Found Phys (2014) 44:1246–1257
DOI 10.1007/s10701-014-9793-1

Quantum Mechanics: Ontology Without Individuals

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Received: 24 November 2013 / Accepted: 6 March 2014 / Published online: 22 March 2014

Abstract The purpose of the present paper is to consider the traditional interpretive problems of quantum mechanics from the viewpoint of a modal ontology of properties. In particular, we will try to delineate a quantum ontology that (i) is modal, because describes the structure of the realm of possibility, and (ii) lacks the ontological category of individual. The final goal is to supply an adequate account of quantum non-individuality on the basis of this ontology.

Keywords Modal ontology · Non-individuality · Indistinguishability

1 Introduction

Under the influence of logical positivism, the philosophy of science of the twentieth century did not bring ontological matters on the forefront, and—despite certain specific debates arisen in the early times of the theory—this situation was reflected in the research about the foundations of quantum mechanics. In fact, the study of the formal properties of the mathematical structure of quantum mechanics led to many results, unknown by the founding fathers of the theory, and this work greatly improved the understanding of the deep obstacles that any interpretation must face. But only in the last decades the interest in the ontological issues has begun to grow in the philosophy of quantum mechanics community. This work is part of this trend.

The purpose of the present paper is to consider the traditional interpretive problems of quantum mechanics from the viewpoint of a modal ontology of properties. This approach follows previous works focused on the Modal-Hamiltonian interpretation

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[1–4] However, here we will take a more general standpoint since not linked to a particular interpretation: the proposal will only require a modal reading of the quantum ontology. In particular, we will try to delineate a quantum ontology that (i) is modal, because describes the structure of the realm of possibility, and (ii) lacks the ontological category of individual. The final goal is to supply an adequate account of quantum non-individuality on the basis of this ontology. With this purpose, the paper is organized as follows. First, the bottom-up building of the ontology, as introduced in previous papers, will be briefly recalled. Then, we will address the problem of indistinguishability from that bottom-up view just presented. This work will allow us to propose a perspective reversion by facing the problem of indistinguishability from a top-down approach, according to which indistinguishability has to be conceived in terms of structural symmetry. On this basis, we will ask about the structure whose symmetry has to be considered in the case of indistinguishability. Finally, we will draw our conclusions and will briefly consider the implications of this approach for the question of non-locality.

2 The Bottom-Up Building of the Ontology

The modal interpretations of quantum mechanics can be viewed as a family that share certain common features but differ in the rule that selects the set of definite-valued observables [5,6]. In fact, all of them are realist, no-collapse interpretations, according to which—following van Fraassen's original ideas [7,8][1]—the quantum state describes possibility rather than actuality. The feature that distinguishes the different modal interpretations from each other is the rule of definite-value ascription, which selects the observables that acquire a definite value without violating the restrictions imposed by the Kochen–Specker theorem [9].

In our previous papers, the ontological problems were approached from the viewpoint of the Modal-Hamiltonian interpretation [1,4], which endows the Hamiltonian with a central role, both in the definition of systems and subsystems and in the identification of the definite-valued observables. However, the present work will not be confined to this particular interpretation, since here we are not interested in the specific rule of definite-value ascription adopted. The only requirement of the discussion will be a modal conception of the ontology, which will allow us to explore the general structure of the domain of possibility and its relation with the domain of actuality.

The departing point of the proposal is to move away from the traditional substances-and-properties view of the ontology, in favor of an ontology of properties, represented in the physical language by the observables of the theory [1,2]. In order to distinguish between the ontological language and the physical language, we will use the symbol $[X]$ to denote the ontological item corresponding to the physical item denoted by X. From this perspective, the elements of the ontology are:

- *Universal type-properties*, symbolized as $[A]$, with their instances $[A^i]$ represented by observables A^i (mathematically, self-adjoint operators of a space of operators).

[1] Although van Fraassen does not endorse scientific realism (from his "constructive empiricism", the aim of science is only empirical adequacy), he admits that a meaningful account of reality is necessary for a scientific theory to be intelligible.

