

Categorizing Different Approaches to the Cosmological Constant Problem

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We have found that proposals addressing the old cosmological constant problem come in various categories. The aim of this paper is to identify as many different, credible mechanisms as possible and to provide them with a code for future reference. We find that they all can be classified into five different schemes of which we indicate the advantages and drawbacks.

KEY WORDS: cosmological constant problem; modifying gravity.

1. STATEMENT OF THE PROBLEM

It is clear that the cosmological constant problem is one of the major obstacles to further progress for both particle physics and cosmology. Actually, after the remarkable discoveries and subsequent confirmations starting in 1997 (SN)^(1–11) (WMAP)^(12,13) (Boomerang)^(14,15) (SdSS)^(16,17) (Hubble)⁽¹⁸⁾ that the universe really is accelerating its expansion, there appear to be at present at least three cosmological constant problems. In a nutshell these are: Why is the cosmological constant so small, why is it then not exactly equal to zero and why is its energy density today of the same order of magnitude as the matter energy density? Although the recent observations concerning the accelerated expansion are usually attributed to a small, non-vanishing Λ , alternatives have been suggested, some of which we briefly discuss.

In this overview however, we will be mainly concerned with the first of these questions, the so-called “old cosmological constant problem”. To

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phrase it more precisely, the question is why is the effective cosmological constant, Λ_{eff} , defined as $\Lambda_{\text{eff}} = \Lambda + 8\pi G \langle \rho \rangle$ so close to zero.² Or, in other words, why is the vacuum state of our universe (at present) so close to the classical vacuum state of zero energy, or perhaps better, why is the resulting four-dimensional curvature so small, or why does Nature prefer a flat spacetime?

The different contributions to the vacuum energy density coming from ordinary particle physics and graviton loops, would naively give a value for $\langle \rho \rangle$ of order M_p^4 (assuming a Planck-scale cutoff for the standard model), which then would have to be (nearly) cancelled by the unknown “bare” value of Λ . Note at this point that only the Λ_{eff} is observable, not Λ .

This cancellation has to be better than about 120 decimal places if we compare the zero-point energy of a scalar field, using the Planck scale as a cut-off, and the experimental value of $\rho_{\text{vac}} = \langle \rho \rangle + \Lambda/8\pi G$, being 10^{-47} GeV^4 . As is well known, even if we take a TeV scale cut-off the difference between experimental and theoretical results still requires a fine-tuning of about 59 orders of magnitude. This magnificent fine-tuning seems to suggest that we miss an important point here. In this paper we give an overview of the main ideas that have appeared in trying to figure out what this point might be.

We have found that proposals addressing this problem come in various categories. The aim of this paper is to identify as many different, credible mechanisms as possible and to provide them with a code for future reference. Our identification code will look as follows, see Table (1).

In other words, an approach examining 6-dimensional supersymmetry for a solution will be coded Type IAa1.

For reviews on the history of the cosmological constant (problem) and many phenomenological considerations, see Refs. 19–28.

2. TYPE 0: FINETUNING

One can set the cosmological constant to any value one likes, by simply adjusting by hand the value of the bare cosmological constant to all, classical and quantum mechanical, contributions to the vacuum energy. No further explanation then is needed. This fine-tuning has to be precise to better than at least 59 decimal places (assuming some TeV scale cut-off), but that is of course not a practical problem. Since we feel some important aspects of gravity are still lacking in our understanding and

²Note that using this definition we use units in which the cosmological constant has dimension GeV^2 throughout. Our metric convention is $(-+++)$.

Table 1. Classification of different approaches

Type 0: Just Finetuning	
Type I: Symmetry	
A: Continuous	(a) Supersymmetry (b) Scale invariance (c) Conformal Symmetry
B: Discrete	(d) Imaginary Space (e) Energy \rightarrow $-$ Energy (f) Holography (g) Sub-super-Planckian (h) Antipodal Symmetry (i) Duality Transformations
Type II: Back-reaction Mechanism	(a) Scalar (b) Gravitons (c) Screening Caused by Trace Anomaly (d) Running CC from Renormalization Group
Type III: Violating Equiv. Principle	(a) Non-local Gravity, Massive Gravitons (b) Ghost Condensation (c) Fat Gravitons (d) Composite graviton as Goldst. boson
Type IV: Statistical Approaches	(a) Hawking Statistics (b) Wormholes (c) Anthropic Principle, Cont. (d) Anthropic Principle, Discrete

Each of them can also be thought of as occurring (1) Beyond 4D, or (2) Beyond Quantum Mechanics, or both.

nothing can be learned from this “mechanism”, we do not consider this to be a physical solution. However, it is a possibility that we can not totally ignore and it is mentioned here just for sake of completeness.

3. TYPE I: SYMMETRY PRINCIPLE

A natural way to understand the smallness of a physical parameter is in terms of a symmetry that altogether forbids any such term to appear. This is also often referred to as “naturalness”: a theory obeys naturalness only if all of its small parameters would lead to an enhancement of its exact symmetry group when replaced by zero. Nature has provided us with several examples of this. Often mentioned in this respect is the example of the mass of the photon. The upper bound on the mass (squared) of the photon from terrestrial measurements of the magnetic field yields

$$m_\gamma^2 \lesssim \mathcal{O}(10^{-50}) \text{ GeV}^2. \quad (1)$$

The most stringent estimates on Λ_{eff} nowadays give

$$\Lambda_{\text{eff}} \lesssim \mathcal{O}(10^{-84}) \text{ GeV}^2 \quad (2)$$

We “know” the mass of the photon to be in principle exactly equal to 0, because due to the $U(1)$ gauge symmetry of QED, the photon has only two physical degrees of freedom (helicities). In combination with Lorentz invariance this sets the mass equal to zero. A photon with only two transverse degrees of freedom can only get a mass if Lorentz invariance is broken. This suggests that there might also be a symmetry acting to keep the effective cosmological constant an extra 34 orders of magnitude smaller.

A perhaps better example to understand the smallness of a mass is chiral symmetry. If chiral symmetry were an exact invariance of Nature, quark masses and in particular masses for the pseudoscalar mesons (π, K, η) would be zero. The spontaneous breakdown of chiral symmetry would imply pseudoscalar Goldstone bosons, which would be massless in the limit of zero quark mass. The octet (π, K, η) would be the obvious candidate and indeed the pion is by far the lightest of the mesons. Making this identification of the pion as a pseudo-Goldstone boson associated with spontaneous breaking of chiral symmetry, we can understand why the pion-mass is so much smaller than for example the proton mass.

3.1. Supersymmetry

One symmetry with this desirable feature is supersymmetry. The quantum corrections to the vacuum coming from bosons are of the same magnitude, but opposite sign compared to fermionic corrections, and therefore cancel each other. The vacuum state in an exactly supersymmetric theory has zero energy. However, supersymmetric partners of the Standard Model particles have not been found, so standard lore dictates that SUSY is broken at least at the TeV scale, which induces a large vacuum energy.

One often encounters some numerology in these scenarios, e.g., Ref. 29, linking the scale of supersymmetry breaking M_{susy} , assumed to be of order TeV, and the Planck mass M_P , to the cosmological constant. Experiment indicates

$$M_{\text{susy}} \sim M_P \left(\frac{\Lambda}{M_P^2} \right)^\alpha \quad \text{with} \quad \alpha = \frac{1}{8}. \quad (3)$$

The standard theoretical result however indicates $M_{\text{susy}} \sim \Lambda^{1/2}$.

However, to discuss the cosmological constant problem, we need to bring gravity into the picture. This implies making the supersymmetry transformations local, leading to the theory of supergravity or SUGRA for short, where the situation is quite different. In exact SUGRA the lowest energy state of the theory, generically has negative energy density: the vacuum of supergravity is AdS.³ This has inspired many to consider so-called no-scale supergravity models. See Ref. 20 or supersymmetry textbooks such as Ref. 31 for excellent reviews.

The important point is that there is an elegant way of guaranteeing a flat potential, with $V = 0$ after susy-breaking, by using a nontrivial form of the Kähler potential G . For a single scalar field z we have

$$\begin{aligned} V &= e^G \left[\frac{\partial_z G \partial_{z^*} G}{\partial_z \partial_{z^*} G} - 3 \right] \\ &= \frac{9e^{4G/3}}{\partial_z \partial_{z^*} G} (\partial_z \partial_{z^*} e^{-G/3}), \end{aligned} \quad (4)$$

where κ^2 , the gravitational constant, has been set equal to one. A flat potential with $V = 0$ is obtained if the expression in brackets vanishes for all z , which happens if

$$G = -3 \log(z + z^*), \quad (5)$$

and one obtains a gravitino mass:

$$m_{3/2} = \langle e^{G/2} \rangle = \langle (z + z^*)^{-3/2} \rangle, \quad (6)$$

which as required is not fixed by the minimization of V . Thus provided we are prepared to choose a suitable, non-trivial form for the Kähler potential G , it is possible to obtain zero CC. Moreover, the gravitino mass is left undetermined; it is fixed dynamically through non-gravitational radiative corrections. The minimum of the effective potential occurs at

$$V_{\text{eff}} \approx -(m_{3/2})^4, \quad (7)$$

where in this case after including the observable sector and soft symmetry-breaking terms we will have $m_{3/2} \approx M_W$. Such a mass is ruled out cosmologically⁽³²⁾ and so other models with the same ideas have been

³This negative energy density can also be forbidden by postulating an unbroken R-symmetry. $D = 11$ SUGRA is a special case; its symmetries implicitly forbid a CC term, see Ref. 30.

constructed that allow a very small mass for the gravitino, also by choosing a specific Kähler potential, see Ref. 33.

That these constructions are possible is quite interesting and in the past there has been some excitement when superstring theory seemed to implicate precisely the kinds of Kähler potential as needed here, see for example Ref. 34. However, that is not enough, these simple structures are not expected to hold beyond zeroth order in perturbation theory.

3.1.1. Unbroken SUSY

To paraphrase Witten:⁽³⁵⁾ “Within the known structure of physics, supergravity in four dimensions leads to a dichotomy: either the symmetry is unbroken and bosons and fermions are degenerate, or the symmetry is broken and the vanishing of the CC is difficult to understand”. However, as he also argues in the same article, in $2 + 1$ dimensions, this unsatisfactory dichotomy does not arise: SUSY can explain the vanishing of the CC without leading to equality of boson and fermion masses, see also Ref. 36.

The argument here is that in order to have equal masses for the bosons and fermions in the same supermultiplet one has to have unbroken global supercharges. These are determined by spinor fields which are covariantly constant at infinity. The supercurrents J^μ from which the supercharges are derived are generically not conserved in the usual sense, but covariantly conserved: $D_\mu J^\mu = 0$. However, in the presence of a covariantly constant spinor ($D_\mu \epsilon = 0$), the conserved current $\bar{\epsilon} J^\mu$ can be constructed and therefore, a globally conserved supercharge:

$$Q = \int d^3x \bar{\epsilon} J^0. \quad (8)$$

But in a $2 + 1$ dimensional spacetime any state of non-zero energy produces a geometry that is asymptotically conical at infinity.⁽³⁷⁾ The spinor fields are then no longer covariantly constant at infinity⁽³⁸⁾ and so even when supersymmetry applies to the vacuum and ensures the vanishing of the vacuum energy, it does not apply to the excited states. This is special to $2 + 1$ -dimensions. Explicit examples have been constructed in Refs. 39–42. Two further ideas in this direction, one in $D < 4$ and one in $D > 4$ are,^(43,44) however the latter later turned out to be internally inconsistent.⁽⁴⁵⁾

In any case, what is very important is to make the statement of “breaking of supersymmetry” more precise. As is clear, we do not observe mass degeneracies between fermions and bosons, therefore supersymmetry, even if it were a good symmetry at high energies between excited states, is broken at lower energies. However, and this is the point, as the example of

Witten shows, the issue of whether we do or do not live in a supersymmetric vacuum state is another question. In some scenarios it is possible to have a supersymmetric vacuum state, without supersymmetric excited states. This really seems to be what we are looking for. The observations of a small or even zero CC could point in the direction of a (nearly) supersymmetric vacuum state.

Obviously the question remains how this scenario and the absence nevertheless of a supersymmetric spectrum can be incorporated in 4 dimensions, where generically spacetime is asymptotically flat around matter sources, instead of asymptotically conical.

3.2. Imaginary Space

So far, the most obvious candidate-symmetry to enforce zero vacuum energy density, supersymmetry, does not seem to work; we need something else. What other symmetry could forbid a cosmological constant term? Einstein's equations are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = -8\pi GT_{\mu\nu}. \quad (9)$$

As was first observed by 't Hooft (unpublished), we can forbid the cosmological constant term by postulating that the transformations:

$$\mathbf{x} \rightarrow i\mathbf{x}, \quad t \rightarrow it, \quad g_{\mu\nu} \rightarrow g_{\mu\nu} \quad (10)$$

are symmetry operations.⁴ The different objects in Einstein's equations transform under this as follows:

$$\begin{aligned} \Gamma_{\mu\nu}^{\lambda} &= \frac{1}{2}g^{\lambda\rho} [\partial_{\mu}g_{\nu\rho} + \partial_{\nu}g_{\mu\sigma} - \partial_{\rho}g_{\mu\sigma}] \rightarrow -i\Gamma_{\mu\nu}^{\lambda}, \\ R_{\mu\nu} &= \partial_{\nu}\Gamma_{\mu\lambda}^{\lambda} - \partial_{\lambda}\Gamma_{\mu\nu}^{\lambda} - \Gamma_{\mu\nu}^{\rho}\Gamma_{\rho\sigma}^{\sigma} + \Gamma_{\mu\sigma}^{\rho}\Gamma_{\nu\rho}^{\sigma} \rightarrow -R_{\mu\nu}, \\ R &= g^{\mu\nu}R_{\mu\nu} \rightarrow -R. \end{aligned}$$

Furthermore we have

$$T_{\mu\nu} \rightarrow -T_{\mu\nu} \quad (11)$$

as long as there are no vacuum terms in the expression for $T_{\mu\nu}$. So Einstein's equations transforms as

$$G_{\mu\nu} - \Lambda g_{\mu\nu} = -8\pi GT_{\mu\nu} \rightarrow -G_{\mu\nu} - \Lambda g_{\mu\nu} = +8\pi GT_{\mu\nu}. \quad (12)$$

⁴ A related suggestion was made in Ref. 46.

Therefore, if we postulate (10) as a symmetry of nature, a CC term is forbidden! Classically $E \sim p^2$ and therefore $E \rightarrow -E$.

However, at first sight, this symmetry does not seem to ameliorate the situation much, since this transformation is not a symmetry of the Standard Model. In particular, we have

$$p^2 = m^2 \quad \text{with} \quad p^\mu = i\partial^\mu \rightarrow -ip^\mu \quad (13)$$

Therefore, imposing (10) as a symmetry of nature, seems to imply that either there exists a copy of all known matter particles with negative mass squared, or that all particles should be massless. In the second case, if we take this symmetry seriously, we should conclude that the smallness of the cosmological constant and the smallness of particle masses (relative to the Planck-scale) although of quite a different order of magnitude, have a common origin.

Another approach is to view this symmetry in combination with boundary conditions. Generally in quantum field theory we Fourier transform our field and impose (often periodic) boundary conditions only on its components in real space. Perhaps the vacuum state does not have to satisfy boundary conditions. In that case it would not matter whether one would impose boundary conditions in either real or imaginary space. Excited states, have to obey boundary conditions, and would violate the symmetry.

Besides, since the transformation (10) effectively changes spacelike dimensions into timelike dimensions and vice versa, a natural playground to study its implications could be a 2+2- or 3+3-dimensional spacetime. The possibility of extra timelike dimensions is not very often considered, because it is assumed that the occurrence of tachyonic modes prevents the construction of physically viable models. However, it was shown in Ref. 47 that these constraints might not be as severe as to rule out this option beforehand. Extra timelike dimensions have been tried before to argue for a vanishing cosmological constant, see Ref. 48.

