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A graph model for conflict resolution with inconsistent preferences among large-scale participants

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Abstract
As a flexible and powerful method to resolve strategy conflicts, the graph model for conflict resolution has drawn much attention. In the graph model for conflict resolution, decision-makers need to provide their preference information for all possible scenarios. Most existing studies assumed that decision-makers adopt quantitative representation formats. However, in some real-life situations, decision-makers may tend to use qualitative assessments due to their cognitive expression habits. In addition, stakeholders involved in a graph model can be a group that is composed of a large number of participants. How to manage these participants' inconsistent preference assessments is also a debatable issue. To fit these gaps, in this study, we propose a graph model for conflict resolution with linguistic preferences, and this model allows participants to use inconsistent assessments. To do this, we first construct a linguistic preference structure, with the necessary concepts being defined. Then, four stability definitions for both a two-decision-maker scenario and an n-decision-maker scenario are introduced. To illustrate the usefulness of the proposed model, an illustrative example regarding the Huawei conflict is provided.

Keywords Decision analysis · Graph model · Conflict resolution · Linguistic preferences · Large-scale participants

1 Introduction

Strategic conflict is a universal phenomenon in the real life of a society (Li et al., 2019). It can be defined as a situation in which two or more parties with their own objectives and preferences interact with each other and make independent choices regarding their individual aims. These parties are also called decision-makers (DMs). From territory, sovereignty, and trade disputes among countries to economic interests among social

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individuals, DMs' strategic conflicts arise in all strata and fields of society (Bashar et al., 2018). To resolve these strategic conflicts, a number of approaches have been developed, such as the graph model for conflict resolution (GMCR) (Hipel & Fang, 2021), game theory (Nash, 1950), drama theory (Howard, 1994), conflict analysis (Fraser & Hipel, 1979), and metagame analysis (Howard, 1971).

Among all these approaches, the GMCR has received much attention from the conflict resolution community because of its flexibility and conciseness (Bashar et al., 2012; Zhao et al., 2019). It is a decision-making method developed from conflict analysis (Fraser & Hipel, 1979) and metagame analysis (Howard, 1971). Formally, the GMCR has the following phases: extracting efficacious information from disputes, structuring mathematical models, analyzing states' stability, and providing useful decision support for DMs. The GMCR provides an efficient framework to study and manage different stability concepts and is convenient for applying to actual scenarios. At present, researches on GMCR mainly include preference information (Bashar et al., 2018; Hipel & Fang, 2021; Li et al., 2019), stability definitions (Zhao et al., 2019), post-stability analysis (Matbouli et al., 2015), and actual applications (He et al., 2017).

If we use the GMCR, generally, two steps are required (Li et al., 2019). One is the modeling process. In this process, three elements should be determined: the DMs' options, a set of states consisting of options, and preference information over states provided by DMs. The second step is the stability analysis in which we need to analyze whether a state is stable for DMs. The stability for a DM relies on the chosen solution concept, specifically under a stability definition and, whether it is advantageous for a DM to leave the current state. In a conflict, DMs' possible patterns of behavior are characterized by stability definitions. Various kinds of stability definitions have been introduced, such as the Nash stability (Nash, 1950), general metarationality (GMR) (Howard, 1994), symmetric metarationality (SMR) (Howard, 1994), and sequential stability (Fraser & Hipel, 1979). In the GMCR, if a state is stable for all DMs under a specific stability definition, then this state can be called an equilibrium.

Preference relation is a powerful tool to assess the stability of a state. Originally, it was assumed that the preference relation was crisp. People used binary relations such as '*(strictly) preferred to* ($\succ$)' and '*indifferent from* ($\sim$)' to characterize the preferences between different states. However, the binary relations have limitations since preferences may be vague and imprecise in real-world disputes. Therefore, many extensions of binary relations have been employed to enhance the representation of preference information in stability analysis (Bashar et al., 2012; Hamouda et al., 2004; Kuang et al., 2015; Rêgo & Santos, 2015, 2018; Xu et al., 2009). Bashar et al. (Bashar et al., 2012) used fuzzy preference relations (FPRs) to deal with DMs' certain and uncertain preference information. With FPRs, DMs can use a numerical value in [0, 1] to express the preference degree of one state over another. There are also other preference structures such as the incomplete FPRs (Li et al., 2019), interval fuzzy preferences (Bashar et al., 2018), probabilistic preferences (Rêgo & Santos, 2015), grey preferences (Kuang et al., 2015), upper and lower preference relations (Rêgo & Santos, 2018), and multiple levels of preferences (Xu et al., 2009).

As we can see, most existing studies used quantitative tools to express preference information. However, in many real-world decision situations, the preference information cannot be assessed precisely in quantitative forms due to limited understanding and human biases. DMs' judgments may depend on some psychological aspects such as emotion, the state of mind and experience. In this sense, it is necessary to use a qualitative tool to express DMs' vague preference information. In addition, the participant in a graph model can be an individual or a party (group) who controls at least one option (Li et al., 2019). In large-scale conflict problems, because of the increasing complexity of decision-making problems, the participator may be a company group, a government, or a whole population that involves large number of stakeholders. Although a participant has common interests, when providing pairwise comparisons over states, different stakeholders within a group may have inconsistent assessments. However, the majority of previous literature assumed that all participants have a unified assessment regarding the preference information.

To model qualitative preference information, Zadeh (Zadeh, 1975) proposed the fuzzy linguistic approach, which assumed that the values of a linguistic variable are not numbers but words or sentences. Because of the flexibility and applicability, the linguistic variable has been used in various fields. Since the linguistic preference (Herrera & Herrera-Viedma, 2000; Herrera-Viedma et al., 2005) is flexible in representing people's perceptions and is close to human's cognitive expression habits, it has been widely used in many decision-making fields (Herrera & Herrera-Viedma, 2000; Herrera-Viedma et al., 2005; Lin et al., 2020). As the linguistic preference has not been adopted in GMCR so far, in this study, we introduce a linguistic preference framework for GMCR in which the linguistic preferences with probabilities are used to characterize different stakeholders' preference information. The main contributions of this study are highlighted as follows:

(1) A GMCR with linguistic preferences is proposed. This model can deal with qualitative preference information of DMs. The concepts of linguistic relative strength of preference (LRSP), linguistic satisficing threshold (LST), and linguistic unilateral improvement (LUI) are provided. Based on these concepts, four linguistic stability definitions are then introduced for two-DM and n-DM graph models.
(2) The proposed GMCR model allows inconsistent preference assessments when the DM is a group with a large number of participants. We use linguistic preferences with probabilities to manage inconsistent preference assessments.
(3) A real-world dispute regarding the ban on Huawei editors and reviewers initiated by IEEE (Institute of Electrical and Electronics Engineers) is provided to illustrate the applicability of our proposed model.

This paper is organized as follows. Section 2 introduces the preliminaries used in this study. In Sect. 3, we construct a linguistic preference structure. Section 4 put forwards four linguistic stability definitions for both two-DM and n-DM graph models. In Sect. 5, an illustrative example is provided. Conclusions are drawn in Sect. 6.

2 Preliminaries

This section introduces some preliminaries used in the rest of this study, including the description of the GMCR and linguistic preferences.

2.1 Graph model for conflict resolution

GMCR is a comprehensive tool to manage strategic conflicts. In a GMCR, DMs control their own options and possible states generated by composing options. Then, DMs express their preference information regarding possible states. A GMCR can be modelled as $V = \{N, S, G, P\}$, in which the four main elements are:

(1) Two or more independent DMs $N = \{1, 2, \cdots, n\}$, who are involved in a conflict. A DM can be an individual or a team comprised of many participants.
(2) A set of possible feasible states $S = \{s_1, s_2, \cdots, s_m\}$, which represent the combination of options of different DMs.
(3) $G_k = (S, \{A_k\}_{k \in N})$ is the directed graph for DM k. $A_k \subseteq S \times S$ represents the arcs that characterize possible moves of states controlled by DM k.
(4) P is the preference structure over S. As mentioned in the Introduction, different kinds of preference structures have been used in GMCR. A summary of typical preference structures is provided in Table 1.