An example of universal type-property is the energy $[H]$, which can be instantiated as the energy $[H^1]$ of this particular system, for instance, an electron, which, in turn, is represented by the Hamiltonian H^1.

- *Possible case-properties* $[a^i_j]$ of the instance $[A^i]$, represented by the possible values a^i_j of the observable A^i (mathematically, eigenvalues of the self-adjoint operator). Following with the previous example, we can talk about the possible case-properties $[\omega^1_j]$ of the energy $[H^1]$ of this particular electron, which are represented by the possible values ω^1_j of the Hamiltonian H^1.
- *Ontological propensities* to actualization for all the possible case-properties, represented by the state ρ (mathematically, a functional on the space of operators).[2] The adoption of a propensity interpretation of quantum probabilities does not imply that we accept Popper's position as a whole. For Popper, propensities are not monadic properties of isolated quantum systems, but relational properties of quantum entities and experimental set-ups. For us, on the contrary, although propensities can only be revealed through measurements, they are independent of such interactions (see [1], Subsection 8.2). As Suárez points out [10], an electron in a one-electron universe may be in a certain quantum state, and thus possesses all the propensities described by that state (for an updated discussion about propensities in physics, see [11]).

On the basis of these elemental items, the rest of the ontological structure can be introduced (see [3]):

- A *bundle* is a collection of instances of universal type-properties, whose physical correlate is the quantum system.[3] According to the Kochen–Specker theorem, it is impossible to simultaneously assign definite values to all the observables of the system. In ontological terms, it is impossible to simultaneously assign definite case-properties to all the type-properties of the system. Therefore, the ontological interpretation of the concept of quantum system proposed here is immune to the challenge represented by this theorem, since it imposes no restriction on type-properties but only on case-properties.
- An *atomic bundle* is a bundle that cannot be decomposed yet in smaller bundles. Its physical correlate is the elementary particle, which is mathematically defined by an irreducible representation of the Galilean group.
- Since, according to the Kochen–Specker theorem, not all the instances may acquire an actual case-property simultaneously, it is necessary to select a "*preferred context*" that contains the instances of the universal type-properties that become actual. For each one of these instances, only one of its possible case-properties becomes actual.

[2] According to the algebraic formalism of quantum mechanics, given a *-algebra $\mathcal{A}$ of operators, (i) the set of the self-adjoint elements of $\mathcal{A}$ is the space $\mathcal{O}$, whose elements represent observables, $O \in \mathcal{O}$, and (ii) states are represented by functionals on $\mathcal{O}$, that is, by elements of the dual space $\mathcal{O}'$, $\rho \in \mathcal{O}'$. In the case of a C*-algebra of operators, it can be represented by a Hilbert space $\mathcal{H}$ (GNS theorem) and $\mathcal{O} = \mathcal{O}'$; therefore, both $\mathcal{O}$ and $\mathcal{O}'$ are represented by $\mathcal{H} \otimes \mathcal{H}$.

[3] Mathematically, a quantum system $\mathcal{S}^1$ is represented by the set of operators $\mathcal{O}^1$, or by the Hilbert space $\mathcal{H}^1$ if $\mathcal{O}^1 = \mathcal{H}^1 \otimes \mathcal{H}^1$.

This formulation supplies a picture of the quantum ontology such that every physical concept, with its mathematical representation, has its ontological correlate. The only element that has to be added as an interpretive complement is the selection of the preferred context, since the theory gives no account of actualization. It is precisely on this point that the different modal interpretations disagree with each other, but this is the point that will not be relevant in our discussion. Therefore, in this framework we have all the ingredients needed to treat the problem of indistinguishability.

3 Indistinguishability from the Bottom-Up Approach

Whereas in the practice of physics indistinguishability takes the form of a restriction on non-symmetric states, during the last decades the problem has been approached from an ontological viewpoint by stressing the difficulties that the category of individual has to face due to indistinguishability: individuality does not fit comfortably into the structure of quantum mechanics [12–14]. In the present context, and following the bottom-up approach just introduced, the indistinguishability between elementary particles begins to be built from the definition of indistinguishability for property items. In particular, two instances $[A^1]$ and $[A^2]$ of a single universal type-property $[A]$ are *indistinguishable* if their respective case-properties $[a_j^1]$ and $[a_j^2]$ are equal, that is, they are mathematically represented by the same number: $a_j^1 = a_j^2$. This means that the indistinguishable instances are only numerically different. Nevertheless, this does not imply a violation of *Leibniz's Principle of Identity of Indiscernibles* (see [14]), since this principle applies to individuals, whereas here we are considering items belonging to the ontological category of property.

