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The Universe as an Eigenstate: Spacetime Paths and Decoherence*

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This paper describes how the entire universe might be considered an eigenstate determined by classical limiting conditions within it. This description is in the context of an approach in which the path of each relativistic particle in spacetime represents a fine-grained history for that particle, and a path integral represents a coarse-grained history as a superposition of paths meeting some criteria. Since spacetime paths are parametrized by an invariant parameter, not time, histories based on such paths do not evolve in time but are rather histories of all spacetime. Measurements can then be represented by orthogonal states that correlate with specific points in such coarse-grained histories, causing them to decohere, allowing a consistent probability interpretation. This conception is applied here to the analysis of the two slit experiment, scattering and, ultimately, the universe as a whole. The decoherence of cosmological states of the universe then provides the eigenstates from which our "real" universe can be selected by the measurements carried out within it.

KEY WORDS: path integrals; relativistic quantum mechanics; quantum cosmology; relativistic dynamics; decoherence; consistent history interpretation.

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1. INTRODUCTION

Before the ascendency of quantum field theory, Stueckelberg proposed an approach to relativistic quantum field theory based on the conception of particle paths in spacetime, parameterized by an invariant fifth

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parameter.[1,2] Feynman later considered this idea as the basis for relativistic path integrals (see the appendices to Feynman[3,4]), a conception which seems to have informed his early work on quantum field theory (though it is not much apparent in his later work).

Since then, a number of authors have further developed the theory of parameterized relativistic quantum physics (see Ref. 5 and references therein), though not necessarily using a path integral approach. However, relativistic path integrals in particular have a natural interpretation in terms of consistent or decoherent histories.[6–8] In this interpretation, the path of a particle in spacetime is considered a *fine-grained* history. A path integral then represents a *coarse-grained* history as a superposition of paths meeting some criteria. When the criteria are properly chosen, the states for these coarse grained histories do not interfere—that is, they are *decoherent*.[9]

Since decoherent histories do not interfere, they can be assigned classical probabilities. Further adopting a "many worlds" interpretation,[10] these histories can be considered to be alternate "branches" in the history of the universe, with associated probabilities for each of the branches to "occur" (for an informal introduction to the ideas of decoherence and emergent classicality, see Ref. 11. For a more extensive survey see Ref. 12).

Relativistic path integrals have also proved useful in the study of quantum gravity and quantum cosmology, because the time coordinate is treated similarly to the space coordinates, rather than as an evolution parameter see, for example, Refs. 9, 13. In quantum cosmological models, the total Hamiltonian annihilates the "wave function of the universe", rather than determining the time evolution of the system. The question is how to extract physical predictions from such a wave function.

Inspired by this, Halliwell and Thorwart recently published a paper with the engaging title "Life in an energy eigenstate"[14] in which they consider the internal dynamics of a simple particle system in an energy eigenstate. In the present paper, I would like to take this idea a bit farther, and describe how the *entire universe* might be considered to be in an eigenstate determined by classical limiting conditions within it. In effect, such an eigenstate is a selection of a specific coarse-grained branch as "the" history of the universe.

Pursuing this idea requires a formalism that allows coarse-grained histories to be expressed as quantum states. I will use the spacetime path formalism proposed in Ref. 15. For completeness, Sec. 2 summarizes the development of this formalism. A particularly important result from this work is that the coarse-grained histories of free particles with fixed three-momentum become on-shell and decoherent in the infinite time limit.

Section 3 then discusses decoherence in the context of the spacetime path formalism. Section 3.1 applies the formalism to the analysis of the familiar scenario of the two slit experiment. Section 3.2 extends the approach to consideration of a scattering process that takes place in a finite region of spacetime. Finally, taking this analysis of scattering as a paradigm, Sec. 3.3 considers the relation of probabilities to measured relative frequencies and Sec. 3.4 presents a heuristic discussion of the decoherence of *cosmological states* of the entire universe.

Throughout, I will use a spacetime metric signature of $(-+++)$ and take $\hbar = c = 1$.

2. SPACETIME PATHS

This section summarizes the spacetime path formalism I will use in the following sections. For further details on the development of this formalism, see Ref. 15.

2.1. Position States

A *spacetime path* is specified by four functions $q^\mu(\lambda)$, for $\mu = 0, 1, 2, 3$, of a *path parameter* λ. Note that such a path is not constrained to be timelike or even to maintain any particular direction in time. The only requirement is that it must be continuous. And, while there is no *a priori* requirement for the paths to be differentiable, we can, as usual, treat them as differentiable within the context of a path integral (see the discussion in Ref. 15.)

It is well known that a spacetime path integral of the form

$$\Delta(x - x_0) = \eta \int_{\lambda_0}^{\infty} d\lambda_1 \int \mathrm{D}^4 q \, \delta^4(q(\lambda_1) - x)\delta^4(q(\lambda_0) - x_0) \exp\left(\mathrm{i} \int_{\lambda_0}^{\lambda_1} d\lambda L(\dot{q}^2(\lambda)) \right), \tag{1}$$

for an appropriate normalization constant η and the Lagrangian function

$$L(\dot{q}^2) = \frac{1}{4}\dot{q}^2 - m^2 ,$$

gives the free-particle Feynman propagator.[3,13,15,16] In the path integral above, the notation $\mathrm{D}^4 q$ indicates that the integral is over the four functions $q^{\mu}(\lambda)$ and the delta functions constrain the starting and ending points of the paths integrated over.

Consider, however, that Eq. (1) can be written

$$\Delta(x - x_0) = \int_{\lambda_0}^{\infty} d\lambda_1\, \Delta(x - x_0; \lambda_1 - \lambda_0),$$

where

$$\Delta(x - x_0; \lambda_1 - \lambda_0) \equiv \eta \int \mathrm{D}^4 q\, \delta^4(q(\lambda_1) - x)\delta^4(q(\lambda_0) - x_0) \exp\left(\mathrm{i} \int_{\lambda_0}^{\lambda_1} d\lambda L(\dot{q}^2(\lambda))\right) \tag{2}$$

now has a similar path integral form as the usual non-relativistic *propagation kernel*,[17,18] except with paths parametrized by λ rather than time. We can, therefore, use the relativistic kernel of Eq. (2) to define a parametrized probability amplitude function in a similar fashion to the non-relativistic case:

$$\psi(x; \lambda) = \int d^4 x_0\, \Delta(x - x_0; \lambda - \lambda_0)\psi(x_0; \lambda_0). \tag{3}$$

These wave functions are just the parametrized probability amplitude functions defined by Stueckelberg.[1] In this sense, the $\psi(x; \lambda)$ represent the probability amplitude for a particle to reach position x at the point along its path with parameter value λ.

The path integral in Eq. (2) can be evaluated to give[13,15]

$$\Delta(x - x_0; \lambda - \lambda_0) = (2\pi)^{-4} \int d^4 p\, \mathrm{e}^{\mathrm{i}p\cdot(x - x_0)} \mathrm{e}^{-\mathrm{i}(\lambda - \lambda_0)(p^2 + m^2)}.$$

Inserting this into Eq. (3), we see that $\psi(x; \lambda)$ satisfies the *Stuekelberg–Schrödinger equation*

$$-\mathrm{i}\frac{\partial}{\partial \lambda}\psi(x; \lambda) = \left(\frac{\partial^2}{\partial x^2} - m^2\right)\psi(x; \lambda).$$

Note that this equation is based on the relativistic Hamiltonian $p^2 + m^2$, and therefore includes the mass term m^2. This is in contrast to most previous authors,[3,19,20] who used a Hamiltonian of the form $p^2/(2m)$, by analogy with non-relativistic mechanics.