Table 1 A summary of typical preference structures

Reference	Preference structure	Mathematical representation
Hamouda et al. (Hamouda et al., 2004)	Strength of preference	$\{\geq, \succ, \sim\}$
Bashar et al. (Bashar et al., 2012)	Fuzzy preference	Numerical value in [0, 1]
Yu et al. (Yu et al., 2017)	Fuzzy strength of preference	Numerical value in $[0, 1] \cup \{-1, 2\}$
Bashar et al. (Bashar et al., 2018)	Interval fuzzy preference	$\tilde{r}_{ij} = [\underline{r}_{ij}, \overline{r}_{ij}] \subseteq [0, 1]$
Li et al. (Li et al., 2019)	Incomplete fuzzy preference	Numerical value in [0, 1] or no value
Kuang et al. (Kuang et al., 2015)	Grey preference	$\otimes x \in \bigcup_{i=1}^{k} [\underline{x}_i, \overline{x}_i]$
Rêgo and Santos (Rêgo & Santos, 2015)	Probabilistic preference	$P_k(s_i, s_j)$
Rêgo and Santos (Rêgo & Santos, 2018)	Upper and lower probabilistic preference	$\underline{P}_k(s_i, s_j)$ and $\overline{P}_k(s_i, s_j)$
Xu et al. (Xu et al., 2009)	Multiple level preference	$\{\sim, \succ, \geq, \cdots, \overset{r}{\succ}\}$
Yu et al. (Yu et al., 2020)	Unknown and fuzzy Preferences	[1/10, 10] and 0

As we can see from Table 1, existing studies used quantitative expression formats to model the preference information of DMs. However, in common daily life, individuals are usually accustomed to convey preference information based on qualitative expressions which are close to human way of thinking and reasoning. Moreover, a DM is represented as an individual or a party with separate opinions. However, with the rapid development of technological paradigms such as social networks and e-democracy, a large number of individuals may take part in a decision-making problem. In this case, it is necessary to collect opinions of those participants so as to make effective decisions. As far as we know, the existing studies on GMCR did not involve these research challenges. Thus, this study dedicates to introducing a linguistic preference framework for GMCR.

2.2 Fuzzy linguistic approach

In some situations, DMs may prefer to use linguistic terms instead of exact numerical values to express preference information. Sometimes, a DM cannot master all valuable information and shows a high degree of hesitancy. For instance, when comparing two universities, a student may not be able to say they prefer one choice moderately or strongly. Therefore, their preference can be '*between moderately preferred and strongly preferred*'.

Definition 1 (Zadeh, 1975). A linguistic variable is characterized by a quintuple $(L, H(L), D, T, M)$, in which L is the name of the variable; $H(L)$ indicates the term set of L, specifically, the set of linguistic values of L, ranging across a universe of discourse D; T is a syntactic rule (usually represented as a grammar) for generating the terms in $H(L)$; M is a semantic rule for relating its meaning $M(X)$ with each L.

Based on Definition 1, we can obtain two methods to select appropriate linguistic descriptors (Rodríguez et al., 2012). One is the ordered structure of linguistic terms; the other is the context-free grammar. In the following, we use the ordered structure method to obtain linguistic descriptors and possibilities for defining their semantics.

The main objective of establishing linguistic descriptors of a linguistic variable is to support the DMs with some natural words by which they can express their cognitive information. To achieve this objective, an appropriate cardinality of a linguistic term set (LTS) should be defined so as to avoid useless precision. Seven or nine is a typical value for the cardinality (Rodríguez et al., 2012). The ordered structure method defines an LTS through considering all terms distributed on a scale by means of supplying the term set directly (Rodríguez et al., 2012). A commonly used 7-value LTS is $\overline{S} = \{\overline{s}_0 = none, \ \overline{s}_1 = very\,low, \ \overline{s}_2 = low, \ \overline{s}_3 = medium, \overline{s}_4 = high, \overline{s}_5 = very\,high, \overline{s}_6 = perfect\}$.

The *semantics based on the ordered structure of the LTS* introduces the semantics from the structure defined over the LTS. DMs can provide their assessments by means of an ordered LTS. A typical symmetrically distributed ordered set of seven linguistic terms with its syntax is presented in Fig. 1. In GMCR, an LTS for comparing two states can be set as $\overline{S} = \{\overline{s}_0 = completely\,less\,preferred, \overline{s}_1 = strongly\,less\,preferred, \overline{s}_2 =$

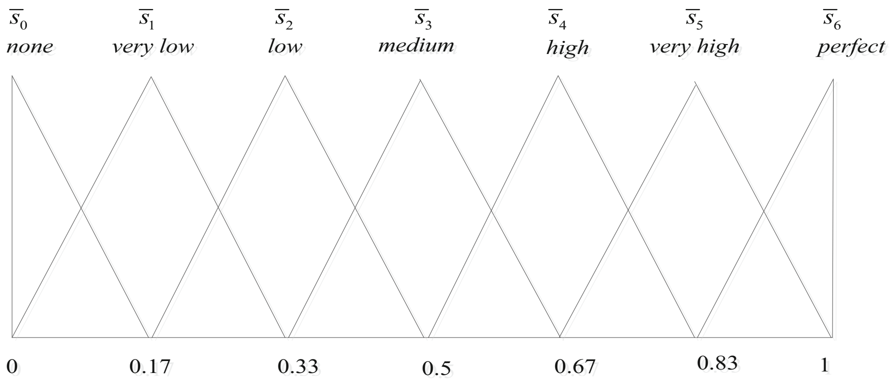

Fig. 1 The set of seven subscript-symmetric terms with its semantics

$less\ preferred, \overline{s}_3 = equally, \overline{s}_4 = preferred, \overline{s}_5 = strongly\ preferred, \overline{s}_6 = completely\ preferred\}$.

Zadeh (Zadeh, 1975) pointed out some properties for any two linguistic terms $\overline{s}_\alpha$ and $\overline{s}_\beta$:

(1) The set is ordered: $\overline{s}_\alpha > \overline{s}_\beta$ if $\alpha > \beta$;
(2) Negation operator: $Neg(\overline{s}_\alpha) = \overline{s}_\beta, \alpha = g - \beta$($g + 1$ is the cardinality);
(3) Maximization operator: $\max(\overline{s}_\alpha, \overline{s}_\beta) = \overline{s}_\alpha$ if $\overline{s}_\alpha \geq \overline{s}_\beta$;
(4) Minimization operator: $\min(\overline{s}_\alpha, \overline{s}_\beta) = \overline{s}_\alpha$ if $\overline{s}_\alpha \leq \overline{s}_\beta$.

Herrera and Herrera-Viedma (Herrera & Herrera-Viedma, 2000) introduced the linguistic preference relation (LPR) to generalize the quantitative FPR in decision-making. The definition of an LPR is provided as follows:

Definition 2 (Herrera & Herrera-Viedma, 2000). Let $S = \{s_1, s_2, \cdots, s_m\}$ be a finite set of states or alternatives. An LPR R is a fuzzy set in S^2, characterized by a membership function, $\mu_R : S \times S \to [0, 1]$, where $\mu_R(s_i, s_j) = r_{ij}$ indicating the linguistic preference degree of state s_i over s_j. r_{ij} should satisfy the following conditions:

$r_{ij} \oplus r_{ji} = s_g$ with $\oplus$ being an operator such that $s_\alpha \oplus s_\beta = s_{\alpha+\beta}, r_{ii} = s_{g/2}$.

Based on Definition 2, a linguistic preference matrix $(R^k)_{m\times m}$ can be constructed to represent the linguistic preferences of DM k over all pairs of states, shown as:

$$(R^k)_{m\times m} = \begin{pmatrix} r^k_{11} & r^k_{12} & \cdots & r^k_{1m} \\ r^k_{21} & r^k_{22} & \cdots & r^k_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ r^k_{m1} & r^k_{m2} & \cdots & r^k_{mm} \end{pmatrix} \tag{1}$$

Next, we use Zhao et al. (Zhao et al., 2019)'s example to interpret the LPR.