Once the indistinguishability of instances is defined, the indistinguishability of atomic bundles is introduced on this basis: *two atomic bundles are indistinguishable* when the respective instances of universal type-properties belonging to them are indistinguishable. Although this characterization seems to agree with the typical use of the terms in the physical discourse when talking about "indistinguishability among elementary particles" , it is necessary to recall that, from this ontological view, bundles are not individual particles; as a consequence, indistinguishability is not a relationship between individuals.

It is worth emphasizing that, although indistinguishability between instances of a single universal type-property may also be found in the classical realm, the difference with the classical case arises when those instances come together to constitute a bundle, which becomes an individual in the classical but not in the quantum case. In fact, as French and Krause [14] stress, the category of individual requires some "principle of individuality" that makes an individual to be that individual and not another. In the traditional bundle theory, individuals are constructed from properties; the principle of individuality is a subset of the properties which, together with some further principle (e.g. impenetrability), ensures that no other individual possesses that subset (see, e.g., van Cleve [15], Loux [16]). But, according to our view, quantum systems are non-individual bundles, because there is no principle of individuality that preserves their identity in different circumstances, for instance, when they combine in a composite entity. This non-individuality of quantum systems is manifested in quantum statistics (Fermi–Dirac and Bose–Einstein), whereas classical systems are individuals that obey the Maxwell–Boltzmann statistics, which takes into account the permutations of individual entities.

If two atomic bundles are indistinguishable, when they aggregate in a composite bundle, it can be expected that the instances belonging to the composite do not distinguish between the component bundles. This ontological feature can be formally expressed as commutativity: the aggregate of indistinguishable atomic bundles is a *commutative* operation, which expresses the ontological fact that, when two indistinguishable bundles combine in an aggregate, no matter which one is picked up first. Moreover, since atomic bundles are not individuals, they do not retain their individuality in the aggregate. In other words, the original non-individual components cannot be re-identified in the non-individual composite. The commutativity of the aggregation of atomic bundles has a direct consequence upon the form of the observables that represent the instances belonging to the new bundle: the instances are represented by *observables symmetric with respect to the permutation* between the atomic bundles.

Up to this point we see that, by beginning with the indistinguishability of the instances of universal type-properties belonging to atomic bundles, we arrive to a structural property of the instances forming the aggregate of those atomic bundles. Or, in the usual language used by physicists, we arrive to a property of the operators representing the observables of the system consisting of indistinguishable particles. The symmetry under permutations of those observables is precisely what makes the states acting on them to behave as *symmetric states*: since any operator can be decomposed into a symmetric part and an anti-symmetric part, the anti-symmetric part of the state-operator has no effect in its application onto symmetric observable-operators (see formal details in [4]). In brief, the states of aggregates of indistinguishable atomic bundles behave as if they were represented by symmetric operators.

In turn, since a symmetric state-operator may be expressed in terms of a symmetric or of an anti-symmetric state-vector, the usual treatment of indistinguishability is obtained as a corollary of the bottom-up strategy. In other words, the symmetry or anti-symmetry of the vectors representing physical pure states of aggregates of "elementary particles" are not the result of an ad hoc symmetrization or anti-symmetrization, but are due to ontological reasons: those symmetry properties of the states are a consequence of the symmetry of the observables of the aggregate, and this symmetry is, in turn, a consequence of the ontological picture supplied by the modal reading of quantum mechanics.

The entire argumentation that led us from the indistinguishability of property items to the symmetry properties of the states was based on the ontological claim that bundles are not individuals and, therefore, the aggregate of bundles is a new bundle where the identity of the components is not retained. This idea suggests the possibility of reversing the direction of the argument by adopting a top-down approach, based on studying the features of the composite bundle in order to understand indistinguishability: this is the strategy to be followed in the rest of the present paper.