The relativistic propagation kernel can also be given a conjugate form as a superposition of particle mass states. For $T > 0$,

$$\begin{aligned}
\theta(T)\Delta(x-x_0;T) &= \mathrm{e}^{-\mathrm{i}Tm^2}\int d^4p\,\mathrm{e}^{\mathrm{i}p\cdot(x-x_0)}\int_0^\infty dT'\,\mathrm{e}^{-\mathrm{i}T'p^2}\delta(T'-T)\\
&= (2\pi)^{-1}\mathrm{e}^{-\mathrm{i}Tm^2}\int d^4p\,\mathrm{e}^{\mathrm{i}p\cdot(x-x_0)}\int_0^\infty dT'\,\mathrm{e}^{-\mathrm{i}T'p^2}\\
&\quad \int dm'^2\,\mathrm{e}^{-\mathrm{i}(T'-T)m'^2}\\
&= (2\pi)^{-1}\mathrm{e}^{-\mathrm{i}Tm^2}\int dm'^2\,\mathrm{e}^{\mathrm{i}Tm'^2}\Delta(x-x_0;m'^2),
\end{aligned} \tag{4}$$

where

$$\begin{aligned}
\Delta(x-x_0;m'^2) &\equiv \int_0^\infty dT'\int d^4p\,\mathrm{e}^{\mathrm{i}p\cdot(x-x_0)}\mathrm{e}^{-\mathrm{i}T'(p^2+m'^2)}\\
&= -\mathrm{i}(2\pi)^{-4}\int d^4p\,\frac{\mathrm{e}^{\mathrm{i}p\cdot(x-x_0)}}{p^2+m'^2-\mathrm{i}\varepsilon}.
\end{aligned}$$

Except for the extra phase factor $\exp(-\mathrm{i}Tm^2)$, this form for $\Delta(x-x_0;T)$ is essentially that of the retarded Green's function derived by Land and Horwitz for parametrized quantum field theory[(21,22)] as a superposition of propagators for different mass states (see also Refs. 23,24).

The value T in $\Delta(x-x_0;T)$ can be thought of as fixing a specific *intrinsic length* for the paths being integrated over in Eq. (2). The full propagator then results from a regular integration over all possible intrinsic path lengths:

$$\Delta(x-x_0) = \int_0^\infty dT\,\Delta(x-x_0;T)\,.$$

As a result of the phase factor $\exp(-\mathrm{i}Tm^2)$ in Eq. (4), the integration over T effectively acts as a Fourier transform, resulting in the Feynman propagator with mass sharply defined at m, $\Delta(x-x_0) = \Delta(x-x_0;m)$.

The functions defined in Eq. (3) form a Hilbert space over four dimensional spacetime, parameterized by λ, in the same way that traditional non-relativistic wave functions form a Hilbert space over three-dimensional space, parameterized by time. We can therefore define a consistent family of *position state* bases $|x;\lambda\rangle$, such that

$$\psi(x;\lambda) = \langle x;\lambda|\psi\rangle, \tag{5}$$

given a single Hilbert space state vector $|\psi\rangle$. These position states are normalized such that

$$\langle x'; \lambda | x; \lambda\rangle = \delta^4(x' - x) .$$

for each value of λ. Further, it follows from Eqs. (3) to (5) that

$$\Delta(x - x_0; \lambda - \lambda_0) = \langle x; \lambda | x_0; \lambda_0\rangle . \tag{6}$$

Thus, $\Delta(x-x_0; \lambda-\lambda_0)$ effectively defines a unitary transformation between the various Hilbert space bases $|x; \lambda\rangle$, indexed by the parameter λ.

The overall state for propagation from x_0 to x is given by the superposition of the states for paths of all intrinsic lengths. If we fix $q^\mu(\lambda_0) = x_0^\mu$, then $|x; \lambda\rangle$ already includes all paths of length $\lambda - \lambda_0$. Therefore, the overall state $|x\rangle$ for the particle to arrive at x should be given by the superposition of the states $|x; \lambda\rangle$ for all $\lambda > \lambda_0$:

$$|x\rangle \equiv \int_{\lambda_0}^{\infty} d\lambda \, |x; \lambda\rangle .$$

Then, using Eq. (6),

$$\langle x | x_0; \lambda_0\rangle = \int_{\lambda_0}^{\infty} d\lambda \, \Delta(x - x_0; \lambda - \lambda_0) = \int_0^{\infty} d\lambda \, \Delta(x - x_0; \lambda) = \Delta(x - x_0) .$$

2.2. On-Shell States

The position states defined in Sec. 2.1 make no distinction based on the time-direction of propagation of particles. Normally, particles are considered to propagate *from* the past *to* the future. Therefore, we can define normal particle states $|x_+\rangle$ such that

$$\langle x_+ | x_0; \lambda_0\rangle = \theta(x^0 - x_0^0)\Delta(x - x_0) , \tag{7}$$

On the other hand, *antiparticles* may be considered to propagate from the *future* into the *past*.[1,2,25] Therefore, antiparticle states $|x_-\rangle$ are such that

$$\langle x_- | x_0; \lambda_0\rangle = \theta(x_0^0 - x^0)\Delta(x - x_0) . \tag{8}$$

Note that the particle/antiparticle distinction proposed here is subtly different than that originally proposed by Stueckelberg.[1,2] Stueckelberg considered the possibility that a single particle path might undergo a dynamical interaction that could change the time direction of its propagation, corresponding to what seemed to be a particle creation or annihilation event when viewed in a time-advancing direction. In contrast, the definitions of particle and antiparticle states given here depend only on whether the *end point* x of the particle path is in the future or past of its starting point x_0. Between these two points, the path may move arbitrarily forward or backwards in time.

This division into particle and antiparticle paths depends, of course, on the choice of a specific coordinate system in which to define the time coordinate. However, if we take the time limit of the end point of the path to infinity for particles and negative infinity for antiparticles, then the particle/antiparticle distinction will be coordinate system independent.

In taking this time limit, one cannot expect to hold the three-position of the path end point constant. However, for a free particle, it is reasonable to take the particle *three-momentum* as being fixed. Therefore, consider the state of a particle or antiparticle with a three-momentum $\boldsymbol{p}$ at a certain time t:

$$|t, \boldsymbol{p}_\pm\rangle \equiv (2\pi)^{-3/2} \int d^3x\, \mathrm{e}^{\mathrm{i}(\mp\omega_{\boldsymbol{p}} t + \boldsymbol{p}\cdot\boldsymbol{x})} |t, \boldsymbol{x}_\pm\rangle\,,$$

where $\omega_{\boldsymbol{p}} \equiv \sqrt{\boldsymbol{p}^2 + m^2}$. Now, as shown in Ref. 15,

$$|t, \boldsymbol{p}_+\rangle = (2\omega_{\boldsymbol{p}})^{-1} \int_{-\infty}^{t} dt_0\, |t_0, \boldsymbol{p}_+; \lambda_0\rangle \quad \text{and}$$

$$|t, \boldsymbol{p}_-\rangle = (2\omega_{\boldsymbol{p}})^{-1} \int_{t}^{+\infty} dt_0\, |t_0, \boldsymbol{p}_-; \lambda_0\rangle\,, \tag{9}$$

where

$$|t, \boldsymbol{p}_\pm; \lambda_0\rangle \equiv (2\pi)^{-3/2} \int d^3x\, \mathrm{e}^{\mathrm{i}(\mp\omega_{\boldsymbol{p}} t + \boldsymbol{p}\cdot\boldsymbol{x})} |t, \boldsymbol{x}; \lambda_0\rangle\,.$$