Example 1 The Chinese government plans to build more paraxylene plants. A local government (LG) has two options: continue (CO) to build a paraxylene project or not (*NO*). From its own perspective, the local government prefers the option of *CO*. However, the residents think that this project involves a high risk to the environment and health. The linguistic preference for *CO* over *NO* is '*strongly preferred*'. The preference matrix can be written as.

$$R^{LG} = \begin{array}{c c} & \begin{array}{cc} CO & NO \end{array} \\ \begin{array}{c} CO \\ NO \end{array} & \begin{pmatrix} \bar{s}_3 & \bar{s}_5 \\ \bar{s} & \bar{s}_3 \end{pmatrix} \end{array}$$

In the qualitative linguistic environment, using the fuzzy linguistic approach still encounters some limitations since it is based on the elicitation of simple terms that express the information provided by DMs concerning a single linguistic variable. There is a need to provide a solution when DMs are hesitant among several linguistic terms. The concept of a hesitant fuzzy linguistic term set (HFLTS) (Rodríguez et al., 2012) was proposed to deal with such situations, and an HFLTS $H_{\overline{S}}$ was defined as an ordered finite subset of the consecutive linguistic terms of S.

The HFLTS is one way to model hesitancy. In some real-world situations, DMs may be limited in terms of having enough information, time, or a sufficient ability to cognitively process preference information. As a result, they cannot acquire a perfect solution. The utilization of HFLTSs within GMCR makes it possible to make decisions in situations involving high hesitancy.

Example 2 (continue to Example 1) After a public opinion poll, the *LG* shows a hesitancy between '*strongly preferred*' and '*preferred*', and cannot provide a precise preference. In such a case, the hesitant preferences are represented by hesitant fuzzy linguistic elements (HFLEs). The hesitant linguistic preference matrix can be written as:

$$R^{LG} = \begin{array}{c c} & \begin{array}{cc} CO & NO \end{array} \\ \begin{array}{c} CO \\ NO \end{array} & \begin{pmatrix} \bar{s}_3 & \{\bar{s}_4, \bar{s}_5\} \\ \{\bar{s}_2, \bar{s}_1\} & \bar{s}_3 \end{pmatrix} \end{array}$$

In GMCR, a DM usually has many participants. For example, the local government has many staff from different departments. When making comparisons over two states, these participants may have different opinions although they share common interests. The probabilistic linguistic term set (PLTS) (Pang et al., 2016) with percentage distribution based on the HFLTS can be used to manage this situation. A PLTS can be defined as $L(p) = \left\{L^{(l)}(p^{(l)}) | L^{(l)} \in \overline{S}, p^{(l)} \geq 0, l = 1, 2, \cdots, \#L(p)\right\}$, where $L^{(l)}(p^{(l)})$ is a linguistic term, $L^{(l)}$ is associated with a probability $p^{(l)}$ and $\#L(p)$ is the number of linguistic terms in $L(p)$.

Example 3 (continue to Example 2) Suppose that 65% of all participants in the local government think the preference degree of *CO* over *NO* is '*strongly preferred*', and 35% is '*preferred*'. Then, the linguistic preference matrix can be further represented as.

$$\delta^{LG} = \begin{array}{c} \\ CO \\ NO \end{array} \begin{array}{cc} CO & NO \\ \left(\begin{array}{c} \bar{s}_3(1) \\ \{\bar{s}_2(0.65),\ \bar{s}_1(0.35)\} \end{array} \right. & \left. \begin{array}{c} \{\bar{s}_4(0.65),\ \bar{s}_5(0.35)\} \\ s_3(1) \end{array} \right) \end{array}$$

To compare and calculate PLTSs, the score of a PLTS was defined (Pang et al., 2016):

$$E(L(p)) = \bar{s}_{\overline{\sigma}} \tag{2}$$

where $\overline{\sigma} = \sum_{l=1}^{\#L(p)} sub^{(l)} p^{(l)}$ with $sub^{(k)}$ being the subscript of the linguistic term $L^{(l)}$. For two PLTSs, if $E(L_1(p)) > E(L_2(p))$, then $L_1(p)$ is superior to $L_2(p)$, which can be denoted as $L_1(p) > L_2(p)$. Note that the calculated result of $E(L(p))$ usually cannot match the semantics in Fig. 1 since decimals may appear. These extended linguistic terms are named as virtual linguistic terms.

To facilitate the understanding of the paper, we provide some of the used notations in Table 2.

3 Linguistic preference structure in the graph model

In this section, we introduce the concepts of LRSP, LST, and LUI, and based on these we can carry out the following stability analysis.

3.1 Linguistic relative strength of preference

In a graph model, a linguistic preference over pairwise states reflects preference uncertainty using linguistic values, denoting linguistic preference degree to which a state is preferred over another. The upper bound of an LTS, $\overline{s}_g$, indicates definite preference. If $r_{ij} < \overline{s}_g$, then the DM does not definitely prefer state s_i over s_j. r_{ji} with $r_{ij} \oplus r_{ji} = s_g$ is the degree to which state s_i is not superior to s_j. Next, we define the intensity of preference with a probability for a state.

Definition 3 Let $L^{(k)}(p)(s_i, s_j) = L_{ij}^{(k)}(p)$ be the preference degree of state s_i over state s_j for DM k. Then, the LRSP $\delta^k(s_i, s_j)$ of state s_i over to state s_j for DM k is.

$$\delta^k(s_i,\ s_j) = L^{(k)}(p)(s_i,\ s_j) \Theta L^{(k)}(p)(s_j,\ s_i) \tag{3}$$

where $L^{(k)}(p)(s_i, s_j) \Theta L^{(k)}(p)(s_j, s_i) = (\overline{\sigma}_i^{(k)} - \overline{\sigma}_j^{(k)})/g$. If a preference value does not have probabilities, then we regard its probability as 1.

Note that for all $i,\ j = 1,\ 2, \cdots, m$ and $k \in N$, we have $-1 \leq \delta^k(s_i, s_j) \leq 1$. We can also obtain the following properties based on Eq. (3):

(1) $\delta^k(s_i, s_j) = 1$ denotes that state s_i is completely superior to state s_j;

Table 2 List of notations

Notations	Meanings
$N = \{1, 2, \cdots, n\}$	The set of DMs
$S = \{s_1, s_2, \cdots, s_m\}$	The set of feasible states
$G_k = (S, \{A_k\}_{k \in N})$	Directed graph for DM k
$\overline{S}$	Linguistic term set
$\overline{s}_\alpha$	Linguistic term
$(R^k)_{m \times m}$	Linguistic preference matrix
$L(p)$	Probabilistic linguistic term set
$E(L(p))$	Score of probabilistic linguistic term set
$L_{ij}^{(k)}(p)$	Preference degree of state s_i over state s_j
$\delta^k(s_i, s_j)$	Linguistic relative strength of preference of state s_i over state s_j
$(\delta^k)_{m \times m}$	Linguistic relative strength of preference matrix
λ_k	Linguistic satisficing threshold
$R_k(s)$	Set of states reachable from state s for DM k
$\tilde{R}^+_{k,\lambda^k}(s)$ or $\tilde{R}^+_k(s)$	Linguistic unilateral improvement list for DM k
Ψ	Coalition containing a set of DMs
$\Omega_\Psi(s, s_1)$	Set of all last DMs in legal sequences of unilateral moves from state s to s_1
$\tilde{R}^+_{\Psi,\lambda_\Psi}(s)$ or $\tilde{R}^+_\Psi(s)$	Linguistic unilateral improvement list of the coalition Ψ
$\tilde{\Omega}^+_{\Psi,\lambda_\Psi}(s, s_1)$	Set of all last DMs in the legal sequences allowable for implementing a unilateral improvement from state s to s_1
$R_{N-k}(s)$	Reachable list of the coalition comprising DM k's opponents
$\tilde{R}^+_{N-k}(s)$	Linguistic unilateral improvement list of the coalition comprising DM k's opponents

(2) $\delta^k(s_i, s_j) = 0$ denotes that state s_i is equal to state s_j;
(3) $\delta^k(s_i, s_j) = -1$ denotes that state s_j is completely superior to state s_i.

