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Group decision making based on DEA cross-efficiency with intuitionistic fuzzy preference relations

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Abstract

The aim of this paper is to investigate a novel approach to group decision making based on DEA cross-efficiency with intuitionistic fuzzy preference relations, which can avoid information distortion and obtain more credible decision making results. An interval transform function is defined, which can transform an intuitionistic fuzzy preference relation into an interval multiplicative preference relation. Then, an interval transform function based data envelopment analysis model is developed to obtain the ranking vector of consistent intuitionistic fuzzy preference relation, in which each of the alternatives is viewed as a decision making unit. Moreover, for any intuitionistic fuzzy preference relations, we propose two DEA cross-efficiency models to get the cross-efficiency values of all alternatives, and we can calculate the normalized intuitionistic fuzzy priority weight vector of the intuitionistic fuzzy preference relation based on the cross-efficiency values. A goal programming model is investigated to derive the weight vector of decision makers. A step-by-step procedure for group decision making approach based on DEA cross-efficiency with intuitionistic fuzzy preference relations is presented. Finally, numerical examples are given to illustrate the validity and applicability of the proposed method. This is the first attempt of employing the DEA cross-efficiency to the group decision making with intuitionistic fuzzy preference relations.

Keywords Group decision making · Intuitionistic fuzzy preference relation · DEA cross-efficiency · Interval transform function

1 Introduction

Group decision making can be simplified as a group of decision makers to make a choice from a set of alternatives for action in accordance with the opinions provided

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by the members. In group decision making process, the decision makers often need to provide their preferences over a set of alternatives or criteria by pairwise comparison. The analytic hierarchy process (AHP) furnishes a convenient framework to derive multiplicative preference relations (MPRs), which uses a 1/9-9 ratio scale to measure the intensity of the pairwise comparison (Saaty 1980). Besides, the 0–1 scale is adopted by the fuzzy preference relation (Herrera-Viedma et al. 2004; Xu 2007a) to express the decision maker's preference information.

However, in many real decision making environment, it is very difficult for the decision makers to express their preferences using a single crisp value due to that (1) the decision makers are lack of knowledge and information associated with the problem (Xu and Cai 2010) and (2) the decision makers are not confident about their judgment, i.e., there exists hesitation regarding their preferences (Wu et al. 2018). To circumvent this issue, Atanassov's intuitionistic fuzzy set (IFS) was introduced in Atanassov (1986), which is an extension of Zadeh's fuzzy set and uses membership degree, non-membership degree and hesitation degree to express decision maker's subjective preference. This IFS provides a powerful framework, which can not only characterize vagueness, uncertainty, but also represent inevitably imprecise or not totally reliable judgments well. From then on, the IFSs have been proven to be very meaningful and practical, and attached great importance by many scholars (Xu and Liao 2015).

Based on IFS, the intuitionistic fuzzy preference relation (IFPR) was defined in Xu (2007b), which is more powerful and useful than multiplicative preference relation and the fuzzy preference relation. Xu (2007b) firstly defined the concepts of consistent IFPR, incomplete IFPR and acceptable IFPR. How to derive priority vector from the fuzzy preference relation has become a hot issue of study. Since then, many prioritization methods for IFPRs have been developed. Wang (2013) proposed a linear goal programming model for deriving intuitionistic fuzzy weights from IFPR by minimizing its deviation from the converted additive consistent IFPR. Besides, a convergent iterative algorithm to improve the consistency of IFPR was developed by Xu and Xia (2014). Liao et al. (2015) and Meng et al. (2017) introduced two consistency adjustment methods to yield the acceptable consistent IFPRs respectively. However, these methods of deriving consistent IFPRs have changed the original evaluation information given by the decision makers, which may distort the given decision making information and make the decision result less reliable.

In addition, Ramanathan (2006) introduced the data envelopment analytic hierarchy process (DEAHP) model for weight derivation from multiplicative preference relation, but the DEAHP model is not applicable for some inconsistent MPRs. Then, Wang et al. (2008) proposed the DEA/AR model, which can overcome the shortcoming of the DEAHP model. Furthermore, Liu et al. (2017) developed a novel output-oriented data envelopment analysis (DEA) model to obtain the priority vector for the consistent fuzzy preference relation. The existing research shows that the DEA cross-efficiency can overcome the problem of extreme weights and incomplete ranking in traditional DEA by mutual evaluation of decision making units (DMUs) (Liang et al. 2008). In fact, an interesting and important issue to be solved is how to develop a DEA cross-efficiency model to derive the priority weight vector from IFPR, which does not require repairing the IFPR. Up to now, there has been no investigation about this issue.

