Physics of Dark Energy Particles

C.G. Böhmer · T. Harko

Abstract We consider the astrophysical and cosmological implications of the existence of a minimum density and mass due to the presence of the cosmological constant. If there is a minimum length in nature, then there is an absolute minimum mass corresponding to a hypothetical particle with radius of the order of the Planck length. On the other hand, quantum mechanical considerations suggest a different minimum mass. These particles associated with the dark energy can be interpreted as the "quanta" of the cosmological constant. We study the possibility that these particles can form stable stellar-type configurations through gravitational condensation, and their Jeans and Chandrasekhar masses are estimated. From the requirement of the energetic stability of the minimum density configuration on a macroscopic scale one obtains a mass of the order of 10^{55} g, of the same order of magnitude as the mass of the universe. This mass can also be interpreted as the Jeans mass of the dark energy fluid. Furthermore we present a representation of the cosmological constant and of the total mass of the universe in terms of 'classical' fundamental constants.

Keywords Gravitation · Dark energy · Minimum mass · Dark energy particles

1 Introduction

Several recent astrophysical observations of distant type Ia supernovae [1–4] have
provided the astonishing result that around 95–96% of the content of the universe

C.G. Böhmer
Department of Mathematics, University College London, Gower Street, London WC1E 6BT, UK
e-mail: c.boehmer@ucl.ac.uk

T. Harko (✉)
Department of Physics and Center for Theoretical and Computational Physics,
The University of Hong Kong, Pok Fu Lam Road, Hong Kong, Hong Kong
e-mail: harko@hkucc.hku.hk

is in the form of dark matter + energy, with only about 4–5% being represented by baryonic matter. More intriguing, around 70% of the total energy-density may be in the form of what is called the dark energy, with the associated density parameter Ω_{DE} of the order of $\Omega_{DE} \sim 0.70$. The dark energy is responsible for the recent acceleration of the Universe. The best candidate for the dark energy is the cosmological constant Λ, which is usually interpreted physically as a vacuum energy, with energy density ρ_Λ and pressure p_Λ satisfying the unusual equation of state $\rho_\Lambda = -p_\Lambda/c^2 = \Lambda/8\pi G/c^2$. Its size is of the order $\Lambda \approx 3 \times 10^{-56}$ cm^{-2} [5, 6].

The existence of the cosmological constant modifies the allowed ranges for various physical parameters, like, for example, the maximum mass of compact stellar objects [7–10], thus leading to a modifications of the "classical" Buchdahl limit [11]. In conjunction with other parameters, like the Schwarzschild radius, the cosmological constant Λ leads to a set of scales relevant not only for cosmological, but also for astrophysical applications. Hence, for example, there exists a lower and an upper cut-off on the possible velocities of test particles traveling over distances of the order of Mpc [12].

Since about 70% of the Universe consists of dark energy, which almost entirely determines its structure and dynamics, it is natural to consider Λ as a fundamental constant and to explore the possibilities which follows from enlarging the set of fundamental constants of nature, which can be considered as being the speed of light c, the gravitational constant G, Planck's constant $\hbar$ and the cosmological constant Λ, respectively [13]. On the other hand, we cannot exclude *a priori* the possibility that the cosmological constant, which may also be interpreted as a manifestation of the vacuum energy, can also play an important role not only at galactic or cosmological scales, but also at the level of elementary particles. Therefore the presence of the cosmological constant may require a drastic modification of the basic laws of physics.

In the presence of a cosmological constant, ordinary Poincaré special relativity is no longer valid, and must be replaced by a de Sitter special relativity, in which Minkowski space is replaced by a de Sitter spacetime [14]. Consequently, the ordinary notions of energy and momentum change, and will satisfy a different kinematic relation. Since the only difference between the Poincaré and the de Sitter groups is the replacement of translations by certain linear combinations of translations and proper conformal transformations, the net result of this change is ultimately the breakdown of ordinary translational invariance [15–17]. From the experimental point of view, therefore, a de Sitter special relativity might be probed by looking for possible violations of translational invariance. If we assume the existence of a connection between the energy scale of an experiment and the local value of the cosmological constant, there would be changes in the kinematics of massive particles which could hopefully be detected in high-energy experiments. Furthermore, due to the presence of a horizon, the usual causal structure of spacetime would be significantly modified at the Planck scale.

