clear way to single out a specific family without relying on an *a-priori* given set of questions one is asking—those associated with the quantities whose spectral family one choses to construct the realm - and this leads to serious interpretational difficulties [93].

In the Mini-Mott experimental setup, we might guide ourselves, in practice, by the questions the experimental set up is "asking" (in fact, this has a close analogy with the use of Bohr's rule in a given experiment or series of experiments). However, in the absence of such guidance (i.e. without *a priori* considering that the experimental set up corresponds to asking certain yes /no questions, as it seems to be required if one does acknowledge the possibility of all superposition states of the apparatus itself, and, in particular, taking the second basis for the discussion of the Mini Mott example) one does not know how to select the family. Note that one is not asking how to select a

particular history within the family, but how to select a particular family from within the collection of all possible decoherent families.

In fact, it is hard to see, in describing the universe, what would dictate the selection of the appropriate projector operators, and thus of the appropriate family, (if we require a description which do not makes use of our own existence and our own asking of certain questions, as part of the input).

This very issue makes its appearance in the cosmological context we are concerned with. In fact, the problem can be seen clearly in the following example: Consider the family of projector operators, where the chosen projectors are not tied to the symmetry, as it is done in [94,95], and consider their results. Those might seem satisfactory in connection to what one needs to understand, namely the shape and a amplitude of the primordial spectrum of cosmological anisotropies and inhomogeneities. However the point is that we could , alternatively, analyze the situation by considering the following family: Construct the projector operator into the space of homogeneous and isotropic states P_{HI}. This is simply the projector into the intersection of the kernels of the generators of translations and rotations. Next, we define $P_{non} \equiv I - P_{HI}$ the orthogonal projector. It is clear that these are projector operators and satisfy $P_{HI} + P_{non} = 1$. Next, we take the initial state for the quantum fluctuations (usually called the vacuum) $|\Phi_0\rangle$, and note that it is homogeneous and isotropic.

Now take any set of values for time $\{t_i\}$ and consider the family associated with that initial state and the two projector operators P_{HI} and P_{non} at all those times. This can easily be seen to define a family of consistent histories, simply because the dynamics preserves the symmetries of homogeneity and isotropy.

Thus, one might consider the following question, what is the probability that (at a given time, characterized in the appropriate relational way), the universe is isotropic and homogeneous?. This can be evaluated using the formula (18) starting with the vacuum state.

It is easy to see that any history containing the orthogonal projector at any time P_{non}, will have zero probability, but the history containing only the operators P_{HI} will have probability one. This leads us to conclude that, at any time, the universe is homogeneous and isotropic. It thus can have no inhomogeneities or anisotropies at all.

Thus, in attempting to take this approach, we would have to face two problems. One, the conclusion that our universe is and has always been homogenous and isotropic, and, thus, we could not be here, but also the fact that the approach has led us to two contradicting conclusions. This later one, and the one obtained in, say, [94,95]. How could we chose trust one of them and not the other, despite the fact that both are obtained by the very same procedure?[12]

In the case of Generalized contexts formalism, we have indicated that this formalism can be considered as an alternative to the theory of consistent histories, having the advantage that some problematic histories of the consistent theory are eliminated. However, concerning the cosmological problem, the formalism of generalized contexts

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[12] According to [92] the posture is that one should believe both, and use the appropriate one in connection with the questions one is asking. This posture is not shared by other authors, for instance [87]. Moreover it seems the implicit views regarding the nature of science are very problematic in general (see for instance [93,96–99]).

has the same problems of the theory of consistent histories: it does not give any rule for selecting a privileged generalized context.

So far, we have argued that none of the widely used interpretations[13] of quantum theory can offer a satisfactory account of the question of the emergence of primordial inhomogeneities out of quantum uncertainties in the inflationary universe.

Let us now discuss briefly one of the paths that seems to offer a promising way to address the problem.

5 Possible Paths to Addressing the Problem

As we have seen, there seems to be no good option,[14] within the existing range of interpretational approaches to quantum theory, capable of addressing the issue at hand. Essentially, we need a scheme that manages to evade the following theorem: Given an initial state possessing a certain symmetry, and evolving autonomously and deterministically with a dynamics respecting said symmetry, one can not end up with a state that fails to have the symmetry in question. Here, the central issue is that, in the cosmological setting in question, there seems to be no way to call upon anything external to disrupt the autonomous evolution. The reader might be surprised to see the word deterministic in connection with quantum theory, but the fact is that Shrödingier's equation is fully deterministic, and the only place where determinism is lost in the context of quantum theory, is at the point where one addresses the connection with the measurements. In the cosmological setting at hand, as we have explained, we simply can not rely on concepts tied to measurements. Thus, we are driven to look for solutions in the class of theories that seek to modify quantum theory by assuming a generic departure from Shrdingier's deterministic evolution, (i.e. even in the absence of measurements). These are known generically as dynamical collapse theories, and have been proposed as means to address the general measurement problem in quantum theory. The most widely known examples are [100–111] and the best known advocate of such ideas has been R. Penrose, joined recently by Weinberg [91].