4 A Top-Down Approach to Indistinguishability

One of the main questions in the philosophy of mathematics is that referred to the nature of mathematical entities. According to Platonism, mathematical entities are objects of a non-empirical but real domain, whose properties must be discovered; conventionalists, on the contrary, conceive mathematics as the activity of developing formal systems and, as a consequence, the question about the nature of mathematical entities makes sense only in this framework. Under the influence of Dedekind's idea that mathematics characterizes its objects up to isomorphism, mathematical structuralism presents itself as an alternative to both positions [17]. According to this view, structures are the primary items of mathematics. In particular, in the *ante rem* version of Shapiro [18], although structures have ontological priority over the places in the structures, those places are treated as genuine, bona fide objects, which constitute the referents of certain terms of the language of the considered theory.

In spite of its appeal, Shapiro's proposal has been challenged by arguing that places in structures may violate Leibniz's Principle of Identity of Indiscernibles. In fact, it can be shown [19] that, in the case of structures having non-trivial automorphisms, some places cannot be distinguished from each other. In a recent paper [20] it has been claimed that this problem in the philosophy of mathematics is analogue to the problem of indistinguishability in quantum mechanics: although the difficulties are apparently unrelated since coming from different disciplines, they are manifestations of a single ontological problem. On this basis, it has been suggested that the solution proposed in one of the domains may be extrapolated and adapted to the other.

The idea of linking indistinguishability with structure symmetry is not new. Already in 1945 José Sebastião e Silva published an important note on the subject (see [21,22]). In 2005, Décio Krause and Antonio Coelho characterize indistinguishability in terms of invariance under automorphisms [23]. The authors begin by defining a *structure* as:

$$\mathcal{A} = \langle D, \{R_i\}_{i\in I}\rangle$$

where D is the domain of the structure, and the R_i are relations defined on the elements of D. In this structure, a *property* is a sub-collection $X \subseteq D$ such that it is invariant under all the automorphisms of $\mathcal{A}$: for all the automorphisms g of $\mathcal{A}$, $g(X) = X$. On this basis, the concept of distinguishability in a structure can be defined as follows:

> For any two elements $a, b \in D$, a and b are $\mathcal{A}$ *-distinguishable* (distinguishable in the structure $\mathcal{A}$) if there exists a property X such that $a \in X$ if and only if $b \notin X$; otherwise, they are $\mathcal{A}$*-indistinguishable*.

The rough idea behind this definition is that two elements of a structure are indistinguishable when they share all the properties definable in that structure. On this basis, it is said that a structure is *rigid* iff its only automorphism is the identity. It can be proved that, in a rigid structure, indistinguishability and identity coincide: a and b are $\mathcal{A}$-indistinguishable if and only if $a = b$ (see [23]).

In this theoretical framework, a new concept is relevant to the discussion about indistinguishability. Given a structure $\mathcal{A} = \langle D, \{R_i\}_{i\in I}\rangle$, another structure $\mathcal{B}$ is an *expansion* of $\mathcal{A}$ if and only if $\mathcal{B} = \langle D, \{R_i\}_{i\in I\cup J}\rangle$, where $I \cap J = \varnothing$. Roughly speaking, $\mathcal{B}$ is an expansion of $\mathcal{A}$ when $\mathcal{B}$ is obtained by adding new relations to $\mathcal{A}$. Moreover, when $\mathcal{B}$ is rigid, it is called a *rigid expansion* of $\mathcal{A}$. Nevertheless, two cases need to be distinguished:

- $\mathcal{B}$ is a *trivial rigid expansion* of $\mathcal{A}$ if and only if $\mathcal{B}$ is a rigid expansion of $\mathcal{A}$ and $\mathcal{B}' = \langle D, \{R_i\}_{i \in J} \rangle$ is rigid. This means that the new relations added to $\mathcal{A}$ are what make $\mathcal{B}$ rigid, independently of the original relations of $\mathcal{A}$.
- $\mathcal{B}$ is a *non-trivial rigid expansion* of $\mathcal{A}$ if and only if $\mathcal{B}$ is a rigid expansion of $\mathcal{A}$ and $\mathcal{B}' = \langle D, \{R_i\}_{i \in J} \rangle$ is not rigid. In this case, the new relations added to $\mathcal{A}$ are not, alone, sufficient to make $\mathcal{B}$ rigid: the original relations of $\mathcal{A}$ in combination with the new relations added in the expansion are responsible for the rigidity of $\mathcal{B}$.