By using dimensional analysis, Wesson [13] has found two different masses, which can be constructed from the set of constants $(c, G, \hbar, \Lambda)$. The mass m_P relevant at the quantum scale is

$$m_P = \left(\frac{\hbar}{c}\right)\sqrt{\frac{\Lambda}{3}} \approx 3.5 \times 10^{-66} \text{ g}, \tag{1}$$

while the mass m_{PE} relevant to the cosmological scale is

$$m_{PE} = \left(\frac{c^2}{G}\right)\sqrt{\frac{3}{\Lambda}} \approx 1 \times 10^{56} \text{ g}. \tag{2}$$

The interpretation of the mass m_{PE} is straightforward: it is the mass of the observable part of the universe, equivalent to 10^{80} baryons of mass 10^{-24} g each. The interpretation of the mass m_P is more difficult. By using the dimensional reduction from higher dimensional relativity and by assuming that the Compton wavelength of a particle cannot take any value, Wesson [13] proposed that the mass is quantized according to the rule $m = (n\hbar/c)\sqrt{\Lambda/3}$. Hence m_P is the minimum mass corresponding to the ground state $n = 1$.

With the use of the generalized Buchdahl identity [7], it can be rigorously proven that the existence of a non-negative Λ imposes a lower bound on the mass M and density ρ for general relativistic objects with radius R, which is given by [18]

$$2GM \geq \frac{\Lambda c^2}{6} R^3, \qquad \rho = \frac{3M}{4\pi R^3} \geq \frac{\Lambda c^2}{16\pi G} =: \rho_{\min}. \tag{3}$$

Therefore, the existence of the cosmological constant implies the existence of an absolute minimum density in the universe. No object present in relativity can have a density that is smaller than $\rho_{\min}$. For $\Lambda > 0$, this result also implies a minimum density for stable fluctuations in energy density. These results have been generalized to compact anisotropic general relativistic objects in [19], where it was shown that in the presence of the cosmological constant, a minimum mass configuration with given anisotropy does exist. For charged general relativistic objects there is also a lower bound for the mass-radius ratio [20]. By considering the total energy (including the gravitational one) and the stability of the objects with minimum mass-radius ratio, a representation of the mass and radius of the charged objects with minimum mass-radius ratio in terms of the charge and vacuum energy only has been obtained.

It is the purpose of the present paper to further explore the possible physical implications of the existence of a minimum mass in the universe, given by (3), and which is a direct consequence of the existence of a non-zero cosmological constant. In particular, we show that if a minimum length does exist in nature, then the condition (3) does imply the existence of an absolute minimum mass. By combining the rigorous result for the minimum mass with the dimensional analysis of Wesson [13], we can obtain an intriguing representation of the vacuum energy as a function of the fundamental constants c, G, $\hbar$ as well as the mass m_e and the radius r_e of the electron. On the other hand, by considering the possibility of the gravitational condensation of the dark energy fluid we obtain the interpretation of the mass m_{PE} as the Jeans mass of the gravitational dark energy condensate. By minimizing the total (matter plus gravitational) energy of a stable configuration consisting of particles with the minimum mass we provide a rigorous derivation of the cosmological mass m_{PE}, given by (2).

The present paper is organized as follows. The physical implications of the existence of a minimum mass are presented in Sect. 2. The gravitational condensation of the dark energy particles is considered in Sect. 3. The total energy (including the gravitational one) for stable configurations of particles with minimum mass is obtained in Sect. 4. We briefly conclude and discuss our results in the last section.

2 Minimum Mass and Radius of Dark Energy Particles

At a microscopic level two basic quantities, the Planck mass m_{Pl} and the Planck length l_{Pl} are supposed to play a fundamental physical role. The Planck mass is derived by equating the gravitational radius $2Gm/c^2$ of a Schwarzschild mass with its Compton wavelength $\hbar/mc$. The corresponding mass $m_{Pl} = (c\hbar/2G)^{1/2}$ is of the order $m_{Pl} \approx 1.5 \times 10^{-5}$ g. The Planck length is given by $l_{Pl} = (\hbar G/c^3)^{1/2} \approx 1.6 \times 10^{-33}$ cm and at about this scale quantum gravity will become important for understanding physics. The Planck mass and length are the only parameters with dimension mass and length, respectively, which can be obtained from the fundamental constants c, G and $\hbar$.

The problem of the physical nature of the cosmological constant/dark energy is one of the most important issues confronting modern physics. A popular interpretation of the cosmological constant is in terms of the vacuum energy $\langle \rho_{vac} \rangle$, which is of the order $\langle \rho_{vac} \rangle \approx 2 \times 10^{71}$ GeV^4. However, astronomical observations indicate that the cosmological constant is many orders of magnitude (around 10^{120}) smaller than the estimate for vacuum energy. Many different approaches to the solution of this problem have been proposed, like the interpretation of the cosmological constant as an integration constant, anthropic considerations, quantum cosmology etc. [21]. Presently, astronomical observations suggests that dark energy could be dynamical and evolving, with the dark-energy density approaching its natural value, zero. The smallness of the dark energy is a result of the expansion of the universe and its old age [5].