Thus, the path seems to require the extension of these dynamical collapse theories to the inflationary cosmology regime, an extension that requires both the application of the ideas to quantum field theory, rather than to non relativistic quantum mechanics, as well as the incorporation of gravitation into the picture. The approach followed in the first treatments of this problem has been rather simplistic: Introduce a one time spontaneous and random collapse per mode of the quantum field taking place during the inflationary regime [38,39]. The main idea has been to consider the predictions emerging from the proposals in order to find the particular assumptions needed so as to obtain a broad agreement with the observational data [40–45] . There are now

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[13] There exists many variants of the major themes we have considered here, and they have not been described in detail because the differences have no bearing on the issue at hand. Namely these variants fail to address the issue we face, for exactly the same reasons as the major ones they are **closely connected** with. However, we acknowledge that there might exist some other proposal we are unaware of, and which fare better in dealing with the problem we have been considering in this work.

[14] With the possible exception of the d' Broglie- Bohm approach, where the source of the primordial asymmetries is found in the initial conditions.

several works that have been based on a particular proposal for the class of dynamics controlling that hypothetical dynamical collapse. In particular, the proposal known as Continuous Spontaneous Localization [108–111] has been adapted, in various forms, to the problem at hand leading to conclusions that depend on the particular scheme of adaptation used [112–114].

Moreover we note, in relation to the Mini Mott example discussed in Sect. 3, that a common feature of these "dynamical collapse theories" is that they privilege position variables (or other closely connected objects tied to localization) over other variables. In fact, the basic objective of these theories could be roughly characterized as modifying quantum theory to prevent the existence (or the extended persistence) of quantum superpositions representing macroscopic objects localized at macroscopically different places. Thus, when applying those theories to the Mini Mott example, one would find that the first basis for the description of the states of the detectors is preferred over the second, symmetric basis, simply because the different relations of the two with the position variables. Thus those theories would lead to collapse in the first basis (or something very close to that) and not in the symmetric basis. Therefore, these theories could account for the breakdown in the symmetry. Such account could be then characterized heuristically by saying that the symmetry in question was incompatible with the localization which is a feature of the states that are broadly stable under collapse. The mathematics of the theories, of course, would reflect that heuristic characterization. The same can not be said of any of the interpretational approaches to quantum theory except for the de-Brogile Bohm proposal that shares with the collapse theories the privileged status given to the position variables.

It is also noteworthy the fact discussed in [115], that "dynamical collapse theories" seem to offer paths to resolve long standing issues afflicting proposals for quantum theories of gravitation, such as the "black hole information paradox" associated with their expected evaporation through Hawking radiation, and "the problem of time" in quantum gravity.

At this point, we should caution the reader that those approaches are still in the development stage. In particular, before any of those can be considered as a serious contender for fully resolving the problem, they would have to be made into a close and self consistent modification of quantum theory in general (i.e. they should cover, in a unified manner, the many particle systems addressed by the GRW or CSL proposals, and those requiring field theoretical and general relativistic treatment such as the cosmological problem we have been considering) , and in connection with its applicability to these filed theoretical and gravitational contexts, it would have to face the difficulties connected with issues of covariance, as well as conservation laws. It is nevertheless worth pointing out that there are promising developments on these topics, such as [116,117] and [118].

6 Discussion

We have reviewed the various issues related to the interpretation of quantum mechanics, and, in particular, the "measurement problem", using as a guide the process of generation of structure from quantum fluctuations in inflationary cosmology. The

discussion of the conceptual issues was facilitated by considering, within the same conditions associated with that cosmological problem, the paradigmatic problem where symmetry serves as a focal point: The quantum decay of a nucleus in a spherically symmetric initial state leading to the well known straight traces in a bubble chamber, a problem studied by Sir.N. F. Mott in the early days of quantum theory.

We have, in fact, focused our attention on a simplified version of Mott's problem, which allows not only an explicit writing of the complete system's (particle and detectors) hamiltonian and quantum mechanical states, but also an explicit solution of the Shrödinger equation. This has made it possible to investigate the problem in all detail. This has offered us a clear way to exhibit the strengths and weaknesses of each one of the proposed interpretations of quantum theory.

