

Bell's Inequality Tests with Meson–Antimeson Pairs

A. Bramon,¹ R. Escribano,^{1,2} and G. Garbarino³

Received June 08, 2005 / Published online February 22, 2006

Recent proposals to test Bell's inequalities with entangled pairs of pseudoscalar mesons are reviewed. This includes pairs of neutral kaons or B-mesons and offers some hope to close both the locality and the detection loopholes. Specific difficulties, however, appear thus invalidating most of those proposals. The best option requires the use of kaon regeneration effects and could lead to a successful test if moderate K^0 and \bar{K}^0 detection efficiencies are achieved.

KEY WORDS: entanglement; Bell's inequalities; neutral kaon and B-meson systems; Loophole-free tests; non-locality without inequalities; quantum measurement.

1. INTRODUCTION

The correlations shown by the distant parts of certain composite systems offer one of the most counterintuitive and subtle aspects of quantum mechanics. This was already evident in 1935, when Einstein, Podolsky and Rosen (EPR),⁽¹⁾ discussing a gedanken experiment with entangled states, arrived at the conclusion that the description of physical reality given by the quantum wave function cannot be complete. Bohr, in his famous response,⁽²⁾ noted that EPR's criterion of physical reality contained an ambiguity if applied to quantum phenomena: an argument using the complementarity point of view led him to conclude that quantum mechanics, in the form restricted to human knowledge, "would appear as a completely rational description of the physical phenomena".

¹Grup de Física Teòrica, Universitat Autònoma de Barcelona, E-08193 Bellaterra, Barcelona, Spain.

²IFAE, Universitat Autònoma de Barcelona, E-08193 Bellaterra, Barcelona, Spain.

³Dipartimento di Fisica Teorica, Università di Torino and INFN, Sezione di Torino, I-10125 Torino, Italy.

For about 30 years the debate triggered by EPR and Bohr remained basically a matter of philosophical belief. Then, in 1964, Bell⁽³⁾ interpreted EPR's argument as the need for the introduction of additional, unobservable variables aiming to restore *completeness*, *relativistic causality* (or *locality*) and *realism* in quantum theory. He established a theorem which proved that any *local hidden-variable* (i.e., *local realistic*⁽⁴⁾) theory is incompatible with some statistical predictions of quantum mechanics. Since then, various forms of Bell's inequalities⁽⁵⁻⁸⁾ have been the tool for an experimental discrimination between local realism (LR) and quantum mechanics (QM).

Many experiments have been performed, mainly with entangled photons⁽⁹⁻¹³⁾ and ions,⁽¹⁴⁾ in order to confront LR with QM. All these tests obtained results in good agreement with QM and showed the violation of *non-genuine* Bell's inequalities. Indeed, because of non-idealities of the apparatus and other technical problems, *supplementary assumptions* not implicit in LR were needed in the interpretation of the experiments. Consequently, no one of these experiments has been strictly *loophole free*,^(10,15,16) i.e., able to test a *genuine* Bell's inequality.

It has been proven^(8,10,17) that for any entangled state one can derive Bell's inequalities without the introduction of (plausible but not testable) supplementary assumptions concerning undetected events. For maximally entangled (non-maximally entangled) states, if one assumes that all detectors have the same overall efficiency η , these genuine inequalities are violated by QM if $\eta > 0.83$ ⁽¹⁸⁾ ($\eta > 0.67$ ⁽¹⁹⁾). Since such thresholds cannot be presently achieved in photon experiments, only non-genuine inequalities have been tested experimentally. They are then violated by QM irrespectively of the detection efficiency values.

Several of these photonic tests violated non-genuine inequalities by the amount predicted by QM but they could not overcome the *detection loophole*. Indeed, local realistic models exploiting detector inefficiencies and reproducing the experimental results can be contrived^(8,20) for these tests. Only the recent experiment with entangled beryllium ions of Ref. 14, for which $\eta \simeq 0.97$, did close the detection loophole. But then the other existing loophole, the *locality loophole*, remains open due to the tiny inter-ion separation. Conversely, an experiment with distant entangled photons⁽¹¹⁾ closed this latter loophole. In this test, the measurements on the two photons were carried out under space-like separation conditions, thus avoiding any exchange of subluminal signals between the two measurement events, but detection efficiencies were too low to close the detection loophole. In other words, no experiment closing simultaneously both loopholes has been performed till now.

Extensions to other kinds of entangled systems are thus important. Over the past 10 years or so there has been an increased interest

on the possibility to test LR versus QM in particle physics, i.e., by using entangled neutral kaons^(21–38) or B -mesons.^(39–43) This is also a manifestation of the desire to go beyond the usually considered spin-singlet case and to have new entangled systems made of massive particles with peculiar quantum–mechanical properties. Entangled $K^0\bar{K}^0$ ($B^0\bar{B}^0$) pairs are produced in the decay of the ϕ resonance⁽⁴⁴⁾ [$\Upsilon(4S)$ resonance⁽⁴⁵⁾] and in proton–antiproton annihilation processes at rest.⁽⁴⁶⁾ For kaons, the strong nature of hadronic interactions should contribute to close the detection loophole, since it enhances the efficiencies to detect the products of kaon decays and kaon interactions with ordinary matter (pions, kaons, nucleons, hyperons, . . .). Moreover, the two kaons produced in ϕ decays or $p\bar{p}$ annihilations at rest fly apart from each other at relativistic velocities and easily fulfill the condition of space-like separation. Therefore, contrary to the experiment with ion pairs of Ref. 14, the locality loophole could be closed with kaon pairs by using equipments able to prepare, very rapidly, the alternative kaon measurement settings.

In this contribution our purpose is to review the Bell's inequalities proposed to test LR versus QM using entangled pairs of neutral pseudoscalar mesons such as $K^0\bar{K}^0$ and $B^0\bar{B}^0$. These proposals will be discussed on the light of the basic requirements necessary to establish genuine Bell's inequalities.

2. NEUTRAL MESON SYSTEMS

2.1. Single Mesons: Time Evolution and Measurements

In this section, we discuss the time evolution of and the kind of measurements on neutral pseudoscalar mesons. We mainly refer to the most known case of neutral kaons, but the modifications which apply to neutral B -mesons are stressed as well. These differences originate from the different values of the meson parameters and turn out to have important consequences when testing LR versus QM.

Neutral kaons are copiously produced by strangeness–conserving strong interaction processes such as $\pi^-p \rightarrow \Lambda K^0$ and $p\bar{p} \rightarrow K^-\pi^+K^0$, $K^+\pi^-\bar{K}^0$ and so they initially appear either as K^0 's (strangeness $S = +1$) or \bar{K}^0 's (strangeness $S = -1$). The distinct strong interactions of the $S = +1$ and $S = -1$ kaons on the bound nucleons of absorber materials project an incoming kaon state into one of these two orthogonal members of the strangeness basis $\{K^0, \bar{K}^0\}$, and permit the measurement of S .⁽²⁶⁾ This strangeness detection is analogous to the projective von Neumann measurements with two-channel analysers for polarized photons or Stern–Gerlach set-ups for spin $-1/2$ particles. Unfortunately, the

detection efficiency for such strangeness measurements is rather limited.⁽⁴⁶⁾ Indeed, it could be close to 1 only for infinitely dense absorber materials or for ultrarelativistic kaons, where, by Lorentz contraction, the absorber is seen by the incoming kaon as extremely dense. In this case, kaon–nucleon strong interactions become much more likely than kaon weak decays. It would be highly desirable to identify very efficient absorbers. Since this does not seem to be viable at present, one has to play with small strangeness detection efficiencies, which originate serious conceptual difficulties when discussing Bell-type tests for entangled kaons.^(34,37)