Due to the fact that in a curved space-time the vacuum is not unique, the phenomenon of particle production occurs in an expanding universe as a typical quantum effect [22]. As the universe evolves, and the curvature changes, the vacuum state also changes, and the initial zero particle vacuum state becomes later a multiparticle state. If the universe is decelerating and is asymptotically Minkowskian at infinity, in the large time limit the particle production does not occur any more. However, observations indicate that we are living in an accelerating universe, and the mechanism of particle production can be very important. The rate of the particle production for a flat universe filled with a fluid with equation of state $p = \alpha \rho$, where α is arbitrary, has been obtained recently in [23]. The calculations were performed for the case of a massless scalar field, for which the corresponding Klein-Gordon equation was solved. The rate of particle production is determined exactly for any value of α, including $\alpha = -1$. When the strong energy condition is satisfied, the rate of particle production decreases as time goes on, in agreement to the fact that the four-dimensional curvature decreases with the expansion; the opposite occurs when the strong energy condition is violated. Hence the "cosmological constant" (with $\rho c^2 = -p$) can be an effective source of particles, during a purely de Sitter evolutionary phase [23].

An alternative approach to particle production from a dark energy vacuum fluid was suggested in [24], by assuming that the vacuum, with the energy density proportional to Λ, gives up energy which corresponds to a particle with rest mass m, so that $d\Lambda = -6(mc/h)^2 ds$. This situation is similar to the Dirac hole theory, in which a positron is regarded as a hole created in an underlying sea of energy. Particle production from dark energy can also be interpreted in geometrical terms. The vacuum

energy/cosmological constant is a "sea of energy" that curves the space-time having a curvature $L = \sqrt{3/\Lambda}$. Locally, a perturbation in the vacuum corresponds to a change in curvature, and a change in the curvature leads to a change in the quantum mechanical vacuum state, resulting in a production of a massive particle.

From (3) one can estimate the numerical value of the minimal density for a positive Λ as $\rho_{\min} = \Lambda c^2/16\pi G = 8.0 \times 10^{-30}$ g cm^{-3}.

Since the Planck length l_{Pl} is a natural minimal length scale in physics, we define the absolute minimal mass which possibly can exist in nature by

$$M_{\min} = \frac{\Lambda c^2}{16\pi G}\frac{4\pi}{3}l_{Pl}^3 = \frac{\Lambda c^2}{12G}l_{Pl}^3 = \frac{\Lambda}{12}\sqrt{\frac{\hbar^3 G}{c^5}} = \frac{\Lambda}{6\sqrt{2}}m_{Pl}l_{Pl}^2 = \frac{\Lambda}{3}\frac{\hbar}{2}\frac{t_{Pl}}{2}, \quad (4)$$

where we denoted by t_{Pl} the Planck time $t_{Pl} = l_{Pl}/c$. The numerical value of $M_{\min}$ is given by

$$M_{\min} \approx 1.4 \times 10^{-127} \text{ g} \approx 7.9 \times 10^{-95} \text{ eV}. \quad (5)$$

If an absolute minimum length does exist in nature, then, via the first of (3), a positive cosmological constant implies the existence of an absolute minimum mass in nature, given by (4).

Hypothetical particles having this value of the mass may be called cosminos. The cosminos could also be interpreted as "quanta" of the dark energy (cosmological constant), and therefore $M_{\min}$ gives the mass of the quantum of the cosmological constant. Compared with the upper bound of the electron neutrino mass $m_{\nu_e} < 1.8$ eV [25], we emphasize the smallness of the minimal mass $M_{\min}$.

By generalizing (4) we propose that the mass is quantized according to the general rule

$$m = n\frac{\Lambda}{3}\frac{\hbar}{2}\frac{t_{Pl}}{2}, \quad n \in N, \quad (6)$$

which is different from Wesson's proposal [13].

