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The Failure to Perform a Loophole-Free Test of Bell's Inequality Supports Local Realism

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It is argued that the long standing failure to show an uncontroversial, loophole-free, empirical violation of a Bell inequality should be interpreted as a support to local realism. After defining realism and locality, this as relativistic causality, the performed experimental tests of Bell's inequalities are commented. It is pointed out that, without any essential modification of quantum mechanics, the theory might be compatible with local realism.

KEY WORDS: Bell's inequality; local realism; quantum nonlocality.

1. INTRODUCTION

Forty years have elapsed since John Bell[(1,2)] discovered his celebrated inequalities. These inequalities, which involve measurable quantities, provide necessary conditions for local realism. Bell also proved that, in some experiments with ideal set-ups, the predictions of quantum mechanics violate the inequalities. During these four decades, hundreds of theoretical papers have pointed out violations of the inequalities in very many different phenomena, but only a few dozens empirical tests have been actually performed. The results of all performed experiments are compatible with local realism and, with few exceptions, agree with the quantum predictions. The standard wisdom derived from these facts is that quantum mechanics has been confirmed and local realism refuted. The latter conclusion because allegedly plausible extrapolations of the empirical results could violate a Bell inequality.

I claim that the current wisdom is misleading and harmful for the progress of science. Misleading because it attempts answering a fundamental scientific question by means of a subjective assessment of plausibility.

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Harmful because it discourages people from making the necessary effort to perform a real, loophole-free, test. The aim of the present paper is to explain the reasons for my claim.

The long time elapsed without a true disproof of local realism may be compared, for instance, with the discovery of parity non-conservation, which required a few months to go from the theoretical paper by Lee-Yang, in 1957, to the uncontroversial (loophole-free) experiment by Wu *et al.* I think that the extreme difficulty to refute local realism, as shown by the unsuccessful effort of 40 years demands a convincing explanation.

Repeated failures at the experimental level have been extremely important in the history of physics. I shall put two examples. After James Watt made his heat engine in 1765, many people attempted to increase the efficiency, but in some sense they failed. In fact, nobody was able to make a *perpetuum mobile* (of the second kind), that is an engine able to produce useful work by just cooling a large reservoir like the sea. It took 60 years to be realized, by Sadi Carnot, that the aim was impossible because a (large) part of the extracted heat should necessarily go to a colder reservoir. Carnot's discovery led soon to the statement of one of the most important principles of physics: *the second law of thermodynamics*. Another example is the question of the absolute motion of the Earth. Several attempts at measuring it failed, the most sophisticated made by Michelson and Morley in 1887. The failure was "explained" less than 20 years later by Einstein with the hypothesis that absolute motion does not exist. Again, a repeated experimental failure led to a fundamental physical law: *the relativity principle*.

My proposal is that the forty-years failure to show a strict (loophole-free) violation of a Bell inequality may be "explained" by another fundamental principle of physics: *nature respects local realism*.

In this paper, after two introductory sections (Secs. 2 and 3) commenting on the interpretation of quantum mechanics and the question of hidden variables, I shall analyze the concept of local realism, its relevance in physics (Sec. 4) and Bell's inequalities (Secs. 5 and 6). After that I shall review the experiments aimed at testing local realism vs. quantum mechanics (Secs. 7 and 8) and propose a local realistic model for the Bell experiments (Sec. 9). Finally, after a digression on philosophy and sociology of science (Sec. 10) I shall discuss the consequences to be drawn (Secs. 11 and 12).

2. THE DEBATE ABOUT QUANTUM THEORY

Since the discovery of quantum mechanics a warm debate has taken place about the interpretation of the theory. With some simplification, we may say that two main philosophical positions have been mantained, namely realism and pragmatism, their early representives being, respectively, Einstein and Bohr. According the pragmatic view, the only purpose of physics is to allow predicting the results of experiments. In contrast realist people demand that, in addition, physics should provide a picture of the natural world, a world whose existence independent of the observers is taken for granted. Of course, neither pragmatists reject realism, in general, nor realists deny the need of an agreement between the predictions of the theory and the experiments, but the words realist or pragmatist capture the focus on the interpretation of physics. During many years Franco Selleri has been one of the most conspicuous representatives of the realist position,[(3)] a position which I strongly support.

It is a fact that today most quantum physicists support a pragmatic attitude towards quantum theory. This fact has been attributed sometimes to cultural reasons or seen as the consequence of living in a highly technological environment. In my opinion, however, the main reason is the intrinsic difficulty which presents the interpretation of the quantum formalism. It is not only that the formalims does not offer any obvious picture of the world, or that it predicts counterintuitive phenomena, but the fact the theory suggests contradictory pictures, like fundamental entities being both localized (particles) and extended (waves). Actually the fact that emerging theories of physics, or natural science in general, give a counterintuitive picture of the world is not new, it has been a constant during the history. The reason is that our intuition rests, to some extent, upon previous theories and a new one changes the picture. This was the case, for instance, when heliocentrism displaced geocentrism, or when time lost an absolute meaning with the arrival of relativity theory. However, I think that the appearance of a theory suggesting contradictory pictures of the world, like quantum mechanics does, had no precedent in the history of physics.

I shall not devote more space to the debate about quantum theory, which is rather well known and has been the subject of many books. Most of these books defend the pragmatic interpretation, but some are more or less critical with it. In this respect I should mention those by Franco Selleri.[(3)] A recent book by Auletta[(4)] reviews most of the interpretational problems of quantum mechanics and the proposed solutions. Many fundamental papers are reprinted in a book by Wheeler and Zurek.[(5)]

3. THE QUESTION OF HIDDEN VARIABLES

Since the early days of quantum mechanics, the realist position has been associated to the search of hidden variables, although the name is somewhat misleading and thus not fully accepted. For instance, Einstein never spoke about hidden variables in his defense of a realist interpretation of (or alternative to) quantum mechanics.

The question of hidden variables was considered seriously during the early times of quantum mechanics as a possible explanation for the statistical character of this theory. However, the possibility was very soon discarded. Indeed John von Neumann studied the subject in his celebrated book[6] and concluded that hidden variables are incompatible with the predictions of quantum mechanics (von Neumann's theorem). Similar "no-hidden-variables theorems" were proved in the years following von Neumann's publication and the impossibility was taken for granted. The situation changed after the explicit hidden-variables theory of Bohm[7] and was finally clarified by Bell[8] in 1966, who showed that "contextual" hidden variables are always possible. Contextual means that the variables of the measuring apparatuses are included and, furthermore, the result of the measurement of every observable may depend on the whole experimental arrangement (see e.g., the review of Mermin.[9])

The proof of the possibility of contextual hidden variables is so simple that I include it here. We consider a typical experiment consisting of the preparation of a system in the quantum state $|\Psi\rangle$ followed, after some evolution, by the measurement of several commuting observables $\{A, B, \ldots, C\}$. As is well known there exists a complete orthonormal set of vectors in the Hilbert space of the system, all of which are simultaneous eigenvectors of $\{A, B, \ldots, C\}$. Let us label these vectors $\{|\lambda\rangle\}$. Thus the expectation value of the product of observables A and B, for instance, may be written (assuming discrete spectrum)

$$\begin{aligned}\langle AB\rangle &= \sum_{\lambda}\langle\Psi|\lambda\rangle\langle\lambda|A|\lambda\rangle\langle\lambda|B|\lambda\rangle\langle\lambda|\Psi\rangle \\ &= \sum_{\lambda} A_{\lambda}B_{\lambda}\,|\langle\lambda|\Psi\rangle|^2\,,\end{aligned}$$

where we have used

$$\sum_{\lambda}|\lambda\rangle\langle\lambda| = 1,\ \ \langle\lambda|A|\lambda'\rangle = \langle\lambda|A|\lambda\rangle\delta_{\lambda\lambda'},$$

and similar for B. The expression for $\langle AB\rangle$ has the structure proposed by Bell as a definition of hidden-variables model (see Eq.(3) below.)