3.3. Energy \rightarrow $-$ Energy

Another approach in which negative energy states are considered has been recently proposed in Ref. 49. Here the discrete symmetry $E \rightarrow -E$ is imposed explicitly on the matter fields by adding to the Lagrangian an identical copy of the normal matter fields, but with an overall minus sign:

$$\mathcal{L} = \sqrt{-g} \left(M_{Pl}^2 R - \Lambda_0 + \mathcal{L}_{\text{matt}}(\psi, D_\mu) - \mathcal{L}_{\text{matt}}(\hat{\psi}, D_\mu) + \dots \right), \quad (14)$$

where Λ_0 is the bare cosmological constant. The Lagrangian with fields $\hat{\psi}$ occurring with the wrong sign is referred to as the ghost sector. The two matter sectors have equal but opposite vacuum energies, and therefore cancelling contributions to the cosmological constant.

Crucial in this reasoning is that there is no coupling other than gravitational between the normal matter fields and their ghost counterparts, otherwise the Minkowski vacuum would not be stable. This gravitational coupling moreover has to be sufficiently small in order to suppress the gravitationally induced interactions between the two sectors and to make sure that the quantum gravitational corrections to the bare cosmological constant are kept very small. It is therefore necessary to impose a UV cutoff on these contributions of order 10^{-3} eV, corresponding to a length scale of about $100 \mu\text{m}$.⁵

Moreover, in order to ensure stability of the vacuum, also some new Lorentz symmetry violating physics is required to suppress processes where normal matter particles and ghosts emerge from the vacuum. In addition, one also has to assume that the ghost sector is rather empty, compared to the normal matter sector, in order not to spoil standard cosmology with such an exotic type dark matter.

3.4. Scale Invariance, e.g., Conformal Symmetry

The above symmetry might be viewed as a specific example of the more general framework of conformal symmetry, $g_{\mu\nu} \rightarrow f(x^\mu)g_{\mu\nu}$. Massless particles are symmetric under a bigger group than just the Lorentz group, namely, the conformal group. This group does not act as symmetries of Minkowski spacetime, but under a (mathematically useful) completion, the “conformal compactification of Minkowski space”. This group is 15-dimensional and corresponds to $SO(2, 4)$, or if fermions are present, the covering group $SU(2, 2)$. Conformal symmetry forbids any term that sets a length scale, so a cosmological constant is not allowed, and indeed also particle masses necessarily have to vanish.

General coordinate transformations and scale invariance, i.e., $g_{\mu\nu} \rightarrow fg_{\mu\nu}$, are incompatible in general relativity. The $R\sqrt{-g}$ term in the Einstein–Hilbert action is the only quantity that can be constructed from the metric tensor and its first and second derivatives only, that is invariant under general coordinate transformations. But this term is not even invariant under a global scale transformation $g_{\mu\nu} \rightarrow fg_{\mu\nu}$ for which f

⁵In Section 5.3 a proposal by one of the authors of Ref. 49 is discussed in which such a cutoff is argued to arise from the graviton not being a point-like particle but having this finite size.

is constant. R transforms with Weyl weight -1 and $\sqrt{-g}$ with weight $+2$. There are two ways to proceed to construct a scale invariant action: introducing a new scalar field,^(50,51) that transforms with weight -1 , giving rise to so-called scalar-tensor theories, or consider Lagrangians that are quadratic in the curvature scalar. We consider the second. Pioneering work in this direction was done in Refs. 52–56. See for example Refs. 57, 58 for some recent studies and many references.

Gravity can be formulated under this bigger group, leading to “Conformal gravity”, defined in terms of the Weyl tensor, which corresponds to the traceless part of the Riemann tensor:

$$\begin{aligned} S_G &= -\alpha \int d^4x \sqrt{-g} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa} \\ &= -2\alpha \int d^4x \sqrt{-g} \left(R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \right) + (\text{boundary terms}), \end{aligned} \quad (15)$$

where $C^{\mu\nu\lambda\kappa}$ is the conformal Weyl tensor, and α is a dimensionless gravitational coupling constant. Thus the Lagrangian is quadratic in the curvature scalar and generates field equations that are fourth-order differential equations. Note that the a cosmological constant term is not allowed, since it violates conformal invariance. Based on the successes of gauge theories with spontaneously broken symmetries and the generation of the Fermi-constant, one may suggest to also dynamically induce the Einstein action with its Newtonian constant as a macroscopic limit of a microscopical conformal theory. This approach has been studied especially by Mannheim and Kazanas, see Refs. 59–64 to solve the CC problem.

These fourth-order equations reduce to a fourth-order Poisson equation:

$$\nabla^4 B(r) = f(r), \quad (16)$$

where $B(r) = -g_{00}(r)$ and the source is given by

$$f(r) = 3(T_0^0 - T_r^r)/4\alpha B(r). \quad (17)$$

For a static, spherically symmetric source, conformal symmetry allows one to put $g_{rr} = -1/g_{00}$ and the exterior solution to (15) can be written:⁽⁶⁴⁾

$$g_{rr} = -1/g_{00} = 1 - \beta(2 - 3\beta\gamma)/r - 3\beta\gamma + \gamma r - kr^2. \quad (18)$$

The non-relativistic potential reads

$$V(r) = -\beta/r + \gamma r/2 \quad (19)$$

which for a spherical source can be completely integrated to yield

$$B(r > R) = -\frac{r}{2} \int_0^R dr' f(r') r'^2 - \frac{1}{6r} \int_0^R dr' f(r') r'^4. \quad (20)$$

Compared to the standard second-order equations:

$$\nabla^2 \phi(r) = g(r) \quad \rightarrow \quad \phi(r > R) = -\frac{1}{r} \int_0^R dr' g(r') r'^2 \quad (21)$$

we see that the fourth-order equations contain the Newtonian potential in its solution, but in addition also a linear potential term that one would like to see dominate over Newtonian gravity only at large distances. The factors β and γ in for example (19) are therefore given by

$$\beta(2 - 3\beta\gamma) = \frac{1}{6} \int_0^R dr' f(r') r'^4, \quad \gamma = -\frac{1}{2} \int_0^R dr' f(r') r'^2. \quad (22)$$

Note in passing that in the non-relativistic limit of GR the (0, 0)-component, where $R_{ij} \simeq (1/2R - \Lambda)g_{ij}$ and therefore $R = g^{\mu\nu}R_{\mu\nu} \simeq R_{00} + 3(1/2R - \Lambda)$, or $R \simeq -2R_{00} + 6\Lambda$ using also that $R_{00} \simeq (-1/2)\nabla^2 g_{00}$ becomes

$$\nabla^2 \phi = 4\pi G \left(\rho - \frac{\Lambda}{4\pi G} \right), \quad (23)$$

the Poisson equation for the normal Newtonian potential modified with a cosmological constant. This can easily be solved to give

$$\phi = -\frac{GM}{r} + \frac{1}{6}\Lambda r^2. \quad (24)$$

However, modifying gravity only at large distances cannot solve the cosmological constant problem. The (nearly) vanishing of the vacuum energy and consequently flat and relatively slowly expanding spacetime is a puzzle already at distance scales of say a meter. We could expect deviations of GR at galactic scales, avoiding the need for dark matter, but at solar system scales GR in principle works perfectly fine. It seems hard to improve on this, since the world simply is not scale invariant.

There is also a more serious problem with the scenario of Mannheim and Kazanas described above. In order for the linear term not to dominate already at say solar system distances, the coefficient γ has to be chosen very small. Not only does this introduce a new kind of fine-tuning, it is simply not allowed to

chose these coefficients at will. The linear term will always dominate over the Newtonian $1/r$ -term, in contradiction with the perfect agreement of GR at these scales. See also Ref. 65 who raised the same objection.

This scenario therefore does not work.

3.4.1. Λ as Integration Constant, Unimodular Theory

Another option is to reformulate the action principle in such a way that a scale dependent quantity like the scalar curvature, remains undetermined by the field equations themselves. These are the so-called “unimodular” theories of gravity, see e.g., Refs. 66, 67. Note that although the action is not globally scale invariant, Einstein’s equations in the absence of matter and with vanishing cosmological constant is. The dynamical equations of pure gravity in other words, are invariant with respect to global scale transformations, and since we have that $R = 0$, they are scale-free, i.e., they contain no intrinsic length scale.

There is a way to keep the scale dependence undetermined also after including matter which also generates a cosmological constant term. This well-known procedure,^(68,69) assumes

$$\sqrt{-g} = \sigma(x) \quad \rightarrow \quad \delta\sqrt{-g} = 0, \quad (25)$$

where $\sigma(x)$ is a scalar density of weight +1. The resulting field equations are

$$R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R = -\kappa \left(T_{\mu\nu} - \frac{1}{4}g_{\mu\nu}T \right). \quad (26)$$

The covariant derivative $D_\mu G_{\mu\nu} = D_\mu T_{\mu\nu} = 0$ still vanishes and from this one obtains

$$R - \kappa T = -4\Lambda, \quad (27)$$

where Λ now appears as an integration constant and the factor of 4 has been chosen for convenience since substituting this back we recover the normal Einstein equations with cosmological constant.

Recently, some arguments have been put forward in which a sort of unimodular theory is supposed to originate more naturally as a result of “the quantum microstructure of spacetime being capable of readjusting itself, soaking up any vacuum energy”, see Refs. 24, 70, 71.

Obviously this does not solve anything, nor does it provide a better understanding of the cosmological constant. The value of the integration

constant Λ has to be inserted by hand in order to arrive at the correct value.

Besides, sometimes it is concluded that there are two inequivalent Einstein equations for gravity, describing two theories that are only equivalent classically, but not quantum mechanically. The group of canonical transformations is much larger than that of unitary transformations in Hilbert space, forcing one to quantize in “preferred” coordinates. We do not agree with this point of view. The constraint $g^{\mu\nu}\delta g_{\mu\nu} = 0$ just reflects a choice of coordinates, a certain gauge.

This issue is closely related to the question of the measure of the quantum gravity functional integral (see discussions by B.S. DeWitt,^(72,73) 't Hooft^(74,75)): Is the integration variable $g_{\mu\nu}$, $g^{\mu\nu}$ or some other function of the metric? The differences in the amplitudes for these theories all appear in the one-loop diagrams, in the form of quartically divergent momentum-independent ghost loops. These all disappear after renormalization and therefore the theories are indistinguishable physically.

3.5. Holography

Gravitational holography⁽⁷⁶⁾ limits the number of states accessible to a system. The entropy of a region generally grows with its covering area (in Planck units) rather than with its volume, implying that the dimension of the Hilbert space, i.e., the number of degrees of freedom describing a region, is finite and much smaller than expected from quantum field theory. Considering an infinite contribution to the vacuum energy is not correct because states are counted that do not exist in a holographic theory of gravity.

It is a symmetry principle since there is a projection from states in the bulk-volume, to states on the covering surface.

In Refs. 77, 78 it is noted that in effective field theory in a box of size L with UV cutoff M the entropy S scales extensively, as $S \sim L^3 M^3$. A free Weyl fermion on a lattice of size L and spacing $1/M$ has $4^{(LM)^3}$ states and entropy⁶ $S \sim (LM)^3$. The corresponding entropy density $s = S/V$ then is $s = M^3$. In $d = 4$ dimensions quantum corrections to the vacuum energy are therefore of order:

$$\rho_{\text{vac}} = \frac{\Lambda}{8\pi G} + \langle \rho \rangle = \frac{\Lambda}{8\pi G} + \mathcal{O}(s^{4/3}), \quad (28)$$

⁶ For bosons the number of states is not limited by a lattice cutoff alone, so in this argument one has to limit oneself to fermions. For bosons there are an infinite number of states, in contradiction to the conjecture of the Holographic Principle.

since both $\langle \rho \rangle$ and s are dominated by ultraviolet modes (see also Ref. 79). Thus finite s implies finite corrections to $\langle \rho \rangle$.

Using a cutoff M , $E \sim M^4 L^3$ is the maximum energy for a system of size L . States with $L < R_s \sim E$, or $L > M^{-2}$ (in Planckian units) have collapsed into a black-hole. If one simply requires that no state in the Hilbert space exists with $R_s \sim E > L$, then a relation between the size L of the region, providing an IR cutoff, and the UV cutoff M can be derived. Under these conditions entropy grows no faster than $A^{3/4} \sim L^{3/2}$, with A the area. If these black hole states give no contribution to $\langle \rho \rangle$, we obtain

$$\langle \rho \rangle \sim s^{4/3} \sim \left(\frac{L^{3/2}}{L^3} \right)^{4/3} \sim L^{-2}. \quad (29)$$

In Ref. 77 this same scaling was obtained by assuming that $S < A$ as usual, but that the delocalized states have typical Heisenberg energy $1/L$:

$$\langle \rho \rangle \sim \frac{s}{L} \sim \frac{L^2}{L^3 L} \sim L^{-2}. \quad (30)$$

Plugging in for L the observed size of the universe today the quantum corrections are only of order 10^{-10} eV^4 .

However, this does not yield the correct equation of state.⁽⁷⁹⁾ During matter dominated epochs, to which WMAP and supernova measurements are sensitive, the horizon size grows as the RW-scale factor, $a(t)^{3/2}$, so the above arguments imply

$$\Lambda_{\text{eff}}(L) \sim a(t)^{-3}, \quad (31)$$

or, $w \equiv p/\rho = 0$ at largest scales, since $\rho(t) \sim a(t)^{-3(1+w)}$. The data on the other hand give $w < -0.78$ (95% CL). In for example^(76,78) $\Lambda(L)$ is at all times comparable to the radiation + matter energy density, which is also argued to give problems for structure formation.⁽⁸⁰⁾

Holography-based scenarios thus naively lead to a cosmological constant that is far less constant than what the data require. This makes a connection between holography and dark energy a lot harder to understand.⁷

⁷In Ref. 81 in a different context a similar relation between the CC and the volume of the universe is derived, thus suffering from the same drawbacks.

More recently however, another proposal was made⁽⁸²⁾ where instead L is taken to be proportional to the size of the future event horizon:

$$L(t) \sim a(t) \int_t^\infty \frac{dt'}{a(t')}. \quad (32)$$

This L describes the size of the largest portion of the universe that any observer will see. This could be a reasonable IR cutoff. It is argued that in this case the equation of state parameter w can be close enough to -1 to agree with the data. This relation is rather *ad hoc* chosen, and its deeper meaning, if any, still has to be discovered.

Another reason to discuss holography in the context of the cosmological constant problem lies in trying to reconcile string theory with the apparent observation of living in a de Sitter spacetime. The discussion centers around the semi-classical result that de Sitter space has a finite entropy, inversely related to the cosmological constant, see for example Ref. 83. Thus one may reason that de Sitter space should be described by a theory with a finite number of independent quantum states and that a theory of quantum gravity should be constructed with a finite dimensional Hilbert space. In this reasoning a cosmological constant should be understood as a direct consequence of the finite number of states in the Hilbert space describing the world. Ergo, the larger the cosmological constant, the smaller the Hilbert space. However, in Ref. 84 it is argued that this relation between the number of degrees of freedom and the CC is not so straightforward.