The linguistic relative preferences over all pairwise of states for DM k can be represented by a matrix $(\delta^k)_{m \times m}$:

$$(\delta^k)_{m \times m} = \begin{pmatrix} \delta^k_{11} & \delta^k_{12} & \cdots & \delta^k_{1m} \\ \delta^k_{21} & \delta^k_{22} & \cdots & \delta^k_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \delta^k_{m1} & \delta^k_{m2} & \cdots & \delta^k_{mm} \end{pmatrix} \tag{4}$$

For the reason that $\delta^k(s_i, s_j) = L^{(k)}(p)(s_i, s_j) \Theta L^{(k)}(p)(s_j, s_i) = -\big(L^{(k)}(p)(s_j, s_i) \Theta L^{(k)}(p)(s_i, s_j)\big) = -\delta^k(s_i, s_j)$, the relative matrix shown as Eq. (4) keeps symmetry.

Example 4 (continue to Example 3) The LRSP of the local government can be represented as:

$$\delta^{LG} = \begin{array}{c} \\ CO \\ NO \end{array} \begin{array}{c} \begin{array}{cc} CO & NO \end{array} \\ \begin{pmatrix} 0 & 0.45 \\ -0.45 & 0 \end{pmatrix} \end{array}$$

Based on the transformation function from LPR to LRSP, we can identify the relative preferences over states intuitively. According to Eq. (4), we can find that a positive value of $\delta_{ij}^{(k)}$ indicates that state s_i is preferred to state s_j while a negative value of $\delta_{ij}^{(k)}$ indicates that state s_j is preferred to state s_i.

3.2 Linguistic satisficing threshold

To analyze a graph model, one crucial thing is to identify that a DM is inclined to move from one state or stay at their current state (Li et al., 2019; Zhao et al., 2019). To learn whether or not a state is worthwhile for a DM to move, an appropriate threshold or criterion is required. Next, we provide the definition of LST.

Definition 4 If DM k would be willing to move from state s to state s_i regarding $\delta^k(s_i, s) \geq \lambda_k$, then λ_k is the LST of DM k.

Note that $\lambda_k \in (0, 1]$. A DM's LST describes the level of LRSP required to find a state that is advantageous in comparison with the current state. A DM only moves from the current state if $\delta^k(s_i, s) \geq \lambda_k$ is satisfied. According to Definition 3, it is necessary to set a reasonable LST. The LST should be positive and does not exceed 1.

3.3 Linguistic unilateral improvement

Based on the concepts of LPSR and LST, we need to determine whether a state is worthwhile for a DM to move from the current state or not.

Definition 5 Let $R_k(s)$ be the set of states reachable from state s for DM k and λ_k be the LST of DM k. A state s_i is an LUI from s for DM k regarding λ_k if and only if (iff) $\delta^k(s_i, s) \geq \lambda^k$. The set of all LUIs from state s for DM k is called the linguistic unilateral improvement list (LUIL), which can be mathematically represented as:

$$\tilde{R}^+_{k,\lambda^k}(s) = \left\{ s_i \in R_k(s) : \delta^k(s_i, s) \geq \lambda^k \right\} \quad (5)$$

For simplicity, $\tilde{R}^+_{k,\lambda_k}(s)$ is also written as $\tilde{R}^+_k(s)$.

4 Stability definition in the GMCR with linguistic preferences

This section introduces four stability definitions for two-DM and n-DM graph models. Subsequently, we investigate the interrelationships among these four definitions.

4.1 Two-DM conflict model

A possible resolution for a strategic conflict means that no DM chooses to deviate from a stable state. The stability analysis in a GMCR is used to determine the states that are stable for DMs. In a graph model, an LUI for a DM is a state to which the DM wants to move. The DM who has the right to move is named a focal DM. However, in applications, the focal DM may not choose the strongly preferred state since sanctions may be imposed by other DMs. As mentioned in the Introduction, four basic stability definitions in a strategic conflict could be given. In this section, we introduce the linguistic Nash stability (LNash), linguistic general metarationality (LGMR), linguistic symmetric metarationality (LSMR), and linguistic sequential stability (LSEQ) for the 2-DM graph model. We set $N = \{p, q\}$. Then, the LSTs and LUIs of these two DMs are denoted as, λ_p and λ_q, and $\tilde{R}_p^+(s)$ and $\tilde{R}_q^+(s)$, respectively.

Definition 6 (LNash) For DM p, a state $s \in S$ is LNash stable iff $\tilde{R}_p^+(s) = \emptyset$, and can be denoted by $s \in S_p^{LNash}$.

Under an LNash, the focal DM will move to a more preferred state only considering their LUIs without taking into account possible responses of other DMs. In other words, state s is an LNash for the focal DM p iff they have no LUI from s according to the satisficing criterion.

Definition 7 (LGMR) For DM p, a state $s \in S$ is LGMR iff there is at least one $s_2 \in R_q(s_1)$ for each $s_1 \in \tilde{R}_p^+(s)$, such that $\delta^p(s_2, s) < \lambda_p$, and can be denoted by $s \in S_p^{LGMR}$.

For an LGMR, the focal DM should not only consider their possible LUIs but need to ask whether the LUIs could be sanctioned subsequently by the opponent q by means of one unilateral move. If DM p decides to move from s to s_1 according to an LUI, the DM q has at least one unilateral movement from s_1 to s_2, which is less preferred by p in comparison with s, then p will stay at s. If DM p has no LUI from state s, then the DM p will stay at s automatically. Therefore, all LNash stable states are LGMR states. One thing to note is that DM q does not consider whether the sanctioning strategy that they adopted is advantageous for them or not.

Definition 8 (LSMR) For DM p, a state $s \in S$ is LSMR iff there is at least one $s_2 \in R_q(s_1)$ for each $s_1 \in \tilde{R}_p^+(s)$, such that $\delta^p(s_2, s) < \lambda_p$ and $\delta^p(s_3, s) < \lambda_p$ for all $s_3 \in R_p(s_2)$, and can be denoted by $s \in S_p^{LSMR}$.

For an LSMR, it is necessary for the focal DM to consider one more step compared with the LGMR stability. If DM q has subsequent unilateral movement to sanction any LUI of DM p from state s, and DM p cannot get away with this sanction, then the state s is an LSMR for DM p. In other words, any LUI s_1 from s of the DM p will be brought to the state s_2 by the DM q, and this state s_2 is not advantageous to move from s for the DM p according to their satisficing criterion. In such case, the DM p will stay at s. One thing to note is that the LSMR adds a restricted consideration to the LGMR. Hence, all LSMR stable states are LGMR states.

Definition 9 (LSEQ) For DM p, a state $s \in S$ is LSEQ iff there is at least one $s_2 \in \tilde{R}_q^+(s_1)$ for each $s_1 \in \tilde{R}_p^+(s)$ such that $\delta^p(s_2, s) < \lambda_p$, and can be denoted by $s \in S_p^{LSEQ}$.

LSEQ is similar to LGMR. The focal DM needs to consider his/her/its possible LUIs and subsequent LUIs of their opponent. Furthermore, DM p cannot escape the sanction. The difference between LSEQ and LGMR is that LSEQ requires the sanction made by DM p's opponent to be credible. Therefore, the concept of LSEQ depends on both the focal DM p's LST λ_p and the opponent's LST λ_q.

If state $s \in S$ is stable for all DMs under a linguistic stability definition, then this stability definition is called a linguistic equilibrium.

4.2 *n*-DM conflict model

In Sect. 4.1, we analyzed four stability definitions for the two-DM conflict model. However, if a graph model has more than two DMs, the opponent of a focal DM will be a coalition. The reachable list that relies on the joint unilateral moves for many DMs is more complex. Therefore, it is necessary to define LUIs by a coalition and carry out the stability analysis for the n-DM conflict model.

Suppose that the coalition containing a set of DMs is denoted by Ψ with $\Psi \subseteq N$ and $|\Psi| \geq 2$. $R_\Psi(s) \subseteq S$ is the set of states that are reachable from state s through a legal sequence of moves by some DMs in Ψ. Note that a legal sequence of moves means no DM in Ψ moves twice consecutively. Let $\Omega_\Psi(s, s_1)$ be the set of all last DMs in legal sequences of unilateral moves from state s to s_1. The concept of a coalition's reachable list is given below.