On this basis, it is easy to see that *every structure has a trivial rigid expansion*. The trivial way to obtain it is by adding to the original structure all the singletons of the elements of its domain.

This formal framework allows us to approach the problem of indistinguishability from a different perspective. Although in the quantum context indistinguishability and identity do not coincide, one can still ask if indistinguishability can be avoided, that is, if there is some way to distinguish elementary particles without violating the principles of the theory. Of course, there is always a trivial rigid expansion that distinguishes particle a from other particles by means of a property like "being identical with a". The relevant question is whether quantum mechanics admits a *non-trivial rigid expansion* that makes possible to distinguish elementary particles. As Krause and Coelho point out, the hypothesis that elementary particles of the same kind are indistinguishable in an ontological sense "*amounts to sustain that a mathematical structure of quantum theory cannot have a non-trivial rigid expansion, which intuitively means that the rigidity of a structure for quantum theory can be achieved only by considering new relations which, by themselves, regardless the quantum nature of the elements of the domain, guarantee such a rigidity*" [23, p. 206].

When discussing weak discernibility—a discernibility that only depends on irreflexive qualitative relations (see [24–26])—in situations with perfect symmetry, Dieks and Versteegh [27] stress the difference between the classical case and the quantum case. In the classical domain, the symmetry of a given situation, such as that of Black's spheres, can be broken by introducing a point of reference or a gauge that distinguishes between the entities involved in the situation: it can be said that the two Black's spheres are *weakly distinguishable*. Although in a different context, two entities related by a symmetry transformation that leaves invariant the whole situation, as the two semi-cones of a light cone or the two spin directions, have been called *formally identical*; when we assign different names to formally identical entities, we introduce a *conventional difference* between them [28,29]. In the formal framework supplied by Krause and Coelho, the introduction of a conventional difference between formally identical entities amounts to introducing a trivial rigid expansion of the original structure. Nevertheless, this does not mean that the formally identical entities collapse into a single entity.

However, as Dieks and Versteegh point out, the quantum case is different because symmetrization is a postulate: symmetry cannot be broken without violating one of the principles of the theory, a violation that would lead to a wrong statistics. In a similar vein, Dieks and Andrea Lubberdink also stress that "[i]*n classical physics perfect symmetry of particle configurations, if it occurs at all, is something contingent; but in quantum mechanics it is a law-like feature that all indices must always occur, in* *any expression and in whatever situation, in a fully symmetrical way.* […] *This is very much different from the case of Black's spheres. In quantum mechanics it is a matter of principle that we can never associate different physical characteristics with different indices in the formalism*" [30, p. 1063]. Furthermore, perfect symmetry in a classical domain is not only contingent, but also extremely improbable: perfectly symmetric configurations have measure zero in the space of all possible configurations (for a proof in a particular case, see [28]). This stands in clear contrast to the quantum case, where symmetry is necessary since imposed by one of the principles of the theory.

But the constraints introduced by indistinguishability are even more serious. In fact, those who appeal to weak discernibility to retain Leibniz's principle in the quantum context need to label entities in order to formulate the argument, in contrast to the fact that quantum elementary particles cannot be labelled. In Steven French and Krause terms: "*Doesn't the appeal to irreflexive relations in order to ground the individuality of the objects which bear such relations involve a circularity? In other words, the worry is that in order to appeal to such relations, one has already had to individuate the particles which are so related and the numerical diversity of the particles has been presupposed by the relation which hence cannot account for it*" [14, pp. 170–171]. In other words, in quantum mechanics we cannot even introduce a conventional difference between indistinguishable particles by assigning them different names. This seems to mean that quantum mechanics does not admit a trivial rigid expansion. But, how to compatibilize this statement with the undoubtedly true claim that every structure has a trivial rigid expansion? In the next section we will see that the structure in which symmetry has to be found is not that defined on a domain of quantum systems, since they are not individuals that can be labeled. On the basis of the modal ontology of properties introduced in the previous sections, we will argue that the structure relevant to treat indistinguishability as symmetry is that defined by the space of operators representing the observables of the composite quantum system.