3.6. “Symmetry” between Sub- and Super-Planckian Degrees of Freedom

This rather speculative reasoning originates from a comparison with condensed matter physics and is due to Volovik, see for example Refs. 85–92. The vacuum energy of superfluid $^4\text{Helium}$, calculated from an effective theory containing phonons as elementary bosonic particles and no fermions is

$$\rho_\Lambda = \sqrt{-g} E_{\text{Debye}}^4 \quad (33)$$

with g the determinant of the acoustic metric, since c is now the speed of sound, and $E_{\text{Debye}} = \hbar c/a$, with a the interatomic distance, which plays the role of the Planck length. However, in the condensed matter case, the full theory exists: a second quantized Hamiltonian describing a collection of a macroscopic number of structureless $^4\text{Helium}$ bosons or $^3\text{Helium}$

fermions, in which the chemical potential μ acts as a Lagrange multiplier to ensure conservation of the number of atoms:

$$H - \mu N = \int d\mathbf{x} \psi^\dagger(\mathbf{x}) \left[-\frac{\nabla^2}{2m} - \mu \right] \psi(\mathbf{x}) + \int d\mathbf{x} d\mathbf{y} V(\mathbf{x} - \mathbf{y}) \psi^\dagger(\mathbf{x}) \psi^\dagger(\mathbf{y}) \psi(\mathbf{y}) \psi(\mathbf{x}). \quad (34)$$

Using this Hamiltonian H to calculate the energy density of the ground state we get

$$E_{\text{vac}} = E - \mu N = \langle \text{vac} | H - \mu N | \text{vac} \rangle. \quad (35)$$

An overall shift of the energy in H is cancelled in a shift of the chemical potential. Exact calculation shows that not only the low energy degrees of freedom from the effective theory, the phonons, but also the higher energy, “trans-Planckian” degrees of freedom have to be taken into account.

Besides, for a liquid of N identical particles at temperature T in a volume V in equilibrium, the relation between the energy E and pressure P is given by the Gibbs–Duhem equation

$$E = TS + \mu N - PV. \quad (36)$$

Therefore at $T = 0$ the energy density of the ground state becomes

$$\rho_{\text{vac}} \equiv \frac{E_{\text{vac}}}{V} = -P_{\text{vac}}, \quad (37)$$

the same equation of state as for the vacuum state in GR. Using just thermodynamic arguments, it is argued that in the infinite volume, zero temperature limit, this gives exactly zero vacuum energy density as long as there are no external forces, i.e., no pressure acting on the quantum liquid. And assuming there is no matter, no curvature and no boundaries which could give rise to a Casimir effect.⁽⁸⁶⁾

The conclusion therefore is that, if these thermodynamic arguments are also valid in a gravitational background for the universe as a whole and up to extremely high energies, one would expect a perfect cancellation between sub- and super-Planckian degrees of freedom contributing to the vacuum energy, resulting in zero cosmological constant.

Moreover, it is also argued that a non-zero cosmological constant arises from perturbations of the vacuum at non-zero temperature. The vacuum energy density would be proportional to the matter energy density, solving the coincidence problem as well.

A similar result is obtained by Ref. 93. In their formulation the world is like a crystal. The atoms of the crystal are in thermal equilibrium and exhibit therefore zero pressure, making the cosmological constant equal to zero.

Both approaches strongly depend on the quantum systems reaching their equilibrium state. However, in the presence of a cosmological constant, the matter in the universe never reaches its equilibrium state.⁽⁹⁴⁾

3.7. Interacting Universes, Antipodal Symmetry

This is an approach developed by Linde^(95,96) arguing that the vacuum energy in our universe is so small because there is a global interaction with another universe where energy densities are negative. Consider the following action of two universes with coordinates x_μ and y_α respectively ($x_\mu, y_\alpha = 0, 1, \dots, 3$) and metrics $g_{\mu\nu}(x)$ and $\bar{g}_{\alpha\beta}(y)$, containing fields $\phi(x)$ and $\bar{\phi}(y)$:

$$S = N \int d^4x d^4y \sqrt{g(x)} \sqrt{\bar{g}(y)} \left[\frac{M_P^2}{16\pi} R(x) + L(\phi(x)) - \frac{M_P^2}{16\pi} R(y) - L(\bar{\phi}(y)) \right], \quad (38)$$

and where N is some normalization constant. This action is invariant under general coordinate transformations in each of the universes separately. The important symmetry of the action is $\phi(x) \rightarrow \bar{\phi}(x)$, $g_{\mu\nu}(x) \rightarrow \bar{g}_{\alpha\beta}(x)$ and under the subsequent change of the overall sign: $S \rightarrow -S$. He calls this an antipodal symmetry, since it relates states with positive and negative energies. As a consequence we have invariance under the change of values of the effective potentials $V(\phi) \rightarrow V(\phi) + c$ and $V(\bar{\phi}) \rightarrow V(\bar{\phi}) + c$ where c is some constant. Therefore nothing in this theory depends on the value of the effective potentials in their absolute minima ϕ_0 and $\bar{\phi}_0$. Note that because of the antipodal symmetry $\phi_0 = \bar{\phi}_0$ and $V(\phi_0) = V(\bar{\phi}_0)$.

In order to avoid the troublesome issues of theories with negative energy states, one has to assume that there can be no interactions between the fields $\phi(x)$ and $\bar{\phi}(y)$. Therefore also the equations of motion for both fields are the same and similarly, also gravitons from both universes do not interact.

However some interaction does occur. The Einstein equations are

$$R_{\mu\nu}(x) - \frac{1}{2}g_{\mu\nu}R(x) = -8\pi GT_{\mu\nu}(x) - g_{\mu\nu} \left\langle \frac{1}{2}R(y) + 8\pi GL(\bar{\phi}(y)) \right\rangle, \quad (39)$$

$$R_{\alpha\beta}(y) - \frac{1}{2}\bar{g}_{\alpha\beta}R(y) = -8\pi GT_{\alpha\beta}(y) - \bar{g}_{\alpha\beta} \left\langle \frac{1}{2}R(x) + 8\pi GL(\phi(x)) \right\rangle. \quad (40)$$

Here $T_{\mu\nu}$ is the energy–momentum tensor of the fields $\phi(x)$ and $T_{\alpha\beta}$ the energy-momentum tensor for the fields $\bar{\phi}(y)$ and the averaging means

$$\langle R(x) \rangle = \frac{\int d^4x \sqrt{g(x)} R(x)}{\int d^4x \sqrt{g(x)}}, \quad (41)$$

$$\langle R(y) \rangle = \frac{\int d^4y \sqrt{\bar{g}(y)} R(y)}{\int d^4y \sqrt{\bar{g}(y)}}, \quad (42)$$

and similarly for $\langle L(x) \rangle$ and $\langle L(y) \rangle$.

Thus there is a global interaction between the universes X and Y . The integral over the whole history of the Y -universe changes the vacuum energy density of the X -universe. Assuming then that at late times the fields settle near the absolute minimum of their potential we have

$$R_{\mu\nu}(x) - \frac{1}{2}g_{\mu\nu}R(x) = -8\pi G g_{\mu\nu} [V(\bar{\phi}_0) - V(\phi_0)] - \frac{1}{2}g_{\mu\nu}R(y), \quad (43)$$

$$R_{\alpha\beta}(y) - \frac{1}{2}\bar{g}_{\alpha\beta}R(y) = -8\pi G g_{\alpha\beta} [V(\phi_0) - V(\bar{\phi}_0)] - \frac{1}{2}g_{\alpha\beta}R(x). \quad (44)$$

Thus at late stages the effective cosmological constant vanishes:

$$R(x) = -R(y) = \frac{32}{3}\pi G [V(\phi_0) - V(\bar{\phi}_0)] = 0, \quad (45)$$

since because of the antipodal symmetry $\phi_0 = \bar{\phi}_0$ and $V(\phi_0) = V(\bar{\phi}_0)$.

This could also be seen as a back-reaction mechanism, from one universe at the other.

3.8. Duality Transformations

3.8.1. S-Duality

A different proposal was considered in Ref. 97, where S-duality acting on the gravitational field is assumed to mix gravitational and matter degrees of freedom. The purpose is to show that whereas the original metric may be (A)dS, the dual will be flat. It is assumed that

$$R^b_a \equiv R^c_a{}^c_b = \Lambda \delta^a_b \quad (46)$$

with Λ the cosmological constant. The mixing between gravitational and matter degrees of freedom is obtained through a new definition of the

gravitational dual of the Riemann tensor, including the field strength F_{abcd} of a 3-form field A_{abc} . which equation of motion is simply $F_{abcd} = \omega \epsilon_{abcd}$, with ω some constant, see also Section 6.1:

$$\begin{aligned}\tilde{R}_{abcd} &= \frac{1}{2} \epsilon_{abef} \left(R^ef_{cd} + F^ef_{cd} \right) + \frac{1}{12} \epsilon_{abcd}, \\ \tilde{F}_{abcd} &= -\frac{1}{2} \epsilon_{abcd} R\end{aligned}\tag{47}$$

such that

$$\begin{aligned}\tilde{\tilde{R}}_{abcd} &= -R_{abcd}, \\ \tilde{\tilde{F}}_{abcd} &= -F_{abcd}.\end{aligned}\tag{48}$$

The equations of motion for the dual tensors become

$$\begin{aligned}\tilde{R}^a_b &= 3\omega \delta^a_b, \\ \tilde{F}_{abcd} &= -\frac{1}{3} \Lambda \epsilon_{abcd} \equiv \tilde{\omega} \epsilon_{abcd}.\end{aligned}\tag{49}$$

Therefore it seems that if the vev ω would vanish, the dual Ricci tensor, in casu the dual cosmological constant would also vanish. Hence the conclusion is that if we would “see” the dual metric, determined by the dual Riemann tensor, we would observe a flat spacetime.

However, with assumption (46), the trace of the left-hand-side of Einstein’s equation vanishes by definition. Hence, also the trace of the energy-momentum tensor should vanish, which in general is not the case. The field equations therefore appear to be inconsistent with the above assumption, unless $\omega = 0$, which makes the addition of the field strength term useless. This scenario, even aside from the other assumptions, therefore cannot work.

Note that S-duality is an important concept in stringtheory. If theories A and B are S-dual then $f_A(\alpha) = f_B(1/\alpha)$. It relates type I stringtheory to the $SO(32)$ heterotic theory, and type IIB theory to itself.

3.8.2. Hodge Duality

This duality between a r -form and a $(D - r)$ -form in D dimensions is studied,⁽⁹⁸⁾ where the cosmological constant is taken to be represented by a 0-form field strength, which is just a constant. This is related to the unimodular approach of Section 3.4.1 in the sense that they try to introduce the cosmological constant in a different way in the Einstein–Hilbert action.

3.9. Summary

A symmetry principle as explanation for the smallness of the cosmological constant in itself is very attractive. A viable mechanism that sets the cosmological constant to zero would be great progress, even if Λ would turn out to be non-zero. Since supersymmetry does not really seem to help, especially some form of scale invariance stands out as a serious option. Needless to say, it is hard to imagine how scale invariance could be used, knowing that the world around us is not scale invariant. Particle masses are small, but many orders of magnitude larger than the observed cosmological constant.

Another option might be that a symmetry condition enforcing ρ_{vac} equal to zero, could be reflected in a certain choice of boundary conditions. In such a scenario, the vacuum state would satisfy different boundary conditions than excited states. The $x \rightarrow ix$ transformation of Section 3.2 could be an example of this.

4. TYPE II: BACK-REACTION MECHANISMS

In this approach it is argued that any cosmological constant will be automatically cancelled, or screened, to a very small value by back-reaction effects on an expanding space. The effective cosmological constant then is small, simply because the universe is rather old. Often these effects are studied in an inflationary background, where a cosmological constant is most dominant. The physical idea of this mechanism can be understood in the context of the energy–time uncertainty principle. For a particle with mass m and co-moving wavevector \mathbf{k} in a spacetime with scalefactor $a(t)$ we have

$$E(\mathbf{k}, t) = \sqrt{m^2 + \|\mathbf{k}\|^2/a^2(t)}. \quad (50)$$

Thus growth of $a(t)$ increases the time a virtual particle of fixed m and \mathbf{k} can exist and, during inflation, virtual particles with zero mass and long enough wavelength can exist forever. The rate (Γ) at which they emerge from the inflationary vacuum depends upon the type of particle. Most massless particles are conformally invariant. In that case, Γ gives the number of particles emerging from the vacuum per unit conformal time η , so the number per unit physical time is

$$\frac{dn}{dt} = \frac{d\eta}{dt} \frac{dn}{d\eta} = \frac{\Gamma}{a}. \quad (51)$$

Their emergence rate thus falls like $1/a(t)$. This means that although those that are produced can exist forever, only very few are created, and their total effect during inflation is negligible, see e.g., Ref. 99.

However, two familiar massless particles are not conformally invariant, massless minimally coupled scalars and gravitons. Therefore in these two sections we consider their effects in more detail.

It should be noted that there exists a no-go theorem, derived by Weinberg, see Ref. 20 for details. The theorem states that the vacuum energy density cannot be cancelled dynamically, using a scalar field, without fine-tuning in any effective four-dimensional theory with constant fields at late times, that satisfies the following conditions:

1. General Covariance.
2. Conventional four-dimensional gravity is mediated by a *massless* graviton.
3. Theory contains a finite number of fields below the cutoff scale.
4. Theory contains no negative norm states.

Under these rather general assumptions the theorem states that the potential for the compensator field, which should adjust the vacuum energy to zero, has a runaway behavior. This means that there is no stationary point for the potential of the scalar field that should realize the adjustment, and thus the mechanism cannot work.

4.1. Scalar Field, Instabilities *in* dS-Space

The first attempts to dynamically cancel a “bare” cosmological constant were made by referring to instabilities in the case of a scalar field in de Sitter space. A massless minimally coupled scalar field ϕ has no de Sitter-invariant vacuum state and the expectation value of ϕ^2 is time-dependent. However, this breaking of de Sitter invariance is not reflected by the energy–momentum tensor, since $T_{\mu\nu}$ only contains derivatives and hence is not sensitive to long-wavelength modes. This changes if one includes interactions. Consider for example a $\lambda\phi^4$. Then

$$\langle T_{\mu\nu} \rangle \sim \lambda \langle \phi^2 \rangle^2 g_{\mu\nu} \propto t^2. \quad (52)$$

So in this case it is possible for $\langle T_{\mu\nu} \rangle$ to grow for some time, until higher order contributions become important. The infrared divergence results in a mass for the field which in turn stops the growth of $\langle T_{\mu\nu} \rangle$, see for example Ref. 100, 101.

Another illustrative, but unsuccessful attempt has been given by Dolgov.⁽¹⁰²⁾ He used a rather simple classical model for back-reaction:

$$\mathcal{L} = \frac{1}{2} \left(\partial_\alpha \phi \partial^\alpha \phi - \xi R \phi^2 \right), \quad (53)$$

where R is the scalar curvature and ξ a *negative* constant. The scalar field energy–momentum tensor at late times approaches the form of a cosmological constant term:

$$8\pi G \langle T_{\mu\nu} \rangle \sim \Lambda_0 g_{\mu\nu} + \mathcal{O}(t^{-2}). \quad (54)$$

$\Lambda_0 = 3H^2$ stands for the effective value of the cosmological constant during a de Sitter phase so the leading back-reaction term cancels this effect. The kinetic energy of the growing ϕ -field acts to cancel the cosmological constant. The no-go theorem of the previous section is circumvented, since the scalar field is not constant at late times.

Unfortunately, not only the cosmological constant term is driven to zero, Newton's constant is also screened:

$$G_{\text{eff}} = \frac{G_0}{1 + 8\pi G |\xi| \phi^2} \sim \frac{1}{t^2}, \quad (55)$$

where G_0 is the “bare” value of G at times where $\phi = 0$. This is a fatal flaw of many of such approaches.

Other models of this kind were also studied by Dolgov, see Refs. 103–105 but these proved to be unstable, leading quickly to a catastrophic cosmic singularity.

As we discussed, Weinberg's no-go theorem is widely applicable to such screening mechanisms. However, it was noted in e.g., Ref. 106, that conformal anomalies might provide a way around this. The Lagrangian obtains an additional term proportional to $\sqrt{g} \phi \Theta_\mu^\mu$, where Θ_μ^μ is the effect of the conformal anomaly.