Bell[1] also introduced the concept of local hidden variables theories and proved that, in some idealized experiments, they are incompatible with quantum mechanics (Bell's theorem). Today it is generally believed that local hidden variables theories have already been disproved by the experiments, but I claim that this state of opinion is wrong, as said in Sec. 1. In any case the question about hidden variables theories appears at present as irrelevant or even nonsense to most quantum physicists. Indeed, from the pragmatic point of view, today dominant, it is believed that the possible existence of hidden variables would not change, in any way, the quantum predictions which are so extremely well confirmed by the experiments. But I do not agree with this opinion. Thus in the rest of the present section I shall argüe for the relevance of the question of hidden variables theories, in particular if they are local. I shall give three arguments:

(1) If a hidden variables theory were found underlying quantum mechanics, it might give predictions more detailed than those provided by quantum mechanics, although not contradicting them. The possibility of new predictions would give rise to unexpected new physics. Indeed the history shows how dangerous is to make a guess about the future, in this case to assume that either such a subquantum theory cannot be found or that nothing interesting will follow from it.

(2) If there are (are not) local hidden variables models for all possible experiments, then nature can (cannot) be interpreted as respecting local realism. This is an important knowledge even if it were considered to belong to philosophy rather than to physics. In fact the question is whether a picture of the world can be devised fitting in the tradition of (classical) physics, as supported e.g., by Einstein.[10] That is, a world view where physical systems have properties independently of any observation ("the moon is there when nobody looks"), actions propagate in space-time at a speed not greater than that of light (without "spooky actions at a distance") and probabilities appear due to ignorance, maybe unavoidable, rather than by an essential indeterminacy, or lack of strictly causal laws (that is "God does not play dice").

(3) If nature respects local realism it is far from obvious that applications of quantum physics relying on "purely quantum phenomena", like entanglement, could produce anything not achievable with devices working according the laws of classical physics. This might be, for instance, the case of quantum computation. If there is a general principle preventing the violation of a Bell inequality (see below, Sec. 10), then it is probable that the same principle might prevent the expected functioning of (large scale) quantum computers. I guess that the principle exists and is related to decoherence and other forms of noise, as will be commented in the last section of the present paper.

4. WHAT IS LOCAL REALISM?

It is not easy to define *realism* with a few words, as is proved by the existence of whole books devoted to the subject. Here I shall give a simple definition appropriate for physics. *Realism is the belief that material bodies have properties independent of any observation, and that the results of any possible measurement depend on these properties.* The said properties are usually called "elements of reality"[(11)] and are frequently identified with the hidden variables. However I think that the latter correspond rather to the parameters used for the description of the said properties and should not be confused with the former.

In order to clarify the point I shall give an example. If I throw upwards a coin, after a while the coin will collide with, say, a table and will soon become at rest on it, with either the head or the tail upwards. The described experiment consists, as is typical, of the *preparation* of the state of a system (the coin thrown upwards) followed by the *evolution* of the system and finishing by the *measurement* of a quantity on it. Our intuition says that the result (head or tail) is determined by the elements of reality of the coin during the fly. Or maybe, taking into account the unavoidable existence of non-idealties (e.g., friction with the air), the elements of reality just determine the probability of the result. In any case we should carefully distinguish between the *observable* (head or tail) and the *elements of reality* (associated to motion of the coin). The relevant lesson of our example is that the result of a measurement depends on both the measured system (the coin) and the measuring apparatus (the table). Sometimes the *observable* (head–tail in our example) is even devoid of sense without the *measuring apparatus* (the table). Therefore it is not so strange that quantum mechanics forbids the "simultaneous existence of definite values for some observables", namely those which cannot be measured toghether. This is the essential content of the Kochen–Specker theorem forbidding non-contextual hidden variables.[(9)] Our example shows that the validity of the Kochen–Specker theorem does not contradict realism as defined above.

Possibly most workers in quantum physics consider that realism is just a philosophical opinion which may or may not be true, but this is not the case. As Einstein put it,[(10)] without accepting the existence of an objective reality, independent of any observation, natural science would be impossible. Actually, even the most pragmatic quantum physicists admit that states of physical systems have some "capabilities" of influencing the results of eventual future measurements on the system. In my view it is a rather semantic question whether we name these capabilities "elements of reality".

Locality is the belief that no influence may be transmitted with a speed greater than that of light. Thus we might identify *locality* with *relativistic causality*. The concept of locality is subtle, however. In fact, quantum mechanics is local in the sense that it forbids the transmission of superluminal signals (say from a human being to another one), but local realism as analyzed here is stronger than that. At a difference with the idea of realism which I consider as an unvoidable requirement for the existence of science, locality derives from our experience at the macroscopic level and might be violated without demolishing the whole building of physics. That is, we might assume that some influences travel at a speed greater than that of light even if this fact does not allow the transmission of superluminal signals. This seemed the position of John Bell[(12)] and also the motivation for Franco Selleri to become interested in the foundations of relativity theory.[(13)]

In spite of this, I think that locality is also important, that is local realism is so fundamental a principle of physics that it should not be rejected without *extremely strong reasons*, an opinion which I believe is quite close to what Einstein maintained until his death. On the other hand the question of local hidden variables is less relevant than the question of local realism. It is true that if local realism were untrue local hidden variables would be impossible, but if local realism is true local hidden variables may still be useless in practice, although possible in principle. Thus I shall refer to local realism, rather than to local hidden variables, in the rest of this article.

5. THE BELL INEQUALITIES

From what we have said it might appear that local realism is a purely philosophical concept. But a *physical* necessary condition for *local realism* was introduced by John Bell[(14)] as follows: *Any correlation between measurements performed at different places should derive from events which happened in the intersection of the past light cones of the measurements.* In order to give an empirical content to the statement Bell considered a generic experiment consisting of the preparation of a pair of particles (or, more generally, physical systems) which are let to evolve in such a way that the two particles go to macroscopically distant regions (the argument that follows has been exposed in more detail elsewhere.[(15)]) Thus Bell searched for the probability, $p(A, a; B, b)$, of getting the result a in the measurement of an observable A of the first particle and the result b in the measurement of the observable B of the second particle. He proposed that, if local realism holds true, the probability could be written

$$p(A, a; B, b) = \int \rho(\lambda) P_1(\lambda; A, a) P_2(\lambda; B, b) d\lambda, \tag{1}$$

where λ is one or several parameters which contain all relevant information about the intersection of the past light cones of the two measurements. An expression similar to (1) for the total probability $p(A, a)$, of getting the result a in the measurement of the observable A on the first particle, follows at once from the fact that it is unity the sum of probabilities associated to particle 2. That is

$$p(A, a) = \sum_b \int \rho(\lambda) P_1(\lambda; A, a) P_2(\lambda; B, b) d\lambda = \int \rho(\lambda) P_1(\lambda; A, a) d\lambda. \tag{2}$$

From now on we shall consider only dichotomic observables, so that the result of the measurement of the observable A may be only 1 (yes) or 0 (not). Thus we shall simplify the notation writing $P_1(\lambda, A)$ (or $P_2(\lambda, B)$) for $P_1(\lambda; A, a)$ (or $P_2(\lambda; B, b)$), and *p(A,B)* (or *p(A)*) for the left side of (1) ((2)) so that Eqs.(1) and (2) will be written

$$p(A) = \int \rho(\lambda) P_1(\lambda, A) d\lambda, \;\; p(A, B) = \int \rho(\lambda) P_1(\lambda, A) P_2(\lambda, B) d\lambda. \tag{3}$$

The functions P and ρ in the formula fulfil the conditions required for probabilities and probability densities, respectively. That is

$$\rho(\lambda) \geqslant 0, \quad \int \rho(\lambda) d\lambda = 1, \tag{4}$$

$$P_1(\lambda, A), P_2(\lambda, B) \geqslant 0, \tag{5}$$

$$P_1(\lambda, A), P_2(\lambda, B) \leqslant 1. \tag{6}$$

It is important to stress that the value of $P_1(\lambda, A)$ is assumed to be independent of B, that is independent on what measurement is performed on the second particle, which is Bell's condition of locality. This independence is sometimes called "parameter independence", which is compatible with a possible "outcome dependence", that is the results of the measurements of A and B may be correlated.[16] Hence, using the notation A', B' for the result 0 in the measurement of A and B, respectively, we obtain a similar independence for the measurable probabilities

$$p(A) = p(A, B) + p(A, B') = p(A, D) + p(A, D') = \ldots$$

Parameter independence holds true also in quantum mechanics and it guarantees that superluminal communication is not possible.