Definition 10 (Reachable list for a coalition) In Definition 10, as long as there is no new state which can be added to $R_\Psi(s)$, and $\left|\Omega_\psi(s, s_1)\right|$ cannot be increased for any $s_1 \in R_\Psi(s)$, then the introduction stops.

(1) If $k \in \Psi$ and $s_1 \in R_k(s)$, then $s_1 \in R_\Psi(s)$ and $k \in \Omega_\Psi(s, s_1)$;
(2) If $s_1 \in R_\Psi(s)$, $s_2 \in R_k(s_1)$, $k \in \Psi$ and $\Omega_\Psi(s, s_1) \neq \{k\}$, then $s_2 \in R_\Psi(s)$ and $k \in \Omega_\Psi(s, s_2)$.

Definition 11 (LUI by a coalition) Let $s \in S$ and $\Psi \subseteq N(|\Psi| \geq 2)$. The subset $R_\Psi(s) \subseteq S$ can be defined as:

(1) If $k \in \Psi$ and $s_1 \in \tilde{R}_k^+(s)$, then $s_1 \in \tilde{R}_{\Psi,\lambda_\Psi}^+(s)$ and $k \in \tilde{\Omega}_{\Psi,\lambda_\Psi}^+(s, s_1)$, where $\tilde{\Omega}_{\Psi,\lambda_\Psi}^+(s, s_1)$ is the set of all last DMs in the legal sequences allowable for implementing a unilateral improvement from state s to s_1;
(2) If $s_1 \in \tilde{R}_{\Psi,\lambda_\Psi}^+(s)$, $s_2 \in \tilde{R}_k^+(s_1)$, $k \in \Psi$ and $\tilde{\Omega}_{\Psi,\lambda_\Psi}^+(s, s_1) \neq \{k\}$, then $s_2 \in \tilde{R}_{\Psi,\lambda_\Psi}^+(s)$ and $k \in \tilde{\Omega}_{\Psi,\lambda_\Psi}^+(s, s_2)$.

Table 3 Linguistic stability definitions in a graph model

Stability definitions	Foresight	Sanction	Strategic risk
LNash	1	No sanction	Ignore
LGMR	2	Unilateral moves	Avoid
LSMR	3	Unilateral moves	Avoid
LSEQ	2	Unilateral improvements	Take risks

Any member of $\tilde{R}^{+}_{\Psi,\lambda_\Psi}(s)$ is an LUI from state s by the coalition Ψ. In Definition 11, as long as there is no new state that can be added to $\tilde{R}^{+}_{\Psi,\lambda_\Psi}(s)$, and $\left|\tilde{\Omega}^{+}_{\Psi,\lambda_\Psi}(s, s_1)\right|$ cannot be increased for any $s_1 \in \tilde{R}^{+}_{\Psi,\lambda_\Psi}(s)$, the introduction stops. For convenience, $\tilde{R}^{+}_{\Psi,\lambda_\Psi}(s)$ is also written as $\tilde{R}^{+}_{\Psi}(s)$.

Next, we introduce linguistic stability definitions for n-DM ($n > 2$) graph model. We should note that the Nash stability cannot be influenced by the responses of opponents. Therefore, the definition of LNash does not change. We present the last three linguistic stability definitions, namely, LGMR, LSMR and LSEQ.

Let $N - k$ be the coalition comprising DM k's opponents. Hence, the reachable list of the coalition and LUIs can be denoted by $R_{N-k}(s)$ and $\tilde{R}^{+}_{N-k}(s)$, respectively.

Definition 12 (LGMR) For DM k, a state $s \in S$ is called an LGMR iff for each $s_1 \in \tilde{R}^{+}_{k}(s)$, there is at least one $s_2 \in R_{N-k}(s_1)$, such that $\delta^k(s_2, s) < \lambda_k$, and can be denoted by $s \in S_k^{LGMR}$.

Definition 13 (LSMR) For DM k, a state $s \in S$ is called an LSMR iff for each $s_1 \in \tilde{R}^{+}_{k}(s)$, there is $s_2 \in R_{N-k}(s_1)$, such that $\delta^k(s_2, s) < \lambda_k$ and $\delta^k(s_3, s) < \lambda_k$ for all $s_3 \in R_k(s_2)$, and can be denoted by $s \in S_k^{LSMR}$.

Definition 14 (LSEQ) For DM k, a state $s \in S$ is called an LSEQ iff for each $s_1 \in \tilde{R}^{+}_{k}(s)$, there is $s_2 \in \tilde{R}^{+}_{N-k}(s_1)$ such that $\delta^k(s_2, s) < \lambda_k$, and can be denoted by $s \in S_k^{LSEQ}$.

Inspired by Kuang et al. (Kuang et al., 2015), a summary of our proposed four linguistic stability definitions is presented in Table 3. These definitions can characterize different reactions of DMs when they respond to potential risks. If a DM has no far-sightedness and considers only rewards and ignores risks, then they will follow LNash. The last three definitions have foresights for a DM who will consider opponents' countermoves. If a DM follows LSMR or LGMR, then they can accept sanctions from the opponents at all costs. Therefore, the DM tends to avoid risks. Conversely, if a DM follows LSEQ, they think that they will take sanctions from the opponent(s) without losing their own benefits. Hence, he/she/it will take some risks (Kuang et al., 2015).

4.3 Interrelationships among stability definitions

In Sect. 4.1, we provided some implicatory links between the stability definitions. For instance, the LNash implies the LGMR, and the LSMR implies the LGMR. Kilgour, Hipel, and Fang (Kilgour et al., 1987) investigated the interrelationships among four basic stability definitions. In this section, we study the interrelationships among these four stability definitions with linguistic preferences based on Kilgour, Hipel, and Fang (Kilgour et al., 1987)'s results.

First, we consider the two-DM graph model.

Theorem 1 For $p \in N$, $S_p^{LNash} \subseteq S_p^{LSMR} \subseteq S_p^{LGMR}$.

Proof If $s \in S_p^{LNash}$, then $\tilde{R}_p^+(s) = \emptyset$. For Definitions 7 and 8, if $\tilde{R}_p^+(s) = \emptyset$, then, there is no $s_1 \in \tilde{R}_p^+(s)$. Thus, we have $s \in S_p^{LGMR}$ and $s \in S_p^{LSMR}$. This implies that $S_p^{LNash} \subseteq S_p^{LGMR}$ and $S_p^{LNash} \subseteq S_p^{LSMR}$.

Next, we prove $S_p^{LSMR} \subseteq S_p^{LGMR}$.

If $s \in S_p^{LSMR}$, there are two cases: (1) If $\tilde{R}_p^+(s) = \emptyset$, then $s \in S_p^{LNash}$. Furthermore, $S_p^{LNash} \subseteq S_p^{LGMR}$. Thus, we have $S_p^{LSMR} \subseteq S_p^{LGMR}$; (2) If $\tilde{R}_p^+(s) \neq \emptyset$, according to Definition 8, for each $s_1 \in \tilde{R}_k^+(s)$, there exists $s_2 \in R_q(s_1)$, such that $\delta^p(s_2, s) < \lambda_p$ and $\delta^p(s_3, s) < \lambda_p$, for all $s_3 \in R_p(s_2)$. If the situation that $s_3 \in R_p(s_2)$ is not considered, then Definition 8 will be satisfied, and thus we have $S_p^{LSMR} \subseteq S_p^{LGMR}$. This completes the proof. ∎

Theorem 2 For $p \in N$, $S_p^{LNash} \subseteq S_p^{LSEQ} \subseteq S_p^{LGMR}$.

Proof If $s \in S_p^{LNash}$, then $\tilde{R}_p^+(s) = \emptyset$. For Definition 9, if $\tilde{R}_p^+(s) = \emptyset$, then, there is no $s_1 \in \tilde{R}_p^+(s)$. Thus, we have $s \in S_p^{LSEQ}$. This implies that $S_p^{LNash} \subseteq S_p^{LSEQ}$.

Next, we prove $S_p^{LSEQ} \subseteq S_p^{LGMR}$.