5 Searching for the Relevant Structure

The concept of structure as characterized by Krause and Coelho includes a domain defined as a set of the standard set theory. But in this theory the elements of a set are individual entities that can be distinguished at least by assigning them different names. This situation opens two alternatives. Some authors consider that the standard set theory is not adequate to deal with the indistinguishable particles of quantum mechanics. The semiextensional quasisets theory, developed by Newton da Costa and Krause [31–33], see also [34,35], and the intensional quasets theory, developed by Maria Luisa dalla Chiara and Giuliano Toraldo di Francia [36,37], describe collections of objects having cardinality but not order type, that is, objects to which the concept of individual of classical logic does not apply. The other alternative is to deny that the elements of the domain are elementary individual particles; we will explore this second option.

The general strategy consists in retaining the idea of linking indistinguishability with structural symmetry. But, if we cannot label identical particles, in what structure should we study symmetry properties? The answer can be found in the modal ontology of properties described above. In this ontology, quantum systems are bundles of instances of universal type-properties, and bundles are not individuals. Therefore, when component bundles combine, the new bundle is a non-individual whole without individual parts. This means that *the structure whose symmetry-properties have to be studied is the composite bundle*. In particular, the instances belonging to the new bundle are represented by observables symmetric with respect to the permutation between the component atomic bundles. Formally, given two indistinguishable atomic bundles $h^1 = \{A_i^1\}$ and $h^2 = \{A_i^2\}$, physically represented by the sets of observables $\mathcal{O}^1$ and $\mathcal{O}^2$ respectively, and mathematically represented by the sets of self-adjoint operators $\mathcal{O}^1$ and $\mathcal{O}^2$ respectively, the structure whose symmetry properties have to be studied is the composite bundle h^c, physically represented by the set of observables $\mathcal{O}^c$, which is mathematically represented by the set of the self-adjoint operators $\mathcal{O}^c \subseteq \mathcal{O}^1 \otimes \mathcal{O}^2$.[4] Since the operators C^c belonging to $\mathcal{O}^c$ are of the form:

$$C^c = \sum_{ij} k_{ij} \left(A_i^1 \otimes A_j^2 \right) \in \mathcal{O}^c \subseteq \mathcal{O}^1 \otimes \mathcal{O}^2$$

where the A_i^1 and the A_j^2 are the operators representing the observables corresponding to the instances belonging to h^1 and h^2 respectively, indistinguishability means that the operators C^c are such that

$$A_i^1 \otimes A_j^2 = A_i^2 \otimes A_j^1$$

In other words, the C^c are *symmetric* with respect to the permutation between the indices 1 and 2.

From this viewpoint, indistinguishability has nothing to do with Leibniz's Principle, since the principle applies to individual entities whereas here there are no individuals involved. Indistinguishability means that the composite system (i) is *a whole* where the components cannot be re-identified, and (ii) has an *internal symmetry* manifested by the invariance under permutation of indices. The first point agrees with the idea expressed by Dieks and Versteegh: "*one possible stance in such discussions is to argue that there is no multiplicity at all: that there is only one undivided physical system*" [27, p. 926]. The second point, in turn, implies denying the usual conception of indices as denoting quantum particles; as Dieks and Lubberdink claim,"*indices in the quantum mechanical formalism of "identical particles" refer to the individual factor spaces from which the total Hilbert space in the formalism is constructed—they are merely mathematical quantities*" [30, p. 1663]. According to the authors, the individual particles encountered in experiments pertain to classical limit situations, and do not correspond to the indices in the quantum formalism.