The kaon time-evolution and decay in free space is governed by the lifetime basis, $\{K_S, K_L\}$, whose states diagonalize the non-Hermitian weak Hamiltonian. The proper time propagation of these short- and long-lived states having well-defined masses $m_{S,L}$ is given by:

$$|K_{S,L}(\tau)\rangle = e^{-im_{S,L}\tau} e^{-\frac{1}{2}\Gamma_{S,L}\tau} |K_{S,L}\rangle, \quad (2.1)$$

where $\Gamma_{S,L} \equiv 1/\tau_{S,L}$ are the kaon decay widths and $\tau_S = (0.8953 \pm 0.0005) \times 10^{-10}$ s and $\tau_L = (5.18 \pm 0.04) \times 10^{-8}$ s⁽⁴⁷⁾ the corresponding lifetimes. Being the dynamics of free kaons governed by strangeness non-conserving weak interactions, $K^0 - \bar{K}^0$ mixing and $K_S - K_L$ interferences will appear thus producing the well known $K^0 - \bar{K}^0$ oscillations in time. Assuming CPT invariance, the relationship between strong and weak interaction eigenstates is provided by:⁽⁴⁸⁾

$$\begin{aligned} |K_S\rangle &= \frac{1}{\sqrt{2(1+|\epsilon|^2)}} [(1+\epsilon)|K^0\rangle + (1-\epsilon)|\bar{K}^0\rangle], \\ |K_L\rangle &= \frac{1}{\sqrt{2(1+|\epsilon|^2)}} [(1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle], \end{aligned} \quad (2.2)$$

ϵ being the CP -violation parameter in the $K^0 - \bar{K}^0$ mixing. Weak interaction eigenstates are related to the CP eigenstates $|K_1\rangle$ ($CP = +1$) and $|K_2\rangle$ ($CP = -1$) by:

$$\begin{aligned} |K_S\rangle &= \frac{1}{\sqrt{1+|\epsilon|^2}} [|K_1\rangle + \epsilon|K_2\rangle], \\ |K_L\rangle &= \frac{1}{\sqrt{1+|\epsilon|^2}} [|K_2\rangle + \epsilon|K_1\rangle]. \end{aligned} \quad (2.3)$$

To observe if a kaon is propagating as a K_S or K_L at time τ , one has to identify at which time it subsequently decays. Kaons, which show a decay between times τ and $\tau + \Delta\tau$ have to be identified as K_S 's, while those decaying later than $\tau + \Delta\tau$ have to be identified as K_L 's. The probabilities for wrong K_S and K_L identification are then given by $\exp(-\Gamma_S \Delta\tau)$ and

$1 - \exp(-\Gamma_L \Delta\tau)$, respectively. With $\Delta\tau = 4.8 \tau_S$, both K_S and K_L misidentification probabilities reduce to $\simeq 0.8\%$. Note that the K_S and K_L states are not strictly orthogonal to each other, $\langle K_S | K_L \rangle = 2 \operatorname{Re} \epsilon / (1 + |\epsilon|^2) \neq 0$, and their identification cannot be exact even in principle. However, ϵ is so small [$|\epsilon| \simeq (2.284 \pm 0.014) \times 10^{-3(47)}$] and the decay probabilities of the two components so different ($\Gamma_S \simeq 579 \Gamma_L$) that the K_S versus K_L identification effectively works in many cases.⁽³⁴⁾ Note also that, contrary to strangeness measurements, lifetime observations can be made with quite high efficiencies; by using detectors with very large solid angles, one can play with almost ideal efficiencies ($\eta_\tau \simeq 1$) for the detection of the kaon decay products.

Apart from this (only approximate) K_S versus K_L identification and the previous (in principle exact) strangeness measurement, no other quantum–mechanical measurement with dichotomic outcomes is possible for neutral kaons.⁽³⁴⁾ Only these two complementary observables can be exploited to establish Bell's inequalities. This is in sharp contrast to the standard spin-singlet case and reduces the possibilities of kaon experiments.

The above methods used to discriminate K^0 versus \bar{K}^0 and K_S versus K_L correspond to *active* measurement procedures since they are performed by exerting the free will of the experimenter. Indeed, at a chosen time, either one places a slab of matter or allows for free space propagation. Contrary to what happens with other two-level quantum systems, such as spin $-1/2$ particles or photons, *passive* measurements of strangeness and lifetime for neutral kaons are also possible⁽⁴⁹⁾ by randomly exploiting the quantum–mechanical dynamics of kaon decays.

The strangeness content of neutral kaon states can indeed be determined by observing their semileptonic decay modes, which obey the well tested $\Delta S = \Delta Q$ rule. This rule allows the modes $K^0 \rightarrow \pi^- + l^+ + \nu_l$ and $\bar{K}^0 \rightarrow \pi^+ + l^- + \bar{\nu}_l$ ($l = e, \mu$) but forbids decays into the respective charge conjugated modes. Obviously, the experimenter cannot induce a kaon to decay semileptonically and not even at a given time: he or she can only sort at the end of the day all observed events in proper decay modes and time intervals. Therefore, this discrimination between K^0 and \bar{K}^0 is called a *passive* measurement of strangeness. As in the case of active strangeness measurements, the detection efficiency for passive strangeness measurements is rather limited—it is given by the K_L and K_S semileptonic branching ratios, which are $\simeq 0.66$ and $\simeq 1.1 \times 10^{-3}$, respectively.⁽⁴⁷⁾ Again, this poses serious problems when testing LR versus QM.

By neglecting the small CP violation effects ($\epsilon = 0$ and thus $\langle K_S | K_L \rangle = 0$), one can discriminate between K_S 's and K_L 's by leaving the kaons to propagate in free space and by observing their distinctive nonleptonic $K_S \rightarrow 2\pi$ or $K_L \rightarrow 3\pi$ decays. This represents a passive measurement of lifetime, since the kaon decay modes—nonleptonic in the present case, instead

of semileptonic as before—as well as the decay times cannot be in any way influenced by the experimenter.

We therefore have two conceptually different experimental procedures to measure each one of the two neutral kaon observables. The active measurement of strangeness is monitored by strangeness conservation while the corresponding passive measurement is assured by the $\Delta S = \Delta Q$ rule. Active and passive lifetime measurements are possible thanks to the smallness of Γ_L/Γ_S and ϵ , respectively. Note that with the passive measurement method, the mere quantum-mechanical dynamics of kaon decays decides if the neutral kaon is going to be measured either in the strangeness or in the lifetime basis. The experimenter remains totally passive in such measurements, which are thus clearly different from the usual, active von Neumann projection measurements.

Both active and passive procedures lead to the same probabilities for strangeness and lifetime measurements.⁽⁴⁹⁾ Considering the evolution of a neutral kaon produced at $\tau = 0$ as a K^0 , in both cases one easily obtains the following transition probabilities:

$$P(K^0(0) \rightarrow K^0(\tau)) = \frac{1}{4}(e^{-\Gamma_S\tau} + e^{-\Gamma_L\tau}) \left[1 + \frac{\cos(\Delta m \tau)}{\cosh(\Delta\Gamma \tau/2)} \right], \quad (2.4)$$

$$P(K^0(0) \rightarrow \bar{K}^0(\tau)) = \frac{1}{4}(e^{-\Gamma_S\tau} + e^{-\Gamma_L\tau}) \left[1 - \frac{\cos(\Delta m \tau)}{\cosh(\Delta\Gamma \tau/2)} \right], \quad (2.5)$$

$$P(K^0(0) \rightarrow K_L(\tau)) = \frac{1}{2}e^{-\Gamma_L\tau}, \quad (2.6)$$

$$P(K^0(0) \rightarrow K_S(\tau)) = \frac{1}{2}e^{-\Gamma_S\tau}, \quad (2.7)$$

where $\Delta m \equiv m_L - m_S$ and $\Delta\Gamma \equiv \Gamma_L - \Gamma_S$ are determined by strangeness oscillation experiments through Eqs. (2.4) and (2.5). The experimental equivalence of active and passive measurement procedures on single kaon states and the agreement with quantum-mechanical predictions have already been established.^(47,50,51)

The existence of the two measurement procedures—active and passive—opens new possibilities for tests of basic principles of QM with kaons⁽⁴⁹⁾—such as quantum erasure and quantitative formulations of Bohr's complementarity—which have no analog for any other two-level quantum system considered up to date. Unfortunately, as we will see in detail in Sec. 3, passive measurements are of no interest when testing Bell's inequalities with kaons, where only *active* measurements must be considered.^(34,35)

Neutral B -mesons are easily produced at asymmetric B -factories using high luminosity and asymmetric e^+e^- colliders operating at the