From the conditions (3)–(6) it is possible to derive inequalities involving only measurable probabilities. We consider an experiment in which we prepare once and again, say $4N$ times ($N \gg 1$), a pair of particles in a given state, the same for all preparations. Here, *the same* means that the parameters which may be controlled in the preparation have the same values. After N preparations, chosen at random amongst the $4N$ made, we measure the dichotomic observables A and B of the two particles. After another N preparations, also chosen at random, we measure the dichotomic observables C and D. Similarly C with B are measured N times, and A with D also N times. We assume that the result of the measurement of any of the observables may be either 0 or 1, and call $p(A, B)$ the probability of getting the result 1 for both observables, A and B (the frequencies measured in the experiment should approach the probabilities if N is large enough). Similarly we may define the probabilities $p(A, D)$, $p(C, B)$ and $p(C, D)$, and also the probability $p(A)$ corresponds to getting the value 1 in the measurement of A and any value (1 or 0) in the measurement of B, or D, performed on the partner particle, and similar for $p(B)$. It is an easy task to derive, from (1) to (6), inequalities involving measurable probabilities. For instance[17]

$$p(A, B) + p(A, D) + p(C, B) - p(C, D) \leqslant p(A) + p(B). \tag{7}$$

This inequality may be related to the existence of a "metric" in the set of propositions associated to the results "yes", "no" in the four measurements. In fact we may define a formal (not measurable) joint probability distribution on the observables $\{A, B, C, D\}$ by means of expressions similar to (3) applied to the four observables, the six pairs $\{AB, AC, AD, BC, BD, CD\}$ and the four triples $\{ABC, ABD, ACD, BCD\}$, in spite of some of them not being actually measurable (e.g., $p(A, C)$ cannot be got empirically because A and C correspond to alternative, incompatible, measurements on the same particle). Now the mere possibility of defining a (formal) joint probability implies the existence of a metric in the set of propositions (yes–no experiments) and the essential property of the metric is the fulfillment of triangle inequalities, which are closely related to the inequality (7). But I shall not pursue the subject here (details may be seen elsewhere.[18])

6. BELL'S vs. TESTED INEQUALITIES. THE CHSH CASE

Soon after Bell's discovery[1] in 1964, it was realized that no performed experiment had shown a violation of local realism. Furthermore, no simple experiment could do the job. In my view, the difficulty is a proof that it is wrong the perceived wisdom according to which quantum mechanics predicts "highly non-local effects". The truth is that non-local effects, if any, are extremely weak and difficult to observe.

In 1969 Clauser, Horne, Shimony and Holt (CHSH)[19] made the first serious proposal for an empirical test of Bell's inequality . They suggested the measurement of the polarization correlation of optical photon pairs. By optical we mean that the corresponding frequencies are in the visible, the near ultraviolet or the near infrared parts of the spectrum. The mentioned authors derived the Bell inequality

$$S \equiv E(A, B) + E(A, D) + E(C, B) - E(C, D) \leqslant 2, \tag{8}$$

where $\{A, C\}$ correspond to two possible positions of a polarization analyzer for the first photon and $\{B, D\}$ for the second. The correlations are defined by

$$E(X, Y) = p_{++}(X, Y) + p_{--}(X, Y) - p_{+-}(X, Y) - p_{-+}(X, Y), \tag{9}$$

with $X = A$ or $C, Y = B$ or D, $p_{++}(X, Y)$ being the probability that the polarization of the first photon is found in the plane X, and that of the second in the plane Y, $p_{+-}(X, Y)$ the probability that the polarization of the first photon is found in the plane X and that of the second is in the plane perpendicular to B, etc.

It is not difficult to see that the (8) inequality is equivalent to (7) provided that the sum of the four probabilities involved is unity, that is

$$p_{++}(X, Y) + p_{--}(X, Y) + p_{+-}(X, Y) + p_{-+}(X, Y) = 1. \tag{10}$$

In fact in this case it is easy to go from (8) to (7) , or viceversa, by repeated use of relations like

$$\begin{aligned} &p_{++}(X, Y) = p(X, Y),\ \ p_{+-}(X, Y) = p(X) - p(X, Y), \\ &p_{--}(X, Y) = 1 - p(Y) - p_{+-}(X, Y). \end{aligned} \tag{11}$$

With respect to the empirical tests, however, the two inequalities look rather different, and only the inequality (7) may be easily adapted to actual experiments. In fact, in the experiments either Eq.(10) is not true, thus (8) not being a true Bell inequality (it cannot be derived from local realism alone) or the quantities $E(X, Y)$ are no longer correlations, as we explain in the following.

In typical experiments there are two arms in the apparatus, each one consisting of a lens system followed by a polarization analyzer (polarizer, for short) and a detector (for the moment we do not consider the case of two-channel analyzers, but see below). Thus we may interpret $p(X, Y)$ as the probability that both photons are detected, after crossing the appropriate polarizers, and $p(X)$ the probability that the "red" photon of the pair is detected, with independence of what happens to the "green" photon (for clarity of exposition we attach fictitious colours, red and green, to the photons of a pair). However, if this interpretation is carried upon the quantities $E(X, Y)$, via the relations (11) , such quantities would be correlations only in the case that both photons of every pair arrive at the polarizers and every photon is detected (with 100% efficiency) whenever it has crossed the corresponding polarizer. But this idealized situation never happens.

The current practice in recent experiments is to use two-channel polarizers, with a detector after each outgoing channel. Attaching the labels $+$ or $-$ to the detectors after the first or second outgoing channel of a polarizer, respectively, it is possible to define p_{++} as the probability that both photons are detected in detectors with label $+$, p_{+-} the probability that the red photon is detected in a detector with label $+$ and the green photon in a detector with label $-$, etc. With this interpretation the quantities $E(X, Y)$ of (9) are indeed true correlations and the inequality (8) is never violated in actual experiments, because all probabilities p_{++}, p_{+-}, etc. are much smaller than unity due to the low collection–detection efficiency (i.e., for most photon pairs only one photon, or none, is detected). The "solution" proposed for this problem has been to renormalize the probabilities defining the correlations by

$$E^*(X, Y) = \frac{p_{++}(X, Y) + p_{--}(X, Y) - p_{+-}(X, Y) - p_{-+}(X, Y)}{p_{++}(X, Y) + p_{--}(X, Y) + p_{+-}(X, Y) + p_{-+}(X, Y)}. \quad (12)$$

Thus people use the inequality (compare with (8))

$$S^* \equiv E^*(A, B) + E^*(A, D) + E^*(C, B) - E^*(C, D) \leqslant 2, \quad (13)$$

in the empirical tests. Indeed, this is the inequality violated in most of the recent experiments. The inequality, however, cannot be derived from Eqs.(4) to (3) alone (without additional assumptions) and therefore it is not a genuine Bell inequality.

7. EXPERIMENTS USING OPTICAL PHOTONS

The first experimental test using optical photons was made by Freedman and Clauser.[20] They used photon pairs produced in the decay of excited calcium atoms via a 0–1–0 cascade. That is, the initial and final atomic states had 0 total angular momentum, so that the two emitted photons were entangled in polarization. The dichotomic observables measured were detection or non-detection of a photon, after it passed through a polarizer . The labels A and C are associated to two different positions of the polarizer for the "red" photon and similarly B and D for the "green" one. The authors were aware that the inequality (7) could not be violated with the technology of the moment because the detection efficiencies of the available detectors were too small (less than 10%.) As the left hand side of the inequality (7) is proportional to the efficiency squared, whilst the right side is proportional to the efficiency, the latter is more than 10 times the former, so that the inequality is very well fulfilled.