However, as already noted by Weinberg,⁽²⁰⁾ this does not provide a loophole to get around the no-go theorem. The reason is that, although the field equation for ϕ now looks like

$$\frac{\partial \mathcal{L}}{\partial \phi} = \sqrt{-g} (T_\mu^\mu + \Theta_\mu^\mu), \quad (56)$$

which may suggest an equilibrium value for ϕ with zero trace, this is not sufficient for a flat space solution. The Einstein equation for a constant metric now becomes

$$0 = \frac{\partial \mathcal{L}_{\text{eff}}}{\partial g_{\mu\nu}} \propto e^{2\phi} \mathcal{L}_0 + \phi \Theta_{\mu}^{\mu}, \quad (57)$$

and the extra factor of ϕ shows that these two conditions are not the same. The reason is that the term Θ_{μ}^{μ} does not simply end up in T_{μ}^{μ} .

4.1.1. Radiative Stability in Scalar Field Feedback Mechanism

Another approach deserves to be mentioned here. This concerns a model that does not solve the cosmological constant problem, but does seem to provide a way to protect a zero or small cosmological constant against radiative corrections, without using a symmetry.^(107,108) This is achieved using a scalar field with a non-standard, curvature dependent kinetic term, such that in the limit where the scalar curvature goes to zero, the kinetic term vanishes.

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa^2} + \alpha R^2 + L_{\text{kin}} - V(\phi) \right),$$

$$L_{\text{kin}} = \frac{\kappa^{-4} K^q}{2q f^{2q-1}}, \quad (58)$$

where q is a constant that has to be $q > 1/2$ for stability reasons, and f is a function of the scalar curvature R , postulated to vanish at $R = 0$ and that behaves near $R = 0$ as

$$f(R) \sim (\kappa^4 R^2)^m \quad (59)$$

with κ the Planck length. The parameter α is assumed to be $\alpha > 0$ to stabilize gravity at low energies, m is an integer that satisfies $2(m-1) > q(2q-1)$ and $K \equiv -\kappa^4 \partial^{\mu} \phi \partial_{\mu} \phi$.

The true value of the vacuum energy in this approach is not zero, but the peculiar dynamics makes the universe settle down to a near zero energy state. The scalar field stops rolling and its kinetic terms diverges.

The two main problems with this scenario are (1) This specific kinetic term is chosen by hand, not motivated by a more fundamental theory, (2) all other fields settle to their ground state faster than the vacuum energy, making the universe empty, and reheating necessary, to thermally populate the universe again.

Other models where some dynamical feedback mechanism is proposed based on a non-standard kinetic term can be found in Refs. 109–112. An interesting conjecture is made on the existence of a conformal fixed point, possibly related to dilatation symmetry.⁽¹¹³⁾ However, these models still

need fine-tuning, and it is unclear whether they are experimentally viable, see Ref. 108.

4.2. Dilaton

A natural scalar field candidate to screen the cosmological constant could be the dilaton, which appears in string theory and compactified supergravity theories. In the presence of a dilaton, all mass scales arise multiplied with an exponential:

$$V_0(\phi) \sim M^4 e^{4\lambda\phi} \quad (60)$$

with ϕ the dilaton, and λ a coupling constant. The minimum of this is obtained for the value $\phi_0 = -\infty$, which is known as the “dilaton runaway problem”: couplings depend typically on ϕ , and these tend to go to zero, or infinity sometimes, in this limit. Moreover, all mass scales have this similar scaling behavior, so particle masses also vanish. Besides, the dilaton itself is nearly massless when it reaches the minimum of its potential, leading to long-range interactions that are severely constrained. Note that quintessence ideas can only be maintained as long as the new hypothetical scalar particle does not couple to the standard model fields, contrary to the dilaton.

In summary, the dynamical cancellation of a cosmological constant term by back-reaction effects of scalar fields is hard to realize. Let's focus therefore on a purely gravitational back-reaction mechanism.

4.3. Gravitons, Instabilities of dS-Space

Gravitational waves propagating in some background spacetime affect the dynamics of this background. This back-reaction can be described by an effective energy–momentum tensor $\tau_{\mu\nu}$.

4.3.1. Scalar-type Perturbations

In Refs. 114, 115 the back-reaction for scalar gravitational perturbations is studied. It is argued this might give a solution to the CC problem.

At linear order, all Fourier modes of the fluctuations evolve independently. However, since the Einstein equations are non-linear, retaining higher order terms in the perturbation amplitude leads to interactions

between the different perturbation modes: they define a gravitational back-reaction.

The idea is first to expand Einstein equations to second order in the perturbations, then to assume that linear terms satisfy equations of motion (and hence cancel). Next the spatial average is taken of the remaining terms and the resulting equations are regarded as equations for a new homogeneous metric $g_{\mu\nu}^{(0,br)}$, where the superscript $(0, br)$ denotes first the order in perturbation theory and the fact that back-reaction is taken into account:

$$G_{\mu\nu} \left(g_{\alpha\beta}^{(0,br)} \right) = -8\pi G \left[T_{\mu\nu}^{(0)} + \tau_{\mu\nu} \right] \quad (61)$$

and $\tau_{\mu\nu}$ contains terms resulting from averaging of the second order metric and matter perturbations:

$$\tau_{\mu\nu} = \left\langle T_{\mu\nu}^{(2)} - \frac{1}{8\pi G} G_{\mu\nu}^{(2)} \right\rangle. \quad (62)$$

In other words, the first-order perturbations are regarded as contributing an extra energy–momentum tensor to the zeroth-order equations of motion; the effective energy–momentum tensor of the first-order equations renormalizes the zeroth-order energy–momentum tensor. This is a somewhat tricky approach and it is not clear whether one can consistently derive the equations of motion in this way, see for example Refs. 116–122.

Now work in longitudinal gauge and take the matter to be described by a single scalar field for simplicity. Then there is only one independent metric perturbation variable denoted $\phi(x, t)$. The perturbed metric is

$$ds^2 = (1 + 2\phi)dt^2 - a(t)^2(1 - 2\phi)\delta_{ij}dx^i dx^j. \quad (63)$$

Calculating the τ_{00} and τ_{ij} elements and using relations valid for the period of inflation, Brandenberger's main result is that the equation of state of the dominant infrared contribution to the energy–momentum tensor $\tau_{\mu\nu}$ which describes back-reaction, takes the form of a negative CC:

$$p_{br} = -\rho_{br}, \quad \rho_{br} < 0. \quad (64)$$

This leads to the speculation that gravitational back-reaction may lead to a dynamical cancellation mechanism for a bare CC since $\tau_0^0 \propto \langle \phi^2 \rangle$, which is proportional to IR phase space and this diverges in a De Sitter universe. Long wavelength modes are those with wavelength longer than H ,

and as more and more modes cross the horizon, $\langle \phi^2 \rangle$ grows. To end inflation this way, however, takes an enormous number of e-folds, see Ref. 123 for a recent discussion.

However, as pointed out in Ref. 122, the spatially averaged metric is not a local physical observable: averaging over a fixed time slice, the averaged value of the expansion will not be the same as the expansion rate at the averaged value of time, because of the non-linear nature of the expansion with time. In other words, locally this “achieved renormalization”, i.e., the effect of the perturbations, is identical to a coordinate transformation of the background equations and not a physical effect. A similar conclusion was obtained in Refs. 124–125.

Brandenberger and co-workers have subsequently tried to improve their analysis by identifying a local physical variable which describes the expansion rate.^(126,127) This amounts to adding another scalar field that acts as an independent physical clock. Within this procedure they argue that back-reaction effects are still significant in renormalizing the cosmological constant.

It is however far from clear whether this scenario is consistent and whether the effects indeed are physical effects. One of the main points is that by performing a coordinate transformation, one can locally always find coordinates such that at a given point P , $g'_{\mu\nu}(x'_P) = \eta_{\mu\nu}$ and $\partial g'_{\mu\nu}/\partial x'_\alpha = 0$ evaluated at $x = x_P$, simply constructing a local inertial frame at the point P . The second and higher order derivatives of the metric can of course not be made to vanish and measure the curvature. The perturbations are small enough that we do not notice any deviation from homogeneity and isotropy, but are argued to be large enough to alter the dynamics of our universe, which sounds contradictory. In Ref. 128 especially, on general grounds these effects are argued to be unphysical and therefore cannot provide a solution to the cosmological constant problem.

Besides, this build-up of infrared scalar metric perturbations (vacuum fluctuations, stretched beyond the Hubble-radius) is set in an inflationary background and since the individual effects are extremely weak a large phase-space of IR-modes, i.e., a long period of inflation, is needed. The influence on today’s cosmological constant is unclear.

4.3.2. Long-Wavelength Back-Reaction in Pure Gravity

Closely related are studies by Tsamis and Woodard, see Refs. 129–135 concerning the back-reaction of long-wavelength gravitational waves in pure gravity with a bare cosmological constant. Leading infrared effects in quantum gravity are, contrary to what is often assumed, similar to those of QED, see Ref. 136.

When $\Lambda \neq 0$, the lowest dimensional self-interaction term is of dimension three, a three-point vertex with no derivatives (corresponding to the $\Lambda\sqrt{-g}$ -term). The IR behavior of the theory with cosmological constant is therefore very different from that without. Tsamis and Woodard christen it Quantum Cosmological Gravity, or QCG for short, and study it on an inflationary background. Here the infrared divergences are enhanced: since the spatial coordinates are exponentially expanded with increasing time, their Fourier conjugates, the spatial momenta, are redshifted to zero. The IR effects originate from the low end of the momentum spectrum, so they are strengthened when this sector is more densely populated.

Since other particles are either massive, in which case they decouple from the infrared, or conformally invariant, and therefore do not feel the de Sitter redshift, gravitons must completely dominate the far IR. The typical strength of quantum gravitational effects during inflation at scale M is

$$G\Lambda = 8\pi \left(\frac{M}{M_P} \right)^4, \quad (65)$$

which for GUT-scale inflation becomes $G\Lambda = 10^{-11}$ and for electroweak-scale inflation $G\Lambda = 10^{-67}$.

The classical background in conformal coordinates is

$$-dt^2 + e^{2Ht} d\mathbf{x} \cdot d\mathbf{x} = \Omega^2 \left(-du^2 + d\mathbf{x} \cdot d\mathbf{x} \right), \quad (66)$$

$$\Omega \equiv \frac{1}{Hu} = \exp(Ht) \quad (67)$$

and $H^2 \equiv \frac{1}{3}\Lambda$. For convenience, perturbation theory is formulated in terms of a pseudo-graviton field $\psi_{\mu\nu}$:

$$g_{\mu\nu} \equiv \Omega^2 \tilde{g}_{\mu\nu} \equiv \Omega^2 (\eta_{\mu\nu} + \kappa \psi_{\mu\nu}), \quad (68)$$

where $\kappa^2 \equiv 16\pi G$.

Because of homogeneity and isotropy of the dynamics and the initial state, the amputated 1-point function, can be written in terms of two functions of conformal time u :

$$D_{\mu\nu}^{\rho\sigma} \langle 0 | \kappa \psi_{\rho\sigma}(x) | 0 \rangle = a(u) \bar{\eta}_{\mu\nu} + c(u) \delta_{\mu}^0 \delta_{\nu}^0, \quad (69)$$

where $D_{\mu\nu}^{\rho\sigma}$ is the gauge fixed kinetic operator, and a bar on $\eta_{\mu\nu}$ indicates that temporal components of this tensor are deleted:

$$\eta_{\mu\nu} = \bar{\eta}_{\mu\nu} \delta_{\mu}^{(0)} \delta_{\nu}^{(0)}. \quad (70)$$

The pseudo-graviton kinetic operator $D_{\mu\nu}^{\rho\sigma}$ splits in two parts, a term proportional to $D_A \equiv \Omega(\partial^2 + \frac{2}{u^2})\Omega$, which is the kinetic operator for a massless minimally coupled scalar, and a part proportional to $D_C \equiv \Omega\partial^2\Omega$, the kinetic operator for a conformally coupled scalar.

After attaching the external legs one obtains the full 1-point function, which has the same form, but with different components:

$$\langle 0|\kappa\psi_{\rho\sigma}(x)|0\rangle = A(u)\bar{\eta}_{\mu\nu} + C(u)\delta_{\mu}^0\delta_{\nu}^0. \quad (71)$$

The functions $A(u)$ and $C(u)$ obey the following differential equations:

$$\begin{aligned} -\frac{1}{4}D_A[A(u) - C(u)] &= a(u), \\ D_C C(u) &= 3a(u) + c(u). \end{aligned} \quad (72)$$

The functions $a(u)$ and $A(u)$ on the one hand, and $c(u)$ and $C(u)$ on the other, are therefore related by retarded Green's functions $G_{A,C}^{ret}$ for the massless minimally coupled and conformally coupled scalars:

$$\begin{aligned} A(u) &= -4G_A^{ret}[a](u) + G_C^{ret}[3a + c](u), \\ C(u) &= G_C^{ret}[3a + c](u) \end{aligned} \quad (73)$$

In terms of these functions $A(u)$ and $C(u)$ the invariant element in comoving coordinates reads

$$\hat{g}_{\mu\nu}(t, \mathbf{x})dx^\mu dx^\nu = -\Omega^2[1 - C(u)]du^2 + \Omega^2[1 + A(u)]d\mathbf{x} \cdot d\mathbf{x}. \quad (74)$$

This gives the following identification:

$$\begin{aligned} R(t) &= \Omega\sqrt{1 + A(u)}, \\ d(t) &= -\Omega\sqrt{1 - C(u)}du, \quad \text{and} \quad d(Ht) = -\sqrt{1 - C(u)}d[\ln(Hu)] \end{aligned} \quad (75)$$

Using this we can find the time dependence of the effective Hubble parameter:

$$H_{\text{eff}}(t) = \frac{d}{dt} \ln(R(t)) = \frac{H}{\sqrt{1 - C(u)}} \left(1 - \frac{\frac{1}{2}u \frac{d}{du} A(u)}{1 + A(u)} \right). \quad (76)$$

The backreaction of the IR gravitons therefore acts to screen the bare cosmological constant, originally present. The improved results⁸ in terms of

$$\epsilon \equiv \left(\frac{\kappa H}{4\pi} \right)^2 = \frac{G\Lambda}{3\pi} = \frac{8}{3} \left(\frac{M}{M_P} \right)^4 \quad (77)$$

turn out to be

$$A(u) = \epsilon^2 \left\{ \frac{172}{9} \ln^3(Hu) + (\text{subleading}) \right\} + \mathcal{O}(\epsilon^3), \quad (78)$$

$$C(u) = \epsilon^2 \left\{ 57 \ln^2(Hu) + (\text{subleading}) \right\} + \mathcal{O}(\epsilon^3). \quad (79)$$

Using (75) we find:

$$Ht = - \left\{ 1 - \frac{19}{2} \epsilon^2 \ln^2(Hu) + \dots \right\} \ln(Hu). \quad (80)$$

This implies that $\ln(Hu) \approx -Ht$ to very good approximation, therefore $A(u)$ can be written:

$$A(u) = -\frac{172}{9} \epsilon^2 (Ht)^3 + \dots \quad (81)$$

and we arrive at

$$\begin{aligned} H_{\text{eff}}(t) &\approx H + \frac{1}{2} \frac{d}{dt} \ln(1 + A), \\ &\approx H \left\{ 1 - \frac{\frac{86}{3} \epsilon^2 (Ht)^2}{1 - \frac{172}{9} \epsilon^2 (Ht)^3} \right\}. \end{aligned} \quad (82)$$

The induced energy density, which acts to screen the original cosmological constant present gives

$$\begin{aligned} \rho(t) &\approx \frac{\Lambda}{8\pi G} \left\{ -\frac{1}{H} \frac{\dot{A}}{1+A} + \frac{1}{4H^2} \left(\frac{\dot{A}}{1+A} \right)^2 \right\} \\ &\approx \frac{\Lambda}{8\pi G} \left\{ -\frac{\frac{172}{3} \epsilon^2 (Ht)^2}{1 - \frac{172}{9} \epsilon^2 (Ht)^3} + \left(\frac{\frac{86}{3} \epsilon^2 (Ht)^2}{1 - \frac{172}{9} \epsilon^2 (Ht)^3} \right)^2 \right\}. \end{aligned} \quad (83)$$

⁸ Papers before 1997 yield different results.