$J^{PC} = 1^{--}\Upsilon(4S)$ resonance.⁽⁴⁵⁾ For these mesons, the strangeness eigenstates are replaced by the *beauty* eigenstates $|B^0\rangle$ and $|\bar{B}^0\rangle$, while the light (m_L) and heavy (m_H) mass eigenstates are $|B_L\rangle$ and $|B_H\rangle$. Experimentally, we know that B_L and B_H have very similar decay widths: $|\Delta\Gamma_B|/\Gamma_B < 0.18$ at 95% CL, where $\Delta\Gamma_B = \Gamma_H - \Gamma_L$ and $\Gamma_B = (\Gamma_L + \Gamma_H)/2 \equiv 1/\tau_B$, with $\tau_B = (1.536 \pm 0.014) \times 10^{-12}$ s.⁽⁴⁷⁾ With these changes, Eqs. (2.1)–(2.7) still hold if ϵ (Δm) is replaced by ϵ_{B^0} ($\Delta m_B = m_H - m_L$), the CP violation parameter in the B^0 – \bar{B}^0 mixing. Contrary to the kaon case, CP violation in the B^0 – \bar{B}^0 mixing has not been observed unambiguously, since $\langle B_L|B_H\rangle = 2\text{Re}\epsilon_{B^0}/(1 + |\epsilon_{B^0}|^2) = (1.0 \pm 6.2) \times 10^{-3}$.⁽⁴⁷⁾ Experimentally one knows that for kaons and B -mesons one has $|\Delta\Gamma| \simeq 2.1\Delta m$ and $|\Delta\Gamma_B| \simeq 0.23\Delta m_B$, respectively; thus, the number of flavour oscillations that one can observe in Eqs. (2.4) and (2.5) is much larger for B -mesons than for K -mesons.

Concerning neutral B -meson measurements, the main difference with respect to the neutral kaon case is that *active* flavour (strangeness or beauty) measurement procedures are only available for kaons.^(42,43) The B -meson beauty can only be determined through a *passive* procedure, by observing the meson decay modes. The series of decay products $f = D^*(2010)^-l^+\nu_l, D^-\pi^+, \dots$, which are forbidden for a \bar{B}^0 , necessarily come from a B^0 , while the opposite is true for the respective charge conjugated modes $\bar{f} = D^*(2010)^+l^-\bar{\nu}_l, D^+\pi^-, \dots$ ($l = e, \mu$). Passive B -meson measurements able to distinguish between B_L 's and B_H 's are almost impossible to perform nowadays, especially if operated in an experiment aiming to test a Bell's inequality, due to the small value of $|\Delta\Gamma_B|/\Gamma_B$. As we discuss in Sec. 3, these limitations play a decisive role when testing LR versus QM with entangled B -mesons.

2.2. Entangled Meson Pairs

Let us now consider two-kaon entangled states, which are analogous to the standard and widely used two-photon entangled states.^(29,31,38,52) From both ϕ -meson resonance decays⁽⁴⁴⁾ or S -wave proton–antiproton annihilation,⁽⁴⁶⁾ one starts at time $\tau = 0$ with the $J^{PC} = 1^{--}$ state:

$$\begin{aligned} |\phi(0)\rangle &= \frac{1}{\sqrt{2}} \left[|K^0\rangle_l |\bar{K}^0\rangle_r - |\bar{K}^0\rangle_l |K^0\rangle_r \right] \\ &= \frac{1}{\sqrt{2}} \frac{1 + |\epsilon|^2}{1 - \epsilon^2} \left[|K_L\rangle_l |K_S\rangle_r - |K_S\rangle_l |K_L\rangle_r \right], \end{aligned} \quad (2.8)$$

where l and r denote the “left” and “right” directions of motion of the two separating kaons and CP–violating effects enter the last equality.

Note that this state is antisymmetric and maximally entangled in the two observable bases.

After production, the left and right moving kaons evolve according to Eq. (2.1) up to times τ_l and τ_r , respectively. This leads to the state:

$$|\phi(\tau_l, \tau_r)\rangle = \frac{1}{\sqrt{2}} e^{-(\Gamma_L \tau_l + \Gamma_S \tau_r)/2} \times \left\{ |K_L\rangle_l |K_S\rangle_r - e^{i\Delta m(\tau_l - \tau_r)} e^{\Delta\Gamma(\tau_l - \tau_r)/2} |K_S\rangle_l |K_L\rangle_r \right\} \quad (2.9)$$

in the lifetime basis, or:

$$|\phi(\tau_l, \tau_r)\rangle = \frac{1}{2\sqrt{2}} e^{-(\Gamma_L \tau_l + \Gamma_S \tau_r)/2} \left\{ (1 - e^{i\Delta m(\tau_l - \tau_r)} e^{\Delta\Gamma(\tau_l - \tau_r)/2}) [|K^0\rangle_l |K^0\rangle_r - |\bar{K}^0\rangle_l |\bar{K}^0\rangle_r] + (1 + e^{i\Delta m(\tau_l - \tau_r)} e^{\Delta\Gamma(\tau_l - \tau_r)/2}) [|K^0\rangle_l |\bar{K}^0\rangle_r - |\bar{K}^0\rangle_l |K^0\rangle_r] \right\} \quad (2.10)$$

in the strangeness basis, where small CP violation effects have been safely neglected.

Note the analogy between state (2.9) and the polarization-entangled two-photon [idler (*i*) plus signal (*s*)] state used in optical tests of Bell's inequalities:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left\{ |V\rangle_i |H\rangle_s - e^{i\Delta\phi} |H\rangle_i |V\rangle_s \right\}, \quad (2.11)$$

where $\Delta\phi$ is an adjustable relative phase. For entangled kaons, the non-vanishing value of Δm plays the same role as $\Delta\phi$ and induces K_S and K_L interferences, as seen from Eq. (2.9), as well as strangeness oscillations in time. These oscillations can be used to mimic the different orientations of polarization analyzers in photonic Bell-tests.^(29,31)

The same- and opposite-strangeness detection probabilities:

$$\begin{aligned} P(K^0, \tau_l; K^0, \tau_r) &= P(\bar{K}^0, \tau_l; \bar{K}^0, \tau_r) \\ &= \frac{1}{8} \left(e^{-(\Gamma_L \tau_l + \Gamma_S \tau_r)} + e^{-(\Gamma_S \tau_l + \Gamma_L \tau_r)} \right) \\ &\quad \times \left\{ 1 - \frac{\cos[\Delta m(\tau_l - \tau_r)]}{\cosh[\Delta\Gamma(\tau_l - \tau_r)/2]} \right\}, \end{aligned} \quad (2.12)$$

$$\begin{aligned}
P(K^0, \tau_l; \bar{K}^0, \tau_r) &= P(\bar{K}^0, \tau_l; K^0, \tau_r) \\
&= \frac{1}{8} \left(e^{-(\Gamma_L \tau_l + \Gamma_S \tau_r)} + e^{-(\Gamma_S \tau_l + \Gamma_L \tau_r)} \right) \\
&\quad \times \left\{ 1 + \frac{\cos[\Delta m(\tau_l - \tau_r)]}{\cosh[\Delta\Gamma(\tau_l - \tau_r)/2]} \right\} \quad (2.13)
\end{aligned}$$

are obtained for both active and passive joint measurements.⁽⁴⁹⁾ Note that for entangled B -meson pairs created in $\Upsilon(4S) \rightarrow B^0 \bar{B}^0$ decays, the same- and opposite-beauty detection probabilities simplify into:

$$\begin{aligned}
P(B^0, \tau_l; B^0, \tau_r) &= P(\bar{B}^0, \tau_l; \bar{B}^0, \tau_r) \\
&= \frac{1}{4} e^{-(\tau_l + \tau_r) \Gamma_B} \{1 - \cos[\Delta m_B(\tau_l - \tau_r)]\}, \quad (2.14)
\end{aligned}$$

$$\begin{aligned}
P(B^0, \tau_l; \bar{B}^0, \tau_r) &= P(\bar{B}^0, \tau_l; B^0, \tau_r) \\
&= \frac{1}{4} e^{-(\tau_l + \tau_r) \Gamma_B} \{1 + \cos[\Delta m_B(\tau_l - \tau_r)]\} \quad (2.15)
\end{aligned}$$

due to the smallness of the B_L and B_H lifetime difference ($\Gamma_L = \Gamma_H = \Gamma_B$). Note also that for $\tau_l = \tau_r$ we have perfect EPR–correlations in $J^{PC} = 1^{--}$ meson–antimeson pairs: the same-flavour probabilities (2.12) and (2.14) vanish and the opposite-flavour probabilities (2.13) and (2.15) take the maximal values.