Recently, Wu et al. (2018) developed the definition of the multiplicative transitivity of IFPR, and discovered the substantial relationship between interval-valued fuzzy preference relation and IFPR. Another interesting issue is to discover the substantial relationship between IFPR and interval multiplicative preference relations (IMPRs).

In this paper, we will propose a new approach to group decision making with IFPRs based on DEA cross-efficiency, which can avoid information distortion and obtain more credible decision making result. An interval transform function is defined to establish the substantial relationship between IFPR and IMPR. Based on the interval transform function, we develop two DEA cross-efficiency models to get the peer-evaluation cross-efficiency values of all alternatives, in which each of the alternatives is viewed as a decision making unit. Then, the normalized intuitionistic fuzzy weight vector for the IFPR is derived based on the cross-efficiency values. Furthermore, a goal programming model is investigated to yield the weights of decision makers. Finally, a novel approach to group decision making based on DEA cross-efficiency with IFPRs is proposed, which does not need consistency adjustments and can yield a normalized intuitionistic fuzzy weight vector. In particular, when the IFPRs given by the experts have a poor consistency, our method has better applicability and can derive more reasonable ranking result than some known methods. This is the first attempt of employing the theory of DEA cross-efficiency for the ranking of preference relations. Hopefully, the proposed work is not only a generalization of existing theory but also an initial step for the development of group decision making based on DEA cross-efficiency.

The paper is organized as follows. Some basic concepts are introduced in Sect. 2, such as the multiplicative preference relation, interval multiplicative preference relation, the intuitionistic fuzzy preference relation and the consistency of MPR, IMPR and IFPR. In Sect. 3, we define the interval transform function, develop two DEA cross-efficiency models to get the peer-evaluation cross-efficiency values of all alternatives and propose the deriving method of the normalized intuitionistic fuzzy weight vector based on cross-efficiency values. In Sect. 4, a goal programming model is investigated to yield the weights of decision makers and a step-by-step procedure for group decision making approach is presented. There are numerical examples and comparison analysis in Sect. 5 to illustrate the advantages, validity and applicability of the proposed methods. Finally, Sect. 6 concludes the paper, and discusses some future research directions.

2 Preliminaries

In this section, we provide a brief review on some basic concepts, including multiplicative preference relation (MPR), interval multiplicative preference relation (IMPR), the intuitionistic fuzzy preference relation (IFPR) and the consistency of MPR, IMPR and IFPR.

For a decision making problem, it is assumed that $X = \{x_1, x_2, \ldots, x_n\}$ is a finite set of alternatives. In the process of decision making, the decision maker usually provides pairwise judgments on any two alternatives over the set X.

Definition 1 (Saaty 1980) A reciprocal multiplicative preference relation R on X is characterized by a matrix $R = (r_{ij})_{n\times n} \subset X \times X$ with

$$r_{ij} \cdot r_{ji} = 1, \quad r_{ij} \geq 0, \quad r_{ii} = 1, \quad i, j = 1, 2, \ldots, n,$$

where r_{ij} is interpreted as the ratio of the preference intensity of alternative x_i to that of x_j.

The reciprocal multiplicative preference relation is also called the multiplicative preference relation. And the consistency of MPR has been proposed by Saaty (1977) as follows:

Definition 2 (Saaty 1977) A MPR $R = (r_{ij})_{n\times n}$ is consistent if

$$r_{ik} \cdot r_{kj} = r_{ij}, \quad i, j, k = 1, 2, \ldots, n. \tag{1}$$

For a MPR $R = (r_{ij})_{n\times n}$, and $w = (w_1, w_2, \cdots, w_n)^T$ is a normalized priority weight vector of R, then R is consistent if and only if

$$r_{ij} = w_i \big/ w_j, \quad i, j = 1, 2, \ldots, n, \tag{2}$$

where $w_i > 0$, $i = 1, 2, \cdots, n$, and $\sum_{i=1}^{n} w_i = 1$.