This can be written more intuitively, to better see the magnitude of the effect as follows:

$$H_{\text{eff}}(t) = H \left\{ 1 - \epsilon^2 \left[\frac{1}{6} (Ht)^2 + (\text{subleading}) \right] + \mathcal{O}(\kappa^6) \right\} \quad (84)$$

and the induced energy density and pressure, in powers of H :

$$\begin{aligned} \rho(t) &= \frac{\Lambda}{8\pi G} + \frac{(\kappa H)H^4}{2^6\pi^4} \left\{ -\frac{1}{2} \ln^2 a + \mathcal{O}(\ln a) \right\} + \mathcal{O}(\kappa^4), \\ p(t) &= -\frac{\Lambda}{8\pi G} + \frac{(\kappa H)H^4}{2^6\pi^4} \left\{ \frac{1}{2} \ln^2 a + \mathcal{O}(\ln a) \right\} + \mathcal{O}(\kappa^4). \end{aligned} \quad (85)$$

The number of e-foldings needed to make the backreaction effect large enough to even end inflation is

$$N \sim \left(\frac{9}{172} \right)^{1/3} \left(\frac{3\pi}{G\Lambda} \right)^{2/3} = \left(\frac{81}{11008} \right)^{1/3} \left(\frac{M_P}{M} \right)^{8/3}, \quad (86)$$

where M is the mass scale at inflation and M_P the Planck mass. For inflation at the GUT scale this gives $N \sim 10^7$ e-foldings. This enormously long period of inflation, much longer than in typical inflation models, is a direct consequence of the fact that gravity is such a weak interaction.

In other words, the effect might be strong enough to effectively kill any cosmological constant present, as long as such a long period of inflation is acceptable. There do exist arguments that the number of e-folds is limited to some 85, see Ref. 137 for details, but these are far from established. Another issue is that these results have been obtained for a very large cosmological constant during inflation. It is unclear what this means for the present day vacuum energy of the universe. Perturbative techniques break down when the effect becomes too strong, making this difficult to answer.

This breaking however is rather soft, since each elementary interaction remains weak. Furthermore, a technique following Starobinski⁽¹³⁸⁾ is used in which non-perturbative aspects are absorbed in a stochastic background that obeys the classical field equations.⁽¹³⁴⁾

It is then argued⁽¹³⁴⁾ that eventually the screening must overcompensate the original bare cosmological constant, leading to a period of deflation. This happens because the screening at any point derives from a coherent superposition of interactions from within the past lightcone and the invariant volume of the past lightcone grows faster as the expansion slows down. Now thermal gravitons are produced that act as a thermal

barrier, that grows hotter and denser as deflation proceeds. Incoming virtual IR modes scatter off this barrier putting a halt to the screening process. The barrier dilutes and the expansion takes over again.

However, discussions are still going on, debating whether these screening effects are real physical effects, or gauge artifacts, see Refs. 128, 139, 140. Especially, the argued cumulative nature of the effect, makes it hard to understand how local physics is affected.

Another objection may be raised in that throughout the above calculation, the “primordial” cosmological constant Λ was used. The mechanism, however, screens the cosmological constant, which implies that the effective cosmological constant should be used instead. The strength of the effect would then be even weaker, since this is controlled by $G\Lambda$. A much larger number of e-folds would then be necessary to stop inflation.

4.4. Screening as a Consequence of the Trace Anomaly

In Refs. 141–143 it is argued that the quantum effects of the trace anomaly of massless conformal fields in 4 dimensions leads to a screening of the cosmological constant. The effective action of 4D gravity yields an extra new spin-0 degree of freedom in the conformal sector, or trace of the metric. At very large distance scales this trace anomaly induced action dominates the standard Einstein action and gives an IR fixed point where scale invariance is restored.

The idea is similar to that in Section 4.3.2. One tries to find a renormalization group screening of the cosmological constant in the IR, but instead of taking full quantum gravity effects, only quantum effects of the conformal factor are considered. See also Ref. 144 for a related earlier study.

The authors conclude that the effective cosmological constant and inverse Newton’s constant in units of Planck mass decreases at large distances and that $G_N\Lambda \rightarrow 0$ at the IR fixed point in the infinite volume limit.

However, the cosmological constant problem manifests itself already at much smaller distances and moreover, it is unclear whether this scenario is compatible with standard cosmological observations. Moreover, like the other approaches in this chapter, it relies heavily on quantum effects having a large impact at enormous distance scales. As argued in the previous section, it is debatable whether these effects can be sufficiently significant.

4.5. Running Λ from Renormalization Group

In Refs. 94, 145–149 a related screening of the cosmological constant is studied, viewing Λ as a parameter subject to renormalization group

running. The cosmological constant then becomes a scaling parameter $\Lambda(\mu)$, where μ is often identified with the Hubble parameter at the corresponding epoch, in order to make the running of Λ smooth enough to agree with all existing data.⁽¹⁵⁰⁾

However, renormalization group equations give logarithmic corrections, which makes it hard to see how this can ever account for the suppression of a factor of 10^{120} needed for the cosmological constant. Although this running is very slow, it could possibly be measured as a quintessence of phantom dark energy and be consistent with all data, as long as $0 \leq |\nu| \ll 1$.⁽¹⁵¹⁾ As a solution to the cosmological constant problem, it obviously cannot help.

In Refs. 149, 152, it is argued that there may be a UV fixed point at which gravity becomes asymptotically free. If there would be an IR fixed point at which $\Lambda_{\text{eff}} = 0$ this could shed some new light on the cosmological constant problem. This scaling also effects G , making it larger at larger distances.

4.5.1. Triviality as in $\lambda\phi^4$ Theory

The Einstein Hilbert action with a cosmological constant can be rewritten as:⁽¹⁵³⁾

$$S = -\frac{3}{4\pi} \int d^4x \sqrt{-\hat{g}} \left(\frac{1}{12} R(\hat{g})\phi^2 + \frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{\lambda}{4!} \phi^4 \right) \quad (87)$$

after rescaling the metric tensor as

$$g_{\mu\nu} = \varphi^2 \hat{g}_{\mu\nu}, \quad ds^2 = \varphi^2 \hat{d}s^2 \quad (88)$$

and defining

$$\phi = \frac{\varphi}{\sqrt{G}}, \quad \Lambda = \frac{\lambda}{4G}. \quad (89)$$

Now it is suggested that the same arguments first given by Wilson,⁽¹⁵⁴⁾ that are valid in ordinary $\lambda\phi^4$ -theory, might also hold here and that this term is suppressed quantum mechanically.

It is noted that perturbative running as in normal $\lambda\phi^4$ -theory is by far not sufficient, but the idea is that perhaps there might be some non-perturbative suppression. Similar ideas have been contemplated by Polyakov.⁽¹⁵⁵⁾

4.6. Summary

Finding a viable mechanism that screens the original possibly large cosmological constant to its small value today, is a very difficult task. Weinberg's no-go theorem puts severe limits on this approach. Back-reaction effects, moreover, are generally either very weak, or lead to other troublesome features like a screened Newton's constant.

The underlying idea however that the effective cosmological constant is small simply because the universe is old, is attractive and deserves full attention.

5. TYPE III: VIOLATING THE EQUIVALENCE PRINCIPLE

An intriguing way to try to shed light on the cosmological constant problem is to look for violations of the equivalence principle of general relativity. The near zero cosmological constant could be an indication that vacuum energy contrary to ordinary matter-energy sources does not gravitate.

The approach is based not on trying to eliminate any vacuum energy, but to make gravity numb for it. This requires a modification of some of the building blocks of general relativity. General covariance (and the absence of ghosts and tachyons) requires that gravitons couple universally to all kinds of energy. Moreover, this also fixes uniquely the low energy effective action to be the Einstein–Hilbert action. If gravity were not mediated by an exactly massless state, this universality would be avoided. One might hope that vacuum energy would then decouple from gravity, thereby eliminating the gravitational relevance of it and thus eliminate the cosmological constant problem.

5.1. Extra Dimensions, Braneworld Models

Since the Casimir effect troubles our notion of a vacuum state, the cosmological constant problem starts to appear when considering distances smaller than a millimeter or so. Therefore, extra dimensions with millimeter sizes might provide a mechanism to understand almost zero 4D vacuum energy, since in these scenarios gravity is changed at distances smaller than a millimeter. This size really is a sort of turn-over scale. Somehow all fluctuations with sizes between a Planck length and a millimeter are cancelled or sum up to zero.

Besides, it is conceivable that the need to introduce a very small cosmological constant or some other form of dark energy to explain an accelerating universe nowadays, is a signal that general relativity breaks down at very large distance scales. General relativity however, works very well on scales from 10^{-1} mm to at least 10^{14} cm, the size of the solar system. This puts severe constraints on alternative theories. Extra-dimensional models, like the early Kaluza–Klein scenarios, generically have additional degrees of freedom, often scalar fields, that couple to the four dimensional energy–momentum tensor and modify four-dimensional gravity. A four dimensional massless graviton has two physical degrees of freedom, a five dimensional one five, just like a massive 4-dimensional graviton.⁹ There are however, strong experimental constraints on such scalar–tensor theories of gravity, see for example Refs. 156, 157.

A lot of research in this direction in recent years has been devoted to braneworld models in $D = 4 + N$ dimensions, with N extra spatial dimensions. In this setting the cosmological constant problem is at least as severe as in any other, but new mechanisms of cancelling a vacuum energy can be thought of. The general idea is that our world is confined on a hypersurface, a brane, embedded in a higher dimensional spacetime. The standard model fields are restricted to live on a 3-brane, while only gravitons can propagate in the full higher dimensional space. To reproduce the correct 4-dimensional gravity at large distances three approaches are known. Usually one takes the extra dimensions to cover a finite volume and compactifies the unseen dimensions. One of the earliest approaches was by Rubakov and Shaposhnikov⁽¹⁵⁸⁾ who unsuccessfully tried to argue that the 4D cosmological constant is zero, since 4D vacuum energy only curves the extra dimensions.

In this chapter we will first briefly review the Randall–Sundrum models and show why they cannot solve the cosmological constant problem. Next we focus on the DGP-model with infinite volume extra dimensions. This is a very interesting setup, but also a good example of the difficulties one faces in deconstructing a higher dimensional model to a viable 4D world meeting all the GR constraints. A rather more speculative but perhaps also more promising approach is subsequently discussed, in which Lorentz invariance is spontaneously broken to yield a Higgs mechanism analog for gravity. Before concluding with a summary, we discuss yet

⁹ In general, the total number of independent components of a rank 2 symmetric tensor in D dimensions is $D(D+1)/2$, however, only $D(D-3)/2$ of those correspond to physical degrees of freedom of a D -dimensional massless graviton; the remaining extra components are the redundancy of manifestly gauge and Lorentz invariant description of the theory.

another option, where one considers the graviton to be a composite particle.

5.1.1. Randall–Sundrum Models, Warped Extra Dimensions

There are in fact two different models known as Randall–Sundrum models, dubbed RS-I and RS-II. We begin with RS-I.

This model consists of two 3-branes at some distance from each other in the extra dimension. One brane, called the “hidden brane” has positive tension, while the other one, the “visible brane”, on which we are supposed to live, has negative tension. Both branes could have gauge theories living on them. All of the Standard Model fields are localized on the brane, and only gravity can propagate through the entire higher dimensional space.

The equation of motion looks as follows:

$$\begin{aligned} & M_* \sqrt{G} \left(R_{AB} - \frac{1}{2} G_{AB} R \right) - M_* \Lambda \sqrt{G} G_{AB} \\ &= T_{\text{hid}} \sqrt{g_{\text{hid}}} g_{\mu\nu}^{\text{hid}} \delta_A^\mu \delta_B^\nu \delta(y) + T_{\text{vis}} \sqrt{g_{\text{vis}}} g_{\mu\nu}^{\text{vis}} \delta_A^\mu \delta_B^\nu \delta(y - y_0) \end{aligned} \quad (90)$$

with notations

$$g_{\mu\nu}^{\text{hid}}(x) = G_{\mu\nu}(x, y = 0), \quad g_{\mu\nu}^{\text{vis}}(x) = G_{\mu\nu}(x, y = y_0). \quad (91)$$

Furthermore, M_* is the 5-dimensional Planck mass, which has to satisfy $M_* \gtrsim 10^8$ GeV, in order not to spoil Newtonian gravity at distances $l \lesssim 0.1$ mm.

The y -direction is compactified on an orbifold S_1/\mathbf{Z}_2 . With the above assumptions for the brane-tensions and bulk CC, it can be shown that there exists the following static solution, with a flat 4D-metric:

$$ds^2 = e^{-|y|/L} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2. \quad (92)$$

The minus sign in the exponential factor occurs because of the assumption that our visible brane has a negative tension. As a result of this “warp-factor”, all masses on the visible brane are suppressed, compared to their natural value. For the Higgs mass for example, one obtains

$$m^2 = e^{-y_0/L} m_0^2 \quad (93)$$

a small hierarchy in y_0/L therefore leads to a large hierarchy between m and m_0 , which would solve the “ordinary” hierarchy problem.

Moreover, despite the fact that the brane tension on the visible brane is negative, it is possible that it still has a flat space solution. Fine-tuning is necessary to obtain this result, and besides, this solution is not unique. Other, non-flat space solutions also exist. Therefore, this cannot help in solving the cosmological constant problem, but it is interesting to see that a 4D cosmological constant can be made to curve only extra dimensions.

Alternatively, the extra dimensions can be kept large, uncompactified, but warped, as in the Randall–Sundrum type-II models, in which there is only one brane. In this case the size of the extra dimensions can be infinite, but their volume $\int dy\sqrt{G}$, is still finite. The warp-factor causes the graviton wavefunction to be peaked near the brane, or, in other words, gravity is localized, such that at large 4D-distances ordinary general relativity is recovered. The same bound as in RS-I applies to the 5D Planck mass.

The action now reads

$$S = \frac{1}{2}M_*^3 \int d^4x \int_{-\infty}^{+\infty} dy\sqrt{G}(R_5 - 2\Lambda_5) + \int d^4x\sqrt{g}(\Lambda_4 + \mathcal{L}_{SM}), \quad (94)$$

where Λ_4 denotes the 4D brane tension and Λ_5 the bulk cosmological constant, which is assumed to be negative. The equation of motion derived from this action, ignoring now \mathcal{L} is

$$M_*\sqrt{G} \left(R_{AB} - \frac{1}{2}G_{AB}R \right) = -M_*^3\Lambda_5\sqrt{G}G_{AB} + \Lambda_4\sqrt{g}g_{\mu\nu}\delta_A^\mu\delta_B^\nu\delta(y), \quad (95)$$

indicating that the brane is located at $y = 0$. This equation has the same flat space solution as above, but, again, at the expense of fine-tuning Λ_5 and Λ_4 .

Gravity in the 4D subspace reduces to GR up to some very small Yukawa-type corrections. Unfortunately however, with regard to the cosmological constant problem, the model suffers from the same drawbacks as RS-I. All fundamental energy scales are at the TeV level, but the vacuum energy density in our 4D-world is much smaller.

5.1.2. Self-Tuning Solutions

Transmitting any contribution to the CC to the bulk parameters, in such a way that a 4D-observer does not realize any change in the 4D geometry seems quite spurious. It would become more interesting if this transmission would occur automatically, without the necessity of re-tuning the bulk quantities by hand every time the 4D vacuum energy changes.

Models that realize this are called *self-tuning* models (see for example Ref. 159). A severe drawback that all these models face is that this scenario does not exclude “*nearby curved solutions*”. This means that in principle there could exist solutions for neighboring values of some bulk parameters, which result in a curved 4D space. Besides, there are additional problems such as a varying effective Planck mass, or varying masses for fields on the brane. So far no mechanism without these drawbacks has been found. See Refs. 160, 161 for recent studies in favor of this approach.