More specifically, the predition of quantum mechanics for the experiment may be summarized as follows, with some simplifications for the sake of clarity. The measurable quantities in the experiment are the single rates, R_1 and R_2, and coincidence rate, $R_{12}(\phi)$, the latter being a function of the angle, ϕ , between the polarizer's planes X and Y. In terms of the production rate, R_0, in the source they are given by

$$R_1(A) = R_2(B) = \frac{1}{2}R_0\eta, \quad R_{12}(X,Y) = \frac{1}{4}R_0\eta^2\alpha(1+V\cos(2\phi)). \quad (14)$$

Here α is an angular correlation parameter and η is the overall detection efficiency of a photon, which includes collection efficiency and quantum efficiency of the detectors (for simplicity we put the same efficiency η for the red and the green photons, which is approximately true in practice, but the generalization would be rather trivial). In actual experiments the quantum prediction (14) is confirmed, except for small deviations which are not considered significant.

The probabilities needed to test the inequality (7) are just the ratios

$$p(A) = \frac{R_1}{R_0}, \quad p(B) = \frac{R_2}{R_0}, \quad p(X,Y) = \frac{R_{12}(\phi)}{R_0}.$$

The production rate, R_0, is not measured but it is not difficult to show that, if we insert (14) into (7) , R_0 cancels out and the inequality becomes

$$\alpha\eta\left[1+\frac{1}{2}V\left(\sum_1^3\cos(2\phi_j)-\cos(2\phi_4)\right)\right] \leqslant 2,$$

where $\{\phi_j\}$ are the angles between the polarization planes of the analyzers, that is between A and B, A and D, C and B, C and D, respectively. These angles fulfil $\phi_1+\phi_4=\phi_2+\phi_3$ and the maximum of $\sum_1^3 \cos(2\phi_j)-\cos(2\phi_4)$ with that constraint is $2\sqrt{2}$. Thus the Bell inequality (15) holds true, for any choice of polarizers positions, whenever

$$\alpha\eta\left(1+\sqrt{2}V\right) \leqslant 2. \tag{15}$$

In the actual experiment[20] $V \simeq 0.85$, but $\eta \simeq 0.0001$, and $\alpha \simeq 1$, so that the inequality was safely fulfilled (η is the product of the quantum efficiency, ζ, of a detector times the collection efficiency of the apertures, see below Eq.(19)).

Freedman and Clauser[20] found a "solution", to circumvent the problem of the low detection efficiency, consisting of the replacement of condition (6) by another one, called "no-enhancement", which they claimed *plausible*. This assumption states that, for any value of the parameter λ, the following inequality holds true:

$$P_1(\lambda, A) \leqslant P_1(\lambda, \infty), \ P_2(\lambda, B) \leqslant P_2(\lambda, \infty), \tag{16}$$

where $P_j(\lambda, \infty)$ are the probabilities of detection of the photon with the corresponding polarizer removed. From inequalities (1)–(5) plus (16), the authors[20] derived the inequality

$$p(A, B) + p(A, D) + p(C, B) - p(C, D) \leqslant p(A, \infty) + p(\infty, B), \tag{17}$$

where $p(A, \infty)$ ($p(\infty, B)$) is the probability of coincidence detection with the polarizer corresponding to the red (green) photon removed. The results of the measurement, and the quantum predictions, for these probabilities are

$$p(A, \infty) = p(\infty, B) = \frac{1}{2}\alpha\eta^2,$$

and the inequality (17) implies

$$\left(1+\sqrt{2}V\right) \leqslant 2 \Leftrightarrow V \leqslant \sqrt{2}/2, \tag{18}$$

to be compared with (15). This was the inequality tested, and violated, in the commented experiment.

Note that, in sharp contrast with the obvious inequality (6), the inequality (16) is not only empirically untestable, it is *counterfactual.*

In fact, as said above, λ is a set of parameters which contains all relevant information about the intersection of the past light cones of the measurements. But the past light cone of one measurement (with a polarizer in place) is necessarily different from the past light cone of a different measurement (with the polarizer removed). In order to give a meaning to the inequality (16) it is necessary to compare a fact (one of the measurements) with a belief (about what would have happened in a different experiment having the same past light cone). For this reason I say that the inequality is counterfactual. Of course, it may be checked empirically that *the average* over λ of the left hand side is not greater than the average of the right hand side, that is for any light beam the detection rate does not increase when we insert a polarizer. (However, it might increase if we insert a polarization rotator plus a polarization analyzer when the incoming light is linearly polarized). In summary, the first alleged empirical disproof of local realism rests upon a counterfactual belief qualified as plausible. Therefore, strictly speaking, it did not test local realism. However, I do not mean that the experiment was useless because it opened an important new line of experimental research.

In the decade that followed the commented experiment, several similar atomic-cascade experiments were performed.[21,22] In addition to the requirement of introducing untestable auxillary assumptions (like (16)), all of them had the problem of being static. That is, the positions of the polarizers were fixed well before the detection events took place. Therefore the experiments could not test locality, in the sense of relativistic causality. In order to solve the problem, Alain Aspect and coworkers[23] performed in 1982 a new atomic-cascade experiment where (in some sense) the polarizers positions were chosen when the photons were already in flight. However the inequality tested was of the type (17) rather than a genuine Bell inequality like (7) .

The experiment of Aspect is usually presented as the definite refutation of local realism. One of the reasons is that, during the preparation of the experiment, Aspect was in close contact with Bell, who approved it. Although Bell was aware that there existed a loophole due to the low efficiency of the available photon detectors, he considered acceptable to make a *fair sampling assumption*. That is, to extrapolate the results actually got in the experiment, with low efficiency detectors, to detectors 100% efficient. This amounts to testing an inequality obtained from (7) by dividing the right hand side by the efficiency, η, and the left side by η^2. The inequality so obtained is practically the same as (17) . The fair sampling assumption was justified by Bell[24] with the frequently quoted sentence: “It is hard for me to believe that quantum mechanics works so nicely for inefficient practical set-ups and is yet going to fail badly when sufficient refinements are made." But this sentence cannot be applied to the commented experiments because *the predictions of quantum mechanics for any atomic-cascade experiment are compatible with local realism even if the experiment is made with ideal set-up*, in particular 100% efficiency detectors, as is shown in the following.[25] Apparently Bell was not aware of this fact before he untimely died in october 1990.

The atomic-cascade decay, giving rise to a photon pair, is a three-body problem with the consequence that the angle, χ, between the directions of emission of the two photons is almost uniformly distributed over the sphere. This implies that the angular correlation parameter α (see (14)) is almost independent of the angle χ, that is $\alpha(\chi) \simeq 1$. On the other hand both the overall detection efficiency, η, and the "visibility", V, of the coincidence curve are functions of the angle, θ, determined by the apertures of the lens system (as seen from the source). The dependence $V(\theta)$ is a loss of polarization correlation when the "red" and "green" photons do not have opposite wavevectors. In the Aspect experiment the functions are[21]

$$\eta = \tfrac{1}{2}(1 - \cos\theta)\zeta, \quad V = 1 - \tfrac{2}{3}(1 - \cos\theta)^2, \quad \alpha \simeq 1, \tag{19}$$

where ζ is the quantum efficiency of the detectors. Using these expressions it is easy to see that the maximum value of the left hand side of (15) is about 0.74 ζ, and the inequality is safely fulfilled even for ideal detectors (i.e., $\zeta = 1$). The figure should be multiplied times 2, giving 1.48 ζ, if we assume that both photons, red and green, may be detected in either detector. But still the inequality (15) holds true for any ζ ($\leqslant 1$). In summary taking into account the low angular correlation of the photon pairs produced in atomic cascades, *these experiments cannot discriminate between local realism and quantum mechanics.* In spite of this fact, the Aspect experiment is quoted everywhere as the definite refutation of local realism.