In order to express the decision makers' uncertain preferences, Saaty and Vargas (1987) introduced the definition of interval multiplicative preference relation.

Definition 3 (Saaty and Vargas 1987; Xia and Chen 2015) An IMPR $\tilde{R}$ is defined as follows:

$$\tilde{R} = (\tilde{r}_{ij})_{n\times n} = \begin{pmatrix} [1,1] & [r_{12}^-, r_{12}^+] & \cdots & [r_{1n}^-, r_{1n}^+] \\ [r_{21}^-, r_{21}^+] & [1,1] & \cdots & [r_{2n}^-, r_{2n}^+] \\ \cdots & \cdots & \cdots & \cdots \\ [r_{n1}^-, r_{n1}^+] & [r_{n2}^-, r_{n2}^+] & \cdots & [1,1] \end{pmatrix},$$

where $r_{ij}^-, r_{ij}^+ > 0$, such that $r_{ij}^- \leq r_{ij}^+$, $r_{ij}^- r_{ji}^+ = 1$ and $r_{ij}^+ r_{ji}^- = 1$, $i, j = 1, 2, \ldots, n$. When $r_{ij}^- = r_{ij}^+$ for all $i, j = 1, 2, \ldots, n$, $\tilde{R}$ degenerates to a MPR.

The consistency of IMPR was defined as follows.

Definition 4 (Wang 2015) An IMPR $\tilde{R} = (\tilde{r}_{ij})_{n\times n}$ with $\tilde{r}_{ij} = [r_{ij}^-, r_{ij}^+]$ $(i, j = 1, 2, \ldots, n)$ is consistent if it satisfies the following transitivity condition:

$$r_{ik}^- r_{ik}^+ r_{kj}^- r_{kj}^+ = r_{ij}^- r_{ij}^+, \quad i, j, k = 1, 2, \ldots, n. \tag{3}$$

According to Definition 4, it is evident that IMPR $\tilde{R} = (\tilde{r}_{ij})_{n\times n}$ with $\tilde{r}_{ij} = [r_{ij}^-, r_{ij}^+]$ $(i, j = 1, 2, \ldots, n)$ is consistent if and only if

$$\rho(\tilde{r}_{ik})\rho(\tilde{r}_{kj}) = \rho(\tilde{r}_{ij}), \quad i, j, k = 1, 2, \ldots, n, \tag{4}$$

where $\rho(\tilde{r}_{ij}) = \sqrt{r_{ij}^- \cdot r_{ij}^+}$.

In 1986, intuitionistic fuzzy set was introduced by Atanassov (1986), which was defined as follows.

Definition 5 (Atanassov 1986) Let $X = \{x_1, x_2, \ldots, x_n\}$ be an ordinary finite non-empty set. An IFS $\tilde{A}$ over X is represented as $\tilde{A} = \{< x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) > | x \in X\}$, where $\mu_{\tilde{A}} : X \to [0, 1]$, $\nu_{\tilde{A}} : X \to [0, 1]$ and $\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1$, for all $x \in X$. The numbers $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ represent the membership degree and non-membership degree of the element $x \in X$ to $\tilde{A}$, respectively. $\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x)$ is called the hesitation degree of x to $\tilde{A}$. Obviously, $0 \leq \pi_{\tilde{A}}(x) \leq 1$ for all $x \in X$. If $\pi_{\tilde{A}}(x) = 0$, we have $\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) = 1$. Then, $\tilde{A}$ is degraded to an ordinary fuzzy set.

For convenience, $\alpha = \langle \mu_\alpha, \nu_\alpha \rangle$ is called an intuitionistic fuzzy value (IFV) (Saaty 1977), where $\mu_\alpha, \nu_\alpha \in [0, 1]$ and $\mu_\alpha + \nu_\alpha \leq 1$.