From a purely quantum mechanical point of view, the value of the minimum mass can be derived with the use of the uncertainty principle for energy and time, which gives

$$m_{\min}c^2 \approx \frac{\hbar}{\Delta t}. \quad (7)$$

By assuming that Δt is of the same order of magnitude as the age of the Universe, $\Delta t \approx 1/H_0$, where $H_0 \approx 3.24 \times 10^{-18}$ s^{-1} is the Hubble constant (the present value of the Hubble function), we obtain for the minimum mass the expression

$$m_{\min} = \frac{\hbar H_0}{c^2} \approx 3.8017 \times 10^{-66} \text{ g}. \quad (8)$$

The numerical value of the minimum mass obtained from quantum mechanical considerations agrees with the value of the mass $m_P = (\hbar/c)\sqrt{\Lambda/3}$ obtained by Wesson [13] by using purely dimensional considerations. Therefore it is natural to assume

that these two masses are the same, thus obtaining

$$\left(\frac{\hbar}{c}\right)\sqrt{\frac{\Lambda}{3}} = \frac{\hbar H_0}{c^2}, \tag{9}$$

which gives

$$H_0 = c\sqrt{\frac{\Lambda}{3}}. \tag{10}$$

We propose to call particles having the mass given by $m_{\min} = m_P$ cosmons[1]. The possibility of the existence of a very light scalar particle, also named cosmon, a dilaton which should essentially decouple from the Standard Matter Lagrangian, but still could mediate new macroscopic forces in the submillimeter range, was proposed in [30]. The mass of the cosmon is given by $m_S^2 = \Lambda_{QCD}^4/M^4$, where $\Lambda_{QCD} \approx 100$ MeV is the intrinsic QCD scale and $M \geq 10^{10}$ GeV is some high energy scale [30]. The mass of this particle is of the order of the neutrino mass, $m_S \approx (10^{-3}–10^{-2})$ eV $\approx 2 \times (10^{-30}–10^{-31})$ g. Therefore it represents a very different particle as compared to the minimum mass particles considered in the present paper.

By assuming however, that the minimum mass in nature is given by $m_P = m_{\min}$ it follows that the radius corresponding to m_P is given by

$$R_P = 48^{1/6}\left(\frac{\hbar G}{c^3}\right)^{1/3}\Lambda^{-1/6} \approx 1.9 l_{Pl}^{2/3}\Lambda^{-1/6}, \tag{11}$$

with the numerical value $R_P = 4.7 \times 10^{-13}$ cm $= 4.7$ fm. This would also imply that *the minimum length in nature could be very different from the Planck length l_{Pl}.*

In fact the radius R_P is of the same order of magnitude as the classical radius of the electron $r_e = e^2/m_e c^2 = 2.81 \times 10^{-13}$ cm. Therefore, by formally equating R_P with r_e and neglecting terms of the order of unity gives a representation of the *cosmological constant in terms of the 'classical' fundamental constants* as

$$\Lambda = \frac{l_{Pl}^4}{r_e^6} = \frac{\hbar^2 G^2 m_e^6 c^6}{e^{12}} \approx 1.4 \times 10^{-56} \text{ cm}^{-2}. \tag{12}$$

Conceptually, the identification of the radius R_P to the electron radius r_e may be based on a "small number hypothesis", representing an extension of the large number hypothesis by Dirac [31], and which proposes that *the numerical equality between two very small quantities with a very similar physical meaning cannot be a simple coincidence.*

---

[1]Cosmons were originally introduced by Peccei, Sola and Wetterich [26] to name scalar fields that could dynamically adjust the cosmological constant to zero, see also [27–29].

3 Gravitational Condensation of Dark Energy Particles

Recently a class of hypothetical compact objects called gravastars (*grav*itational *va*cuum stars) have been proposed as potential alternatives to explain the astrophysical phenomenology traditionally associated to black holes [32]. According to this scenario, the quantum vacuum undergoes a phase transition at or near the location the event horizon is expected to form. Hence the gravastar consists of an interior de Sitter condensate, governed by an equation of state $\rho c^2 = -p$, matched to a shell of finite thickness with an equation of state $\rho c^2 = p$. The latter is then matched to an exterior Schwarzschild solution. Dark energy stars, for which the interior vacuum energy is much larger than the cosmological energy, have also been investigated [33, 34]. Hence the possibility that condensation processes, like, for example, Bose-Einstein condensation, could play an essential role in astrophysical and cosmological situations cannot be excluded *a priori*.

Generally, Bose-Einstein condensation processes take place in a Bose gas consisting of particles with mass m and number density n when the thermal de Broglie wave length $\lambda_{dB} = \sqrt{2\pi\hbar^2/mkT}$, where k is Boltzmann's constant and T is the temperature, exceeds the mean inter-particle distance $n^{1/3}$, and the wave packets percolate in space. The critical condensation temperature is $T \leq 2\pi\hbar^2 n^{2/3}/mk$ [35]. If we assume an adiabatic cosmological expansion of the universe, the temperature dependence of the number density of the particle is $T \propto n^{2/3}$. Hence Bose-Einstein condensation occurs if the mass of the particle satisfies the condition $m < 1.87$ eV [36], a condition which is obviously satisfied by both cosminos and cosmons. Hence these particles may Bose-Einstein condense to form large scale astrophysical or cosmological structures.