The problem of the lack of angular correlation might be solved if the recoil atom were detected[26] but that experiment would be extremely difficult. A more simple solution is to use optical photon pairs produced in the process of parametric down conversion, and this has been the source common in all experiments since about 1984. At a difference with atomic-cascade experiments, here the photons have a good angular correlation. In fact the parameter α of (14) as a function of the angle, χ, between the wavevectors of the two photons is such that the probability of detection of the green photon conditional to the detection of the red one is just the quantum efficiency ζ (or close to it.) Thus putting the detectors in appropriate places we may rewrite (15) with ζ substituted for $\alpha\eta$, that is

$$\zeta\left(1+\sqrt{2}V\right)\leqslant 2. \tag{20}$$

This inequality might be violated if V is close to 1 (which is achievable in actual experiments) and $\zeta > 2\left(\sqrt{2}-1\right)\simeq 0.82$. But such a high value of the detection efficiency has not yet been achieved and the low efficiency of detectors remains as a persistent loophole for the disproof of local realism.

This difficulty has led to the use of the modified CHSH inequality (13) as the standard inequality tested in practically all recent experiments with optical photons. These experiments use two-channel polarizers, and the prediction of quantum mechanics for them may be summarized in terms of four coincidence detection rates as follows

$$\begin{aligned} R_{++}(\phi) &= R_{--}(\phi) = \frac{1}{2}\eta R_0\left[1+V\cos(2\phi)\right], \\ R_{+-}(\phi) &= R_{-+}(\phi) = R_{++}\left(\phi+\frac{\pi}{2}\right), \end{aligned} \tag{21}$$

whilst the single rates are usually not measured (or, at least, not reported as relevant). In the actual experiments there are small departures from (21) which are not considered significant, but may be relevant for the reasons to be explained in the next section. As said above the inequality tested is (13), and the probabilities involved may be obtained from (21) as ratios between the measured coincidence rates and the production rate. That is, putting (21) into (12) we get

$$E^{*} = V\cos(2\phi). \tag{22}$$

If this is used in (13), steps similar to those leading to (18) give

$$S^{*} = 2\sqrt{2}V \leqslant 2 \Leftrightarrow V \leqslant \sqrt{2}/2. \tag{23}$$

This inequality looks the same as (18), but here V is obtained from measurements using two-channel polarizers. In practice V may be got by three different procedures:

(1) From the best fit of the measured correlation, $E(\phi)$, to the theoretical curve (9), where the probabilities, $p_{++}(\phi)$, etc., are the ratios of measured rates, $R_{++}(\phi)$, etc., to the production rate, R_0. It is easy to see that the fitting does not require the measurement of R_0. We shall label just V the quantity so obtained.

(2) As half the "visibility" of the empirical curve $E(\phi)$, that is the difference between the maximum and the minimum values divided by the sum. Again the value of R_0 is not required. I shall label V_A this quantity.

(3) From the value of S^* measured for the angles $\phi_1 = \phi_2 = \phi_3 = \pi/8, \phi_4 = 3\pi/8$, using the first equality (23) . These angles provide the maximum value of S^* if the empirical data agree with (21) . This value will be labelled V_B and it is the quantity commonly used in the test of the inequality (23). Indeed in recent times it has become standard practice to claim that local realism is refuted whenever $V_B > \sqrt{2}/2$.

According to quantum predictions the equality $V = V_A = V_B$ should hold true, but in actual experiments there are small differences between them. In the next section we will exhibit a local model which also predicts differences when the detection efficiency is high enough.

8. OTHER EXPERIMENTS AIMED AT TESTING LOCAL REALISM

In his pioneer work[(1)] Bell used the example of an Einstein–Podolsky–Rosen–Bohm[(27)] system, that is a pair of spin-1/2 particles with zero total spin. Thus it is not strange that some experiments have been proposed consisting of the measurement of the spin correlation of two spin-1/2 particles. The use of massive particles has the advantage that they may be quite reliably detected, so that such experiments do not suffer from the detection loophole. The proposed experiments use non-relativistic particles. As far as I know, no experiment of this kind has been proposed using relativistic particles. The reason is probably the difficulty for producing a pair with zero total spin if we take into account that spin of relativistic particles is not strictly conserved (only the total angular momentum of a free particle is strictly conserved in Dirac's theory).

The non-relativistic particles present the problem that it is difficult to guarantee the space-like separation of the measurements. As an example, we may consider the experiment proposed by Lo and Shimony.[(28)] It consists of the dissociation of molecules with two sodium atoms followed by the measurement of their spins by means of a Stern–Gerlach apparatus. The typical velocity of the sodium atoms, after dissociation, is about 3000 m/s and the length of the measuring magnets 0.25 m this giving a measurement time about 10^{-4} s. Thus, in order that the measurements were space-like separated, the Stern–Gerlach apparatused should be distant by more than 30 km. It is rather obvious that such experiment could not be a practical test of local realism as defined above. Similar problems appear in the proposed experiment by Adelberger and Jones[(29)] using neutron pairs. The neutrons should collide at low energy in order to insure a pure S-wave scattering so that, by Pauli's principle, the total spin should be zero. Again the distance between the spin measurements (by scattering with magnetized material) should be extremely large.

In addition there are fundamental constraints, derived from Heisenberg uncertainty principle, on experiments using non-relativistic particles.[30] For instance, let us assume that the particle detectors are static and placed on opposite sides and at a distance L from the source each, so that the distance between detectors is $2L$. If the particles have mass m (the same for both, for simplicity) and travel at a velocity v, then the initial position and velocity are uncertain by, at least, $\Delta x \Delta v = \hbar/2m$. Thus the arrival time at the detectors will be uncertain by, at least, $\sqrt{2\hbar L/(mv^3)}$. We may be sure that the measurements are space-like separated only if this quantity is smaller than $2L/c$, which leads to the constraint $L \geqslant 2\hbar c^2/(mv^3)$, a macroscopic quantity (for instance, in the experiment proposed by Lo and Shimony[28] this gives about 1 m). The quantity is not so big as to put unsurmountable practical difficulties, but it shows that a Bell test using non-relativistic particles requires measurements made at quite macroscopic distances.

An experiment using the scattering of non-relativistic protons was performed by Lamehi–Rachti and Mittig in 1976.[31] The spin components of the protons were measured by scattering on carbon foils. The experimental results agreed with quantum predictions, but the auxiliary assumptions needed for the experiment to be a test of a Bell inequality were stronger than in experiments with optical photons. An experiment has been recently performed[32] using two $^9Be^+$ ions in a trap, each of which behaves as a two-state systems. It has been claimed, and widely commented, that the experiment "has closed the detection loophole" because the atoms may be detected with 100% efficiency. However, the distance between ions in the trap, 3 μm, was very small. Although this distance is about 100 times the size of an ion wavepacket, it is 10^6 times smaller than the wavelength of the photons involved in the atomic transitions between the two levels (compare with the fundamental constraints commented in the previous paragraph). In these conditions the experiment cannot test locality in the sense of relativistic causality.

A loophole-free experiment involving spin measurements of atoms has also been proposed. It consists of the dissociation of mercury molecules followed by the measurement of nuclear spin correlation of the atoms.[33] In order to make the measurement time very short, the idea was to use a polarized pulse of laser light, which would induce selectively the ionization of the atom when it is in one specific spin state (say up) but not in the other possible state (down). After several years of preparation, the detailed proposal of the experiment was published in 1995, but 9 years later no results have been reported. (In the Oviedo Conference, held in July 2002, Fry reported that important difficulties had been found. Fry's talk was not published.)

Many other experimental tests of a Bell inequality have been performed or proposed, each one suffering from one or several loopholes. For instance, several experiments have been performed measuring the polarization correlation of gamma rays produced in the decay of positronium, one of the experiments violating the quantum prediction.[22] These experiments have the difficulty that the polarization cannot be measured with high enough precision.