Definition 6 (Meng et al. 2017) Let $\alpha = \langle \mu_\alpha, \nu_\alpha \rangle$ be an IFV, then the score function and accuracy function of α are defined by $S(\alpha) = u_\alpha - v_\alpha$ and $H(\alpha) = u_\alpha + v_\alpha$, respectively. Suppose α_1 and α_2 are two IFVs, then

(1) If $S(\alpha_1) > S(\alpha_2)$, then α_1 is larger than α_2, denoted by $\alpha_1 > \alpha_2$;
(2) If $S(\alpha_1) = S(\alpha_2)$, then

   (a) If $H(\alpha_1) > H(\alpha_2)$, then $\alpha_1 > \alpha_2$;
   (b) If $H(\alpha_1) = H(\alpha_2)$, then $\alpha_1 = \alpha_2$, which means that α_1 is equal to α_2.

Based on IFV, Xu (2007b) gave a standard definition of intuitionistic fuzzy preference relation, which is shown as follows:

Definition 7 (Xu 2007b) An IFPR over a finite set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$ is represented by a matrix $\tilde{P} = (\tilde{p}_{ij})_{n\times n}$, where $\tilde{p}_{ij} =< (x_i, x_j), \mu(x_i, x_j), \nu(x_i, x_j), \pi(x_i, x_j) >$ for all $i, j = 1, 2, \ldots, n$. For convenience, let $\tilde{p}_{ij} = \langle \mu_{ij}, \nu_{ij} \rangle$ be an IFV, where μ_{ij} defines the certainty degree that the alternative x_i is preferred to x_j, ν_{ij} indicates the certainty degree that the alternative x_i is non-preferred to x_j and $0 \leq \mu_{ij}, \nu_{ij} \leq 1$, $0 \leq \mu_{ij} + \nu_{ij} \leq 1$, $\mu_{ij} = \nu_{ji}$,$\mu_{ji} = \nu_{ij}$,$\mu_{ii} = \nu_{ii} = 0.5$ for all $i, j = 1, 2, \cdots, n$.

Definition 8 (Wu et al. 2018) An IFPR $\tilde{P} = (\tilde{p}_{ij})_{n\times n}$ with $\tilde{p}_{ij} = \langle \mu_{ij}, \nu_{ij} \rangle$ $(i, j = 1, 2, \ldots, n)$ is multiplicative consistent (multiplicative transitive) if and only if $\forall i, j, k = 1, 2, \ldots, n$:

$$\begin{cases} \mu_{ij}\mu_{jk}\mu_{ki} = \mu_{ik}\mu_{kj}\mu_{ji}, \\ (1 - \nu_{ij})(1 - \nu_{jk})(1 - \nu_{ki}) = (1 - \nu_{ik})(1 - \nu_{kj})(1 - \nu_{ji}). \end{cases}$$

Definition 9 (Wang 2013) An intuitionistic fuzzy weight vector $\tilde{w} = (\tilde{w}_1, \tilde{w}_2, \cdots, \tilde{w}_n)^T$ with $\tilde{w}_i = \langle w_i^\mu, w_i^\nu \rangle^T$, $0 \leq w_i^\mu, w_i^\nu \leq 1$ and $w_i^\mu + w_i^\nu \leq 1$ for $i = 1, 2, \ldots, n$ is said to be normalized if it satisfies the following conditions:

$$\sum_{j=1, j\neq i}^{n} w_j^{\mu} \leq w_i^{\nu}, \quad w_i^{\mu} + n - 2 \geq \sum_{j=1, j\neq i}^{n} w_j^{\nu}, \quad i = 1, 2, \ldots, n.$$

3 Deriving priority vector of IFPR based on DEA cross-efficiency

In this section, we investigate a new approach of deriving the normalized intuitionistic fuzzy priority weight vector from IFPR based on DEA cross-efficiency. Unless otherwise mentioned, it is assumed that Ω is the set of interval values and Ψ is the set of intuitionistic fuzzy values (IFVs).

3.1 Interval transform function of IFPR

In order to extract effective information of the intuitionistic fuzzy value sufficiently, we propose the definition of interval transform function, which can be defined as follows.