If $s \in S_p^{LSEQ}$, there are two cases: (1) If $\tilde{R}_p^+(s) = \emptyset$, then $s \in S_p^{LNash}$. Furthermore, $S_p^{LNash} \subseteq S_p^{LGMR}$. Thus, we have $S_p^{LSEQ} \subseteq S_p^{LGMR}$; (2) If $\tilde{R}_p^+(s) \neq \emptyset$, according to Definition 9, for each $s_1 \in \tilde{R}_k^+(s)$, there exists $s_2 \in \tilde{R}_q^+(s_1)$, such that $\delta^p(s_2, s) < \lambda_p$. Since $s_2 \in \tilde{R}_q^+(s_1) \subseteq R_q(s_1)$, then Definition 7 is satisfied. Thus, we have $S_p^{LSEQ} \subseteq S_p^{LGMR}$. This completes the proof. ∎

The proof for n-DM graph model is similar to the proof for two-DM graph model. The difference is that the subsequent unilateral movement $s_2 \in R_k(s_1)$ for $s \in S_k^{LSMR}$ and $s \in S_k^{LSMR}$, and LUI $s_2 \in \tilde{R}_k^+(s_1)$ for $s \in S_k^{LSEQ}$, should be replaced by $s_2 \in R_{N-k}(s_1)$ and $s_2 \in \tilde{R}_{N-k}^+(s_1)$, respectively.

The two theorems above provide the logic interrelationships among four stability definitions. The Venn diagram is presented in Fig. 2. The LNash covers the minimum range. Therefore, it is the strongest stability. The LGMR and LSEQ do not have a clear inclusion relationship.

4.4 Decision-making procedure

Step 1. Based on the above analyses, we provide the following decision-making procedure for the ease of understanding.

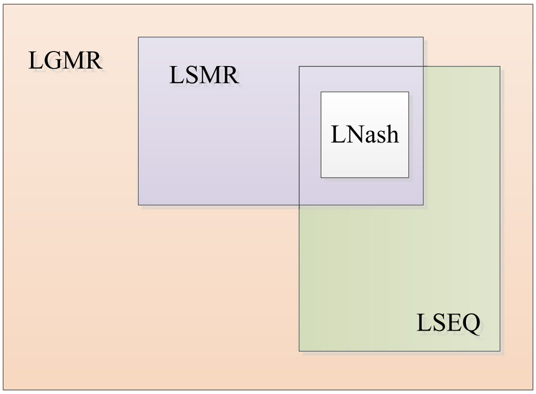

Fig. 2 Interrelationships among four linguistic stability definitions

Step 2. Identify key DMs and their corresponding options;
Step 3. Identify feasible states and draw the integrated graph of the conflict;
Step 4. Provide linguistic preferences of DMs;
Step 5. Calculate LRSPs of DMs based on Eq. (3);
Step 6. Carry out linguistic stability analysis;
Step 7. Identify the optimal solution of the conflict.

5 An application case

In this section, we use a real-world conflict regarding the ban on Huawei editors and reviewers by IEEE to illustrate the applicability of the four proposed linguistic stability definitions.

5.1 Background description

The interdependence among countries is strengthening in international economic exchanges under the background of globalization. However, the wave of modern globalization has narrowed comparative advantage and intensified competition. As a result, trade frictions happen frequently between countries. Furthermore, all countries will attach importance to their own benefit distribution. However, the economic situation and national conditions of different countries vary greatly, hence the disputes in the process of business transaction. In the filed of international trade, trade frictions have a long history. Early well-known trade wars include Franco-Italian, Anglo-Hanse, and Hawley-Smoot conflicts. After 1990s, new trade protectionism arose. After entering the twenty-first century, trade protectionism has been developing in some countries. Under this background, trade frictions are becoming increasingly frequent and their main scopes are expanding. Trade frictions between developed and developing countries are noticeable, especially the China-US trade conflict.

Since the formal establishment of diplomatic relations between China and the USA in 1979, bilateral trade has developed rapidly. According to statistics data from the

Ministry of Commerce of China, the two-way trade between China and the USA increased from \$ 2.5 billion in 1979 to \$ 633.52 billion in 2018. China has become the largest trading partner of the USA. However, due to the imbalance in bilateral trade frictions have worsened, the USA would like to use its huge deficits to gain more economic and political interests.

On May 16, 2019, the Bureau of Industry and Security (BIS) of the U.S. Department of Commerce added Huawei Technologies Co. Ltd. to the Bureau's Entity List (https://www.commerce.gov/news/press-releases/2019/05/department-commerce-announces-addition-huawei-technologies-co-ltd). Huawei and its 68 affiliates were added into the entity list by the Export Administration Regulations (EAR) according to Supplement NO. 4 to EAR Part 744. Subsequently, Google and some Chip designers and suppliers such as Intel, ARM (Advanced RISC Machine), and Qualcomm announced the termination of cooperation with Huawei. Some industrial technology alliances have also cancelled Huawei's membership, including SDA (Secure Digital Card Association) and PCIe (Peripheral Component Interconnect express). On May 30, 2019, IEEE also claimed that it will not use colleagues including editors and reviewers from Huawei for the peer-review process according to the FAQ document. IEEE is the most famous and largest non-profit transnational academic organization in the field of electronic, electrical, computer, communication and automation engineering technology. Soon later, many societies from China such as CIC (China Institute of Communication) and CIE (Chinese Institute of Electronics) and some scholars from all over the world opposed the announcement and thought that this ban will have a negative effect on the normal orders of academic exchange, academic independence, and scientific community values.

Based on the background description above, in the following subsection, we use our proposed linguistic GMCR to model and solve this conflict.

5.2 Preference modeling

Step 1. Identify key DMs and their corresponding options.

This conflict involves three DMs, which are summarized as follows:

(1) The US Department of Commerce's Bureau of Industry and Security (BIS). BIS has three options. It can continue its prohibition on Huawei. However, in such case, some American manufacturers and enterprises such as Flex and NeoPhotonics will lose a lot of economic interests. A considerable portion of income of these manufactures and enterprises comes from Huawei. BIS can also adjust the ban and allow American enterprises to continue the commercial dealings with Huawei. Note that in such case, Huawei is still in the entity list. Furthermore, the content of transactions between American companies and Huawei needs to be audited by BIS. The third option is to cancel all restrictions on Huawei directly.

(2) IEEE. IEEE's option is to lift the ban on Huawei editors and reviewers. IEEE thinks that if it continues to use Huawei editors and reviewers, severe legal implications will arise. Then, there will be a dispute between IEEE and BIS. However, the opposition of academic circles also cause great pressure.

(3) Academic organizations and individuals (AOI), consisting of Chinese societies and scholars who have academic posts in IEEE as well as some international academic organizations. The option of AOI is to oppose the ban.

Step 2. Identify feasible states and draw the integrated graph of the conflict;

There are five options in this conflict. These five options are combined to form $2^5 = 32$ possible states. However, some of these options are infeasible because of mutual exclusiveness. For instance, BIS will not adjust the prohibition and continue it at the same time. Consequently, only 7 feasible states are identified and listed in Table 4.

Figure 3 presents the integrated graph of the conflict. In Fig. 3, a node represents a feasible state; an arc with one or two arrowheads denotes an allowable moving direction; the labels on the arcs are the DMs. We can see that there are both bidirectional and unidirectional moves. For instance, BIS would not withdraw its decision once it decides to cancel the ban. Therefore, the moves from s_1, s_2, s_3, s_4, s_5, and s_6 to s_7 are unidirectional.