Since

$$\langle t', \boldsymbol{p}'_\pm; \lambda_0 | t, \boldsymbol{p}_\pm; \lambda_0\rangle = \delta(t' - t)\delta^3(\boldsymbol{p}' - \boldsymbol{p})\,,$$

we have, from Eq. (9),

$$\langle t, \boldsymbol{p}_\pm | t_0, \boldsymbol{p}_{0\pm}; \lambda_0\rangle = (2\omega_{\boldsymbol{p}})^{-1}\theta(\pm(t - t_0))\delta^3(\boldsymbol{p} - \boldsymbol{p}_0)\,.$$

If we now define the time limit particle and antiparticle states

$$|\boldsymbol{p}_{\pm}\rangle \equiv \lim_{t \to \pm\infty} |t, \boldsymbol{p}_{\pm}\rangle \,, \tag{10}$$

then

$$\langle \boldsymbol{p}_{\pm} | t_0, \boldsymbol{p}_{0\pm}; \lambda_0 \rangle = (2\omega_{\boldsymbol{p}})^{-1} \delta^3(\boldsymbol{p} - \boldsymbol{p}_0) \,, \tag{11}$$

for *any* value of t_0.

Equation (11) is a natural introduction of an "induced" inner product, in the sense of Refs. 16,26. To see how this induced inner product may be used, consider, the two Hilbert-space subspaces spanned by the normal particle states $|t, \boldsymbol{p}_+; \lambda_0\rangle$ and the antiparticle states $|t, \boldsymbol{p}_-; \lambda_0\rangle$, for each time t. States in these subspaces have the form

$$|t, \psi_{\pm}; \lambda_0\rangle = \int d^3p \, \psi(\boldsymbol{p}) |t, \boldsymbol{p}_{\pm}; \lambda_0\rangle \,,$$

for any square-integrable function $\psi(\boldsymbol{p})$, with

$$\psi(\boldsymbol{p}) = (2\omega_{\boldsymbol{p}}) \langle \boldsymbol{p}_{\pm} | t, \psi_{\pm} \rangle \,.$$

Similarly, consider the dual subspaces spanned by the bra states $\langle \boldsymbol{p}_+|$ and $\langle \boldsymbol{p}_-|$, such that

$$\langle \psi_{\pm}| \equiv \int d^3p \, \psi(\boldsymbol{p})^* \langle \boldsymbol{p}_{\pm}|$$

and

$$\psi(\boldsymbol{p})^* = \langle \psi_{\pm} | t, \boldsymbol{p}_{\pm} \rangle (2\omega_{\boldsymbol{p}}) \,. \tag{12}$$

As a result of Eq. (11), we get the traditional inner product

$$(\psi', \psi) \equiv \langle \psi'_{\pm} | t, \psi_{\pm} \rangle = \int \frac{d^3p}{2\omega_{\boldsymbol{p}}} \psi'(\boldsymbol{p})^* \psi(\boldsymbol{p}) \,. \tag{13}$$

With the inner product given by Eq. (13), the spaces of the $|t, \psi_{\pm}\rangle$ can be considered "reduced" Hilbert spaces in their own right, with the dual Hilbert space being the spaces of the $\langle \psi_{\pm}|$. Equation (11) can then be seen as a *bi-orthonormality* relation (see Ref. 27 and App. A.8.1 of Ref. 28) expressing the orthonormality of the $|t, \boldsymbol{p}; \lambda\rangle$ basis with respect to this inner product and allowing for the resolution of the identity

$$\int d^3p\,(2\omega_{\boldsymbol{p}})|t, \boldsymbol{p}_\pm; \lambda_0\rangle\langle \boldsymbol{p}| = 1\,. \tag{14}$$

This can be used to reproduce the usual probabilistic interpretation of quantum mechanics over three-space for each time t (for further details, see Ref. 15).

Further, writing

$$|t_0, \boldsymbol{p}_\pm; \lambda_0\rangle = (2\pi)^{-1/2} e^{\mp i\omega_{\boldsymbol{p}} t_0} \int dp^0\, e^{ip^0 t_0} |p; \lambda_0\rangle\,,$$

where

$$|p; \lambda_0\rangle \equiv (2\pi)^{-2} \int d^4x\, e^{ip\cdot x} |x; \lambda_0\rangle$$

is the corresponding four-momentum state, it is straightforward to see from Eq. (9) that the time limit of Eq. (10) is

$$|\boldsymbol{p}_\pm\rangle \equiv \lim_{t\to\pm\infty} |t, \boldsymbol{p}_\pm\rangle = (2\pi)^{1/2}(2\omega_{\boldsymbol{p}})^{-1}|\pm\omega_{\boldsymbol{p}}, \boldsymbol{p}; \lambda_0\rangle\,.$$

Thus, a normal particle (+) or antiparticle (−) that has three-momentum $\boldsymbol{p}$ as $t \to \pm\infty$ is *on-shell*, with energy $\pm\omega_{\boldsymbol{p}}$. Such on-shell particles are unambiguously normal particles or antiparticles, independent of choice of coordinate system, and, because of the bi-orthonormality relation of Eq. (11), we can assign classical probabilities for them to have specific three-momenta.

2.3. Fields and Interactions

Multiple particle states can be straightforwardly introduced as members of a Fock space over the Hilbert space of position states $|x; \lambda\rangle$. First, in order to allow for multiparticle states with different types of particles, extend the position state of each individual particle with a *particle type index* n, such that

$$\langle x', n'; \lambda | x, n; \lambda\rangle = \delta_n^{n'} \delta^4(x' - x)\,.$$

Then, construct a basis for the Fock space of multiparticle states as symmetrized products of N single particle states:

$$|x_1, n_1, \lambda_1; \ldots ; x_N, n_N, \lambda_N\rangle \equiv (N!)^{-1/2} \sum_{\text{perms } \mathcal{P}} |x_{\mathcal{P}1}, n_{\mathcal{P}1}; \lambda_{\mathcal{P}1}\rangle \cdots |x_{\mathcal{P}N}, n_{\mathcal{P}N}; \lambda_{\mathcal{P}N}\rangle ,$$

where the sum is over all permutations $\mathcal{P}$ of $1, 2, \ldots , N$ (since, for simplicity, I am only considering scalar particles in the present work, only Bose-Einstein statistics need be accounted for).