It is tempting to assume that the cosmons with mass M_{min} or $m_{min} = m_P$ could *condense gravitationally* to form stellar type stable compact objects. To study the cosmological implications of the condensation process we assume that the cosmon fluid, with an initial density $\rho_0 = \Lambda c^2/8\pi G$ and pressure p_0, satisfying the equation of state $\rho_0 c^2 + p_0 = 0$, condenses into a non-relativistic, dissipationless fluid, which can be characterized by a density ρ, a pressure p, a velocity $\mathbf{v}$ and a gravitational acceleration $\mathbf{g}$. The dynamics of the system is described by the continuity equation, the hydrodynamical Euler equation and the Poisson equation, which can be written as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \qquad \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \mathbf{g}, \qquad (13)$$

$$\nabla \times \mathbf{g} = 0, \qquad \nabla \cdot \mathbf{g} = -4\pi G\rho. \qquad (14)$$

We take as the initial (unperturbed) state of the system the state characterized by the absence of the "real" gravitational forces, $\mathbf{g} = \mathbf{g}_0 = 0$, of the hydrodynamical flow, $\mathbf{v} = \mathbf{v}_0 = 0$, and by constant values of the density and pressure, $\rho = \rho_0$ and $p = p_0$, respectively, with $\rho_0 c^2 + p_0 = 0$. The condensation process leads to the appearance of the gravitational interaction in the system, as well as to small perturbations of the hydrodynamical quantities, so that

$$\rho = \rho_0 + \rho_1, \qquad p = p_0 + p_1, \qquad \mathbf{v} = \mathbf{v}_0 + \mathbf{v}_1, \qquad \mathbf{g} = \mathbf{g}_0 + \mathbf{g}_1, \qquad (15)$$

so that $-1 \ll \rho_1/\rho_0 \ll 1$ and $-1 \ll p_1/p_0 \ll 1$, respectively. In the first order approximation equations (13) and (14) take the form

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \nabla \cdot \mathbf{v}_1 = 0, \qquad \frac{\partial \mathbf{v}_1}{\partial t} = -\frac{v_s^2}{\rho_0} \nabla \rho_1 + \mathbf{g}_1, \qquad (16)$$

$$\nabla \times \mathbf{g}_1 = 0, \qquad \nabla \cdot \mathbf{g}_1 = -4\pi G \rho_1, \qquad (17)$$

where we have introduced the adiabatic speed of sound v_s in the condensed cosmon fluid, defined as $v_s = \sqrt{p_1/\rho_1} = \sqrt{\partial p/\partial \rho}$. By taking the partial derivative with respect to the time of the continuity equation in (16), we obtain the propagation equation of the density perturbation in the cosmon fluid in the form

$$\frac{\partial^2 \rho_1}{\partial t^2} = v_s^2 \nabla^2 \rho_1 + \frac{\Lambda c^2}{2} \rho_1. \qquad (18)$$

By looking for a solution of the form $\rho_1 \propto \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$, we obtain the following dispersion relation for ω

$$\omega^2 = v_s^2 \mathbf{k}^2 - \frac{\Lambda c^2}{2}. \qquad (19)$$

From the dispersion equation one can see that for $k < k_J$, where

$$k_J = \sqrt{\frac{\Lambda c^2}{2v_s^2}}, \qquad (20)$$

is the Jeans wave number, the angular frequency ω becomes an imaginary quantity, which corresponds to an instability of the fluid-ρ_1 can increase (or decrease) exponentially, leading to a gravitational condensation (or rarefaction). Therefore, for $k < k_J$, $\omega = \pm v_s \sqrt{k^2 - k_J^2} = i \operatorname{Im} \omega$, where $\operatorname{Im} \omega = \pm v_s \sqrt{k_J^2 - k^2}$, and consequently $\rho_1 \propto \exp[\pm |\operatorname{Im} \omega| t]$.