Another serious problem is that in many proposals, the 4D brane tension creates a deficit angle in the bulk, which easily becomes larger than 2π . The cosmological constant problem rises again in a different fine-tuning problem. For a recent review of this approach and many references, see Ref. 162.

A related approach, considering a warped higher dimensional geometry, is studied in Refs. 163–166. It is argued that once a cosmological constant vanishes in the UV, there exist solutions such that it will not be regenerated along the renormalization group flow. Any vacuum energy is cancelled by a decreasing warp factor, ensuring a flat space solution on the brane. However, these are not the solutions and there exists no argument why they should be preferred. Note however, that this is quite contrary to ordinary renormalization group behavior, as studied in Section 4.5.

5.1.3. Infinite Volume Extra Dimensions

In Refs. 167–172, a model based on infinite volume extra dimensions is presented. Embedding our spacetime in infinite volume extra dimensions has several advantages. If they are compactified, one would get a theory approaching GR in the IR, facing Weinberg’s no-go theorem again. Details of how these large dimension models circumvent the no-go theorem can be found in Ref. 170. Moreover, often the assumption is made that the higher-dimensional theory is supersymmetric and that susy is spontaneously broken on the brane. These breaking effects can be localized on the brane only, without affecting the bulk, because the infinite volume gives a large enough suppression factor. Apart from that, an unbroken R-parity might be assumed to forbid any negative vacuum energy density in the bulk.

They start with the following low-energy effective action:

$$S = M_*^{2+N} \int d^4x d^N y \sqrt{G} \mathcal{R} + \int d^4x \sqrt{g} \left(\mathcal{E}_4 + M_P^2 R + \mathcal{L}_{SM} \right), \quad (96)$$

where M_*^{2+N} is the $(4 + N)$ -dimensional Planck mass, the scale of the higher dimensional theory, G_{AB} the $(4 + N)$ -dimensional metric, y are

the “perpendicular” coordinates and $\mathcal{E}_4 = M_{Pl}^2 \Lambda$, the brane tension, or 4D cosmological constant. Thus the first term is the bulk Einstein–Hilbert action for $(4 + N)$ -dimensional gravity and the $M_p^2 R$ term is the induced 4D-Einstein–Hilbert action. So there are two free parameters: M_* and \mathcal{E} . M_* is assumed to be very small, making gravity in the extra dimensions much stronger than in our 4D world. The 4D-Planck mass in this setup is a derived quantity.⁽¹⁷³⁾

Gravity on the brane can be recovered either by making a decomposition into Kaluza–Klein modes, or by considering the 4D graviton as a resonance, a metastable state with a mass given by $m_g \sim M_*^3/M_{Pl}^2$.

The higher dimensional graviton can be expanded in 4D Kaluza–Klein modes as follows:

$$h_{\mu\nu}(x, y_n) = \int d^N m \epsilon_{\mu\nu}^m(x) \sigma_m(y_n), \tag{97}$$

where $\epsilon_{\mu\nu}^m(x)$ are 4D spin-2 fields with mass m and $\sigma_m(y_n)$ are their wave-function profiles in the extra dimensions. Each of these modes gives rise to a Yukawa-type gravitational potential, the coupling-strength to brane sources of which are determined by the value of σ_m at the position of the brane, say $y = 0$:

$$V(r) \propto \frac{1}{M_*^{2+N}} \int_0^\infty d m m^{N-1} |\sigma_m(0)|^2 \frac{e^{-rm}}{r}. \tag{98}$$

However, in this scenario there is a cut-off of this integral; modes with $m > 1/r_c$ have suppressed wavefunctions, where r_c is some cross-over scale, given by $r_c = M_{Pl}^2/M_*^3 \sim H_0^{-1}$. For $r \ll r_c$ the gravitational potential is $1/r$, dominated by the induced 4D kinetic term, and for $r \gg r_c$ it turns to $1/r^2$, in case of one extra dimension. In ordinary extra dimensional gravity, all $|\sigma_m(0)| = 1$, here however:

$$|\sigma_m(0)| = \frac{4}{4 + m^2 r_c^2}, \tag{99}$$

which decreases for $m \gg r_c$. Therefore, the gravitational potential interpolates between the 4D and 5D regimes at r_c . Below r_c almost normal 4D gravity is recovered, while at larger scales it is effectively 5-dimensional and thus weaker. This could cause the universe’s acceleration.

The question now is, whether there exist solutions such that the 4D induced metric on the brane is flat: $g_{\mu\nu} = \eta_{\mu\nu}$. Einstein’s equation from (96) now becomes (up to two derivatives):

$$\begin{aligned}
M_*^{2+N} \left(\mathcal{R}_{AB} - \frac{1}{2} G_{AB} \mathcal{R} \right) + \delta^{(N)} M_P^2 \left(R - \frac{1}{2} g_{\mu\nu} R \right) \delta_A^\mu \delta_B^\nu \\
= \mathcal{E}_4 \delta^{(N)}(y) g_{\mu\nu} \delta_A^\mu \delta_B^\nu.
\end{aligned} \tag{100}$$

In case of one extra dimension it is not possible to generate a viable dynamics with a flat 4D metric. For $N \geq 2$, however, solutions of the theory can be parameterized as

$$ds^2 = A^2(y) g_{\mu\nu}(x) dx^\mu dx^\nu - B^2(y) dy^2 - C^2(y) y^2 d\Omega_{N-1}^2, \tag{101}$$

where $y \equiv \sqrt{y_1^2 + \dots + y_n^2}$ and the functions A, B, C depend on \mathcal{E}_4 and M_* :

$$A, B, C = \left(1 - \left(\frac{y_g}{y} \right)^{N-2} \right)^{\alpha, \beta, \gamma}, \tag{102}$$

where α, β, γ correspond to A, B, C respectively, and depend on dimensionality and y_g is the gravitational radius of the brane:

$$y_g \sim M_*^{-1} \left(\frac{\mathcal{E}_4}{M_*^4} \right)^{1/(N-2)} \quad \text{for } N \neq 2. \tag{103}$$

Most importantly, one explicitly known solution, with $N = 2$, generates a flat 4D Minkowski metric and $R(g) = 0$.⁽¹⁷⁴⁾ The 4D brane tension is spent on creating a deficit angle in the bulk. However, one has to fine-tune this tension in order not to generate a deficit angle larger than 2π . So also the $N = 2$ model does not work.

For $N > 2$ consistent solutions possibly do exist with a flat 4D metric. However, these are not the only solutions, and besides, their interpretation is rather complicated because of the appearance of a naked singularity. Spacetime in $4+N$ dimensions looks like $\mathfrak{H}_4 \times S_{N-1} \times R_+$, where \mathfrak{H}_4 denotes flat spacetime on the brane, and $S_{N-1} \times R_+$ are Schwarzschild solutions in the extra dimensions.

They argue that the final physical result is

$$H \sim M_* \left(\frac{M_*^4}{\mathcal{E}_4} \right)^{1/(N-2)}. \tag{104}$$

According to the 4D result, $N = 0$, the expansion rate grows as \mathcal{E}_4 increases, but for $N > 2$ the acceleration rate H decreases as \mathcal{E}_4 increases.

In this sense, vacuum energy can still be very large, it just gravitates very little; 4D vacuum energy is supposed to curve mostly the extra dimensions.

This scenario has been criticized for different reasons, which we will come to in Section 5.1.5. The most important issue raised is that, since gravity has essentially become massive in this scenario, the graviton has five degrees of freedom, and especially the extra scalar degree of freedom, often leads to deviations of GR at small scales.

5.1.4. Non-local Gravity

From a 4D-perspective, this approach can also be viewed as to make the effective Newton's constant frequency and wavelength dependent, in such a way that for sources that are uniform in space and time it is tiny:⁽¹⁷⁵⁾

$$M_{Pl}^2 \left(1 + \mathcal{F}(L^2 \nabla^2)\right) G_{\mu\nu} = T_{\mu\nu}. \quad (105)$$

Here $\mathcal{F}(L^2 \nabla^2)$ is a filter function:

$$\begin{aligned} \mathcal{F}(\alpha) &\rightarrow 0 && \text{for } \alpha \gg 1, \\ \mathcal{F}(\alpha) &\gg 1 && \text{for } \alpha \ll 1. \end{aligned} \quad (106)$$

L is a distance scale at which deviations from general relativity are to be expected and $\nabla^2 \equiv \nabla_\mu \nabla^\mu$ denotes the covariant d'Alembertian. Thus (105) can be viewed as Einstein's equation with $(8\pi G_N^{\text{eff}})^{-1} = M_{Pl}^2(1 + \mathcal{F})$. It is argued that for vacuum energy $\mathcal{F}(0)$ is large enough, such that it will barely gravitate, resulting in a very small curvature radius R :

$$M_P^2 (1 + \mathcal{F}(0)) G_{\mu\nu} = \left(M_P^2 + \bar{M}^2\right) G_{\mu\nu} \quad \text{and} \quad R = -\frac{4\mathcal{E}_4}{M_P^2 + \bar{M}^2}. \quad (107)$$

To reproduce the observed acceleration a value \bar{M} is needed $\bar{M} \sim 10^{48}$ GeV for a vacuum energy density of TeV level, and a $\bar{M} \sim 10^{80}$ GeV for \mathcal{E}_4 of Planck mass value, which is about equal to the mass of the universe.

In terms of the graviton propagator, it gets an extra factor $(1 + \mathcal{F}(k^2 L^2))^{-1}$ and therefore goes to zero when $\mathcal{F}(0) \rightarrow \infty$, instead of generating a tadpole.

In the limit $L \rightarrow \infty$ one arrives at

$$M_{Pl}^2 G_{\mu\nu} - \frac{1}{4} \bar{M}^2 g_{\mu\nu} \bar{R} = T_{\mu\nu}, \quad (108)$$

just the zero mode part of $G_{\mu\nu}$, which is proportional to $g_{\mu\nu}$, where

$$\bar{R} \equiv \frac{\int d^4x \sqrt{g} R}{\int d^4x \sqrt{g}}, \quad (109)$$

\bar{R} thus is the spacetime averaged Ricci curvature, which vanishes for all localized solutions, such as stars, black holes and also for FRW models. For de Sitter space however, $\bar{R} \neq 0$, but a constant and equal to $\bar{R} = R_\infty$, with R_∞ the asymptotic de Sitter curvature.

At the price of losing 4D-locality and causality, the new averaged term is both non-local and acausal, a model is constructed in which a huge vacuum energy does not lead to an unacceptably large curvature. The Planck scale is made enormous for Fourier modes with a wavelength larger than a size L . Such sources would feel gravity only due to their coupling with the graviton zero mode. This zero mode however, is very weakly coupled to brane sources since it is suppressed by the volume of the extra dimensions.

It is argued that the acausality has no other observable effect. Moreover, it has been claimed that non-locality should be an essential element in any modification of GR in the infrared that intends to solve the cosmological constant problem.⁽¹⁶⁷⁾ The argument is that it takes local, causal physics a time $1/L$ to respond to modifications at scale $L \sim 10^{28}$ cm, and thus in particular to sources which have characteristic wavelength larger than H_0^{-1} , “such as vacuum energy”.⁽¹⁷¹⁾

The non-localities in this case appear in the four dimensional truncation of the $4 + N$ -dimensional theory of Section 5.1.3. There is an infinite number of degrees of freedom below any non-zero energy scale. Therefore, in order to rewrite the model as an effective four dimensional field theory, and infinite number of degrees of freedom have to be integrated out. This results in the appearance of non-local interactions, despite the fact that the full theory is local.

Another idea based on a model of non-local quantum gravity and field theory due to Moffat,^(176,177) also suppresses the coupling of gravity to the vacuum energy density and also leads to a violation of the Weak Equivalence Principle.

5.1.5. Massive Gravitons

A much studied approach to change general relativity in the infrared which is not simply a variety of a scalar–tensor theory, is to allow for tiny masses for gravitons, like in the Fierz–Pauli theory of massive gravity,⁽¹⁷⁸⁾ and in the example above. Note in passing that due to mass terms, gravitons might become unstable and could possibly decay into lighter particles, for example photons. If so, gravity no longer obeys the standard

inverse-square law, but becomes weaker at large scales, leading to accelerated cosmic expansion.

Of course, the extra degrees of freedom, extra polarizations of a massive graviton, could also become noticeable at much shorter distances, putting severe constraints on such scenarios. In the UV the new scalar degrees of freedom become strongly coupled, where the effective theory breaks down and the physics becomes sensitive to the unknown UV-completion of the theory.

A severe obstacle massive gravity theories have to overcome is something known as the Van Dam, Veltman, Zakharov, or (vDVZ), discontinuity.^(179,180) vDVZ argued that in the massive case, even with extremely small graviton mass, the bending of light rays passing near the sun would be too far off from experimental results in the massive case, that the mass of the graviton has to be exactly equal to zero. The physical reason indeed being, that even in the limit where the mass of the graviton goes to zero, there is an additional scalar attraction, which distinguishes the theory from Einstein's GR.

In the DGP model, the extra dimensions are infinitely large, and in the literature, there is an ongoing discussion whether this model is experimentally viable and capable of avoiding the massive gravity difficulties, see Refs. 181–186 for criticism. It appears that indeed also this model suffers from strong interactions at short distances due to the scalar polarization of the massive graviton, that can be understood in terms of a propagating ghosts-like degree of freedom.

The deviations of GR are argued to take place at distances set by $r_c \equiv M_p^2/M_*^3$. The one-graviton exchange approximation breaks down at distances $R_* \sim (R_S r_c^2)^{1/3}$,⁽¹⁸¹⁾ called the Vainshtein scale, with R_S the Schwarzschild radius of the source. R_* is very large for astrophysical sources, which suggests that the DGP model may describe our universe. For distances larger than R_* gravity deviates significantly from GR, yet for smaller distances it should yield (approximately) the same results. However, quantum effects become important at much smaller distances scales, given by

$$r_{\text{crit}} = \left(\frac{r_c^2}{M_p^2} \right)^{1/3}, \quad (110)$$

which can be as small as a 1000 km, for $r_c \sim H \sim 10^{28}$ cm. These strong interactions can be traced back to the appearance of a negative norm state. This is however a controversial result, also argued for in Ref. 182, yet waived away in Ref. 187. Further studies are necessary to settle this question.

The Schwarzschild solutions in the DGP model are also heavily debated and it is not yet clear what the correct way is to calculate these,

and whether they will eventually lead to consistent phenomenological behavior. For a recent study and references, see Ref. 188.

In the next section we will consider an alternative, that does not suffer from this “strong coupling problem”.

In Ref. 189 bounds on graviton masses are discussed using the LISA space interferometer.

5.2. Ghost Condensation or Gravitational Higgs Mechanism

In this framework gravity is modified in the infrared as a result of interactions with a “ghost condensate”, leading among other things to a mass for the graviton, see Ref. 190.

Assume that for a scalar field ϕ we have

$$\langle \phi \rangle = M^2 \rightarrow \phi = M^2 t + \pi \quad (111)$$

and that it has a shift symmetry $\phi \rightarrow \phi + a$ so that it is derivatively coupled, and that its kinetic term enters with the wrong sign in the Lagrangian:

$$\mathcal{L}_\phi = -\frac{1}{2} \partial^\mu \phi \partial_\mu \phi + \dots \quad (112)$$

The consequence of this wrong sign is that the usual background with $\langle \phi \rangle = 0$ is unstable and that after vacuum decay, the resulting background will break Lorentz invariance spontaneously.