There have been also proposals using high energy particles. For instance, the strangeness oscillations of pairs $K^0 - \overline{K}^0$ have been the subject of many papers,[34,35] but no loophole-free violation of a Bell inequality seems possible in this case due to the small decay time of the short K^0 in comparison with the oscillation period. Also an experiment has been recently performed using B^0 mesons, but here also the damping made impossible the violation of a Bell inequality, and only a normalization of the correlation function to the undecayed pair leads to a violation.[36]

In recent years a lot of effort has been devoted to the so-called "tests without inequalities".[37] The idea is to prepare a system in some state and perform a measurement such that the quantum prediction is definite (say "yes") but the prediction of any local realistic model is the opposite ("no"). For a proof of the incompatibility between local realism and quantum mechanics, in ideal experiments, the proposal is very appealing but from a practical point of view the possible experiments are less reliable than those resting upon Bell's inequalities. In particular they require an extreme control of the purity of the prepared state, which is not the case in the Bell's proof (see Sec. 4). An experimental test of local realism resting upon the idea has been performed,[38] but the experiment is not conclusive, as is shown be the existence of a local model reproducing the results.[39]

In summary, no performed experiment has been able to test a genuine Bell inequality with the condition that the measurments are performed at space-like separation. And, as far as I know, only a detailed proposal for a loophole-free experiment with available technology exists,[33] but this experiment seems to present unsurmountable difficulties. In consequence local realism has not been refuted. Furthermore it is the case that, strictly speaking, *local realism has not yet been tested against quantum mechanics. That is no experiment has been performed able to discriminate between local realism and quantum mechanics.*

9. A LOCAL REALISTIC MODEL FOR BELL EXPERIMENTS

In the early papers on the subject it was frequent to assert that local realistic models for the performed experiments, although possible, would be necessarily contrived.(17,21) Also, it is generally believed that the predictions of any local realistic model would depart dramatically from the quantum predictions when the measuring devices become close to ideal (e.g., using high efficiency detectors). The purpose of the present section is to show that both prejudices are false. In fact, we shall exhibit a model which appears as quite natural and is rather close to quantum mechanics in the predictions even in situations close to the ideal.

As said above most of the recent experimental tests of Bell's inequalities use photon pairs produced by parametric down-conversion and the quantity measured is the polarization correlation of photon pairs, by means of two-channel polarizers. I shall consider only these experiments here, although the arguments that follow apply more generally. In the case of using one-channel polarizers the local model should reproduce Eq.(14), and should consist of three functions, $\rho(\lambda)$, $P_1(\lambda, \phi_1)$ and $P_2(\lambda, \phi_2)$, fulfilling the conditions (4) to (6) and such that

$$\frac{R_1}{R_0} = \int \rho(\lambda) P_1(\lambda, \phi_1) d\lambda, \quad \frac{R_2}{R_0} = \int \rho(\lambda) P_2(\lambda, \phi_2) d\lambda, \tag{24}$$

$$\frac{R_{12}(\phi)}{R_0} = \int \rho(\lambda) P_1(\lambda, \phi_1) P_2(\lambda, \phi_2) d\lambda, \quad \phi_1 - \phi_2 = \phi. \tag{25}$$

Here ϕ_1 (ϕ_2) is the angle of the polarization plane of the first (second) analyzer with respect to the horizontal. For experiments with two channel polarizers we may use two pairs of functions so that $P_{j-}(\lambda, \phi_j) = P_{j+}(\lambda, \phi_j + \pi/2)$. Many local models have been exhibited in the past, for instance the 1984 Marshall–Santos–Selleri one,(40) a variant of which is the following. In order to show that the model is not artificial, I shall compare it with the quantum calculation.

In quantum mechanics, a pair of photons entangled in polarization may be represented by the state vector

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(a_H^\dagger b_V^\dagger + a_V^\dagger b_H^\dagger \right) |\text{vacuum}\rangle, \tag{26}$$

where H (V) labels horizontal (vertical) polarization and $a^\dagger (b^\dagger)$ are creation operators of photons in the first (second) beam. If we search for a local realistic model, it is natural to attach two-dimensional polarization vectors (in the plane perpendicular to the wave-vectors) to the incoming light signals, such as

$$\boldsymbol{\alpha} \equiv (\alpha_H, \alpha_V), \quad \boldsymbol{\beta} \equiv (\beta_V, \beta_H). \tag{27}$$

The scalar product of these two vectors is the model analog of the probability amplitude associated to (26), whence the probability should be the square of that scalar product. This suggests a probability function, ρ (see Eq.(6)) of the form

$$\rho \propto (\boldsymbol{\alpha} \cdot \boldsymbol{\beta})^2 \propto \cos^2(\lambda_1 - \lambda_2), \tag{28}$$

where the hidden variables λ_1 and λ_2 are angles, say with the horizontal, defining the directions of the polarization vectors of the photons. It is easy to see that, after normalization, this gives

$$\rho = \frac{1}{\pi^2}[1 + \cos(2\lambda_1 - 2\lambda_2)]. \tag{29}$$

Now we shall choose the probabilities, $P_j(\lambda, \phi_j)$, of detection of a photon after passage through the corresponding polarizer. In a classical theory an electromagnetic signal, polarized in the plane forming an angle λ_j with respect to the horizontal, would be divided in the polarizer according to Malus' law, a large intensity going to the upper channel if λ_j is close to the angle, ϕ_j, of the polarizer (and similarly to the lower channel if λ_j is close to $(\phi_j + \pi/2)$). Thus it is natural to assume that a detection event would be more probable if the intensity is large, which suggests that $P_j(\lambda, \phi_j)$ should decrease with increasing value of the angle $|\lambda_j - \phi_j|$. A simple expression which fits in these conditions is the following one

$$\begin{aligned} P_j(\lambda_j, \phi_j) = \beta \quad & \text{if } |\lambda_j - \phi_j| \leqslant \gamma \pmod{\pi} \\ 0 \quad & \text{otherwise.} \end{aligned} \tag{30}$$

By the nature of polarizers the angle ϕ_j is equivalent to $\phi_j + \pi$, whence the periodicity π. For the validity of the model it is crucial that $\beta \leqslant 1$, so that conditions (6) are fulfilled. In order to extend the model to experiments with two-channel polarizers it is necessary to define functions P_{j+} and P_{j-}. I propose the following, in terms of those defined by (30)

$$P_{j+}(\lambda_j, \phi_j) = P_j(\lambda_j, \phi_j), \quad P_{j-}(\lambda_j, \phi_j) = P_j(\lambda_j, \phi_j + \frac{\pi}{2}). \tag{31}$$

In this form we obtain a model, not too artificial, reproducing the quantum predictions, Eq. (1), for all performed experiments involving optical photons.

In fact, inserting Eqs. (29) and (30) in (25), it is easy to see that the model leads to

$$\frac{R_{++}(\phi)}{R_1} = \frac{2\beta\gamma}{\pi}\left[1 + \frac{\sin^2(2\gamma)}{4\gamma^2}\cos(2\phi)\right] = \frac{R_{--}(\phi)}{R_1}, \tag{32}$$

$$\frac{R_{+-}(\phi)}{R_1} = \frac{R_{-+}(\phi)}{R_1} = \frac{R_{++}(\phi + \pi/2)}{R_1}, \tag{33}$$

which reproduces the quantum prediction (21) if we assume

$$V = \frac{\sin^2(2\gamma)}{4\gamma^2}, \quad \eta = \frac{4\gamma\beta}{\pi}. \tag{34}$$

(It may be realized that using functions P_j different from (30), but depending only on $|\lambda_j - \phi_j|$, would change just the constant factor in front of $\cos(2\phi)$ in (32)). The constraint $\beta \leqslant 1$ shows that, in the typical range of values considered here, the model may agree with the quantum predictions only if

$$V \leqslant \frac{\sin^2(\pi\eta/2)}{(\pi\eta/2)^2} \simeq 1 - \frac{\pi^2\eta^2}{12}. \tag{35}$$

Typical values in actual experiments are $\eta \lesssim 0.2$, $V \lesssim 0.97$ (if raw data are used, see next section), so that the model is compatible with the empirical results. For instance the experiment by Kurtsiefer *et al.*[41] reports a value $S^* = 2.6979 \pm 0.00012$, so that $V_B = 0.9539 \pm 0.00004$ (see (13)) with $\eta = 0.214$. The values V and V_A were measured with less precision, but they agree with V_B within statistical errors. The right hand side of (35) takes the value 0.962 so that the inequality is fulfilled, that is the results of the experiment are compatible with our model. Note that the experiment violates the alleged Bell inequality (23) but it does not violate a genuine Bell inequality like (20).