When the mass of the condensate exceeds the mass of a sphere with radius $2\pi/k_J$, a gravitational instability occurs in the cosmon fluid, and the cloud of particles would collapse. The critical mass is the Jeans mass $M_J = (4\pi/3)(2\pi/k_J)^3 \rho_0$, and for the cosmon fluid it is given by

$$M_J = \frac{8\sqrt{2}}{3} \pi^3 \left(\frac{v_s}{c}\right)^3 \frac{c^2}{G} \Lambda^{-1/2} \approx 1.6 \times 10^{30} \times (\Lambda \, \mathrm{cm}^{-2})^{-1/2} \left(\frac{v_s}{c}\right)^3 \mathrm{g}. \qquad (21)$$

For $\Lambda = 3 \times 10^{-56} \, \mathrm{cm}^{-2}$ we obtain $M_J = 9.24 \times 10^{57} (v_s/c)^3$ g. By taking into account the representation of the cosmological constant in terms of the "classical constants" given by (12), we obtain for the critical Jeans mass of the cosmon fluid the expression

$$M_J = \frac{8\sqrt{2}}{3} \pi^3 \left(\frac{v_s}{c}\right)^3 \frac{e^6}{\hbar G^2 m_e^3 c}. \qquad (22)$$

The effective radius R_J of the stable cosmon configuration is given by

$$R_J = 2^{3/2}\pi \frac{v_s}{c} \Lambda^{-1/2} \approx 2^{3/2}\pi \frac{v_s}{c} \frac{e^6}{\hbar G m_e^3 c^3}. \tag{23}$$

The theoretical value of the maximum mass M_{Ch} of the stable compact astrophysical type objects, like white dwarfs and neutron stars, was found by Chandrasekhar and Landau and is given by the Chandrasekhar limit,

$$M_{Ch} \approx \left[\left(\frac{\hbar c}{G}\right)m_B^{-4/3}\right]^{3/2}, \tag{24}$$

where m_B is the mass of the particles giving the main contribution to the mass (baryons in the case of the white dwarfs and neutron stars) [37]. Thus, with the exception of some composition-dependent numerical factors, the maximum mass of a degenerate star depends only on fundamental physical constants.

The Jeans mass for cosmons can also be written in the form of a Chandrasekhar limiting mass, if we assume that the cosmons have an effective mass m_{eff} given by

$$m_{eff} = (G\hbar^5 c^5)^{1/4}\frac{m_e^{3/2}}{e^3}, \tag{25}$$

so that $M_J = [8\sqrt{2}\pi^3(v_s/c)^3/3][(\hbar G/c)m_{eff}^{-4/3}]^{3/2}$. The value of the effective mass of the cosmon is $m_{eff} \approx 8 \times 10^{-20}$ g.

On the other hand, one can also assume that the cosminos or the cosmons could form stellar type objects with the limiting mass given by the Chandrasekhar limit, (24). The mass of such an hypothetical super-massive object formed from cosminos with mass $M_{\min}$ is of the order of $M_{Ch}^{(1)} = 8 \times 10^{237}$ g, which exceeds by around 180 orders of magnitude the mass of our universe. Therefore, it follows that cosminos did not condense gravitationally, and hence the particles associated with dark energy fail to represent dark matter, which is in complete agreement with the present standard model of cosmology. On the other hand, in the case of cosmons for the Chandrasekhar limiting mass $M_{Ch}^{(2)}$ we obtain $M_{Ch}^{(2)} = 2.7782 \times 10^{116}$ g, which also shows that degenerate cosmon stars, having masses much larger than the mass of the universe, are very unlikely to exist.

4 Gravitational Energy of Stable Cosmon Configurations

The total energy (including the gravitational field contribution) inside an equipotential surface S of radius R can be defined, according to [38], to be

$$E = E_M + E_F = \frac{c^4}{8\pi G}\xi_s \int_S [K]dS, \tag{26}$$

where ξ^i is a Killing vector field of time translation, ξ_s its value at S and $[K]$ is the jump across the shell of the trace of the extrinsic curvature of S, considered as

embedded in the 2-space $t = $ constant. $E_M = \int_S T_i^k \xi^i \sqrt{-g} \, dS_k$ and E_F are the energy of the matter and of the gravitational field, respectively. This definition is manifestly coordinate invariant.

For a static spherically symmetric system in a Schwarzschild-de Sitter space-time the total energy is

$$E = \frac{c^4}{G} R \left[1 - \left(1 - \frac{2GM}{c^2 R} - \frac{\Lambda}{3} R^2 \right)^{1/2} \right] \left(1 - \frac{2GM}{c^2 R} - \frac{\Lambda}{3} R^2 \right)^{1/2}. \qquad (27)$$

For the minimum mass particle the total energy can be expressed in terms of the radius and cosmological constant only as

$$E = \frac{c^4}{G} R \left[1 - \left(1 - \frac{\Lambda}{2} R^2 \right)^{1/2} \right] \left(1 - \frac{\Lambda}{2} R^2 \right)^{1/2}. \qquad (28)$$