Definition 10 Let $\alpha = \langle \mu_\alpha, \nu_\alpha \rangle$ be an IFV with $\mu_\alpha, \nu_\alpha > 0$. Then, an interval transform function of IFV is a mapping $\tau : \Psi^+ \to \Omega^+$, such that

$$\tau(\alpha) = \tau(\langle \mu_\alpha, \nu_\alpha \rangle) = \left[\frac{\mu_\alpha}{1-\mu_\alpha}, \frac{1-\nu_\alpha}{\nu_\alpha}\right], \tag{5}$$

where $\Psi^+ = \{\langle \mu_\alpha, \nu_\alpha \rangle | \mu_\alpha > 0, \nu_\alpha > 0, \mu_\alpha + \nu_\alpha \leq 1\}$ is the set of IFVs, in which both the membership degree and the non-membership degree of IFVs are all positive.

The interval transform function defined by Definition 10 can transform an IFV into an interval value. Then, we show a characterized theorem for the interval transform function.

Theorem 1 *Let* $\tilde{P} = (\tilde{p}_{ij})_{n\times n} = \left(\langle \mu_{ij}, \nu_{ij} \rangle\right)_{n\times n}$ *be an IFPR and* $\mu_{ij}, \nu_{ij} > 0$.

(1) $\tilde{H} = \left([h_{ij}^-, h_{ij}^+]\right)_{n\times n} = \left(\tau(\tilde{p}_{ij})\right)_{n\times n}$ *is an IMPR.*

(2) *If* $\tilde{P}$ *is multiplicative consistent, then* $\tilde{H} = \left([h_{ij}^-, h_{ij}^+]\right)_{n\times n} = \left(\tau(\tilde{p}_{ij})\right)_{n\times n}$ *is a consistent IMPR.*

Proof Let $\tilde{P} = (\tilde{p}_{ij})_{n\times n} = \left(\langle \mu_{ij}, \nu_{ij} \rangle\right)_{n\times n}$ be an IFPR, $\mu_{ij}, \nu_{ij} > 0$ and $\tilde{H} = \left(\tau(\tilde{p}_{ij})\right)_{n\times n}$. From Definition 10, we have $[h_{ij}^-, h_{ij}^+] = \tau(\tilde{p}_{ij}) = [\frac{\mu_{ij}}{1-\mu_{ij}}, \frac{1-\nu_{ij}}{\nu_{ij}}]$. Since $\mu_{ij}, \nu_{ij} > 0$, $\mu_{ij} + \nu_{ij} \leq 1$, $\mu_{ij} = \nu_{ji}$, $\mu_{ji} = \nu_{ij}$ and $\mu_{ii} = \nu_{ii} = 0.5$ for all $i, j = 1, 2, \ldots, n$, we have $h_{ij}^- = \frac{\mu_{ij}}{1-\mu_{ij}}$,$h_{ij}^+ = \frac{1-\nu_{ij}}{\nu_{ij}} > 0$, $1 - \mu_{ij} \geq \nu_{ij}$ and $1 - \nu_{ij} \geq \mu_{ij}$. Thus, $h_{ij}^- = \frac{\mu_{ij}}{1-\mu_{ij}} \leq \frac{1-\nu_{ij}}{\nu_{ij}} = h_{ij}^+$,

$$h_{ij}^{-}h_{ji}^{+} = \frac{\mu_{ij}}{1-\mu_{ij}} \cdot \frac{1-\nu_{ji}}{\nu_{ji}} = \frac{\mu_{ij}}{1-\mu_{ij}} \cdot \frac{1-\mu_{ij}}{\mu_{ij}} = 1,$$
$$h_{ji}^{-}h_{ij}^{+} = \frac{\mu_{ji}}{1-\mu_{ji}} \cdot \frac{1-\nu_{ij}}{\nu_{ij}} = \frac{\mu_{ji}}{1-\mu_{ji}} \cdot \frac{1-\mu_{ji}}{\mu_{ji}} = 1, \tag{6}$$

and

$$\left[h_{ii}^{-}, h_{ii}^{+}\right] = \tau(\tilde{p}_{ii}) = \left[\frac{\mu_{ii}}{1-\mu_{ii}}, \frac{1-\nu_{ii}}{\nu_{ii}}\right] = \left[\frac{0.5}{1-0.5}, \frac{1-0.5}{0.5}\right] = [1, 1]. \tag{7}$$

According to Eqs. (6), (7) and Definition 3, it is obvious that $\tilde{H} = \left([h_{ij}^{-}, h_{ij}^{+}]\right)_{n\times n}$ is an IMPR.