The low energy effective action for the π has the form:

$$S \sim \int d^4x \left[\frac{1}{2} \dot{\pi}^2 - \frac{1}{2M^2} (\nabla^2 \pi)^2 + \dots \right], \quad (113)$$

so that the π 's have a low energy dispersion relation like

$$\omega^2 \sim \frac{k^4}{M^2} \quad (114)$$

instead of the ordinary $\omega^2 \sim k^2$ relation for light excitations. Time-translational invariance is broken, because $\langle \phi \rangle = M^2 t$ and as a consequence there are two types of energy, a “particle physics” and a “gravitational” energy which are not the same. The particle physics energy takes the form

$$\mathcal{E}_{\text{pp}} \sim \frac{1}{2} \dot{\pi}^2 + \frac{(\nabla^2 \pi)^2}{2M^2} + \dots, \quad (115)$$

whereas the gravitational energy is

$$\mathcal{E}_{\text{grav}} = T_{00} \sim M^2 \dot{\pi} + \dots \tag{116}$$

Although time-translation- and shift-symmetry are broken in the background, a diagonal combination is left unbroken and generates new “time” translations. The Noether charge associated with this unbroken symmetry is the conserved particle physics energy. The energy that couples to gravity is associated with the broken time translation symmetry. Since this energy begins at linear order in $\dot{\pi}$, lumps of π can either gravitate or anti-gravitate, depending on the sign of $\dot{\pi}$! The π thus maximally violate the equivalence principle.

If the standard model fields would couple directly to the condensate there would be a splitting between particle and anti-particle dispersion relations, and a new spin-dependent inverse-square force, mediated by π exchange, which results from the dispersion relation (114). In the non-relativistic limit, with \mathbf{S} the spin:

$$\Delta\mathcal{L} \sim \frac{1}{F} \mathbf{S} \cdot \nabla\pi, \tag{117}$$

where F is some normalization constant. Because of the k^4 dispersion relation, the potential between two sources with spin \mathbf{S}_1 and spin \mathbf{S}_2 , will be proportional to $1/r$:

$$V \sim \frac{M^4}{\tilde{M}^2 F^2} \frac{\mathbf{S}_1 \cdot \mathbf{S}_2 - 3(\mathbf{S}_1 \cdot \hat{\mathbf{r}})(\mathbf{S}_2 \cdot \hat{\mathbf{r}})}{r}, \tag{118}$$

when using only static sources, ignoring retardation effects.

Moreover, not only Lorentz invariance, but also CPT is broken if the standard model fields would couple directly to the condensate. The leading derivative coupling is of the form

$$\Delta\mathcal{L} = \sum_{\psi} \frac{c_{\psi}}{F} \bar{\psi} \bar{\sigma}^{\mu} \psi \partial_{\mu} \phi. \tag{119}$$

As noted in Ref. 190, field redefinitions $\psi \rightarrow e^{i c_{\psi} \phi / F} \psi$ may remove these couplings, but only if such a $U(1)$ symmetry is not broken by mass terms or other couplings in the Lagrangian. If the fermion field ψ has a Dirac mass term $m_D \psi \psi^c$, then the vector couplings, for which $c_{\psi} + c_{\psi^c} = 0$, still can be removed, but the axial couplings remain

$$\Delta\mathcal{L} \sim \frac{1}{F} \bar{\Psi} \gamma^{\mu} \gamma^5 \Psi \partial_{\mu} \phi. \tag{120}$$

After expanding $\phi = M^2 t + \pi$ this becomes

$$\Delta\mathcal{L} \sim \mu \bar{\Psi} \gamma^0 \gamma^5 \Psi + \frac{1}{F} \bar{\Psi} \gamma^\mu \gamma^5 \Psi \partial_\mu \pi \quad (121)$$

with $\mu = M^2/F$. This first term violates both Lorentz invariance and CPT, leading to different dispersion relations for particles and their anti-particles. A bound on μ is obtained by considering the earth to be moving with respect to spatially isotropic condensate background. The induced Lorentz and CPT violating mass term then looks like

$$\mu \bar{\Psi} \gamma \gamma^5 \Psi \cdot \mathbf{v}_{\text{earth}}, \quad (122)$$

which in the non-relativistic limit gives rise to an interaction Hamiltonian:

$$\mu \mathbf{S} \cdot \mathbf{v}_{\text{earth}}. \quad (123)$$

The experimental limit on μ for coupling to electrons is $\mu \leq 10^{-25} \text{ GeV}^{(191)}$ assuming $|\mathbf{v}_{\text{earth}}| \sim 10^{-3}$. For other limits on CPT and Lorentz invariance, see Refs. 192–194.

If there is no direct coupling, the SM fields would still interact with the ghost sector through gravity. Interestingly, IR modifications of general relativity could be seen at relatively short distances, but only after a certain (long) period of time! Depending on the mass M and the expectation value of ϕ , deviations of Newtonian gravity could be seen at distances 1000 km, but only after a time $t_c \sim H_0^{-1}$ where H_0 is the Hubble constant. More general, the distance scale at which deviations from the Newtonian potential are predicted is $r_c \sim M_{Pl}/M^2$ and their time scale is $t_c \sim M_{Pl}^2/M^3$.

To see the IR modifications to GR explicitly, let us consider the effective gravitational potential felt by a test mass outside a source $\rho_m(r, t) = \delta^3(r)\theta(t)$, i.e., a source that turns on at time $t = 0$. This potential is given by:

$$\Phi(r, t) = -\frac{G}{r} [1 + I(r, t)], \quad (124)$$

where $I(r, t)$ is a spatial Fourier integral over momenta k , evaluated using an expansion around flat space; a bare cosmological constant is set to zero.

$$I(r, t) = \frac{2}{\pi} \left\{ \int_0^1 du \frac{\sin(uR)}{(u^3 - u)} \left(1 - \cosh(Tu\sqrt{1 - u^2}) \right) + \int_1^\infty du \frac{\sin(uR)}{(u^3 - u)} \left(1 - \cos(Tu\sqrt{u^2 - 1}) \right) \right\}. \quad (125)$$

Here $u = k/m$, $R = mr$, $T = \alpha M^3/2M_{Pl}^2$, where $m \equiv M^2/\sqrt{2}M_{Pl}$ and α is a coefficient of order 1. For late times, $t \gtrsim t_c$, or $T \gtrsim 1$, the first integrand will dominate and $I(r, t)$ can be well approximated by

$$I(r, t) \simeq \frac{2}{\sqrt{\pi T}} \exp\left(-\frac{R^2}{8T} + \frac{T}{2}\right) \sin\left(\frac{R}{\sqrt{2}}\right). \quad (126)$$

For $R \ll T$, there is indeed an oscillatory behavior for the gravitational potential, growing exponentially as $\exp(T/2)$, while for $R \gg T$ the modification vanishes.

More general gravitational effects have been studied in Ref. 195, where moving sources were considered, and in Ref. 196 where inflation was studied in this context. Moreover, the quantum stability of the condensate was studied in Ref. 197.

This highly speculative scenario opens up a new way of looking at the cosmological constant problem, especially because of the distinction between particle physics energy, \mathcal{E}_{pp} and gravitational energy, \mathcal{E}_{grav} . It has to be developed further to obtain a better judgement.

5.3. Fat Gravitons

A proposal involving a sub-millimeter breakdown of the point-particle approximation for gravitons has been put forward by Sundrum.⁽¹⁹⁸⁾ In standard perturbative gravity, diagrams with external gravitons and SM-particles in loops (see Fig. 1) give a contribution to the effective CC of which the dominant part diverges as Λ_{UV}^4 where Λ_{UV} is some ultraviolet cutoff. This leads to the enormous discrepancy with experimental results for any reasonable value of Λ_{UV} . However, one might wonder what the risks are when throwing away these diagrams from the effective theory $\Gamma_{\text{eff}}[g_{\mu\nu}]$, when $|k^2|$, the momentum of the external gravitons, is larger than some low energy cutoff. Properties at stake are: Unitarity, General Coordinate Invariance (GCI) and locality. In standard effective theory one also has diagrams where soft gravitons give corrections to the SM self energy diagrams (Fig. 1). These cannot be thrown away, since they are crucial in maintaining the equivalence principle between inertial and gravitational masses. However, locally these diagrams are indistinguishable in spacetime, only globally can we discern their topological difference. Thus given locality of the couplings of the point particles in the diagrams, we cannot throw the first diagram away and keep the other. Therefore, it seems progress can be made by considering a graviton as an extended

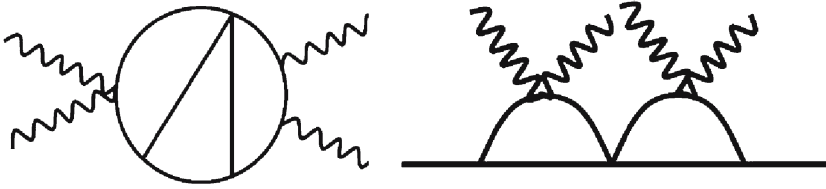


Fig. 1. On the left-hand-side, a typical Standard Model contribution to $\Gamma_{\text{eff}}[g_{\mu\nu}]$. On the right, soft gravitons coupled to loop-correction to SM self-energy. Wiggly lines are gravitons and smooth lines are SM particles.

object. Define the graviton size:

$$l_{\text{grav}} \equiv \frac{1}{\Lambda_{\text{grav}}}. \tag{127}$$

Such a “fat graviton” does not have to couple with point-like locality to SM loops, but with locality up to l_{grav} . Thus a fat graviton can distinguish between the two types of diagrams, possibly suppressing the first while retaining the second.

The value of the CC based on usual power counting would then be

$$\Lambda_{\text{eff}} \sim \mathcal{O}(\Lambda_{\text{grav}}^4/16\pi^2). \tag{128}$$

Comparing with the observational value this gives a bound on the graviton size of

$$l_{\text{grav}} > 20 \mu\text{m} \tag{129}$$

which would indicate a short-distance modification of Newton’s law below $20 \mu\text{m}$. This is however not enough to suppress standard model contributions to the cosmological constant. A new model by the same author has been proposed to take into account also these effects, see Section 3.3.

5.4. Composite Graviton as Goldstone boson

Another approach is to consider the possibility that the graviton appears as a composite Goldstone boson. There exists a theorem by Weinberg and Witten,⁽¹⁹⁹⁾ stating that a Lorentz invariant theory, with a Lorentz covariant energy–momentum tensor does not admit a composite graviton. It is therefore natural to try a mechanism where the graviton appears as a Goldstone boson associated with the spontaneous breaking

of Lorentz invariance. Being a Goldstone boson, the graviton would not develop a potential, and hence the normal cosmological constant problem is absent, see for example Refs. 200, 201.

However, besides difficulties erasing the traces of broken Lorentz invariance to make the model agree with observations, also new fine-tunings are introduced.

A composite structure of the graviton has also been contemplated in Refs. 202, 203, based on more intuitive ideas.

5.5. Summary

Since General Relativity has only been thoroughly tested on solar system distance scales it is a very legitimate idea to consider corrections to GR at galactic and/or cosmological distance scales. However, often these models are not so harmless as supposed to be. The laws of gravity are also significantly changed at shorter scales, or the changes lead to violations of locality. The scenarios described in this section do not directly solve the cosmological constant problem, but offer new ways of looking at it.

On the more positive side, many theories that predict modifications of GR in the IR, reproduce Einstein gravity at smaller distances, but up to some small corrections. These corrections are discussed in Ref. 204 and could be potentially observable at solar system distance scales. At the linearized level gravity is of the scalar–tensor type, because the graviton has an extra polarization that also couples to conserved energy–momentum sources. If these models are correct, an anomalous perihelion precession of the planets is expected to be observed in the near future.

Besides, submillimeter experiments of Newtonian gravity set ever more stringent bounds on both extra dimensional approaches and composite graviton scenarios. It would be very exciting to see a deviation of Newtonian gravity at short distances. On the other hand, observing no change at all, will seriously discourage the hopes that such a mechanism might help in solving the cosmological constant problem.

6. TYPE IV: STATISTICAL APPROACHES

6.1. Hawking Statistics

If the cosmological constant could *a priori* have any value, appearing for example as a constant of integration as in Section 3.4.1, or would become a dynamical variable by means of some other mechanism, then

in quantum cosmology the state vector of the universe would be a superposition of states with different values of Λ_{eff} . The path integral would include all, or some range of values of this effective cosmological constant. The observable value of the CC in this framework is not a fundamental parameter. Different universes with different values of Λ_{eff} contribute to the path integral. The probability of observing a given field configuration will be proportional to $\mathcal{P} \propto \exp(-S(\Lambda_{\text{eff}}))$ in which Λ_{eff} is promoted to be a quantum number.

Eleven dimensional supergravity contains a three-form gauge field, with a four-form field strength $F_{\mu\nu\rho\sigma} = \partial_{[\mu} A_{\nu\rho\sigma]}$.⁽²⁰⁵⁾ When reduced to four dimensions, this gives a contribution to the cosmological constant.^(206–210) Hawking⁽²¹¹⁾ used such a three-form gauge field to argue that the wave function of the universe is peaked at zero cosmological constant. It is the first appearance of the idea that the CC could be fixed by the shape of the wave function of the universe.

The three-form field $A_{\mu\nu\lambda}$ has gauge transformations:

$$A_{\mu\nu\rho} \rightarrow A_{\mu\nu\rho} + \nabla_{[\mu} C_{\nu\rho]} \quad \text{with} \quad F_{\mu\nu\rho\sigma} = \nabla_{[\mu} A_{\nu\rho\sigma]}. \quad (130)$$

This field would contribute an extra term to the action:

$$I = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} (R + 2\Lambda_B) - \frac{1}{48} \int d^4x \sqrt{-g} F_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma}. \quad (131)$$

The field equation for $F^{\mu\nu\rho\sigma}$ is

$$D_\mu F^{\mu\nu\rho\sigma} = 0 \quad \rightarrow \quad \sqrt{-g} F^{\mu\nu\rho\sigma} = \omega \epsilon^{\mu\nu\rho\sigma} \quad (132)$$

Such a field F has no dynamics, but the F^2 term in the action behaves like an effective cosmological constant term, whose value is determined by the unknown scalar field ω , which takes on some arbitrary value. If we substitute the solution (132) back into the Einstein equation, we find, using that $\epsilon^{\mu\nu\rho\sigma} \epsilon_{\mu\nu\rho\sigma} = \pm 4!$:

$$T^{\mu\nu} = \frac{1}{6} \left(F^{\mu\alpha\beta\gamma} F_{\alpha\beta\gamma}^\nu - \frac{1}{8} g^{\mu\nu} F^{\alpha\beta\gamma\delta} F_{\alpha\beta\gamma\delta} \right) = \pm \frac{1}{2} \omega^2 g^{\mu\nu}, \quad (133)$$

where the sign depends on the metric used: in Euclidean metric $\epsilon^{\mu\nu\rho\sigma} \epsilon_{\mu\nu\rho\sigma}$ is positive, whereas in Lorentzian metric it is negative. In the Euclidean action Hawking used:

$$R = -4\Lambda_{\text{eff}} = -4(\Lambda_B - 8\pi G\omega^2), \quad (134)$$

where Λ_B is the bare cosmological constant in Einstein’s equation. It follows that

$$S_{\text{Hawking}} = -\Lambda_{\text{eff}} \frac{V}{8\pi G}. \tag{135}$$

The maximum value of this action is given when V is at its maximum, which Hawking takes to be S^4 , with radius $r = (3\Lambda_{\text{eff}}^{-1})^{1/2}$ and proper circumference $2\pi r$. Then

$$V = \frac{24\pi^2}{\Lambda_{\text{eff}}^2} \rightarrow S(\Lambda) = -3\pi \frac{M_P^2}{\Lambda_{\text{eff}}} \tag{136}$$

and thus the probability density:

$$\mathcal{P} \propto \exp\left(3\pi \frac{M_P^2}{\Lambda_{\text{eff}}}\right) \tag{137}$$

is peaked at $\Lambda = 0$.

Note that we have used here that the probability is evaluated as the exponential of minus the effective action at its stationary point. That is, stationary in $A_{\mu\nu\lambda}$, meaning vanishing covariant derivative of $F_{\mu\nu\lambda\rho}$, in matter fields ϕ and in $g_{\mu\nu}$. The latter condition simply means that $g_{\mu\nu}$ has to satisfy the Einstein equations. Equation (135) is the effective action at the stationary point. It is a good thing that we only need the effective action at its stationary point, so that we do not have to worry about the Euclidean action not being bounded from below, see for example Ref. 20.