The commented experiment[41] shows that recent tests are close to the limits of validity of the simple model given by (29)–(31). It is possible to get models of the same general type which depart but slightly from the quantum prediction for higher values of η and/or V. A good model consists of using (30) again for the detection probabilities but using, instead of (29), the probability function

$$\rho = \frac{1}{\sqrt{2\pi^3}\sigma} \exp\left(-(\lambda^2/2\sigma^2)\right), \quad \lambda = \lambda_1 - \lambda_2 \in [-(\pi/2), (\pi/2)]. \quad (36)$$

Defining $\rho(\lambda + n\pi) = \rho(\lambda), n = \pm 1, \pm 2, \ldots$ this becomes a periodic function which may be expanded in Fourier series to give

$$\rho = \frac{1}{\pi^2}\left[1 + 2\sum_{n=1} \exp\left(-2n^2\sigma^2\right)\cos(2n\lambda)\right], \quad (37)$$

provided that σ is small enough so that the Gaussian function in Eq. (36) is negligible for $\lambda = \pi/2$. Hence we get a prediction (with $\beta = 1$)

$$\frac{R_{++}}{R_1} = \frac{2\gamma}{\pi}\left[1 + 2\sum_{n=1} \exp\left(-2n^2\sigma^2\right)\frac{\sin^2(2n\gamma)}{(2n\gamma)^2}\cos(2n\phi)\right], \quad (38)$$

$$R_{+-}(\phi) = R_{-+}(\phi) = R_{++}\left(\phi + \frac{\pi}{2}\right), \; R_{--}(\phi) = R_{++}(\phi). \quad (39)$$

The expression obtained for $E^*(\phi)$, which I shall not write explicitly, is very close to the quantum prediction (22), even for quite high detection efficiency, using $\sigma = \pi/18$, a value recently proposed.[42] For this choice it is possible to get $V = 1$ with efficiencies up to $\eta \simeq 0.848$ (that is $\gamma = \pi\eta/4 \simeq 2/3$, see (34)) and still the departures from the quantum prediction are only of a few percent. In fact we get

$$V_A = 0.980, \quad V_B = 0.957.$$

Even closer is the prediction for the function $E^*(\phi)$, see Eq.(12), where the discrepancy corresponds to a term in $\cos(6\phi)$ with a coefficient of order 0.02. This shows that the agreement between the function $E^*(\phi)$, empirically got, and the pure cosinus curve, predicted by quantum mechanics, is not an argument against local realism, contrary to what has been sometimes claimed.

For higher efficiencies the model cannot reproduce the (ideal) quantum prediction $V = 1$. In particular, for 100% efficiency, i.e., $\eta = 1 \Rightarrow \gamma = \pi/4$, we get $V \leqslant 8/\pi^2$, whatever is the value of σ. But, even in this limit, the model does not depart too much from quantum mechanics. For instance, with the same choice $\sigma = \pi/18$, we get

$$V = 0.7627, \quad V_A = 0.8225, \quad V_B = 0.7043,$$

in comparison with the ideal quantum prediction $V = V_A = V_B = 1$. It may be realized that the value given by that model for the left side of the inequality (8) is 1.992, very close to the Bell limit, 2, although not so close to the quantum prediction with ideal set ups, 2.414.

In summary, the proposed family of models agrees exactly with quantum predictions for low enough detection efficiency, and departs when the efficiency increases. The departure manifests in that the model prediction for $E^* (\phi)$, when Fourier analyzed, either contains terms in $\cos(2n\phi)$ with $n > 1$ or has a coefficient for the $\cos(2\phi)$ term lower than the quantum prediction, or both. The deviations slowly increase with the detection efficiency up to a few percent for about 85% efficiency and more rapidly for higher efficiencies, up to about 20% for 100% efficiency.

Our conclusion from this section is that the accuracy of the agreement with quantum mechanical predictions of the experiments (allegedly) testing Bell's inequalities is not an argument against local realism, because local models exist also able to reproduce exactly the results of performed experiments and quite accurately the predictions of quantum mechanics for (future) experiments with higher efficiency detectors. Only the loophole-free violation of a genuine Bell inequality would disprove local realism.

10. DIGRESSION ON PHILOSOPHY AND SOCIOLOGY OF SCIENCE

For the analysis of significance of the results obtained in the performed tests of local realism it is convenient to make a digression on philosophy and sociology of science. The pragmatic approach to quantum mechanics, commented in Sec. 2, has led to an "antimetaphysical" attitude, that is the idea that science should not be constrained at all by any philosophical principle. I think that this position is not completely correct. Of course, the philosophy of the natural world should rest upon knowledge derived from science, and not viceversa, but it is also true that science itself rests upon some philosophical principles.

One of the central principles of the philosophy of science is that, although a single experiment may refute a theory, no theory can ever be absolutely confirmed by experiments, a principle stressed by Karl Popper.(43) The reason is that, if an experiment is compatible with a theory, say A, it is sure that there are many other theories, B, C, . . . , also compatible with the experiment. Thus the only possibility to increase the degree of confidence in a theory is to perform many experiments able to refute it. If the results of these experiments are compatible with the theory, it becomes increasingly supported. This is precisely what has happened with local realism. All experiments attempting at violating it have failed during 40 years.

Another philosophical point which is required in any serious discussion of the present status of local realism is that established theories are protected, as was stressed by Imre Lakatos.[44] That is, when a new discovery seems to contradict the theory, it is always possible to introduce some auxillary hypotheses which allow interpreting the new finding within the accepted theory. It is well known the example put by Lakatos on the hypothetical observation of an anomaly in the motion of a planet. It could be explained, without rejecting Newton's gravitational theory, by the existence of another, unknown, planet. If this is not found by observation in the predicted place, it might be assumed that there are two planets instead of one, etc. Indeed, it is a historical fact that no theory has been rejected by its contradiction with a single or even several experiments (e.g., Newton's gravity by the anomaly in the motion of Mercury). The theory survives until a new, superior, theory is available (e.g., Newton's gravity survived until the appearance of general relativity). The consequence of this sociological fact is that any argument *for* a established theory is accepted without too much discussion, but any argument *against* the theory is carefully analyzed in order to discover a flaw. Thus, even a honest experimentalist will devote much more care searching for possible errors if an experiment contradicts the assumed predictions of quantum mechanics than if it confirms the theory.

A good example of this behaviour has happened in the early, atomic-cascade, tests of Bell's inequalities. As said in Sec. 7 the first experiment of that kind was performed by Freedman and Clauser[20] and the results agreed with quantum predictions. The second experiment was made by Holt and Pipkin (see, e.g., the reviews by Clauser and Shimony[21] or by Duncan and Kleinpoppen.[22]) The results of the experiment disagreed with quantum predictions but did not violate the inequality (18) tested. The consequence is that the experimental results were never formally published and many people (including the authors) made a careful search for possible sources of error. The Holt–Pipkin experiment had two main differences with the Freedman–Clauser one: (1) the use of a cascade of atomic mercury, instead of calcium, and (2) the use of calcite polarizers, instead of polarizers made of piles of plates. In order to clarify the anomaly, Clauser[45] "repeated" the Holt–Pipkin experiment, that is performed a new experiment using mercury but, again, piles of plates as polarizers. This time the results agreed with quantum predictions and violated the tested inequality (18). However, there are arguments[46] suggesting that it is the use of calcite what is very relevant, because it has an extremely good extintion ratio, less than 10^{-4} to be compared with 0.02 for typical piles of plates. In contrast calcite possesses bad efficiency for maximum transmission of linear polarized light, about 80% to be compared with 98% for typical piles of plates. But there are arguments supporting the opinion that it is the minimal, and not the maximal, transmission of the polarizer what matters.[46] In spite of this fact, the Holt–Pipkin experiment has never been repeated in the sense of using calcite polarizers.