It is then convenient to introduce a *creation field* operator $\hat{\psi}^{\dagger}(x, n; \lambda)$ such that

$$\hat{\psi}^{\dagger}(x, n; \lambda)|x_1, n_1, \lambda_1; \ldots ; x_N, n_N, \lambda_N\rangle = |x, n, \lambda; x_1, n_1, \lambda_1; \ldots ; x_N, n_N, \lambda_N\rangle ,$$

with the corresponding annihilation field $\hat{\psi}(x, n; \lambda)$ having the commutation relation

$$[\hat{\psi}(x', n'; \lambda), \hat{\psi}^{\dagger}(x, n; \lambda_0)] = \delta_n^{n'} \Delta(x' - x; \lambda - \lambda_0) .$$

Further, define

$$\hat{\psi}(x, n) \equiv \int_{\lambda_0}^{\infty} d\lambda \, \hat{\psi}(x, n; \lambda) ,$$

so that

$$[\hat{\psi}(x', n'), \hat{\psi}^{\dagger}(x, n; \lambda_0)] = \delta_n^{n'} \Delta(x' - x) .$$

Now, an individual interaction vertex can be considered an event at which some number of incoming particles are destroyed and some number of outgoing particles are created (note that I am using the qualifiers "incoming" and "outgoing" here in the sense of the path evolution parameter λ, not time—which means that we are *not* separately considering particles and antiparticles at this point). Such an interaction can be modeled using a *vertex operator* constructed from the appropriate number of annihilation and creation operators.

For example, consider the case of an interaction with two incoming particles, one of type n_A and one of type n_B, and two outgoing particles of the same types. The vertex operator for this interaction is

$$\hat{V} \equiv g \int d^4x \, \hat{\psi}^{\dagger}(x, n_A; \lambda_0)\hat{\psi}^{\dagger}(x, n_A; \lambda_0)\hat{\psi}(x, n_A)\hat{\psi}(x, n_A), \tag{15}$$

where the coefficient g represents the relative probability amplitude of the interaction.

In the following, it will be convenient to use the special adjoint $\hat{\psi}^{\ddagger}$ defined by

$$\hat{\psi}^{\ddagger}(x,n) = \hat{\psi}^{\dagger}(x,n;\lambda_0) \text{ and } \hat{\psi}^{\ddagger}(x,n;\lambda_0) = \hat{\psi}^{\dagger}(x,n)\,.$$

With this notation, the expression for $\hat{V}$ becomes

$$\hat{V} = g \int d^4x\, \hat{\psi}^{\ddagger}(x,n_A)\hat{\psi}^{\ddagger}(x,n_B)\hat{\psi}(x,n_A)\hat{\psi}(x,n_B)\,.$$

To account for the possibility of any number of interactions, we just need to sum up powers of $\hat{V}$ to obtain the *interaction operator*

$$\hat{G} \equiv \sum_{m=0}^{\infty} \frac{(-\mathrm{i})^m}{m!} \hat{V}^m = \mathrm{e}^{-i\hat{V}}\,, \tag{16}$$

where the $1/m!$ factor accounts for all possible permutations of the m identical factors of $\hat{V}$. Note that, unlike the usual scattering operator, there is no time ordering in the summation here (more on this in Sec. 3.2).

The $-\mathrm{i}$ factors are introduced in Eq. (16) so that $\hat{G}$ is unitary relative to the special adjoint (that is, $\hat{G}^{\ddagger}\hat{G} = \hat{G}\hat{G}^{\ddagger} = 1$), so long as $\hat{V}$ is self-adjoint relative to it (that is, $\hat{V}^{\ddagger} = \hat{V}$). The self-adjointness of $\hat{V}$ implies that an interaction must have the same number of incoming and outgoing particles, of the same types, at least when only one possible type of interaction is involved [as is the case with the example of Eq. (15)]. The formalism can be easily extended to allow for multiple types of interactions by adding additional terms to the definition of $\hat{V}$. In this case, only the overall operator $\hat{V}$ needs to be self-adjoint, not the individual interaction terms.

Now, clearly we can also construct a Fock space from the three-momentum representation states $|t,\boldsymbol{p};\lambda_0\rangle$ and $|t,\boldsymbol{p}\rangle$. We can then define the multiparticle time-limit states

$$\begin{aligned} \langle \boldsymbol{p}'_{1\pm}, n'_1; \ldots | &\equiv \lim_{t'_i \to \pm\infty} \langle t'_1, \boldsymbol{p}_{1\pm}, n'_1; \ldots |\,, \\ |\boldsymbol{p}_{1\pm}, n_1; \ldots; \lambda_0\rangle &\equiv \lim_{t_i \to \mp\infty} |t_1, \boldsymbol{p}_{1\pm}, n_1, \lambda_0; \ldots\rangle\,. \end{aligned}$$

In these states, each particle is *either* a normal particle (+) *or* and antiparticle (−). Note that the limit is taken to $+\infty$ for outgoing particles, but to $-\infty$ for outgoing antiparticles (and vice versa for incoming particles).

These multiparticle three-momentum states can be used with the interaction operator $\hat{G}$ to compute multipoint interaction amplitudes. For example, the four point amplitude for one incoming particle, one incoming antiparticle, one outgoing particle and one outgoing antiparticle is given by

$$\begin{aligned} G\ &(\boldsymbol{p}'_{1+}, n'_1; \boldsymbol{p}'_{2-}, n'_2 | \boldsymbol{p}_{1+}, n_1; \boldsymbol{p}_{2-}, n_2; \lambda_0) \\ &= (2\omega_{\boldsymbol{p}'_1} 2\omega_{\boldsymbol{p}'_2} 2\omega_{\boldsymbol{p}_1} 2\omega_{\boldsymbol{p}_2})^{1/2} \langle \boldsymbol{p}'_{1+}, n'_1; \boldsymbol{p}'_{2-}, n'_2 | \hat{G} | \boldsymbol{p}_{1+}, n_1; \boldsymbol{p}_{2-}, n_2; \lambda_0 \rangle. \end{aligned} \tag{17}$$

The $2\omega_{\boldsymbol{p}}$ factors are required by the resolution of the identity for the multiparticle three-momentum states, generalizing the single particle case of Eq. (11). Expanding $\hat{G}$ as in Eq. (16) gives a sum of Feynman diagrams for possible number of interactions. The time-limited three-momentum states give the correct truncated amplitudes for the external legs of the diagrams.(15)

3. DECOHERENCE

The bi-orthonormality condition of Eq. (11) already provides an example of decoherence. The operator $(2\omega_{\boldsymbol{p}})|t, \boldsymbol{p}_{\pm}; \lambda_0\rangle\langle \boldsymbol{p}|$ represents the quantum proposition that a particle or antiparticle has a coarse-grained history in which it is free with three-momentum $\boldsymbol{p}$. The fact that these operators are orthogonal by Eq. (11) and resolve the identity by Eq. (13) indicates that these histories are decoherent and classical probabilities can be assigned as to whether a particle is in one such history or another.(29)

In this section I will explore further this concept of decohering histories of particle paths. I will start with the familiar case of the two slit experiment, to provide a heuristic example of the analysis of measurement-induced decoherence using the spacetime path formalism. This is followed by consideration of scattering experiments and then, finally, extension of these ideas to the universe as a whole.

3.1. Two Slit Experiment

The canonical two-slit experiment has, of course, been analyzed several times previously, both in terms of path integrals and decoherence (see, for example, Refs. 9, 11, 18, 30). Nevertheless, it is still instructive to use this familiar case as a means for introducing the application of the formalism defined in Sec. 2.

Presume that incoming particles are prepared to have a fixed three-momentum $\boldsymbol{p}$. Then, we can take a particle emitted at time t_0 to be in the three-momentum state $|t_0, \boldsymbol{p}_+; \lambda_0\rangle$. Further, assume that the flight time is long enough that, when the particles reach the slits, they can be considered to be in the on-shell state $|\boldsymbol{p}_+\rangle$.