For a stable configuration, the energy should have a minimum, $\partial E / \partial R = 0$, a condition which determines R as

$$R_{BC} = \frac{1}{3} \sqrt{11 + \sqrt{13}} \Lambda^{-1/2} \approx 1.3 \times \Lambda^{-1/2}. \qquad (29)$$

Therefore the mass of the stable configuration can be obtained as

$$M_{BC} = \frac{1.15}{6} \frac{c^2}{G} \Lambda^{-1/2} \approx 0.2 \frac{c^2}{G} \frac{r_e^3}{l_{Pl}^2} \approx 0.2 m_{Pl} \left(\frac{r_e}{l_{Pl}} \right)^3, \qquad (30)$$

which gives a mass of the order $M_{BC} \approx 8.2 \times 10^{54}$ g, a value which is close to the mass m_{PE}, which follows from dimensional considerations, and is of the same order of magnitude as the total mass of the observable universe. Therefore we may regard the observable universe as a dark energy dominated object with minimum density.

For the second derivative of the energy, evaluated for $R = R_{BC}$, we obtain the expression $(\partial^2 E / \partial R^2)|_{R=R_{BC}} = -6.89\sqrt{\Lambda}$, which shows that indeed the configuration is in a state of minimum total energy.

5 Discussions and Final Remarks

In the present paper we have investigated some of the possible consequences of the existence of a minimum mass and density for stable general relativistic objects, which is a direct result of the existence of the cosmological constant. The existence of a fundamental length, assumed to be the Planck scale, leads to an absolute minimum mass in nature, which could be the mass of the quanta of the dark energy (the cosminos), with radius of the order of the Planck length. However, the application of the quantum uncertainty principle for the energy shows that the mass of the elementary particles associated to the dark energy (the cosmons) is given by $m_P = \hbar H_0 / c^2 = (\hbar/c)\sqrt{\Lambda/3}$. If this is indeed the case, then the radius of such a particle is of the same order of magnitude as the classical electron radius. This leads *to the intriguing possibility of the*

electron charge and radius, or, more generally, of the electromagnetic processes, as playing an essential role in the dark energy related phenomena.

We also propose that "dark energy particles" may condensate, either Bose-Einstein or gravitationally, to form compact super-massive objects, formed of cosminos or cosmons, respectively. The mass of this condensation, which is gravitationally stable, was derived using two independent methods. Firstly, we have assumed that the dark energy fluid condenses into a dissipationless, non-relativistic fluid. The corresponding Jeans mass is proportional to $\Lambda^{-1/2}$, and (except some numerical factors) is the same as the mass m_{PE} introduced from dimensional considerations. Its numerical value is of the same order of magnitude as the total mass of the universe.

The requirement that the total energy of the stable configuration formed from the particle satisfying the relation $2GM = \Lambda/6R^3$ is a minimum leads to a second, rigorous derivation of the mass m_{PE}, which is of the same order of magnitude as the mass of the universe. This also shows that the only energetically stable dark energy dominated general relativistic objects must have a mass of the same order of magnitude as our universe. Therefore the general relativistic condition (3) as combined with the thermodynamic condition of energetic stability may explain the actual value of the mass of the universe. Moreover, the total mass of the universe can also be obtained in terms of the elementary constants $c, \hbar, e, m_e, G$.

Therefore, these two independent results may imply that our universe was born as the result of the dark energy condensation, which took place at a very high temperature and density. Hence the initial constituents of our universe may have been the cosmons. We also obtain the physical interpretation of the masses m_{PE} and M_{BC} as the critical Jeans mass of the Universe, that is, the mass of the gravitationally stable dark particles clouds. This result also gives a new physical interpretation of the cosmological constant. From (20), by assuming that the speed of sound in the gravitationally condensed dark energy fluids is equal to the speed of light, $v_s = c$, it follows that $\Lambda \approx k_J^2$, that is, *physically the cosmological constant represents the square of the Jeans wave number of a dark energy fluid*. Alternatively, one can express the cosmological constant as $\Lambda = 8\pi^2/\lambda_J^2$, where $\lambda_J = 2\pi/k_J$ is the Jeans wavelength. Moreover, the mass of the universe can be expressed in terms of three fundamental quantities, the Planck mass m_{Pl}, the Planck length l_{Pl}, and the classical electron radius r_e, respectively.

On the other hand, even that the estimation of the limiting Chandrasekhar masses [37] for cosmons suggests the possible existence of super-massive stable degenerate dark energy objects, the existence of such stars with masses much larger than the mass of the universe is impossible in the universe we are living in.

Finally, it would be very interesting to recall the cosmological constant problem again here. If it is interpreted as a measure of the vacuum energy density and from a particles physics point of view, the cosmological constant Λ is 120 orders of magnitude too small than expected [5].