I presume that something similar is happening with the efficiency of photon counters. Apparently there are already detectors with efficiency above 80% (which might allow loophole-free tests of local realism), but no report exists of an experiment performed, or even planned, in recent years with these detectors. I suposse that the reason is that the said detectors do not possess the necessary qualities. That is a low enough dark rate and a linear response (i.e., a quantum efficiency which does not change with the detection rate). I would guess that some loophole-free experiments have been attempted, but the results have not been published because they do not support the standard paradigm, namely that a (genuine) Bell inequality could be violated when the detection efficiency is high enough.

11. PRESENT STATUS OF LOCAL REALISM AT THE EMPIRICAL LEVEL

Now we arrive at the crucial question: Is local realism a valid principle of physics? The current wisdom is that it has been definitely refuted by the optical experiments already performed, modulo some loopholes due to nonidealities which, it is added, are quite common in experimental physics. But, as explained in Sec. 7, this is not true for the atomic-cascade experiments because they do not discriminate between local realism and quantum mechanics. Indeed the *ideal* predictions of the latter are compatible with the former. We are left with experimental tests involving optical photons produced in the process of parametric down-conversion (e.g., the mentioned experiment by Kurtsiefer *et al.*[41]). As discussed in Sec. 7, these experiments cannot tests (genuine) Bell inequalities due to the low efficiency of available photon detectors, and other non-idealities. If we exclude the down-conversion experiments, the evidence against local realism is meager because all other tests present greater difficulties. It is true that the efficiency loophole has been closed in experiments with atoms,[32] what has been used as an argument *against* the validity of local realism.[47] In my opinion the fact that different loopholes appear in different experiments is an argument *for* it. Indeed, it suggests that nature preserves local realism in every case.

The assumption behind the claim that Bell's inequalities have already been violated is that photons, like electrons, or atoms, or molecules, could be treated as particles. If this were true there would be no reason why detectors could not be manufactured having 100% efficiency (without too much noise). But I think that it is closer to the truth, and less difficult to understand, the assumption that atoms are (localized) particles which for yet unknown reasons behave sometimes like waves, but optical photons are (extended) wavepackets which behave, in some cases, like particles. There are two arguments, at least, against optical photons being particles. First there is no "position operator" for photons in quantum mechanics, and second the photon number is usually not well defined. That is, common states of light, like laser light or thermal light, have an indefinite number of photons. A photon is (or should be associated to) a wavepacket in the form of a needle with a length of the order of the coherence length, which for atomic emissions is at least of order of centimeters, and several wavelengths in transverse dimensions. This associates to a typical optical photon a volume about 10^{18} atomic volumes. In sharp contrast, a gamma ray photon may be associated to a volume smaller than that of an atom. If we take the atomic volume as standard, we are led to say that high energy photons are localized entities (behaving mainly as particles) whilst optical photons are not localized (behaving mainly as waves). Thus, with reference to the sentence of Bell, quoted in Sec. 3 above, it might not be the case that quantum mechanics would fail in experiments with highly efficient photon counters, but that counters of *optical* photons working with high efficiency and good performance may not exist.

In any test of local realism using photons, it is necessary to measure both, the position of the photon and other quantity like polarization or phase. The former may be called a particle property whilst the latter is a wave property. Thus, if we remenber the Bell inequality (20), it is natural to associate the parameters ζ (detection efficiency) and V (visibility of the polarization correlation curve) to those two quantities and conclude that the Bell inequality forbids that a photon behaves as a particle and as a wave at the same time. In contrast, the tested inequality (23) just constrains the "amount of wave behaviour, V". Thus its violation means that we cannot dismiss the wave character of optical photons. On the other hand, tests using gamma rays do not have any problem with the position measurement (i.e., the efficiency of detection), but there are difficulties for a precise measurement of polarization, as commented in Sec. 8. I conclude that, in tests using photons, a trade-off exists between measurability of position and measurability of polarization, a trade-off quantified by the Bell inequality (20). The "corpuscular" property (position), may be accurately measured only in photons much smaller than atoms, like gammas, the "wave" property (polarization), in those much larger than atoms, like optical photons.

In conclusion, I claim that *local realism is such a fundamental principle which should not be dismissed without extremely strong arguments*. It is a fact that there is no direct empirical evidence at all for the violation of local realism. Only when the existing evidence is combined with theoretical arguments (or prejudices) it might be argüed that local realism is refuted. But, in my opinion, this combination is too weak for such a strong conclusion.

12. QUANTUM MECHANICS, NOISE AND THE SECOND LAW

The scientific community dismisses the mentioned loopholes as unplausible explanations (see, e.g., the relatively recent article by Laloë,[48] excellent in many other respects). I think that this opinion rests upon the theoretical prejudice mentioned above, namely that the validity of local realism would imply that quantum mechanics is false. And, for good reasons, nobody is willing to accept that quantum mechanics is wrong. But I think that *a violation of local realism is no more acceptable than a violation of quantum mechanics*. Thus I claim that there exists a real problem whose only good solution is the compatibility of local realism with quantum mechanics, or some "small" modification of this theory.

Quantum mechanics consists of two quite different ingredients: the formalism (including the equations) and the theory of measurement. We must assume that the equations are correct, because the extremely accurate agreement of measured quantities with the theoretical values, for instance in quantum electrodynamics, cannot be explained otherwise. However, the quantum theory of measurement is objectable from many points of view, and only a minimal fraction is really required for the interpretation of most experiments. For instance, the postulate about position measurements (i.e., Born's rule) is enough for the interpretation of scattering experiments, including spectroscopy (which may be seen as photon scattering). On the other hand the standard proofs of "Bell's theorem" rest upon the theory of measurement. Consequently I guess that a weakening of the standard measurement theory, without touching the formalism, might make quantum mechanics compatible with local realism. As is well known, quantum mechanics puts some bounds to the possible preparations and measurements in the form of Heisenberg inequalities (uncertainty relations). I guess that stronger bounds should exist preventing the violation of local realism in any case.

Quantum mechanics is intrinsically stochastic, in the sense that its predictions are typically about probabilities of events. In classical physics the essential ingredient of any stochastic theory is noise. Thus the natural assumption would be that the probabilistic character of quantum theory is due to noise, but this is not the standard interpretation. It is believed that the probabilistic character of quantum predictions is a manifestation of the lack of strict causality of nature. In my opinion what happens is that we do not yet understand correctly noise in the quantum domain, in particular quantum vacuum fluctuations. My guess is that quantum noise is what may put fundamental constraints to the accuracy of some measurements, in such a way as to prevent the violation of local realism.

Ian Percival[49] has pointed out that, in classical physics, the second law of thermodynamics does not contradict the laws of (Newtonian) mechanics, but nevertheless it restrict the possible evolutions of physical systems. He proposed that a similar physical principle might prevent the violation of local realism without actually contradicting quantum mechanics. In my view this is an interesting observation, because I presume that it is the second law, with quantum noise taken into account, what may prevent the violation of local realism in the quantum domain. I think that a better understanding of the laws of thermodynamics at the quantum level is required. Indeed, the traditional interpretation of the third law (zero entropy at zero Kelvin) seems difficult to be reconciled with the existence of (non-thermal) quantum vacuum fluctuations. In summary, a serious attention to the loopholes in the empirical tests of the Bell inequalities, rather than their uncritical dismissal, may improve our understanding of nature.

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