Entanglement in the flavour quantum number has been tested experimentally, over macroscopic distances, for kaons at CPLEAR,⁽⁴⁶⁾ using active strangeness measurements, and for B -mesons at Belle,⁽⁴¹⁾ using passive measurements of beauty. The non–separability of the meson–antimeson $J^{PC} = 1^{--}$ state could be also observed at the DaΦne ϕ -factory,⁽⁴⁴⁾ using passive⁽⁵³⁾ and (with some modification of the set-up) active strangeness measurements.

3. BELL'S INEQUALITY TESTS WITH MESON-ANTIMESON PAIRS

3.1. Requirements to Establish Genuine Bell's Inequality Tests

The requirements for deriving from LR a Bell's inequality which could contradict quantum–mechanical predictions can be summarized as follows:

- (1) A non-factorizable or entangled state must be used;
- (2) Alternative (mutually exclusive) measurements corresponding to two non-commuting observables must be chosen at will both on the left and right side;
- (3) Dichotomic outcomes should correspond to each single measurement (or trichotomic if the possibility of undetected events is considered as a third outcome);
- (4) Measurement events must be space-like separated.

The first requirement poses no problem. As previously stated, entanglement has been confirmed experimentally for meson–antimeson pairs. It is then important to explore the possibility to derive genuine Bell's inequalities for such systems.

Difficulties appear with requirement number (2). Indeed, among the differences between the singlet-spin state of entangled photons and the $K^0\bar{K}^0$ entangled state previously considered, the most important one is that while for photons one can measure the linear polarization along *any* space direction chosen at will, measurements on neutral kaons are only of *two* kinds: one can choose to measure either strangeness or lifetime. This reduces considerably the possibilities of Bell-tests with neutral kaons. For entangled $B^0\bar{B}^0$ pairs the situation is even more unfortunate: indeed, the lack of active measurement procedures for B -mesons makes impossible the derivation of genuine Bell's inequalities.⁽⁴³⁾

Also, in order to establish the feasibility of a real test, one has to derive the detection efficiencies necessary for a meaningful quantum–mechanical violation of the considered Bell's inequality. In addition, decay events are known to further complicate the issue. With all this in mind and in the light of the basic requirements (1)–(4), we proceed now to analyse various proposals of Bell-tests with entangled meson–antimeson pairs.

3.2. Proposals with Passive Measurements

A recent paper⁽⁴¹⁾ claims that a violation of a Bell's inequality has been observed for the first time in particle physics using the particle–antiparticle correlations in semileptonic B -meson decays. Other authors⁽²⁷⁾ proposed an analogous test with neutral kaons. In the following we show that, since B - or K -decays serve to identify flavour passively, the inequalities considered in Refs. 27 and 41 cannot be considered genuine Bell's inequalities.

To exemplify, let us consider in some detail the recent test of Ref. 41, where an entangled B -meson state analogous to that of Eqs. (2.9) and

(2.10) was employed. The experiment measured the joint probabilities of Eqs. (2.14) and (2.15). The flavour of each member of the pair was identified by observing its semileptonic decay. The decay channel $f = D^*(2010)^- l^+ \nu_l$, which is forbidden for a B^0 , unambiguously comes from a B^0 , while the opposite is true for the respective charge conjugated mode $\bar{f} = D^*(2010)^+ l^- \bar{\nu}_l$ ($l = e, \mu$). The corresponding partial decay widths satisfy $\Gamma_{B^0 \rightarrow f} = \Gamma_{\bar{B}^0 \rightarrow \bar{f}}$.⁽⁴⁷⁾ Experimentally, one counts the number of joint B -meson decay events into the distinct decay modes $f_{l,r}$ and in the appropriate time intervals $[\tau_{l,r}, \tau_{l,r} + d\tau_{l,r}]$; then the joint decay probabilities $\mathcal{P}(f_l, \tau_l; f_r, \tau_r)$ are obtained after dividing these numbers by the total number of initial $B^0 \bar{B}^0$ pairs. Finally, the corresponding joint decay rates $\Gamma(f_l, \tau_l; f_r, \tau_r)$ are derived as:

$$\begin{aligned} \Gamma(f_l, \tau_l; f_r, \tau_r) &\equiv \frac{d^2 \mathcal{P}(f_l, \tau_l; f_r, \tau_r)}{d\tau_l d\tau_r} \\ &= P(B_l, \tau_l; B_r, \tau_r) \Gamma_{B_l \rightarrow f_l} \Gamma_{B_r \rightarrow f_r} \end{aligned} \quad (3.1)$$

from which the joint probabilities $P(B_l, \tau_l; B_r, \tau_r)$ of Eqs. (2.14) and (2.15) immediately follow. The data of Ref. 41 are found to be in good agreement with the quantum–mechanical predictions in Eqs. (2.14) and (2.15). This is a convincing proof of the entanglement between the two members of each B -meson pair, but is it a meaningful test confronting LR versus QM?

In our view and because of the lack of active measurements, the Clauser, Horne, Shimony and Holt (CHSH)⁽⁷⁾ inequality tested in Ref. 41 is not a genuine Bell's inequality. The conventional and most convincing procedure to demonstrate this consists in constructing a local model of hidden variables which agrees with the quantum–mechanical predictions and thus with the experimental data of Ref. 41. In the present case, this is easily achieved⁽⁴³⁾ by simply adapting an original argument introduced by Kasday⁽⁵⁴⁾ in another context. Each $B^0 \bar{B}^0$ pair is assumed to be produced at $\tau = 0$ with a set of hidden variables $\{\tau_l, f_l, \tau_r, f_r\}$ deterministically specifying *ab initio* the future decay times and decay modes of its two members. Different B -meson pairs are then supposed to be produced with a probability distribution coinciding precisely with the joint decay probability $\mathcal{P}(f_l, \tau_l; f_r, \tau_r)$ entering Eq. (3.1). Note that the conventional normalization in the hidden variable space, $\int d\lambda \rho(\lambda) = 1$, is now similarly given by $\int \Sigma_{f_l, f_r} \int d\tau_l \int d\tau_r \Gamma(f_l, \tau_l; f_r, \tau_r) = 1$, where the time integrals extend from 0 to ∞ and the sum to all B^0 and \bar{B}^0 decay modes. Note also that our proposed hidden variable distribution function $\mathcal{P}(f_l, \tau_l; f_r, \tau_r)$ reproduces the successful quantum—mechanical description of all the measurements in Ref. 41. More importantly, our *ad hoc* local realistic model also violates the inequality measured there. This proves that the inequality tested in Ref. 41 is

not a genuine Bell-inequality, which, by definition, has to be satisfied in any local realistic approach. A similar criticism applies to the inequality derived in Ref. 27 for entangled $K^0\bar{K}^0$ pairs. The failure of both discussions is due to the lack of an active intervention of the experimenter.

3.3. Proposals with Active Measurements in Free Space

The analogy between strangeness and linear polarization measurements has been exploited by several authors. In the analysis by Ghirardi *et al.*⁽²¹⁾ one considers the $K^0\bar{K}^0$ state (2.10) and performs active joint strangeness measurements at two different times on the left beam (τ_1 and τ_2) and at other two different times on the right beam (τ_3 and τ_4). The detection times should be chosen at will and in accordance with the locality requirement. The proposed inequality is again in the CHSH form:⁽⁷⁾

$$|E_{\text{LR}}(\tau_1, \tau_3) - E_{\text{LR}}(\tau_1, \tau_4) + E_{\text{LR}}(\tau_2, \tau_3) + E_{\text{LR}}(\tau_2, \tau_4)| \leq 2, \quad (3.2)$$

where $E(\tau_l, \tau_r)$ is a correlation function which takes the value +1 when either two \bar{K}^0 's or no \bar{K}^0 's are found in the left (τ_l) and right (τ_r) measurements, and -1 otherwise:

$$E(\tau_l, \tau_r) \equiv P(Y, \tau_l; Y, \tau_r) + P(N, \tau_l; N, \tau_r) - P(Y, \tau_l; N, \tau_r) - P(N, \tau_l; Y, \tau_r). \quad (3.3)$$

The probabilities entering this correlation function, where Y (Yes) and N (No) answer to the question whether a K^0 is detected at the considered time, can be obtained in QM from Eqs. (2.12) and (2.13), and $E_{\text{QM}}(\tau_l, \tau_r) = -\exp\{-(\Gamma_L + \Gamma_S)(\tau_l + \tau_r)/2\} \cos[\Delta m(\tau_l - \tau_r)]$.