If $\tilde{P}$ is multiplicative consistent, from Definition 8, we have

$$\begin{cases} \mu_{ij}\mu_{jk}\mu_{ki} = \mu_{ik}\mu_{kj}\mu_{ji} \\ (1-\nu_{ij})(1-\nu_{jk})(1-\nu_{ki}) = (1-\nu_{ik})(1-\nu_{kj})(1-\nu_{ji}) \end{cases} \text{for } all \quad i, k, j = 1, 2, \ldots, n. \tag{8}$$

Thus, one can obtain

$$\mu_{ij}\mu_{jk}\mu_{ki}(1-\nu_{ij})(1-\nu_{jk})(1-\nu_{ki}) = \mu_{ik}\mu_{kj}\mu_{ji}(1-\nu_{ik})(1-\nu_{kj})(1-\nu_{ji}). \tag{9}$$

Equation (9) can be rewritten as

$$\frac{\mu_{ik}}{1-\nu_{ki}} \cdot \frac{1-\nu_{ik}}{\mu_{ki}} \cdot \frac{\mu_{kj}}{1-\nu_{jk}} \cdot \frac{1-\nu_{kj}}{\mu_{jk}} = \frac{\mu_{ij}}{1-\nu_{ji}} \cdot \frac{1-\nu_{ij}}{\mu_{ji}}. \tag{10}$$

By Eq. (10), it is obvious that

$$h_{ik}^{-} \cdot h_{ik}^{+} \cdot h_{kj}^{-} \cdot h_{kj}^{+} = h_{ij}^{-}h_{ij}^{+}, \quad i, k, j = 1, 2, \ldots, n. \tag{11}$$

According to Definition 4, $\tilde{H} = \left(\tau(\tilde{p}_{ij})\right)_{n\times n}$ is a consistent IMPR. □

Theorem 1 shows that the interval transform function can not only transform the IFPR into IMPR, but also ensure the consistency of preference relation.

3.2 DEA model of IFPR

DEA is a powerful tool to evaluate the relative efficiency of a set of homogenous decision making units (DMUs) with multiple inputs and multiple outputs by establishing a nonparametric linear programming model. In this section, we propose a DEA model to evaluate the relative efficiency of each alternative from IFPR.

In decision making process, decision maker offers an IFPR $\tilde{P} = (\tilde{p}_{ij})_{n\times n} = \left(\langle\mu_{ij}, \nu_{ij}\rangle\right)_{n\times n}$ over the alternative set $X = \{x_1, x_2, \ldots, x_n\}$, where $\mu_{ij}, \nu_{ij} > 0$. Each alternative x_i can be viewed as an independent DMU, $i = 1, 2, \ldots, n$. When the

Table 1 Inputs and outputs of DEA based on $\tilde{P}=(\tilde{p}_{ij})_{n\times n}=\left(\langle\mu_{ij},\nu_{ij}\rangle\right)_{n\times n}$

		Output 1	Output 2	···	Output n	Dummy input
x_1	DMU_1	$\tau(\tilde{p}_{11})$	$\tau(\tilde{p}_{12})$	···	$\tau(\tilde{p}_{1n})$	$\tau(\tilde{p}_0)$
x_2	DMU_2	$\tau(\tilde{p}_{21})$	$\tau(\tilde{p}_{22})$	···	$\tau(\tilde{p}_{2n})$	$\tau(\tilde{p}_0)$
···	···	···	···	···	···	···
x_n	DMU_n	$\tau(\tilde{p}_{n1})$	$\tau(\tilde{p}_{n2})$	···	$\tau(\tilde{p}_{nn})$	$\tau(\tilde{p}_0)$