In Sect. 4, we have seen how each one of these interpretations fare in the face of the cosmological problem we had described:

a. The Classical Apparatuses Interpretation does not solve the cosmological problem because it needs the introduction of external (classical) detectors.
b. The Copenhagen Interpretation faces the same situation.
c. The Statistical Interpretation does not solve the cosmological problem because, in this context, there are no measuring apparatuses, and no measurements. These interpretations fail to deal with our problem, basically from the start simply because in the cosmological context we need to account for the emergence of the primordial inhomogeneities and anisotropies which are the seeds of all cosmic structure, including galaxies, stars, planets, life, humans (or other sapient beings), and instruments. Thus we have to do without instruments at the stage where we want to understand the emergence of those primordial features.
d. In Modal interpretations the symmetric initial state evolves with a symmetric Hamiltonian, and then the symmetry will be present in the state at all times. In addition:
    i. The Atomic Modal Interpretation does not offer a choice of the privileged base.
        (a) The Biorthogonal-decomposition modal interpretation requires for its application a privileged basis selected beforehand.
        (b) The Perspectival modal interpretation faces the same problems as those appearing when attempting to rely on discussion based on decoherence.
        (c) The Modal-Hamiltonian interpretation can not be applied to systems with a time-dependent Hamiltonian.
e. The de-Brogile Bohm Interpretation.Using it we can not really argue that it leads to a breakdown of the initial symmetry, either in the Mini-Mott problem, or in the cosmological context where one wants to explain the emergence to the seeds of cosmic structure, because, with n this approach the symmetry was never there to start with. That is, even though the wave function is symmetric, the initial values for the position variables are not symmetric. However, we must acknowledge that although one does not have the right to argue that this approach explains the emerge of asymmetry, it seems to be able to account for the seeds of structure in the universe (provided that one assumes that the preferential variable is something like the field amplitude and that the initial condition corresponds, in some particular sense, to something close to the equilibrium distribution [77,78]. With such additions, this

approach seems to be the only competition to the one described in the previous section.

f. In the case of Many Worlds Interpretation we saw that the schemes posseses no elements allowing one to select the basis, or context in which the ensemble must be described, or in which the splitting of the world takes place, and that depending on the arbitrary choice that one makes, one could be led to argue in favor or against the breakdown of the initial symmetry, and thus in favor or against of the emergence of structure in our universe.

g. Something similar happens with the Interpretations based on histories. One simply does not have anything like an unambiguous rule indicating which kind of realm to consider, and depending on the choice, one might end up assigning non-vanishing probabilities to non symmetric histories, or an exactly vanishing probability for all except the symmetric ones.

Thus we conclude that none of the existing interpretational frameworks for quantum theory offers a satisfactory account leading to the desired breaking of the initial symmetry in the problem at hand, and leading to what we think are the appropriate characterization of the late time situations where the symmetry is gone.

The analysis presented here indicates that something new is required, and we have briefly scketched what we feel is a promising path in the search for a clear characterization of that novel aspect of physics: dynamical collapse theories.

Appendix

In this appendix, we discuss some specific issues that arise in the attempt to use decoherence related arguments in the context of the problem at hand.

The first issue is that connected to the implication of symmetry regarding the choice of a preferential basis or so called pointer states.

The simplest example exhibiting this problem is provided by a standard EPR-R setup: Consider the decay of a spin 0 particle into two spin 1/2 particles. Take the direction of the decay as being the x axis (the particles momenta are $\vec{P} = \pm P\hat{x}$ with $\hat{x}$ the unit vector in the $\vec{x}$ direction)[15]. Now, we characterize the two particle states, that emerges after the decay in terms of the $\vec{z}$ polarization states of the two Hilbert spaces of individual particles. As it is known, the conservation of the angular momentum of the system indicates that the state must be:

$$|\chi\rangle = \frac{1}{\sqrt{2}}(|+\rangle_z^{(1)}|-\rangle_z^{(2)} + |+\rangle_z^{(2)}|-\rangle_z^{(1)}) \tag{19}$$

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[15] We are ignoring here the issue of how this decay became actualized into that particular direction, as the point here is to exemplify a specific technical issue.

The state is clearly invariant under rotations around the x axis (simply because it is an eigen-state with zero angular momentum along that axis). The density matrix for the system is thus $\rho = |\chi\rangle\langle\chi|$. Now assume we decide we are not interested in one of the particles (call it 1), and thus we regard it as an environment for the system of interest (particle 2). The reduced density matrix is then:

$$\rho^{(2)} = \rho_{Reduced} = Tr_{(1)}\rho = \frac{1}{2}(|+\rangle_z^{(2)}\langle+|_z^{(2)} + |-\rangle_z^{(2)}\langle-|_z^{(2)}) \qquad (20)$$

Now suppose we want to say that as the reduced density matrix is diagonal, we have found the pointer basis and that somehow the particle must be considered as having its spin along the z axis defined to be either $+1/2$ or $-1/2$.