Step 3. Provide linguistic preferences of DMs;

DMs' linguistic preferences are presented in Table 5. In the linguistic preference

Table 4 Options and feasible states in the conflict

BIS							
1.Continue	Y	Y	Y	N	N	N	–
2.Adjust	N	N	N	Y	Y	Y	–
3.Cancel	N	N	N	N	N	N	Y
IEEE							
4.Lift	N	N	Y	N	N	Y	–
AOI							
5.Oppose	Y	N	N	Y	N	N	–
States	s_1	s_2	s_3	s_4	s_5	s_6	s_7

In Table 3, "Y" means that the option is selected by the DM and "N" means the option is not chosen

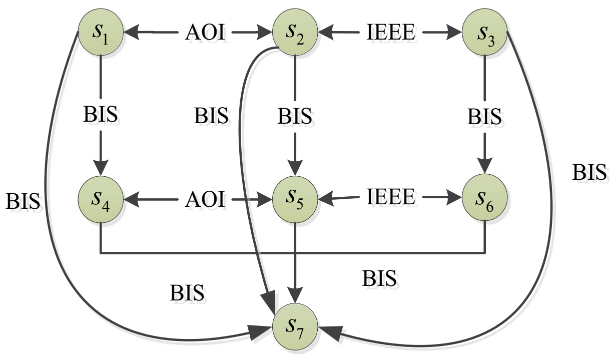

Fig. 3 An integrated graph model for the Huawei conflict

Table 5 Linguistic preferences of three DMs

$$
R^{BIS} = \begin{array}{c c}
 & \begin{array}{ccccccc} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 \end{array} \\
\begin{array}{c} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \end{array} &
\begin{bmatrix}
\bar{s}_3 & \bar{s}_0 & \bar{s}_0 & \bar{s}_6 & \bar{s}_0 & \bar{s}_0 & \bar{s}_6 \\
\bar{s}_6 & \bar{s}_3 & \bar{s}_0 & \bar{s}_6 & \bar{s}_0 & \bar{s}_0 & \bar{s}_6 \\
\bar{s}_6 & \bar{s}_6 & \bar{s}_3 & \bar{s}_6 & \bar{s}_0 & \bar{s}_0 & \bar{s}_6 \\
\bar{s}_0 & \bar{s}_0 & \bar{s}_0 & \bar{s}_3 & \bar{s}_0 & \bar{s}_0 & \bar{s}_1 \\
\bar{s}_6 & \bar{s}_6 & \bar{s}_6 & \bar{s}_6 & \bar{s}_3 & \bar{s}_0 & \bar{s}_6 \\
\bar{s}_6 & \bar{s}_6 & \bar{s}_6 & \bar{s}_6 & \bar{s}_6 & \bar{s}_3 & \bar{s}_6 \\
\bar{s}_0 & \bar{s}_0 & \bar{s}_0 & \bar{s}_5 & \bar{s}_0 & \bar{s}_0 & \bar{s}_3
\end{bmatrix}
\end{array}
$$

$$
R^{IEEE} = \begin{array}{c c}
 & \begin{array}{ccccccc} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 \end{array} \\
\begin{array}{c} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \end{array} &
\begin{bmatrix}
\bar{s}_3 & \bar{s}_0 & \bar{s}_0 & \bar{s}_6 & \bar{s}_0 & \bar{s}_0 & \bar{s}_5 \\
\bar{s}_6 & \bar{s}_3 & \bar{s}_5 & \bar{s}_6 & \bar{s}_6 & \{\bar{s}_0, \bar{s}_1, \bar{s}_2\} & \bar{s}_6 \\
\bar{s}_6 & \bar{s}_1 & \bar{s}_3 & \bar{s}_6 & \{\bar{s}_5, \bar{s}_6\} & \bar{s}_0 & \bar{s}_6 \\
\bar{s}_0 & \bar{s}_0 & \bar{s}_0 & \bar{s}_3 & \bar{s}_0 & \bar{s}_0 & \{\bar{s}_0, \bar{s}_1\} \\
\bar{s}_6 & \bar{s}_0 & \{\bar{s}_1, \bar{s}_0\} & \bar{s}_6 & \bar{s}_3 & \bar{s}_0 & \bar{s}_6 \\
\bar{s}_6 & \{\bar{s}_6, \bar{s}_5, \bar{s}_4\} & \bar{s}_6 & \bar{s}_6 & \bar{s}_6 & \bar{s}_0 & \bar{s}_6 \\
\bar{s}_1 & \bar{s}_0 & \bar{s}_0 & \{\bar{s}_6, \bar{s}_5\} & \bar{s}_0 & \bar{s}_0 & \bar{s}_3
\end{bmatrix}
\end{array}
$$

$$
R^{AOI} = \begin{array}{c c}
 & \begin{array}{ccccccc} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 \end{array} \\
\begin{array}{c} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \end{array} &
\begin{bmatrix}
\bar{s}_3(1) & \bar{s}_6(1) & \{\bar{s}_5(0.8), \bar{s}_6(0.2)\} & \bar{s}_6(1) & \bar{s}_6(1) & \bar{s}_0(1) & \bar{s}_0(1) \\
\bar{s}_0(1) & \bar{s}_3(1) & \bar{s}_0(1) & \bar{s}_0(1) & \bar{s}_6(1) & \bar{s}_0(1) & \bar{s}_0(1) \\
\{\bar{s}_1(0.8), \bar{s}_0(0.2)\} & \bar{s}_6(1) & \bar{s}_3(1) & \bar{s}_6(1) & \bar{s}_6(1) & \{\bar{s}_5(0.1), \bar{s}_6(0.9)\} & \{\bar{s}_2(0.5), \bar{s}_3(0.5)\} \\
\bar{s}_0(1) & \bar{s}_6(1) & \bar{s}_0(1) & \bar{s}_3(1) & \bar{s}_6(1) & \bar{s}_0(1) & \bar{s}_0(1) \\
\bar{s}_0(1) & \bar{s}_6(1) & \bar{s}_0(1) & \bar{s}_0(1) & \bar{s}_3(1) & \bar{s}_0(1) & \bar{s}_0(1) \\
\bar{s}_6(1) & \bar{s}_6(1) & \{\bar{s}_1(0.1), \bar{s}_0(0.9)\} & \bar{s}_6(1) & \bar{s}_6(1) & \bar{s}_3(1) & \bar{s}_0(1) \\
\bar{s}_6(1) & \bar{s}_6(1) & \{\bar{s}_4(0.5), \bar{s}_3(0.5)\} & \bar{s}_6(1) & \bar{s}_6(1) & \bar{s}_6(1) & \bar{s}_3(1)
\end{bmatrix}
\end{array}
$$

of IEEE, possible hesitancies are considered. For instance, the preference of state s_2 to state s_6 shows a high degree of hesitation. In these two states, IEEE adopts the same strategy with BIS. IEEE would prefer not to be in a conflict with BIS. Furthermore, s_6 is relatively preferred to s_2 in R^{IEEE} since lifting the ban is beneficial to academic exchanges and reputation. Because AOI contains a large number of societies and scholars, there will be some inconsistent assessments. Therefore, the linguistic preference of AOI is provided based on PLTSs. For instance, the preference of state s_1 to s_3 is $\{\bar{s}_5(0.8), \bar{s}_6(0.2)\}$, which denotes that 80% of participants of AOI think that the preference degree should be $\bar{s}_5$ and 20% think that the preference degree should be $\bar{s}_6$.

5.3 Stability analysis

Step 4. Calculate LRSPs of DMs based on Eq. (3);

Based on the linguistic preferences of three DMs, we can calculate their LRSPs by Eq. (3), which are shown in Table 6.

Step 5. Carry out linguistic stability analysis;

Next, to carry out the linguistic stability analysis, we need to apply the solution concepts for n-DM graph model introduced in this study to identify the states that have high degree of stability. Table 7 presents the results of the linguistic stability analysis. To analyze the influence of the LST, four sets of LSTs are provided. These four sets are: 1) $\lambda_{BIS} = 1$, $\lambda_{IEEE} = 0.4$, $\lambda_{AOI} = 0.1$; 2) $\lambda_{BIS} = 1$, $\lambda_{IEEE} = 0.4$, $\lambda_{AOI} = 0.3$; 3) $\lambda_{BIS} = 1$, $\lambda_{IEEE} = 0.7$, $\lambda_{AOI} = 0.1$; 4) $\lambda_{BIS} = 1$, $\lambda_{IEEE} = 0.7$, $\lambda_{AOI} = 0.3$.