Let us therefore assume that the cosmological constant were indeed 120 orders of magnitude larger. This would have drastic consequences for the minimal mass and we would find $M_{\min} \approx 10^{19}$ eV, in which case the minimal mass would exceed the masses of all elementary particles. From this point of view, we would like to also argue that because of the resulting problems, the interpretation of the cosmological

constant as the vacuum energy density may raise some conceptual contradictions with the results derived in the present paper.

References

1. Riess, A.G., et al.: Astron. J. **116**, 109 (1998)
2. Perlmutter, S., et al.: Astrophys. J. **517**, 565 (1999)
3. de Bernardis, P., et al.: Nature **404**, 995 (2000)
4. Hanany, S., et al.: Astrophys. J. **545**, L5 (2000)
5. Peebles, P.J.E., Ratra, B.: Rev. Mod. Phys. **75**, 559 (2003)
6. Padmanabhan, T.: Phys. Rep. **380**, 235 (2003)
7. Mak, M.K., Dobson Jr., P.N., Harko, T.: Mod. Phys. Lett. A **15**, 2153 (2000)
8. Böhmer, C.G.: Gen. Relativ. Gravit. **36**, 1039 (2004)
9. Böhmer, C.G.: Ukr. J. Phys. **50**, 1219 (2005)
10. Balaguera-Antolinez, A., Böhmer, C.G., Nowakowski, M.: Int. J. Mod. Phys. D **14**, 1507 (2005)
11. Buchdahl, H.A.: Phys. Rev. **116**, 1027 (1959)
12. Balaguera-Antolinez, A., Böhmer, C.G., Nowakowski, M.: Class. Quantum Gravity **23**, 485 (2006)
13. Wesson, P.S.: Mod. Phys. Lett. A **19**, 1995 (2004)
14. Aldrovandi, R., Beltran Almeida, J.P., Pereira, J.G.: arXiv:gr-qc/0702065 (2007)
15. Aldrovandi, R., Beltran Almeida, J.P., Mayor, C.S.O., Pereira, J.G.: arXiv:0709.3947 (2007)
16. Aldrovandi, R., Beltran Almeida, J.P., Mayor, C.S.O., Pereira, J.G.: arXiv:0710.0610 (2007)
17. Aldrovandi, R., Beltran Almeida, J.P., Pereira, J.G.: Class. Quantum Gravity **24**, 1385 (2007)
18. Böhmer, C.G., Harko, T.: Phys. Lett. B **630**, 73 (2005)
19. Böhmer, C.G., Harko, T.: Class. Quantum Gravity **23**, 6479 (2006)
20. Böhmer, C.G., Harko, T.: Gen. Relativ. Gravit. **39**, 757 (2007)
21. Weinberg, S.: Rev. Mod. Phys. **61**, 1 (1989)
22. Birrel, N.D., Davies, P.C.W.: Quantum Fields in Curved Space. Cambridge University Press, Cambridge (1982)
23. Batista, A.B., Fabris, J.C., Houndjo, S.: arXiv:0710.0999 (2007)
24. Wesson, P.S.: Found. Phys. Lett. **19**, 285 (2006)
25. Trinczek, M., et al.: Phys. Rev. Lett. **90**, 012501 (2003)
26. Peccei, R.D., Sola, J., Wetterich, C.: Phys. Lett. B **195**, 183 (1987)
27. Wetterich, C.: Nucl. Phys. B **302**, 668 (1988)
28. Sola, J.: Phys. Lett. B **228**, 317 (1989)
29. Sola, J.: Int. J. Mod. Phys. A **5**, 4225 (1990)
30. Shapiro, I.L., Sola, J.: Phys. Lett. B **475**, 236 (2000)
31. Dirac, P.A.M.: Proc. R. Soc. A **333**, 439 (1974)
32. Mazur, P.O., Mottola, E.: Proc. Natl. Acad. Sci. **101**, 9545 (2004)
33. Chapline, G.: arXiv:astro-ph/0503200 (2005)
34. Lobo, F.S.N.: Class. Quantum Gravity **23**, 1525 (2006)
35. Böhmer, C.G., Harko, T.: J. Cosmol. Astropart. Phys. **06**, 025 (2007)
36. Fukuyama, T., Morikawa, M., Tatekawa, T.: arXiv:0705.3091 (2007)
37. Shapiro, S.L., Teukolsky, S.A.: Black Holes, White Dwarfs, and Neutron Stars. Wiley, New York (1983)
38. Katz, J., Lynden-Bell, D., Israel, W.: Class. Quantum Gravity **5**, 971 (1988)