Because of strangeness oscillations in free space along both kaon paths, choosing among four different times corresponds to four different choices of measurement directions in the photon case. In this sense, there is a total analogy and CHSH inequality (3.2) is a strict consequence of LR. Unfortunately, this inequality is never violated by QM because strangeness oscillations proceed too slowly and cannot compete with the more rapid kaon weak decays. The conclusion is the same for the CHSH inequalities derivable for the $B^0-\bar{B}^0$ and $D^0-\bar{D}^0$ meson systems.^(36,42) On the contrary, genuine CHSH inequalities violated by QM could be derived for $B_s^0\bar{B}_s^0$ pairs if active flavour measurements were possible for these mesons. As discussed in Refs. 31 and 32, Bell's inequalities exploiting strangeness measurements at four different times can be violated by QM only if a normalization of the observables to undecayed kaon pairs is employed.

Unfortunately, the Bell's inequalities obtained with such a normalization procedure are non-genuine.⁽⁴²⁾

In Ref. 24, Uchiyama derived the following Wigner-like inequality:⁽⁵⁾

$$P_{\text{LR}}(K_S, K^0) \leq P_{\text{LR}}(K_S, K_1) + P_{\text{LR}}(K_1, K^0) \quad (3.4)$$

for the entangled kaon state of Eq. (2.8). The joint probabilities are assumed to be measured at a proper time $\tau = \tau_l = \tau_r$ very close to the instant of the pair creation, $\tau \rightarrow 0$; therefore, the inequality would eventually test non-contextuality rather than locality. Inserting the quantum–mechanical probabilities into Eq. (3.4), one obtains $\mathcal{R}e\epsilon \leq |\epsilon|^2$, which is violated by the presently accepted value of ϵ . Note that the proposed inequality involves passive measurements along a new, third basis consisting of the two CP eigenstates (K_1 and K_2). But the smallness of $|\epsilon|$ and Eq. (2.3) preclude any realistic attempt of discriminating between lifetime (K_S versus K_L) and CP (K_1 versus K_2) eigenstates. In this sense, the interest of inequality (3.4) reduces to that of a clear and well-defined gedanken experiment.

3.4. Proposals with Active Measurements and Regenerators

The authors of Refs. 29 and 30, while insisting on the convenience of performing only unambiguous strangeness measurements, have substituted the use of different times (as in Ref. 21) by the possibility of choosing among different kaon regenerators to be inserted along the kaon path(s). The well-known regeneration effect can be interpreted as producing adjustable “rotations” in the kaon “quasi-spin” space analogous to the strangeness oscillations (i.e., quasi-spin oscillations in vacuum) in Ref. 21, without requiring additional time intervals. One can thus derive Bell's inequalities, violated by QM, for simultaneous left–right strangeness measurements. The drawback of these analyses is that, up to now, they only refer to thin regenerators and the predicted violations of Bell's inequalities (below a few per cent) are hardly observable.

Eberhard⁽²²⁾ considered the alternative option, based on K_S versus K_L identification, for establishing a Bell's inequality. He combined such measurements in four experimental set-ups. In a first set-up, the state (2.9) is allowed to propagate in free space; its normalization is lost because of weak decays, but its perfect antisymmetry is maintained. In the other three set-ups, thick regenerators are asymmetrically located along one beam, or along the other or along both. An interesting inequality relating the number of K_L 's detected downstream from the production vertex and in each experimental set-up is then derived from LR. It turns out to be

significantly violated by quantum-mechanical predictions. Unfortunately, these successful predictions have some practical limitations, as already discussed by the author.⁽²²⁾ In particular, they are valid for asymmetric ϕ -factories (where the two neutral kaon beams form a small angle), whose construction is not foreseen.

New forms of Bell's inequalities for neutral kaons not affected by the drawbacks we have just mentioned have been derived in Ref. 34. Here, two kinds of active measurements, K^0 versus \bar{K}^0 and K_S versus K_L , have been considered in various alternative experimental set-ups with a thin regenerator fixed on the right beam as close as possible to the kaon-pair creation point. The proper time $\Delta\tau_r$ required by the neutral kaon to cross the regenerator is assumed to be short enough ($\Delta\tau_r \ll \tau_S$) to neglect weak decays. Then free space propagation is allowed up to a proper time T , with $\tau_S \ll T \ll \tau_L$. The normalization to surviving pairs leads then to the non-maximally entangled state:

$$|\Phi\rangle = \frac{1}{\sqrt{2 + |R|^2}} [|K_S\rangle_l |K_L\rangle_r - |K_L\rangle_l |K_S\rangle_r + R |K_L\rangle_l |K_L\rangle_r], \quad (3.5)$$

where

$$R \equiv -r e^{-i(\Delta m - \frac{1}{2}\Delta\Gamma)T} \quad (3.6)$$

and

$$r \equiv i \frac{\pi\nu}{m_K} (f - \bar{f}) \Delta\tau_r = i \frac{\pi\nu}{p_K} (f - \bar{f}) d \quad (3.7)$$

is the regeneration parameter. In Eq. (3.7), m_K is the average neutral kaon mass, p_K the kaon momentum, $f(\bar{f})$ the K^0 -nucleus (\bar{K}^0 -nucleus) forward scattering amplitude, ν the density of scattering centres of the homogeneous regenerator whose total thickness is d . The state (3.5) describes all kaon pairs with both left and right partners surviving up to a common proper time T .

At this point, alternative measurements of strangeness or lifetime will be performed on each one of these kaon pairs (3.5) according to the strategies for active measurement procedures illustrated in Sec. 2. Care has to be taken to choose T large enough to guarantee the space-like separation between left and right measurements. Locality excludes then any influence from the experimental set-up encountered by one member of the kaon pair at time T on the behaviour of its other-side partner between T and $T + \Delta\tau$. For kaon pairs from ϕ decays, moving at $\beta \simeq 0.22$, and using an interval time $\Delta\tau = 4.8\tau_S$ for the lifetime identification, this implies

$T > (\beta^{-1} - 1)\Delta\tau/2 = 8.7 \tau_S$, with a considerable reduction of the total kaon sample; a reduction which is much more moderate for more relativistic kaons as in $p\bar{p}$ annihilations.

The requirements (1)–(4) of Sec. 3.1 for deriving genuine Bell's inequalities are thus fulfilled and one can write several inequalities. Among these, we first discussed⁽³⁴⁾ a Clauser and Horne (CH) inequality⁽⁸⁾ which was substantially violated by QM. More recently, in Ref. 35 we have improved the analysis of Ref. 34 by applying Hardy's proof without inequalities of Bell's theorem⁽⁵⁵⁾ to the state (3.5).

Let us concentrate on the proof of Ref. 35. Neglecting for the moment CP -violation and K_L – K_S misidentification effects, from state (3.5) with $R = -1$ (called Hardy's state) one obtains the following quantum–mechanical predictions:

$$P_{QM}(K^0, \bar{K}^0) = \frac{\eta \bar{\eta}}{12}, \tag{3.8}$$

$$P_{QM}(K^0, K_L) = 0, \tag{3.9}$$

$$P_{QM}(K_L, \bar{K}^0) = 0, \tag{3.10}$$

$$P_{QM}(K_S, K_S) = 0, \tag{3.11}$$

where η ($\bar{\eta}$) is the K^0 (\bar{K}^0) overall detection efficiency. It is found that the necessity to reproduce, under LR, equalities (3.8)–(3.10) requires:

$$P_{LR}(K_S, K_S) \geq P_{LR}(K^0, \bar{K}^0) = \frac{\eta \bar{\eta}}{12}, \tag{3.12}$$

which contradicts Eq. (3.11). In principle, this allows for an “all-or-nothing” Hardy-like test of LR versus QM. In Ref. 35 it was concluded that, by requiring a perfect discrimination between K_S and K_L states, an experiment measuring the joint probabilities of Eqs. (3.8)–(3.11) closes the efficiency loophole even for infinitesimal values of the strangeness detection efficiencies η and $\bar{\eta}$. However, since K_L – K_S misidentifications (due to the finite value of $\Gamma_S/\Gamma_L \simeq 579$) do not permit an ideal lifetime measurement even when the detection efficiency η_τ for the kaon decay products is 100%,⁽³⁷⁾ the original proposal must be reanalysed paying particular attention to the inefficiencies involved in the real test.