For the purposes of the discussion here, it is sufficient to further idealize the experiment by considering the slits to be single points at positions $\boldsymbol{x}_i$, for $i = 1, 2$. The state for the particle to reach one or the other of the slits is then

$$|\boldsymbol{x}_{i+}\rangle = (2\pi)^{-3/2} \int d^3 p\, e^{-\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{x}_i} |\boldsymbol{p}_+\rangle \,.$$

From Eq. (12), the corresponding probability amplitudes are

$$\phi_i = \langle \boldsymbol{x}_{i+} | t_0, \boldsymbol{p}_+; \lambda_0\rangle (2\omega_{\boldsymbol{p}}) = e^{\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{x}_i} ,$$

corresponding to an incoming plane wave. Taking the plane of the slits to be perpendicular to the direction of $\boldsymbol{p}$ results in $\phi_1 = \phi_2 = 1$, corresponding to the equal probability of the particle reaching any point on that plane. Since the particle is blocked from passing except through the slits, we can clearly renormalize the ϕ_i so that

$$\phi_1 = \phi_2 = \frac{1}{\sqrt{2}} \,.$$

Suppose the particle passes through the slit at $\boldsymbol{x}_i$ at some time t_i. One can now consider its remaining path separately, starting at $(t_i, \boldsymbol{x}_i)$ and ending at some position $\boldsymbol{x}$ on the final screen of the experiment. Qualitatively, the amplitude for this can be given by

$$\psi_i(\boldsymbol{x}) = \langle \boldsymbol{x}_+ | t_i, \boldsymbol{x}_{i+}; \lambda_0\rangle \,.$$

The amplitude for passing through either slit and reaching $\boldsymbol{x}$ is then

$$\psi(\boldsymbol{x}) = \phi_1\psi_1(\boldsymbol{x}) + \phi_2\psi_2(\boldsymbol{x}) = \frac{1}{\sqrt{2}}(\langle \boldsymbol{x}_+ | t_1, \boldsymbol{x}_{1+}; \lambda_0\rangle + \langle \boldsymbol{x}_+ | t_2, \boldsymbol{x}_{2+}; \lambda_0\rangle). \tag{18}$$

The result of the experiment is a measurement made of the final position $\boldsymbol{x}$. This measurement is represented by a measuring instrument eigenstate $|m(\boldsymbol{x})\rangle$ such that

$$\langle m(\boldsymbol{x}') | m(\boldsymbol{x})\rangle = \delta^3(\boldsymbol{x}' - \boldsymbol{x}) \,. \tag{19}$$

The measurement eigenstate $|m(\boldsymbol{x})\rangle$ must be weighted by the amplitude $\psi(\boldsymbol{x})$ for the particle to reach position $\boldsymbol{x}$. From the point of view of particle paths, each state $\psi(\boldsymbol{x})|m(\boldsymbol{x})\rangle$ can be viewed as representing the entire coarse-grained history of a particle being emitted, passing through one or the other of the slits and being measured as arriving at position $\boldsymbol{x}$. Due to the orthogonality condition of Eq. (19), these coarse-grained history states do not interfere with each other—that is, the histories *decohere*, so a classical probability of $|\psi(\boldsymbol{x})|^2$ can be assigned to them. From Eq. (18), it is clear that this probability will, however, include interference effects between the slit-specific amplitudes ψ_1 and ψ_2.

We can, of course, also represent the less-coarse-grained histories for the particle passing through *just* one slit as $\psi_i(\boldsymbol{x})|m(\boldsymbol{x})\rangle$, for $i = 1, 2$. But these histories do *not* decohere, since

$$\psi_1^*(\boldsymbol{x})\psi_2(\boldsymbol{x})\langle m(\boldsymbol{x})|m(\boldsymbol{x})\rangle,$$

is not zero (actually, with the delta function normalization of Eq. (19) this value is infinite, but that would not be so for a more realistic instrument with finite resolution).

Suppose, however, that we add a measuring device that measures whether the particle passes through slit 1 or slit 2. This device has two eigenstates denoted $|s(i)\rangle$, for $i = 1, 2$, such that

$$\langle s(i)|s(j)\rangle = \delta_{ij}\,.$$

The coarse-grained history for a particle being measured as passing through slit i then being measured as reaching position $\boldsymbol{x}$ is now $\psi_i(\boldsymbol{x})|s(i)\rangle$ $|m(\boldsymbol{x})\rangle$. These histories now *do* decohere, since

$$\psi_i^*(\boldsymbol{x}')\psi_j(\boldsymbol{x})\langle s(i)|s(j)\rangle\langle m(\boldsymbol{x}')|m(\boldsymbol{x})\rangle = |\psi_i(\boldsymbol{x})|^2\delta_{ij}\delta^3(\boldsymbol{x}' - \boldsymbol{x})$$

and they can be given the individual probabilities $|\psi_i(\boldsymbol{x})|^2$.

The results of this analysis are, of course, as would be expected. Notice, however, that, rather than the usual approach of time evolving states, the approach here constructs states representing entire coarse-grained particle histories. Measurements are modeled as being coupled to specific points in these histories. Thus, rather than modeling some initial state of a measuring instrument evolving into a state with a specific measurement, the states $|s(i)\rangle$ and $|m(\boldsymbol{x})\rangle$ represent the occurrence of specific measurement values *as part of* the overall history of the experiment.

The occurrence of a specific measurement value places a constraint on the possible particle paths that can be included in any coarse-grained history consistent with that measurement. Thus, $|s(i)\rangle$ places the constraint that paths must pass through slit i, while $|m(\boldsymbol{x})\rangle$ places the constraint that the paths end at position $\boldsymbol{x}$. If a coarse-grained history includes *all* possible paths consistent with the constraints for specific measurement values, and no others, then the orthogonality of the measurement states causes such a history to decohere from other similar histories for different measurement values.

In this sense, the tensor product of the measurement eigenstates provides a complete, orthogonal basis for decoherent coarse-grained histories of the experiment. Given observations of certain measurement values, the experiment, as a whole, can be said with certainty to be "in" the specific history eigenstate selected by those measurement values. Nothing definitive, however, can be said about finer-grained histories, since these histories do not decohere.

The important point here is that the experiment is not modeled as "evolving" into a decoherent state. Rather it is *entire coarse-grained histories* of the experiment that decohere, with observed measurements simply identifying which actual history was observed.

3.2. Scattering

We now turn to the more general problem of multiparticle scattering, with the goal of providing an analysis similar to that provided for the two slit experiment in Sec. 3.1. Clearly, we can base this on the multiple particle interaction formalism discussed in Sec. 2.3.

However, the formulation of Eq. (17) is still not that of the usual scattering matrix, since the incoming state involves particles at $t \to +\infty$ but antiparticles at $t \to -\infty$, and vice versa for the outgoing state. To construct the usual scattering matrix, it is necessary to have incoming multiparticle states that are composed of individual asymptotic particle states that are all consistently for $t \to -\infty$ and outgoing states with individual asymptotic states all for $t \to +\infty$. That is, we need to shift to considering "incoming" and "outgoing" in the sense of *time*.

To do this, we can take the viewpoint of considering antiparticles to be positive energy particles traveling forwards in time, rather than negative energy particles traveling backwards in time. Since both particles and their antiparticles will then have positive energy, it becomes necessary to explicitly label antiparticles with separate (though related) types from their corresponding particles. Let n_+ denote the type label for a normal particle type and n_- denote the corresponding antiparticle type.