However, Hawking’s argument is not correct, since one should not plug an ansatz for a solution back into the action, but rather vary the unconstrained action.⁽²¹²⁾ This differs a minus sign in this case, the same minus sign as going from a Lorentzian to a Euclidean metric, $\Lambda_{\text{eff}} = (\Lambda_B \pm 8\pi G\omega^2)$, but now between the coefficient of $g^{\mu\nu}$ in the Einstein equations, and the coefficient of $(8\pi G)^{-1}\sqrt{g}$ in the action. The correct action becomes:⁽²¹²⁾

$$S = (-3\Lambda_{\text{eff}} + 2\Lambda_B) \frac{-3\pi M_P^2}{\Lambda_{\text{eff}}^2} = -3\pi M_P^2 \frac{\Lambda_B - 12\pi G\omega^2}{(\Lambda_B - 4\pi G\omega^2)^2} \tag{138}$$

now for $\Lambda_{\text{eff}} \rightarrow 0$, the action becomes large and positive and consequently, $\Lambda_{\text{eff}} = 0$ becomes the *least* probable configuration.

Besides, in Ref. 213 it is shown that this approach has also other serious limitations. It is argued that it can only work in the “Landscape” scenario that we discuss in Section 6.3. The reason is that the four-form flux should be subject to Dirac quantization and the spacing in Λ then only becomes small enough with an enormous number of vacua.

6.2. Wormholes

In a somewhat similar approach Coleman⁽²¹⁴⁾ argued that one did not need to introduce a 3-form gauge field, if one includes the topological effects of wormholes. This also transforms the cosmological constant into a dynamical variable. The argument is that on extremely small scales our universe is in contact, through wormholes, with other universes, otherwise disconnected, but governed by the same physics as ours. Although the two ends of a wormhole may be very far apart, in the effective theory of just our universe, the only effect of wormholes is to add *local* interactions, one for each type of wormhole.

The extra term in the action has the form:

$$S_{\text{wormhole}} = - \sum_i (a_i + a_i^\dagger) \int d^4x \sqrt{g} e^{-S_i} K_i, \tag{139}$$

where a_i and a_i^\dagger are the annihilation and creation operators for a type i baby universe, S_i is the action of a semi-wormhole (one that terminates on a baby universe), and K_i is some function of fields on the manifold, with an important exponential factor that suppresses the effects of all wormholes, except those of Planckian size.^(215–217)

The coefficients of these interaction terms are operators $A_i = a_i + a_i^\dagger$ which only act on the variables describing the baby universes, and commute with everything else. Written in terms of A -eigenstates, the effective action becomes:

$$S_{\text{wormhole}} = - \sum_i \int d^4x \sqrt{g} \alpha_i e^{-S_i} K_i \tag{140}$$

with α_i the eigenvalues of the operators A , which would be interpreted as constants of nature, by an observer doing experiments at distance scales larger than the wormhole scale, i.e., for an observer who cannot detect the baby universes.

This way, the effective cosmological constant becomes a function of the a_i . Moreover, on scales larger than the wormhole scale, manifolds that appear disconnected will really be connected by wormholes, and therefore are to be integrated over.

The sum of all vacuum-to-vacuum graphs is the exponential of the sum of connected graphs, which gives the probability density \mathcal{P} :

$$\mathcal{P} \propto \exp \left[\sum_{\text{CCM}} e^{-S_{\text{eff}}(\alpha)} \right], \tag{141}$$

where CCM stands for closed connected manifolds. The sum can be expressed as a background gravitational field effective action, Γ . The sum over closed connected manifolds can then be written as a sum over topologies:

$$\sum_{\text{CCM}} e^{-S_{\text{eff}}(\alpha)} = \sum_{\text{topologies}} e^{-\Gamma(g)} \quad (142)$$

with g the background metric on each topology and each term on the right is again to be evaluated at its stationary point. This is progress, since the leading term in Γ for large, smooth universes is known, and is the cosmological constant term:

$$\Gamma = \Lambda(\alpha) \int d^4x \sqrt{g} + \dots, \quad (143)$$

$\Lambda(\alpha)$ being the fully renormalized cosmological constant. Plugging this back into (141) gives the final result:

$$\mathcal{P} \propto \exp \left[\exp \left(3\pi \frac{M_P^2}{\Lambda_{\text{eff}}} \right) \right], \quad (144)$$

and thus is even sharper peaked at $\Lambda = 0$ than in Hawking's case. For positive CC the maximum volume is taken, like in Hawking's case, the 4-sphere with $r = (3\Lambda_{\text{eff}}^{-1})^{1/2}$. Furthermore, the higher order terms in (143) are neglected.

An advantage of Coleman's approach is that he is able to sidestep many technical difficulties Hawking's approach suffers from. In particular, he uses the Euclidean path integral, which is a solution to the Wheeler–DeWitt equation, only to calculate expectation values of some scalar field. These are independent of x , because the theory is generally covariant. It includes an average over the time in the history of the universe that the expectation value for this operator was measured. This circumvents many issues related to the notion of time in quantum gravity.

However, both Hawking's and Coleman's proposal rely strongly on using a Euclidean path integral and it is unclear whether this is suitable for a theory of quantum gravity.

There is also a more direct problem with Coleman's idea, as put forward by Fishler, Susskind and Polchinski,^(218,219) also see Refs. 220, 221. The problem is that in Coleman's scenario wormholes of every size will materialize in the vacuum with maximum kinematically allowed density,

leading to a universe packed with wormholes of every size. The exponential suppression factor in (139) is inconsistent with the other assumptions that quantum gravity is described by a Euclidean path integral, which is dominated by large scale spherical universes connected by wormholes, where the amplitude of a large scale universe is of order $\exp(M_P^2/\Lambda)$. In particular, taking into account the higher order terms in (143), leads to a violation of the dilute gas approximation, used by Coleman.

In conclusion, wormholes should be left out of the functional integral of quantum gravity. Rather, their effect is that they renormalize the values of physical constants in our universe. Most importantly, if for some reason it is valid to only take Planck-scale wormholes into account, this would make the wavefunction of the universe in the Euclidean formalism, peak at zero value of the cosmological constant.

6.3. Anthropic Principle

One of the first to use anthropic arguments related to the value of the cosmological constant was Weinberg,⁽²²²⁾ see also Refs. 223, 224. He even made the prediction in 1987 that, since the anthropic bound was just a few orders of magnitude larger than the experimental bounds, a non-zero cosmological constant would soon be discovered, which indeed happened.

One can rather easily set anthropic bounds on the value of the cosmological constant. A large positive CC would very early in the evolution of the universe lead to an exponentially expanding de Sitter phase, which then lasts forever. If this would happen before the time of formation of galaxies, at redshift $z \sim 4$, clumps of matter would not become gravitationally bound, and galaxies, and presumably intelligent life, would not form. Therefore

$$\Omega_\Lambda(z_{\text{gal}}) \leq \Omega_M(z_{\text{gal}}) \rightarrow \frac{\Omega_{\Lambda 0}}{\Omega_{M 0}} \leq a_{\text{gal}}^3 = (1 + z_{\text{gal}})^3 \sim 125. \quad (145)$$

This implies that the cosmological constant could have been larger than observed and still not be in conflict with galaxy formation (note that in these estimates everything is held fixed, except Ω_Λ which is allowed to vary, unless stated otherwise).

A typical observer therefore would measure $\rho_\Lambda \sim \tilde{\rho}_\Lambda$, with $\tilde{\rho}_\Lambda$ the value for which the vacuum energy density dominates at about the epoch of galaxy formation. This is the anthropic prediction and it peaks at $\Omega_\Lambda \sim 0.9$, in agreement with the experimental value $\Omega_\Lambda \sim 0.7$ at the 2σ level.⁽²²⁵⁾ It is argued that the agreement can be increased to the 1σ level, by allowing for non-zero neutrino masses.⁽²²⁶⁾ Neutrino masses

would slow down the growth of density fluctuations, and hence influence the value of $\bar{\rho}_\Lambda$. The sum of the neutrino masses would have to be $m_\nu \sim 1 - 2 \text{ eV}$.

On the other hand, a large negative cosmological constant would lead to a rapid collapse of the universe and (perhaps) a big crunch. To set this lower anthropic bound, one has to wonder how long it takes for the emergence of intelligent life. If 7 billion years is sufficient, the bound for a flat universe is $\Lambda \gtrsim -18.8 \rho_0 \sim -2 \times 10^{-28} \text{ g/cm}^3$, if 14 billion years are needed, the constraint is $\Lambda \gtrsim -4.7 \rho_0 \sim -5 \times 10^{-29} \text{ g/cm}^3$.⁽²²⁷⁾

It makes more sense however, to ask what the most likely value of the cosmological constant is, the value that would be experienced by the largest number of observers. Vilenkin's "Principle of Mediocrity",⁽²²⁸⁾ stating that we should expect to find ourselves in a big bang that is typical of those in which intelligent life is possible, is often used. In order for such statistics to be meaningful, it is necessary that there are alternative conditions where things are different. Therefore, it is usually assumed that there is some process that produces an ensemble of a large number of universes, or different, isolated pockets of the same universe, with widely varying properties. Several inflationary scenarios,^(229–232) quantum cosmologies,^(211,228,233–235) and string theory^(213,236–240) predict different domains of the universe, or even different universes, with widely varying values for the different coupling constants. In these considerations it is assumed that there exists many discrete vacua with densely spaced vacuum energies.

The probability measure for observing a value ρ_Λ , using Bayesian statistics, can be written as

$$d\mathcal{P}(\rho_\Lambda) = N(\rho_\Lambda)\mathcal{P}_*(\rho_\Lambda)d\rho_\Lambda, \quad (146)$$

where $\mathcal{P}_*(\rho_\Lambda)d\rho_\Lambda$ is the *a priori* probability of a particular big bang having vacuum energy density between ρ_Λ and $\rho_\Lambda + d\rho_\Lambda$ and is proportional to the volume of those parts of the universe where ρ_Λ takes values in the interval $d\rho_\Lambda$. $N(\rho_\Lambda)$ is the average number of galaxies that form at a specified ρ_Λ ,⁽²²⁾ or, the average number of scientific civilizations in big bangs with energy density ρ_Λ ,⁽²³⁾ per unit volume. The quantity $N(\rho_\Lambda)$ is often assumed to be proportional to the number of baryons, that end up in galaxies.

Given a particle physics model which allows ρ_Λ to vary, and a model of inflation, one can in principle calculate $\mathcal{P}_*(\rho_\Lambda)$, see the above references for specific models and Ref. 241 for more general arguments. $\mathcal{P}_*(\rho_\Lambda)d\rho_\Lambda$ is sometimes argued to be constant,⁽²⁴²⁾ since $N(\rho_\Lambda)$ is only non-zero for a narrow range of values of ρ_Λ . Others point out that there may be a significant departure from a constant distribution.⁽²⁴³⁾ Its value is fixed by the

requirement that the total probability should be one:

$$d\mathcal{P}(\rho_\Lambda) = \frac{N(\rho_\Lambda)d\rho_\Lambda}{\int N(\rho'_\Lambda)d\rho'_\Lambda}. \quad (147)$$

The number $N(\rho_\Lambda)$ is usually calculated using the so-called “spherical infall” model of Gunn and Gott.⁽²⁴⁴⁾ Assuming a constant $\mathcal{P}_*(\rho_\Lambda)$, it is argued that the probability of a big bang with $\Omega_\Lambda \lesssim 0.7$ is roughly 10%, depending on some assumptions about the density of baryons at recombination.^(23,245)

However, it has been claimed that these successful predictions would not hold, when other parameters, such as the amplitude of primordial density fluctuations are also allowed to vary.^(246,247) These arguments are widely debated and no consensus has been reached.^(248,249)

However, it has been very difficult to calculate the *a priori* distribution. The dynamics, leading to a “multiverse” in which there are different pocket universes with different values for the constants of nature, is claimed to be well understood, for example in case of eternal inflation,^(230,232,260) but the problem is that the volume of these thermalized regions with any given value of the constants is infinite. Therefore, to compare them, one has to introduce some cutoff and the results tend to be highly sensitive to the choice of cutoff procedure.^(251–253) In a recent paper a different method is proposed to find this distribution.⁽²⁵⁴⁾

It should be stressed that this approach to the cosmological constant problem is especially used within string theory, where one has stumbled upon a wide variety of possible vacuum states, rather than a unique one.^(213,236–240,255,256) By taking different combinations of extra-dimensional geometries, brane configurations, and gauge field fluxes, a wide variety of states can be constructed, with different local values of physical constants, such as the cosmological constant. These are the 3-form RR and NS fluxes that can be distributed over the 3-cycles of the Calabi Yau manifold. The number of independent fluxes therefore is related to the number of 3-cycles in the 6-dimensional Calabi Yau space, and can be several hundred. In addition, the moduli are also numerous and also in the hundreds, leading to a total number of degrees of freedom in a Calabi Yau compactification of order 1000 or more. The number of metastable vacua for a given Calabi Yau compactification therefore could be 10^{1000} , and the spacing between the energy levels $10^{-1000} M_P^4$, of which some 10^{500} would have a vacuum energy that is anthropically allowed. The states with (nearly) vanishing vacuum energy tend to be those where one begins with a supersymmetric state with a negative vacuum energy, to which supersymmetry breaking adds just the right amount of positive vacuum energy.

This picture is often referred to as the “Landscape”. The spectrum of ρ_Λ could be very dense in this ‘discretuum’ of vacua, but nearby values of ρ_Λ could correspond to very different values of string parameters. The prior distribution would then no longer be flat, and it is unclear how it should be calculated.

A review of failed attempts to apply anthropic reasoning to models with varying cosmological constant can be found in Ref. 257. See Ref. 258 for a recent critique. Another serious criticism was given in Ref. 259, where it is argued that very different universes than our own could also lead to a small cosmological constant, long-lived stars, planets and chemistry based life, for example a cold big bang scenario. An analysis of how to make an anthropic prediction is made in Ref. 260.

A not very technical and almost foundational introduction to the anthropic principle is given by Ref. 96.

6.3.1. Discrete Anthropic Principle

It might be worthwhile to make a distinction between a continuous anthropic principle and a discrete version. Imagine we have a theory at our hands that describes an ensemble of universes (different possible vacuum solutions) with different discrete values for the fine structure constant:

$$\frac{1}{\alpha} = n + \mathcal{O}\left(\frac{1}{n}\right) \quad (148)$$

such that the terms $1/n$ are calculable. An anthropic argument could then be used to explain why we are in the universe with $n = 137$. Such a version of the anthropic principle might be easier to accept than one where all digits are supposed to be anthropically determined. Note that we are already very familiar with such use of an anthropic principle. In a finite universe, there is a finite number of planets and we live on one of the (very few?) inhabitable ones. Unfortunately, we have no theory at our hands to determine the fine structure constant this way, let alone the cosmological constant.

6.4. Summary

This very much discussed approach offers a new line of thought, but so far, unfortunately, predictions for different constants of Nature, like the cosmological constant and the fine-structure constant, are not interrelated. We try to look for a more satisfying approach.

7. CONCLUSIONS

In this paper we categorized the different approaches to the cosmological constant problem. The many different ways in which it can be phrased often blurs the road to a possible solution and the wide variety of approaches makes it difficult to distinguish real progress.

So far we can only conclude that in fact none of the approaches described above is a real outstanding candidate for a solution of the “old” cosmological constant problem. Most effort nowadays is in finding a physical mechanism that drives the Universe’s acceleration, but as we have seen these approaches, be it by modifying general relativity in the far infrared, or by studying higher dimensional braneworlds, generally do not convincingly attack the old and most basic problem.

Since even the sometimes very drastic modifications advocated in the proposals we discussed do not lead to a satisfactory answer, this seems to imply that the ultimate theory of quantum gravity might very well be based on very different grounds than imagined so far. The only way out could be the discovery of a symmetry that forbids a cosmological constant term to appear.

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