Retaining the effects due to the K_S – K_L misidentification, from Eq. (3.5) with $R = -1$ one obtains (see the Appendix for details):

$$P_{\text{QM}}(K^0, \bar{K}^0) = \frac{\eta\bar{\eta}}{12}, \quad (3.13)$$

$$P_{\text{QM}}(K^0, K_L) = 6.77 \times 10^{-4} \eta \eta_\tau, \quad (3.14)$$

$$P_{\text{QM}}(K_L, \bar{K}^0) = 6.77 \times 10^{-4} \bar{\eta} \eta_\tau, \quad (3.15)$$

$$P_{\text{QM}}(K_S, K_S) = 1.19 \times 10^{-5} \eta_\tau^2, \quad (3.16)$$

which replace the results of Eqs. (3.8)–(3.11) and where η_τ is the efficiency for the detection of the kaon decay products. In the standard Hardy-like proof of non-locality,⁽⁵⁵⁾ the probabilities corresponding to our (3.14)–(3.16) are perfectly vanishing. In our realistic case, they are very small but not zero. Nevertheless, this does not prevent us from deriving a contradiction between LR and QM. Indeed, as proved in Ref. 56, the well-known criterion of physical reality of EPR⁽¹⁾ can be generalized to include predictions made with *almost* certainty, as it is required in our case due to the non-vanishing values of probabilities (3.14)–(3.16).

According to this generalization, the following Eberhard's inequality⁽¹⁹⁾ must be used to demonstrate the incompatibility between LR and QM:

$$H_{\text{LR}} \equiv \frac{P_{\text{LR}}(K^0, \bar{K}^0)}{P_{\text{LR}}(K^0, K_L) + P_{\text{LR}}(K_S, K_S) + P_{\text{LR}}(K_L, \bar{K}^0) + P(K^0, U_{\text{Lif}}) + P(U_{\text{Lif}}, \bar{K}^0)} \leq 1. \quad (3.17)$$

Note that this inequality involves the probabilities of Eqs. (3.13)–(3.16) together with two probabilities containing lifetime undetection, whose expressions in QM are:

$$P_{\text{QM}}(K^0, U_{\text{Lif}}) = \frac{1}{6} \eta (1 - \eta_\tau), \quad (3.18)$$

$$P_{\text{QM}}(U_{\text{Lif}}, \bar{K}^0) = \frac{1}{6} \bar{\eta} (1 - \eta_\tau). \quad (3.19)$$

Note also that the use of an inequality⁽⁵⁷⁾ allows for small deviations (existing in real experiments) around the value $R = -1$ required to prepare our Hardy's state. Essentially, inequality (3.17) is a different writing of the following inhomogeneous CH inequality:⁽⁸⁾

$$Q_{\text{LR}} \equiv \frac{P_{\text{LR}}(K_S, \bar{K}^0) - P_{\text{LR}}(K_S, K_S) + P_{\text{LR}}(K^0, \bar{K}^0) + P_{\text{LR}}(K^0, K_S)}{P_{\text{LR}}(K^0, *) + P_{\text{LR}}(*, \bar{K}^0)} \leq 1, \quad (3.20)$$

where

$$\begin{aligned} P_{\text{LR}}(K^0, *) &= P_{\text{LR}}(K^0, K_S) + P_{\text{LR}}(K^0, K_L) + P_{\text{LR}}(K^0, U_{\text{Lif}}), \\ P_{\text{LR}}(*, \bar{K}^0) &= P_{\text{LR}}(K_L, \bar{K}^0) + P_{\text{LR}}(K_S, \bar{K}^0) + P_{\text{LR}}(U_{\text{Lif}}, \bar{K}^0) \end{aligned} \quad (3.21)$$

and inhomogeneous refers to the appearance of both coincidence and single probabilities in the various terms of the inequality, as discussed by Santos.⁽²⁰⁾ Both Eberhard's and CH inequalities are actually derivable from LR for any value of R . However, Hardy's proof leads to inequality (3.17) only for Hardy's state ($R = -1$). It is important to stress that the previous Eberhard's and CH inequalities have been obtained *without invoking supplementary assumptions* on undetected events. They are both genuine Bell's inequalities and provide the same restrictions on the efficiencies η , $\bar{\eta}$ and η_τ required for a detection loophole free experiment.

In order to discuss the feasibility of such an experiment, let us start considering a few ideal cases. Assume first that perfect discrimination between K_S and K_L were always possible ($\eta_\tau = 1$ and $p_L = p_S = 1$, see Appendix); one could then make a conclusive test of LR for any nonvanishing values of η and $\bar{\eta}$: $H_{\text{QM}}^{\eta_\tau=p_L=p_S=1} \rightarrow \infty, \forall \eta, \bar{\eta} \neq 0$. In a second ideal case with no undetected events, i.e. with $\eta = \bar{\eta} = \eta_\tau = 1$, the inequalities are strongly violated by QM: $H_{\text{QM}}^{\eta=\bar{\eta}=\eta_\tau=1} \simeq 60.0$, $Q_{\text{QM}}^{\eta=\bar{\eta}=\eta_\tau=1} \simeq 1.25$, even if one allows for unavoidable K_S and K_L misidentifications. Finally, assuming that only the detection efficiency of kaon decay products is ideal ($\eta_\tau = 1$), for $\eta = \bar{\eta}$ ($\eta = \bar{\eta}/2$), Eberhard's and CH inequalities are contradicted by QM whenever $\eta > 0.023$ ($\eta > 0.017$).

Let us now consider more realistic situations with small and achievable values of η and $\bar{\eta}$. This implies that we have to consider large, but still realistic, decay-product detection efficiencies such as $\eta_\tau = 0.97, 0.98, 0.99$ and, ideally, 1. For each η_τ , the values of η and $\bar{\eta}$ that permit a detection loophole free test ($H_{\text{QM}}, Q_{\text{QM}} > 1$) lie above the corresponding curve plotted in Fig. 1. As expected, when η_τ decreases, the region of η and $\bar{\eta}$ values which permits a conclusive test diminishes and larger values of η and $\bar{\eta}$ are required. Note, however, that the strangeness detection efficiencies required for a conclusive test of LR versus QM with neutral kaons are considerably smaller than the limit ($\eta_0 = 0.67$) deduced by Eberhard⁽¹⁹⁾ for non-maximally entangled photon states. The values for η and $\bar{\eta}$ required by the test we have proposed seem to be not far from the present experimental capabilities.

4. CONCLUSIONS

A series of recent proposals aiming to perform Bell's inequality tests with entangled pairs of pseudoscalar mesons have been discussed. This

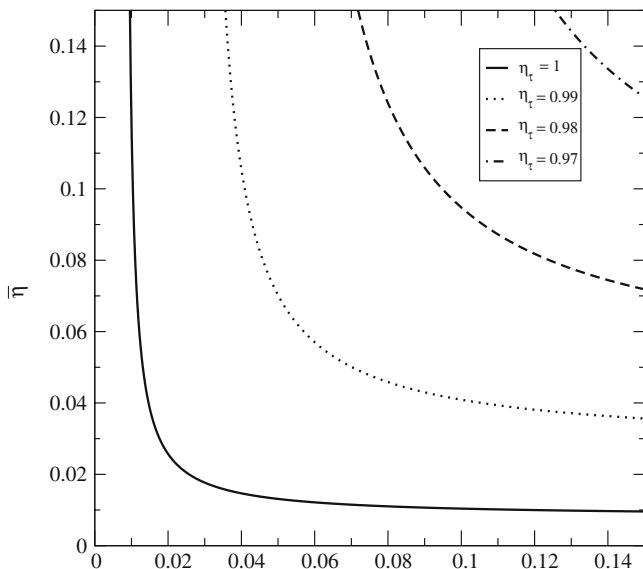


Fig. 1. The four curves (corresponding to $\eta_\tau = 1, 0.99, 0.98$ and 0.97) provide the values of η and $\bar{\eta}$ for which $H_{QM} = Q_{QM} = 1$ using Hardy's state. The QM violates inequalities (3.17) and (3.20) for values of η and $\bar{\eta}$ situated above the corresponding curve.

includes, in particular, pairs of neutral kaons or B -mesons. The relativistic velocities of these mesons and their strong interactions seem to offer the possibility of simultaneously closing the so-called locality and detection loopholes. The real situation, however, is not a simple one.