The problem is that the symmetry of the state $|\chi\rangle$ regarding rotations around the x axis is reflected in the fact that we could have written this density matrix also as

$$\rho^{(2)} = \frac{1}{2}(|+\rangle_y^{(2)}\langle+|_y^{(2)} + |-\rangle_y^{(2)}\langle-|_y^{(2)}) \qquad (21)$$

leading, this time, to the conclusion that the particle must be considered as having its spin along the y axis defined to be either $+1/2$ or $-1/2$.

In fact, as the density matrix is proportional to the identity (i.e. $\rho^{(2)} = \frac{1}{2}I$) it would have the same form in any orthogonal basis.

One might be inclined to consider that this problem occurs only in very simple situations, such as the one of the above example, and that, in general, we will not encounter such difficulty. However that consideration is mistaken as can be seen from the general result encapsulated in the following:

Theorem

Consider a quantum system made of a subsystem S and an environment E, with corresponding Hilbert spaces H_S and H_E so that the complete system is described by states in the product Hilbert space $H_S \otimes H_E$. Let G be a symmetry group acting on the Hilbert space of the full system in a way that does not mix the system and environment. That is, the unitary representation O of G on $H_S \otimes H_E$ is such that $\forall g \in G, \hat{O}(g) = \hat{O}^S(g) \otimes \hat{O}^E(g)$, where $\hat{O}^S(g)$ and $\hat{O}^E(g)$ act on H_S and H_E respectively.

Let the system be characterized by a density matrix $\hat{\rho}$ which is invariant under G. Then the reduced density matrix of the subsystem is a multiple of the identity in each invariant subspace of H_S.

Proof

The reduced density matrix $\hat{\rho}_S = Tr_E(\hat{\rho})$. The trace over the environment of any operator $\hat{A}$ in $H_S \otimes H_E$ is obtained by taking any orthonormal basis $\{|e_j\rangle\}$ of H_E and evaluating $\Sigma_j \langle e_j|\hat{A}|e_j\rangle$.

Now, by assumption, we have $\hat{\rho} = \hat{O}(g)^{\dagger}\hat{\rho}\hat{O}(g)$, $\forall g \in G$. Then, for all $g \in G$, we have $\hat{\rho}_S = \Sigma_j \langle e_j|\hat{\rho}|e_j\rangle = \Sigma_j \langle e_j|\hat{O}^S(g)^{\dagger} \otimes \hat{O}^E(g)^{\dagger}\hat{\rho}\hat{O}^S(g) \otimes \hat{O}^E(g)|e_j\rangle = \Sigma_j \hat{O}^S(g)^{\dagger}\langle e'_j|\hat{\rho}|e'_j\rangle\hat{O}^S(g)$, where $|e'_j\rangle \equiv O^E(g)|e_j\rangle$. However, the fact that the operator $\hat{O}^E(g)$ is unitary implies that the transformed states $\{|e'_j\rangle\}$ form also an orthonormal basis of H_E.

Thus we have $\hat{\rho}_S = \hat{O}^S(g)^{\dagger}(\Sigma_j \langle e'_j|\hat{\rho}|e'_j\rangle)\hat{O}^S(g) = \hat{O}^S(g)^{\dagger}\hat{\rho}_S\hat{O}^S(g)$ or equivalently $\hat{\rho}_S\hat{O}^S(g) = \hat{O}^S(g)\hat{\rho}_S$. So we have found that $[\hat{\rho}_S, \hat{O}^S(g)] = 0$, $\forall g \in G$, and thus by Schur's lemma it follows that $\hat{\rho}_S$ must be a multiple of the identity in each invariant subspace of H_S, QED.

In particular, this result indicates that, if we start with a pure state invariant under the symmetry group, the reduced density matrix must be a multiple of the identity in each invariant subspace of H_S. This is exemplified by the well known case of a standard EPR setting, where a spinless particle decays into two photons, and where one considers the photons' spin degrees of freedom. The reduced density matrix describing one of the photons is a multiple of the identity, and thus the decoherence that results from tracing over the first photon's spin does not determine a preferential basis for the characterization of the spin of the second photon. Decoherence then fails under these conditions to provide a well defined preferential context for the interpretation of the reduced density matrix, as representing the various alternatives for the state of the subsystem after decoherence.

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