For normal particles of type n_+, position states are defined as in Eq. (7):

$$\langle x, n_+ | x_0, n_+; \lambda_0 \rangle = \theta(x^0 - x_0^0)\Delta(x - x_0) \,.$$

For antiparticles of type n_-, however, position states are now defined such that

$$\langle x, n_- | x_0, n_-; \lambda_0 \rangle = \theta(x^0 - x_0^0)\Delta(x_0 - x) \,.$$

Note the reversal with respect to Eq. (8) of x_0 and x on the right side of this equation.

Carrying through the derivation for antiparticle three-momentum states based on the new antiparticle states $|x, n_-\rangle$ does, indeed, give positive energy states, but with reversed three momentum:[15]

$$|t, \boldsymbol{p}, n_-\rangle = (2\omega_{\boldsymbol{p}})^{-1} \int_{-\infty}^{t} dt_0 \, |t_0, \boldsymbol{p}, n_-; \lambda_0\rangle \,,$$

where

$$|t_0, \boldsymbol{p}, n_-; \lambda_0\rangle = |t_0, -\boldsymbol{p}_+, n; \lambda_0\rangle \,.$$

Further, taking the limit $t \to +\infty$ gives the on-shell states

$$|\boldsymbol{p}, n_-\rangle \equiv \lim_{t \to +\infty} |t, \boldsymbol{p}, n_-\rangle = (2\pi)^{1/2}(2\omega_{\boldsymbol{p}})^{-1}| + \omega_{\boldsymbol{p}}, -\boldsymbol{p}; \lambda_0\rangle \,.$$

We can now reasonably construct Fock spaces with single time, multiparticle basis states

$$|t; \boldsymbol{p}_1, n_{1\pm}; \ldots ; \boldsymbol{p}_N, n_{N\pm}; \lambda_0\rangle \equiv |t, \boldsymbol{p}_1, n_{1\pm}; \ldots ; t, \boldsymbol{p}_N, n_{N\pm}; \lambda_0\rangle \,,$$

over all combinations of particle and antiparticle types and, similarly,

$$|t; \boldsymbol{p}_1, n_{1\pm}; \ldots ; \boldsymbol{p}_N, n_{N\pm}\rangle \equiv |t, \boldsymbol{p}_1, n_{1\pm}; \ldots ; t, \boldsymbol{p}_N, n_{N\pm}\rangle \,.$$

We can then take consistent time limits for particles and antiparticles alike to get the incoming and outgoing states

$$\begin{aligned} |\boldsymbol{p}_1, n_{1\pm}; \ldots ; \boldsymbol{p}_N, n_{N\pm}; \lambda_0\rangle &= \lim_{t \to -\infty} |t; \boldsymbol{p}_1, n_{1\pm}; \ldots ; \boldsymbol{p}_N, n_{N\pm}; \lambda_0\rangle \,, \\ |\boldsymbol{p}_1, n_{1\pm}; \ldots ; \boldsymbol{p}_N, n_{N\pm}\rangle &= \lim_{t \to +\infty} |t; \boldsymbol{p}_1, n_{1\pm}; \ldots ; \boldsymbol{p}_N, n_{N\pm}\rangle \,. \end{aligned}$$

Reorganizing the interaction amplitude of Eq. (17) in terms of these new asymptotic states gives the more usual form using the scattering operator $\hat{S}$. Showing explicitly the asymptotic time limit used for each particle:

$$\begin{aligned}
&\langle +\infty, \boldsymbol{p}'_{1+}, n'_1; -\infty, \boldsymbol{p}'_{2-}, n'_2|\hat{G}| - \infty, \boldsymbol{p}_{1+}, n_1; +\infty, \boldsymbol{p}_{2-}, n_2; \lambda_0\rangle \\
&\qquad = \langle +\infty, \boldsymbol{p}'_1, n'_{1+}; +\infty, \boldsymbol{p}_2, n_{2-}|\hat{S}| - \infty, \boldsymbol{p}_1, n_{1+}; -\infty, \\
&\qquad\quad \boldsymbol{p}'_2, n'_{2-}; \lambda_0\rangle \qquad (20) \\
&\qquad = \langle \boldsymbol{p}'_1, n'_{1+}; \boldsymbol{p}_2, n_{2-}|\hat{S}|\boldsymbol{p}_1, n_{1+}; \boldsymbol{p}'_2, n'_{2-}; \lambda_0\rangle\,.
\end{aligned}$$

More generally, consider applying $\hat{S}$ to an incoming state of N particles, giving $\hat{S}|\boldsymbol{p}_1, n_{1\pm}; \ldots; \boldsymbol{p}_N, n_{N\pm}; \lambda_0\rangle$. Using the resolution of the identity

$$\begin{aligned}
&\sum_{N=0}^{\infty} \sum_{n_{i\pm}} \int d^3p_1 \cdots d^3p_N \left[\prod_{i=1}^{N} 2\omega_{\boldsymbol{p}_i}\right] \\
&\qquad \times|\boldsymbol{p}_1, n_{1\pm}; \ldots; \boldsymbol{p}_N, n_{N\pm}; \lambda_0\rangle\langle \boldsymbol{p}_1, n_{1\pm}; \ldots; \boldsymbol{p}_N, n_{N\pm}| = 1\,, \qquad (21)
\end{aligned}$$

expand the state $\hat{S}|\boldsymbol{p}_1, n_{1\pm}; \ldots; \boldsymbol{p}_N, n_{N\pm}; \lambda_0\rangle$ as

$$\begin{aligned}
&\hat{S}|\boldsymbol{p}_1, n_{1\pm}; \ldots; \boldsymbol{p}_N, n_{N\pm}; \lambda_0\rangle \\
&\quad = \sum_{N'=0}^{\infty} \sum_{n_{i\pm}} \int d^3p'_1 \cdots d^3p'_{N'} \left[\prod_{i=1}^{N'} 2\omega_{\boldsymbol{p}'_i}\right] |\boldsymbol{p}'_1, n'_{1\pm}; \ldots; \boldsymbol{p}'_{N'}, n'_{N'\pm}; \lambda_0\rangle \\
&\qquad \times\langle \boldsymbol{p}'_1, n'_{1\pm}; \ldots; \boldsymbol{p}'_{N'}, n'_{N'\pm}|\hat{S}|\boldsymbol{p}_1, n_{1\pm}; \ldots; \boldsymbol{p}_N, n_{N\pm}; \lambda_0\rangle\,.
\end{aligned}$$

This shows how $\hat{S}|\boldsymbol{p}_1, n_{1\pm}; \ldots; \boldsymbol{p}_N, n_{N\pm}; \lambda_0\rangle$ is a superposition of possible out states, with the square of the scattering amplitude giving the probability of a particular out state for a particular in state.

Note that each operator

$$|\boldsymbol{p}_1, n_{1\pm}; \ldots; \boldsymbol{p}_N, n_{N\pm}; \lambda_0\rangle\langle \boldsymbol{p}_1, n_{1\pm}; \ldots; \boldsymbol{p}_N, n_{N\pm}|$$

represents not the proposition that the particles have the three-momenta $\boldsymbol{p}_i$ at any one point in time, but, rather, that they have these momenta *for their entire history*. Since, by Eq. (21), these operators orthogonally resolve the identity, these histories do not interfere with each other and are thus trivially decoherent. This is why the square of the scattering amplitude gives a classical probability.