In several proposals, the measurements required to perform a Bell-test consist in identifying the flavour of each meson via its observed decay mode. The inequalities so derived are not a consequence of LR and, in this sense, cannot provide Bell-tests of LR versus QM. The reason is that the observed meson decays correspond to passive flavour measurements—with no choice for the experimenter—in such a way that a local realistic model can always be constructed reproducing all the probabilities predicted by QM.

Other proposals suffer from the difficulties coming from the fact that the number of different complementary measurements on pseudoscalar mesons is very small. For neutral kaons, for instance, they essentially reduce to strangeness and lifetime measurements. A situation which can be improved if the well known effects of kaon regeneration are taken into account.

Indeed, a series of papers have proposed Bell-tests with neutral kaons using kaon regeneration. On the one hand, this amounts to an effective

increase of the number of mutually exclusive measurements one can perform. On the other, by changing or removing the regenerators the active presence of the experimenter is guaranteed. A final difficulty could still remain: the low efficiency of some of these neutral kaon measurements. A detailed analysis suggests that a Bell-test with neutral kaons free from the detection loophole would require a few percentage of strangeness detection efficiencies and very high efficiencies for the detection of the kaon decay products. Both requirements seem achievable with present day technology.

ACKNOWLEDGMENTS

We would like to thank Prof. Emilio Santos for his help, encouragement and clarifications along several years. Work partly supported by EURIDICE HPRN-CT-2002-00311, MIUR 2001024324_007 and DGICYT BFM-2002-02588, as well as by the Ramon y Cajal program (R.E.).

APPENDIX

In Ref. 35, K_S 's states at time T are identified through decay events taking place between times T and $T + \Delta\tau$; similarly, K_L 's states are identified as kaons decaying after time $T + \Delta\tau$. For $\Delta\tau = 4.8 \tau_S$, the probabilities for correct K_S and K_L identifications are:

$$p_S \equiv 1 - \exp(-4.8) = p_L \equiv \exp(-4.8/579) = 0.9918. \quad (4.1)$$

and misidentifications are thus at the level of some eight per thousand.

One can further reduce these misidentifications by considering not only the kaon decay time but also the decay channel. Neglecting K_S and K_L branching ratios smaller than 10^{-5} , decays into $\pi\pi\pi$ identify K_L 's and only semileptonic and $\pi\pi$ channels are accessible to both K_S and K_L .⁽⁴⁷⁾ $\text{BR}(K_L \rightarrow \pi e \nu_e \text{ or } \pi \mu \nu_\mu) = 0.6600$, $\text{BR}(K_L \rightarrow \pi\pi) = 0.0030$, $\text{BR}(K_S \rightarrow \pi e \nu_e \text{ or } \pi \mu \nu_\mu) = 0.0011$ and $\text{BR}(K_S \rightarrow \pi\pi) = 0.9989$. However, semileptonic decays have to be assigned to K_L 's decays for any decay time (this implies that a fraction equal to $\text{BR}(K_S \rightarrow \pi e \nu_e \text{ or } \pi \mu \nu_\mu) = 1.1 \times 10^{-3}$ of the K_S 's is going to be misidentified). Indeed, the probability that a K_L decays semileptonically in a time interval $\Delta\tau$ after T is larger than the probability corresponding to a K_S , for any value of $\Delta\tau$. A decay into $\pi\pi$ occurring between T and $T + 5.82 \tau_S$ (after $T + 5.82 \tau_S$) has to be assigned to a K_S (K_L). In fact, the probability that a K_S [K_L], which is alive at time

T , decays into $\pi\pi$ after $T + \Delta\tau$ is $P_S(\Delta\tau) = \exp(-\Delta\tau/\tau_S)\text{BR}(K_S \rightarrow \pi\pi)$ [$P_L(\Delta\tau) = \exp(-\Delta\tau/\tau_L)\text{BR}(K_L \rightarrow \pi\pi)$] and $P_L(\Delta\tau)$ is larger (smaller) than $P_S(\Delta\tau)$ for $\Delta\tau > 5.82\tau_S$ ($\Delta\tau < 5.82\tau_S$). The probabilities that K_S 's and K_L 's are actually identified as K_S 's and K_L 's are thus:

$$\begin{aligned} p_S &= 1 - \text{BR}(K_S \rightarrow \pi e \nu_e \text{ or } \pi \mu \nu_\mu) - \text{BR}(K_S \rightarrow \pi\pi) \exp(-5.82) \\ &= \text{BR}(K_S \rightarrow \pi\pi)[1 - \exp(-5.82)] = 0.99594, \\ p_L &= 1 - \text{BR}(K_L \rightarrow \pi\pi)[1 - \exp(-5.82/579)] = 0.99997 \end{aligned} \quad (4.2)$$

thus improving the lifetime identification with respect to the method of Eq. (4.1).

Retaining the effects due to the K_S - K_L misidentification (CP-violation and the non-orthogonality of $|K_L\rangle$ and $|K_S\rangle$ can indeed be neglected), from Eq. (3.5) with $R = -1$ we obtain:

$$P_{\text{QM}}(K^0, \bar{K}^0) = \frac{\eta\bar{\eta}}{12}, \quad (4.3)$$

$$P_{\text{QM}}(K^0, K_L) = |\langle K^0 K_S | \Phi \rangle|^2 \eta \eta_\tau (1 - p_S) = \frac{1}{6} \eta \eta_\tau (1 - p_S), \quad (4.4)$$

$$P_{\text{QM}}(K_L, \bar{K}^0) = |\langle K_S \bar{K}^0 | \Phi \rangle|^2 \bar{\eta} \eta_\tau (1 - p_S) = \frac{1}{6} \bar{\eta} \eta_\tau (1 - p_S), \quad (4.5)$$

$$\begin{aligned} P_{\text{QM}}(K_S, K_S) &= \frac{2}{3} \eta_\tau^2 \left\{ p_S(1 - p_L) - \text{BR}(K_S \rightarrow \pi\pi) \text{BR}(K_L \rightarrow \pi\pi) \right. \\ &\quad \times \frac{\Gamma_S \Gamma_L}{\Gamma^2 + \Delta m^2} \times \left[1 - 2e^{-5.82 \frac{\Gamma}{\Gamma_S}} \cos\left(5.82 \frac{\Delta m}{\Gamma_S}\right) \right. \\ &\quad \left. \left. + e^{-2 \times 5.82 \frac{\Gamma}{\Gamma_S}} \right] \right\} \end{aligned} \quad (4.6)$$

from which the numerical values of Eqs. (3.14)–(3.16) follow via Eq. (4.2) and Ref. 47.

In Eq. (4.4) [(4.5)] semileptonic K_S decay events on the right (left) and K_S states surviving up to $T + 5.82\tau_S$ are wrongly assumed as coming from K_L 's. The derivation of Eq. (4.6) deserves some comment. Since K_S 's are identified through their $\pi\pi$ decays occurring between times T and $T + 5.82\tau_S$, experimentally one has to measure the following double differential rate:

$$\Gamma(\pi\pi, \tau_l; \pi\pi, \tau_r) = \int d\Omega_l \int d\Omega_r |A(\pi\pi, \tau_l; \pi\pi, \tau_r)|^2, \quad (4.7)$$

where the integrations are over the phase space for the decay product states and $0 \leq \tau_l, \tau_r \leq 5.82\tau_S$. The corresponding amplitude is obtained from Eq. (3.5) with $R = -1$ as:

$$A(\pi\pi, \tau_l; \pi\pi, \tau_r) = \frac{1}{\sqrt{3}} \langle \pi\pi | T | K_S \rangle \langle \pi\pi | T | K_L \rangle \\ \times [e^{-i\lambda_S \tau_l - i\lambda_L \tau_r} - e^{-i\lambda_L \tau_l - i\lambda_S \tau_r}], \quad (4.8)$$

where we have neglected the small contribution coming from the $|K_L\rangle_l |K_L\rangle_r$ part of the state and $\lambda_{L,S} = m_{L,S} - i\Gamma_{L,S}/2$. The joint probability (4.6) is then computed with the following relation:

$$P_{QM}(K_S, K_S) = \int_0^{5.82 \tau_S} d\tau_l \int_0^{5.82 \tau_S} d\tau_r \Gamma(\pi\pi, \tau_l; \pi\pi, \tau_r). \quad (4.9)$$