According to Table 7, states s_6 and s_7 have a high degree of stability. Under four linguistic stability definitions of LNash, LGMR, LSMR, and LSEQ, for each set of LTSs, they are stable. The number of stable states increases as the value of the IEEE's LST increases from 0.4 to 0.7, and as the value of the BIS's LST increases from 0.6

Table 6 LRSPs of three DMs

$$\delta^{BIS} = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 \end{array} \\ \begin{bmatrix} 0 & -1 & -1 & 1 & -1 & -1 & 1 \\ 1 & 0 & -1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 0 & 1 & -1 & -1 & 1 \\ -1 & -1 & -1 & 0 & -1 & -1 & -0.67 \\ 1 & 1 & 1 & 1 & 0 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ -1 & -1 & -1 & 0.67 & -1 & -1 & 0 \end{bmatrix} \end{array}$$

$$\delta^{IEEE} = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 \end{array} \\ \begin{bmatrix} 0 & -1 & -1 & 1 & -1 & -1 & 0.67 \\ 1 & 0 & 0.67 & 1 & -1 & -1 & 1 \\ 1 & -0.67 & 0 & 1 & 0.83 & -1 & 1 \\ -1 & -1 & -1 & 0 & -1 & -1 & -0.83 \\ 1 & -1 & -0.83 & 1 & 0 & -1 & 1 \\ 1 & 0.67 & 1 & 1 & 1 & 0 & 1 \\ -0.67 & -1 & -1 & 0.83 & -1 & -1 & 0 \end{bmatrix} \end{array}$$

$$\delta^{AOI} = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 \end{array} \\ \begin{bmatrix} 0 & 1 & 0.73 & 1 & 1 & -1 & -1 \\ 1 & 0 & -1 & -1 & 1 & -1 & -1 \\ -0.73 & 1 & 0 & 1 & 1 & 0.97 & -1 \\ -1 & 1 & -1 & 0 & 1 & -1 & -0.17 \\ -1 & -1 & -1 & -1 & 0 & -1 & -1 \\ 1 & 1 & -0.97 & 1 & 1 & 0 & -1 \\ 1 & 1 & 0.17 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Table 7 Linguistic stability results of the conflict

LSTs	States	LNash				LGMR				LSMR				LSEQ			
		BIS	IEEE	AOI	LE	BIS	IEEE	AOI	LE	BIS	IEEE	AOI	LE	BIS	IEEE	AOI	LE
$\lambda_{BIS} = 1$	s_1		√	√			√	√			√	√			√	√	
$\lambda_{IEEE} =$ 0.4	s_2		√			√	√			√	√				√		
$\lambda_{AOI} = 0.1$	s_3			√			√	√				√			√	√	
	s_4	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
	s_5	√				√	√			√	√			√	√		
	s_6	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
	s_7	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
$\lambda_{BIS} = 1 = 1$	s_1		√	√			√	√			√	√			√	√	
$\lambda_{IEEE} =$ 0.4	s_2		√			√	√			√	√				√		
$\lambda_{AOI} = 0.3$	s_3			√			√	√				√			√	√	
	s_4	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
	s_5	√				√	√			√	√			√	√		
	s_6	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
	s_7	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
$\lambda_{BIS} = 1$	s_1		√	√			√	√			√	√			√	√	
$\lambda_{IEEE} =$ 0.7	s_2		√			√	√			√	√				√		
$\lambda_{AOI} = 0.1$	s_3		√	√			√	√			√	√			√	√	
	s_4		√	√			√	√			√	√			√	√	
	s_5	√				√	√			√	√			√	√		
	s_6	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
	s_7	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√

Table 7 (continued)

LSTs	States	LNash				LGMR				LSMR				LSEQ			
		BIS	IEEE	AOI	LE	BIS	IEEE	AOI	LE	BIS	IEEE	AOI	LE	BIS	IEEE	AOI	LE
$\lambda_{BIS} = 1$	s_1		√	√			√	√			√	√			√	√	
$\lambda_{IEEE} = 0.7$	s_2		√			√	√			√	√				√		
$\lambda_{AOI} = 0.3$	s_3		√	√			√	√			√	√			√	√	
	s_4		√	√			√	√			√	√			√	√	
	s_5	√				√	√			√	√			√	√		
	s_6	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
	s_7	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√

LE: linguistic equilibrium. In Table 7, "√" indicates that the state is stable for the DM, or a linguistic equilibrium under the indicated linguistic stability definition to 1. A higher value of LST indicates that the DM is more conservative since they only would move to a state when a high threshold is satisfied. On the contrary, a lower value of LST denotes that the DM is more aggressive.

In state s_7, BIS cancels the ban on Huawei thoroughly. Therefore, other two DMs do not need to choose options. However, this state is one of the least preferred state for BIS and IEEE. The USA and China have many trade disputes for a long time. BIS also holds that Huawei's activities have influence on the national security. Moreover, as shown in Fig. 3, the movement of other states to state s_7 is controlled by BIS on its own. Thus, state s_7 seems unlikely to happen in real-world situations.

As the value of the BIS's LST increases from 0.6 to 1, state s_4 is added to the equilibrium list. In state s_4, BIS decides to adjust the prohibition while IEEE does not lift the ban. However, s_4 needs strong condition and is also one of the least preferred state for BIS and IEEE. Hence, it seems unlikely that it will be the end result. State s_5 is similar to state s_4. The difference is that AOI does not oppose the prohibition of IEEE in state s_5. Other states like s_1, s_2, and s_3 are not in the equilibrium list.

Step 6. Identify the optimal solution of the conflict.

Based on the above discussions, state s_6, where BIS adjusts the ban, IEEE lifts the prohibition, and AOI does not continue to oppose, is the recommended and reasonable resolution. In this state, American enterprises and suppliers can cooperate with Huawei and obtain more profits. Furthermore, normal academic exchanges and cooperation will not be hindered. Actually, on June 3, 2019, IEEE lifted the ban on Huawei reviewers and editors, and on July 10, 2019, BIS also adopted the second option.

These results indicate that linguistic stability analysis can predict realistic resolutions of the conflict, something which quantitative analysis cannot well describe. Moreover, four extensions of stability definitions represent four different ways that consider linguistic preference relations when modeling human behavior. Interrelationships of the stability and equilibrium results are also consistent with the theorems provided in Sect. 4.3.

6 Conclusions

In real-world conflicts, DMs may tend to use qualitative representation formats to express their preference information over states. In this study, a GMCR with linguistic preferences was introduced. In order to analyze linguistic stability definitions, the concepts of LRSP, LST, and LUI were first defined. Then, four linguistic stability definitions for two-DM and n-DM graph models were investigated in detail. We also presented the interrelationships among these four definitions. Another important contribution of this study is that a DM's inconsistent assessments were considered by using PLTSs if this DM contains a large number of participants.

An illustrative example of the Huawei conflict with three DMs was carried out to demonstrate the effectiveness and usefulness of the proposed model. In this example, DMs used linguistic terms to express their preferences irrespective of whether they were hesitant, could not master exact information, or large-scale participants were involved. The results showed that, as the LST was increased, the scope of the equilibria became larger. From the results, it is suggested that BIS adjusted its ban, IEEE lifted the prohibition, and AOI did not continue to oppose. This resolution result showed that our model was applicable.

Based on the main body of this study, we highlight the advantages of our model from a theoretical perspective and emphasized how they differentiate from the contributions already reported in the literature as follows:

(1) With the increasing complexity of decision-making activities, DMs usually have vagueness or ambiguity in their thinking. Thus, it may be not enough to use quantitative representation formats to express DM's preference information. The proposed model differentiates from other related studies because it first considered linguistic expressions in GMCR to capture vague cognition of DMs;
(2) The rapid expansion of societal and technological paradigms makes it a reality for a large number of individuals to participant in decision-making processes. This study considered how to incorporate preference information of a large group of participants into GMCR.

This study also has limitations. On the one hand, we assumed that DMs move sequentially. No DM was allowed to move twice continuously. On the other hand, DMs can only use pairwise comparisons. If there are many states, then it is time-consuming to provide a complete pairwise comparison matrix. In the future, we shall relax the assumption that DMs cannot move twice continuously and study more complex behaviors of DMs. Furthermore, it is necessary to investigate how to construct a GMCR if there are multiple states.

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