It should also be noted that both $|\boldsymbol{p}_1, n_1\pm; \ldots; \boldsymbol{p}_N, n_N\pm; \lambda_0\rangle$ and $\hat{S}|\boldsymbol{p}_1, n_1\pm; \ldots; \boldsymbol{p}_N, n_N\pm; \lambda_0\rangle$ represent states of the entire "universe" under consideration. The state $|\boldsymbol{p}_1, n_1\pm; \ldots; \boldsymbol{p}_N, n_N\pm; \lambda_0\rangle$ represents a universe in which all particles remain free and there are no interactions. This free particle state does not evolve into $\hat{S}|\boldsymbol{p}_1, n_1\pm; \ldots; \boldsymbol{p}_N, n_N\pm; \lambda_0\rangle$. Rather, $\hat{S}|\boldsymbol{p}_1, n_1\pm; \ldots; \boldsymbol{p}_N, n_N\pm; \lambda_0\rangle$ is the state of a *different* universe, in which interactions *do* occur. The operator $\hat{S}$ simply provides a convenient method for constructing the states of the interacting particle universe from the states of the free particle universe.

3.3. Probabilities

The decoherence of coarse-grained histories allows for a mathematically consistent assignment of probabilities. Physically, the concept of "probability" here is to be interpreted as meaning the likelihood that an arbitrary selection from the population of all possible coarse-grained histories will yield a specific history. In other words, the greater the probability assigned to a history, the more likely it is that it is actually the history of the "universe" under consideration.

Of course, it is not immediately clear how the assignment of probabilities to entire histories relates to the statistics of physical results of measurement processes occuring within those histories. Before continuing, I would like to briefly consider this point.

To simplify further discussion, let a single Greek letter, say α, represent an entire configuration $\boldsymbol{p}_1, \boldsymbol{p}_2, \ldots$ of on-shell particle three-momenta. In this notation, incoming states $|\boldsymbol{p}_1, n_1\pm; \ldots; \boldsymbol{p}_N, n_N\pm; \lambda_0\rangle$ are denoted as simply $|\alpha; \lambda_0\rangle$ and outgoing states $|\boldsymbol{p}'_1, n'_1\pm; \ldots; \boldsymbol{p}'_{N'}, n'_{N'}\pm\rangle$ become $|\alpha'\rangle$. The resolution of the identity from Eq. (21) is then

$$\int \mathrm{d}\alpha\, |\alpha; \lambda_0\rangle\langle\alpha| = 1\,,$$

where $\int \mathrm{d}\alpha$ denotes the entire set of integrals and summations.

Suppose the same scattering experiment is repeated, independently, n times. Let $|\psi_i; \lambda_0\rangle$ be the asymptotic free incoming state for the ith repetition. Considered all together, the overall free particle state of this "universe" of experiments is

$$|\psi; \lambda_0\rangle = |\psi_1; \lambda_0\rangle \cdots |\psi_n; \lambda_0\rangle\,.$$

The state $\hat{S}|\psi; \lambda_0\rangle$ is then the superposition of all possible histories of interactions among the incoming particles. At a large enough time after all the experiments take place, the outgoing particles should be on-shell in a state $\langle\alpha| = \langle\alpha_1, \ldots, \alpha_n|$, where each $\langle\alpha_i|$ is the outgoing state for the ith repetition, and the probability for this overall result is $|\langle\alpha|\hat{S}|\psi; \lambda_0\rangle|^2$.

If we can neglect interactions between each experiment repetition, then the scattering amplitude should approximately factor:

$$\langle\alpha|\hat{S}|\psi; \lambda_0\rangle \approx \langle\alpha_1|\hat{S}|\psi_1; \lambda_0\rangle \cdots \langle\alpha_n|\hat{S}|\psi_n; \lambda_0\rangle .$$

If the repetitions are widely spacelike separated, then this follows from the cluster decomposition of $\hat{S}$.[31,32] Thus, the overall probability for scattering into α is approximately the product of the scattering probabilities for each cluster.

Now, consider a measurement $m(\alpha_i)$ taken of each experimental result. Suppose the measurement determines in which member of a disjoint partition of values α_i lies. The probability amplitude for a measurement of α_i to have the specific (discrete) value m_i is

$$\psi_i(m_i) \equiv \int_{m_i} \mathrm{d}\alpha_i \, \langle\alpha_i|\hat{S}|\psi_i; \lambda_0\rangle ,$$

where the integration is over the subset of values corresponding to the measurement result m_i. Assuming identical preparation for the experiments, the ψ_i should all be the same function $\psi(m)$.

The overall weighted measurement state is then

$$\psi(m_1) \cdots \psi(m_n)|m_1\rangle \cdots |m_n\rangle , \tag{22}$$

where $|m_i\rangle$ is the measuring instrument eigenstate for the measurement of the ith experimental result. Once again, this overall state represents a specific coarse-grained history in which the specific measurement results $m_1, \ldots, m_n$ are obtained for the n repetitions of the scattering experiment. The question to be asked is how the relative frequency of any given result in this set compares to the quantum mechanically predicted probabilities $|\psi(m_i)|^2$ (see also Refs. 33,34 for discussions of this question in the context of traditional and many-worlds interpretations of quantum mechanics).

Define the *relative frequency* of some specific measurement result ℓ within the set $m_1, \ldots, m_n$ to be

$$f_\ell(m_1, \ldots, m_n) \equiv \frac{1}{n} \sum_{i=1}^{n} \delta_{m_i \ell} . \tag{23}$$

Since this relative frequency is itself an observable, a relative frequency operator $\hat{F}_\ell$ can be defined which has relative frequencies as its eigenvalues:

$$\hat{F}_\ell |m_1\rangle \cdots |m_n\rangle = f_\ell(m_1, \dots, m_n)|m_1\rangle \cdots |m_n\rangle \,.$$

Define the average

$$\langle \hat{F}_\ell \rangle \equiv \sum_{m_1 \dots m_n} f_\ell(m_1, \dots, m_n)|\psi(m_1)|^2 \cdots |\psi(m_n)|^2 \,.$$

Substituting Eq. (23) and using the normalization $\sum |\psi(m_i)|^2 = 1$ then gives[33,34]

$$\langle \hat{F}_\ell \rangle = |\psi(\ell)|^2 \,. \tag{24}$$

Equation (24) is mathematically consistent with the probability interpretation of quantum mechanics. However, this mathematical average still needs to be connected to physical results. To do this, consider a further measurement, this time of the relative frequency $\hat{F}_\ell$. Note that this is a measurement *of the previous measurements* m_i, perhaps simply by counting the records of the results of those measurements. The new measurement results are thus the functions $f_\ell(m_1, \dots, m_n)$, with corresponding eigenstates $|f_\ell(m_1, \dots, m_n)\rangle$.

The overall state

$$\psi(m_1) \cdots \psi(m_n)|m_1\rangle \cdots |m_n\rangle |f_\ell(m_1, \dots, m_n)\rangle, \tag{25}$$

then represents the history in which a specific relative frequency is measured for a specific set of scattering results. Since these history states are still decoherent due to the original set of measurement states, the total probability for observing a certain relative frequency f_ℓ is given by the sum of the probabilities for each of the states for which nf_ℓ of the m_i have the value ℓ. This probability is

$$p(f_\ell) = \binom{n}{nf_\ell} p_\ell^{nf_\ell}(1 - p_\ell)^{n(1-f_\ell)} \,,$$

where $p_\ell = |\psi(\ell)|^2$.