REFERENCES

1. A. Einstein, B. Podolsky, and N. Rosen, *Phys. Rev.* **47**, 777 (1935).
2. N. Bohr, *Phys. Rev.* **48**, 696 (1935).
3. J. Bell, *Physics* **1**, 195 (1964).
4. M. Redhead, *Incompleteness, Non–Locality and Realism* (Oxford University Press, Oxford, 1990).
5. E. P. Wigner, *Am. J. Phys.* **38**, 1005 (1970).
6. J. S. Bell, *Speakable and Unsayable in Quantum Mechanics (Collected Papers on Quantum Philosophy)* (Cambridge University Press, Cambridge, 1987).
7. J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, *Phys. Rev. Lett.* **23**, 880 (1969).
8. J. F. Clauser and M. A. Horne, *Phys. Rev. D* **10**, 526 (1974).
9. A. Aspect, J. Dalibard, and G. Roger, *Phys. Rev. Lett.* **49**, 1804 (1982).
10. J. F. Clauser and A. Shimony, *Rep. Prog. Phys.* **41**, 1881 (1978).
11. G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, and A. Zeilinger, *Phys. Rev. Lett.* **81**, 5039 (1998).
12. W. Tittel, J. Brendel, N. Gisin, and H. Zbinden, *Phys. Rev. A* **59**, 4150 (1999); W. Tittel, J. Brendel, H. Zbinden, and N. Gisin, *Phys. Rev. Lett.* **81**, 3563 (1998).
13. R. A. Bertlmann and A. Zeilinger (eds), *Quantum (Un)speakables—From Bell to Quantum Information* (Springer, Berlin, 2002).
14. M. A. Rowe, D. Kielpinski, V. Meyer, C. A. Sackett, W. M. Itano, C. Monroe, and D. J. Wineland, *Nature* **409**, 791 (2001).
15. L. Vaidman, *Phys. Lett. A* **286**, 241 (2001).
16. E. Santos, *Stud. History Philos. Modern Phys.* **36**(3), 554–565 (2005).
17. P. Pearle, *Rep. Rev. D* **2**, 1418 (1970).
18. A. Garg and N. D. Mermin, *Phys. Rev. D* **35**, 3831 (1987).
19. P. H. Eberhard, *Phys. Rev. A* **47**, R747 (1993).
20. E. Santos, *Phys. Rev. A* **46**, 3646 (1992); *Phys. Lett. A* **212**, 10 (1996); N. Gisin and B. Gisin, *Phys. Lett. A* **260**, 323 (1999).
21. G. C. Ghirardi, R. Grassi, and T. Webern, in *Proceedings of the Workshop on Physics and Detectors for DaΦne*, G. Pancheri, ed. (INFN, Laboratori Nazionali di Frascati, Frascati, Italy, 1991) p. 261.
22. P. H. Eberhard, *Nucl. Phys. B* **398**, 155 (1993).
23. A. Di Domenico, *Nucl. Phys. B* **450**, 293 (1995).
24. F. Uchiyama, *Phys. Lett. A* **231**, 295 (1997).

25. F. Selleri, *Phys. Rev. A* **56**, 3493 (1997); R. Foadi and F. Selleri, *Phys. Lett. B* **461**, 123 (1999); *Phys. Rev. A* **61**, 012106 (2000).
26. A. Afriat and F. Selleri, *The Einstein, Podolsky and Rosen Paradox in Atomic, Nuclear and Particle Physics* (Plenum Press, New York, 1998).
27. F. Benatti and R. Floreanini, *Phys. Rev. D* **57**, R1332 (1998); *Eur. Phys. J. C* **13**, 267 (2000).
28. R. A. Bertlmann, W. Grimus, and B. C. Hiesmayr, *Phys. Rev. D* **60**, 114032 (1999); *Phys. Lett. A* **289**, 21 (2001); B. C. Hiesmayr, *Found. Phys. Lett.* **14**, 231 (2001); R. A. Bertlmann, and B. C. Hiesmayr, *Phys. Rev. A* **63**, 062112 (2001); R. A. Bertlmann, K. Durstberger, and B. C. Hiesmayr, *Phys. Rev. A* **68**, 012111 (2003).
29. A. Bramon and M. Nowakowski, *Phys. Rev. Lett.* **83**, 1 (1999).
30. B. Ancochea, A. Bramon, and M. Nowakowski, *Phys. Rev. D* **60**, 094008 (1999).
31. N. Gisin and A. Go, *Am. J. Phys.* **69**, 264 (2001).
32. R. Dalitz and G. Garbarino, *Nucl. Phys. B* **606**, 483 (2001).
33. M. Genovese, C. Novero, and E. Predazzi, *Phys. Lett. B* **513**, 401 (2001); *Found. Phys.* **32**, 589 (2002).
34. A. Bramon and G. Garbarino, *Phys. Rev. Lett.* **88**, 040403 (2002).
35. A. Bramon and G. Garbarino, *Phys. Rev. Lett.* **89**, 160401 (2002).
36. B. C. Hiesmayr, Ph.D. Thesis, University of Vienna, 2002.
37. M. Genovese, *Phys. Rev. A* **69**, 022103 (2004).
38. R. A. Bertlmann, *Lecture Notes in Physics, Springer Verlag*.
39. A. Datta and D. Home, *Phys. Lett. A* **119**, 3 (1986).
40. A. Pompili and F. Selleri, *Eur. Phys. J. C* **14**, 469 (2000).
41. A. Go, *J. Mod. Optics* **51**, 991 (2004); quant-ph/0310192.
42. R. A. Bertlmann, A. Bramon, G. Garbarino, and B. C. Hiesmayr, *Phys. Lett. A* **332**, 355 (2004).
43. A. Bramon, R. Escribano, and G. Garbarino, *J. Mod. Opt.* **52**, 1681 (2005).
44. L. Maiani, G. Pancheri, and N. Paver, eds., *The Second DaΦne Physics Handbook* (INFN, Laboratori Nazionali di Frascati, Frascati, Italy, 1995).
45. S. Kurokawa and E. Kikutani, *Nucl. Instrum. Meth. A* **499**, 1 (2003).
46. A. Apostolakis *et al.*, *Phys. Lett. B* **422**, 339 (1998).
47. S. Eidelman *et al.*, (Particle Data Group), *Phys. Lett. B* **592**, 1 (2004).
48. P. K. Kabir, *The CP Puzzle* (Academic Press, London, 1968).
49. A. Bramon, G. Garbarino, and B. C. Hiesmayr, *Phys. Rev. A* **69**, 062111 (2004).
50. A. Angelopoulos *et al.*, *Phys. Rept.* **374**, 165 (2003); *Phys. Lett.* **503**, 49 (2001); **444**, 38 (1998).
51. A. Bramon, G. Garbarino, and B. C. Hiesmayr, *Eur. Phys. J. C* **32**, 377 (2004).
52. A. Bramon, G. Garbarino, and B. C. Hiesmayr, *Phys. Rev. Lett.* **92**, 020405 (2004).
53. A. Di Domenico, hep-ex/0312032, published in eConf C0309101:THWP007 (2003).
54. L. Kasday, in *Foundations of Quantum Mechanics*, B. d'Espagnat ed. (Academic Press, New York, 1971), p. 195. *Proceedings of the International School of Physics 'Enrico Fermi'*, Course, IL.
55. L. Hardy, *Phys. Rev. Lett.* **68**, 2981 (1992); *Phys. Rev. Lett.* **71**, 1665 (1993).
56. P. H. Eberhard and P. Rosselet, Universite de Lausanne Report No. IPNL-93-3, 1993; *Found. Phys.* **25**, 91 (1995).
57. L. Hardy, *Phys. Rev. Lett.* **73**, 2279 (1994); N. D. Mermin, *Am. J. Phys.* **62**, 880 (1994); A. Garuccio, *Phys. Rev. A* **52**, 2535 (1995).