The idea of a realm without individuals has been proposed by other authors for different reasons. A very active case in present-day literature is structural realism, which arose in its epistemological version in the context of the problem of theory

4 In the case of working with C*-algebras of operators, $\mathcal{O}^1$ and $\mathcal{O}^2$ are represented by $\mathcal{H}^1 \otimes \mathcal{H}^1$ and $\mathcal{H}^2 \otimes \mathcal{H}^2$ respectively. Therefore, $\mathcal{O}^c = \mathcal{O}^1 \otimes \mathcal{O}^2 = \mathcal{H}^1 \otimes \mathcal{H}^1 \otimes \mathcal{H}^2 \otimes \mathcal{H}^2$.

change [38]. This view turned into an ontic version in the hands of Ladyman [39], who was moved by the ontological problems derived from quantum mechanics to an eliminativist stance about individual objects. Ontic structural realism proposes, then, a reconceptualization of ontology at the most basic metaphysical level, which effects a shift from objects to structures [40]. These realists follow Cassirer in the claim that elemental particles are not individuals but "points of intersection" of certain relations: physical objects are "*reduced to mere 'nodes' of the structure, or 'intersections' of the relevant relations*" [41, p. 173]. They also stress the formal limitations to express their position, by pointing out "*the descriptive inadequacies of modern logic and set theory which retains the classical framework of individual objects represented by variables and which are the subject of predication or membership respectively*" [40, p. 41]. Therefore, our proposal can be easily compatibilized with this view: "*structuralism is certainly closely related to various forms of the bundle theory*" [42, p. 34]. Ontic structural realism has been widely articulated from a conceptual viewpoint in response to the debates generated by its theses in the philosophy of science community [39–43]. Nevertheless, although mainly motivated by the quantum challenges, this ontological perspective has not been applied to quantum mechanics with the detail sufficient to characterize the particular structure of the ontology in terms of the elements of the theory. Our proposal tries to complete the picture by identifying the items belonging to the category of property that constitute the structure of the quantum realm.

Summing up, the modal ontology of properties proposed here allows us to conclude that indistinguishability is not a relation between individual particles belonging to a certain domain, but a symmetry internal to a non-individual and indivisible whole. When the problem is formulated in these terms, the replacement of the standard set theory is not necessary but, at the same time, many of the ontological perplexities derived from of considering individuals without individuality suddenly vanish.

6 Conclusions

Traditionally, quantum indistinguishability is interpreted as a relation between quantum elementary particles conceived as individuals. In this context, discussions usually focus on the possibility of distinguishing those particles by finding some property or relation that applies to one of them but not to the others (see, for instance, claims of weak distinguishability in [24–26]). The present paper moves away from that traditional view from the very beginning. By taking a modal ontology of properties as the conceptual starting point, our proposal puts forward a solution to the problem of indistinguishability from two perspectives, which can be seen as two faces of the same coin:

– From a bottom-up approach, the indistinguishability of atomic bundles is the result of the indistinguishability of the instances of the universal type-properties that constitute them.
– From a top-down approach, the solution is based on two factors: holism and symmetry. Indistinguishability is not a relation between individuals but an internal symmetry of a holistic non-individual entity.

This view immediately suggests the connection between quantum indistinguishability and quantum non-separability. In fact, if one of the ingredients in the explanation of indistinguishability is holism, the holistic interpretation of non-separability seems to be a necessary consequence. This means that the modal ontology of properties might supply a framework adequate also to give a response to the problem of non-separability that does not rely on non-locality. Additionally, this response could help to re-conceptualize certain issues studied in the context of quantum information, as the phenomenon of teletransportation. But this task is beyond the limits of the present paper and will be the subject matter of a future work. Moreover, some questions concerning invariance and automorphisms, treated by da Costa and Alexandre Rodrigues [22], will also be developed in future works.

Acknowledgments The authors want to thank Décio Krause for his organization of the Workshop "*Logical Quantum Structures*" in the context of the *4th World Congress and School on Universal Logic*, and also the attendants to the workshop for their constructive comments to the oral version of this work. This paper was partially supported by grants of the Buenos Aires University (UBA), the National Research Council (CONICET) and the National Research Agency (FONCYT) of Argentina.

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