The probability $p(f_\ell)$ is just a Bernoulli distribution. By the de Moivre–Laplace theorem, for large n, this distribution is sharply peaked about the mean $f_\ell = p_\ell = \langle \hat{F}_\ell \rangle$. Thus, the probability becomes almost certain that a choice of one of the histories (25) will be a history in which the observed relative frequency will be near the prediction given by the usual Born probability interpretation. Of course, for finite n, there is still the possibility of a "maverick" universe in which f_ℓ is arbitrarily far from the expected value—but it would seem that (in most cases, at least) our universe is simply not one of these.

There have been a number of criticisms in the literature of using relative frequency as above as the basis for the quantum probability interpetation (see, for example, Refs. 35, 36). However, these criticisms relate to attempts to actually justify the Born probability interpretation itself. My goal here is more modest: to simply show that, assuming the Born probability rule applies for history states, that the statistics of repeated measurement results within such a history would be expected to follow a similar rule. In this regard, criticisms of, e.g., circularity and the need for additional assumptions, do not apply here (for justification of the Born rule itself for quantum states, the arguments of Zurek based on "environment-assisted invariance"[37,38] would seem to be relevent, but I will not pursue this further here).

3.4. Cosmological States

Extending the ideas from Sec. 3.3, let $|\Psi;\lambda_0\rangle$ be the *cosmological state* representing the free-particle evolution of the universe from the initial condition of the big bang. Then $\hat{S}|\Psi;\lambda_0\rangle$ is a superposition of all possible interacting particle histories of the universe. Obviously, this really should also include interactions leading to bound states, not just scattering. For the purposes of the present discussion, however, it is sufficient to simply allow that some of the products of the scattering interactions may be composite particles rather than fundamental.

A specific coarse-grained history in this superposition can be identified by a specific configuration α of all classically observable particles throughout the life of the universe. (For the present discussion, assume that this is a large but finite number of particles.) In this case, $\Psi(\alpha) = \langle\alpha|\hat{S}|\Psi;\lambda_0\rangle$ might reasonably be called the "wave function of the universe," since $|\Psi(\alpha)|^2$ is the probability of the universe having the configuration α given its cosmological state $\hat{S}|\Psi;\lambda_0\rangle$ (clearly, for this to be the true wave function of the universe, $\hat{S}$ would need to include the effects of all the actual types of interactions, including gravity[39]). Further, given that the universe can be decomposed into approximately isolated subsystems, the overall probability $|\Psi(\alpha)|^2$ will approximately factor into a product of probabilities for the histories of each of the subsystems.

Now, consider that any classically measurable quantity should be a function of some subset of the classical configuration α. Divide α into $\alpha_1, \alpha_2, \ldots$ (this division need not be complete or disjoint), and let $m_i(\alpha_i)$ represent the result of a measurement made on the subset α_i. We can then represent a measuring instrument for m_i as having a set of orthogonal states $|m_i(\alpha_i)\rangle$ representing the various possible measurement outcomes.

Of course, a measuring instrument is, itself, a part of the universe being measured. And a complete theory of measurement would have to account for how such an instrument, as a subsystem of the universe, becomes correlated with some other part of the universe and itself decoheres into non-interfering states. However, it is not the intent of this paper to present such a complete theory (for a discussion of related issues in a non-relativistic context, see Refs. 37,40 and the references given there).

For our purposes here, it is sufficient to consider a "measurement process" to be a process that produces a persistent record of distinguishable results correlated with the measured subsystem, based on classical variables. By definition, such a process can be abstracted into a representation by orthogonal result states. We can then extend the kind of analysis used in Sec. 3.1 for the two slit experiment, and consider the complete measurement state of the universe to be

$$\Psi(\alpha_1, \alpha_2, \ldots)|m_1(\alpha_1)\rangle|m_2(\alpha_2)\rangle \ldots , \tag{26}$$

in which the measurement results are correlated with the corresponding configuration of the universe with probability amplitude given by the wave function $\Psi(\alpha_1, \alpha_2, \ldots)$.

Further, suppose some of the measurements are of relative frequencies of results of repeated experiments. Then, by extension of the argument in Sec. 3.3, for a large enough number of repetitions within a "typical" history, the observed relative frequency will accurately reflect the probabilities as predicted by quantum theory.

It is worth emphasizing again that the universe does not "evolve into" the state (26). Rather, this state represents a *complete* coarse-grained history of the universe, in which the measurement values $m_1(\alpha_1), m_2(\alpha_2), \ldots$ are observed, implying the corresponding classical configuration $\alpha_1, \alpha_2, \ldots$ for the universe. The correlation of the measurements with the configuration of the universe means that the measurement results effectively provide information on which coarse-grained history the universe is "really in."

It is in exactly this sense that the universe can be represented as the eigenstate (26) of the measurements made within it.

4. CONCLUSION

I would like to conclude with some remarks on the interpretational implications of the concept of cosmological states defined in Sec. 3.4.

Each cosmological state $|\alpha_1, \alpha_2, \ldots\rangle$, with corresponding measurement state (26), represents a possible, complete, coarse-grained history of the universe. Of course, each such course-grained history is still a quantum superposition of many fine-grained histories. However, if we include in the m_i all the measurements made in the entire history of the universe, then the corresponding measurement states are the finest-grained possible that can be determined by inhabitants of the universe.

The measurement states themselves are decoherent and orthogonal, but the distribution of measurement results in any specific coarse-grained history will still show the effects of interference of the superposed fine-grained histories (as we saw in the simple case of the two slit experiment in Sec. 3.1). This reflects the fact that such interference effects really are observed in our universe.

Now, all measurements ever made so far determine only some very small portion of a configuration α of the universe. Nevertheless, in principle, it is consistent to consider all such measurements to be, indeed, made on a portion of some overall α, selecting a specific classical history from the family given by $\hat{S}|\Psi; \lambda_0\rangle$, and that this is the "real" history of the universe. The formalism here allows for no further judgement on the "real" history of the universe beyond the coarse-grained superpositions determined by the measurement results.

This conception is very much in the spirit of the original work by Everett[10] on what has become known as the "many worlds" interpretation. The key point is that there is no need to consider any sort of observation by observation "collapse of the wave function." Rather, consistent measurement results are determined by appropriately decohering histories,[9,11,12] and known measurement results constrain the possible histories.

However, Everett and his successors[41] generally considered the dynamic evolution of states in time. In this formulation, a measurement process at a certain time causes a state to "branch" into orthogonal components, one for each possible measurement result. This leads almost inevitably to the conception of the continual dynamic creation of "many worlds," only one of which is ever really apparent to any observer.

In contrast, in the approach presented here, entire coarse-grained histories of the universe decohere for all time. It is only necessary to consider one of these to be the "real" history of the actual universe, though we have only very partial information on which history this actually is. There is no need to consider the other histories to have any "real" existence at all. Nevertheless, within the "real" history of our universe, all observations made at the classical level will be distributed according to the probabilistic rules of quantum theory.

Instead of a "no collapse" interpretation, this is, in a sense, a "one collapse" interpretation—the single collapse of the wave function of the universe into the cosmological state of the entire coarse-grained history of the universe. It is as if God did indeed play dice with the universe, but that He threw very many dice just once, determining the fate of the universe for all space and time.

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