decision maker believes that alternative x_i is better than x_j, for any $k\in\{1,2,\ldots,n\}$, the certainty degree that the alternative x_i is preferred to x_k is greater than the certainty degree that the alternative x_j is preferred to x_k, i.e., $\mu_{ik}\geq\mu_{jk}$, and certainty degree that the alternative x_i is non-preferred to x_k is smaller than the certainty degree that the alternative x_j is non-preferred to x_k, i.e., $\nu_{ik}\leq\nu_{jk}$. Thus, we have $\tilde{p}_{ik}=\langle\mu_{ik},\nu_{ik}\rangle\geq\tilde{p}_{jk}=\langle\mu_{jk},\nu_{jk}\rangle$. Furthermore, since $\frac{\mu_{ik}}{1-\mu_{ik}}\geq\frac{\mu_{jk}}{1-\mu_{jk}}$ and $\frac{1-\nu_{ik}}{\nu_{ik}}\geq\frac{1-\nu_{jk}}{\nu_{jk}}$, one can obtain $\tau(\tilde{p}_{ik})\geq\tau(\tilde{p}_{jk})$. Hence, each row of the IFPR corresponds to a DMU, and each column of the IFPR can be viewed as an output. Since DEA calculations cannot be made without inputs, to ensure the fairness of decision making, each decision making unit is given the same intuitionistic fuzzy input variable $\tilde{p}_0=\langle0.5,0.5\rangle$. Then, the relationships between inputs, outputs and IFPR $\tilde{P}=(\tilde{p}_{ij})_{n\times n}=\left(\langle\mu_{ij},\nu_{ij}\rangle\right)_{n\times n}$ are shown in Table 1.

Based on above analysis and the interval transform function τ given by Definition 10, we can develop an output-oriented CCR DEA model to evaluate the relative efficiency of alternative x_k $(k=1,2,\cdots,n)$ as follows:

$$\begin{aligned}&\underset{\mu,\phi}{\text{Max}}\ \phi_k\\&s.t.\begin{cases}\sum_{i=1}^{n}\mu_i\tau(\tilde{p}_{ij})\geq\phi_k\tau(\tilde{p}_{kj}), & j=1,2,\ldots,n,\\ \sum_{i=1}^{n}\mu_i\tau(\tilde{p}_0)\leq\tau(\tilde{p}_0),\\ \mu_i\geq0, & i=1,2,\ldots,n.\end{cases}\end{aligned}\tag{12}$$

For an IFPR $\tilde{P}=(\tilde{p}_{ij})_{n\times n}=\left(\langle\mu_{ij},\nu_{ij}\rangle\right)_{n\times n}$ with $\mu_{ij},\nu_{ij}>0$, if $\mu_{ij}+\nu_{ij}=1$, then $\frac{\mu_{ij}}{1-\mu_{ij}}=\frac{1-\nu_{ij}}{\nu_{ij}}$. It follows that $\tau(\tilde{p}_{ij})=[h_{ij}^-,h_{ij}^+]=[\frac{\mu_{ij}}{1-\mu_{ij}},\frac{1-\nu_{ij}}{\nu_{ij}}]$ degenerates to a real number. Denoting $h_{ij}=h_{ij}^-=h_{ij}^+=\frac{\mu_{ij}}{1-\mu_{ij}}=\frac{1-\nu_{ij}}{\nu_{ij}}$, $\tilde{H}=\left(\tau(\tilde{p}_{ij})\right)_{n\times n}$ degenerates to a MPR $H=(h_{ij})_{n\times n}$. Based on Model (12), we have the following Theorem.

Theorem 2 *Let $\tilde{P}=(\tilde{p}_{ij})_{n\times n}=\left(\langle\mu_{ij},\nu_{ij}\rangle\right)_{n\times n}$ be a multiplicative consistent IFPR with $\mu_{ij},\nu_{ij}>0$, $\mu_{ij}+\nu_{ij}=1$, and ϕ_k^* be the optimal objective function value of Model (12). Then, the normalized priority vector of $\tilde{H}=\left(\tau(\tilde{p}_{ij})\right)_{n\times n}$ can be derived by*

$$w_k=(\phi_k^*)^{-1}\Big/\sum_{l=1}^{n}(\phi_l^*)^{-1},\quad k=1,2,\ldots,n.\tag{13}$$

Proof Let $\tilde{P} = (\tilde{p}_{ij})_{n\times n} = \left(\langle\mu_{ij}, \nu_{ij}\rangle\right)_{n\times n}$ be a multiplicative consistent IFPR with $\mu_{ij}, \nu_{ij} > 0$, $\mu_{ij}+\nu_{ij}= 1$. According to Theorem 1, $\tilde{H} = \left([h^-_{ij}, h^+_{ij}]\right)_{n\times n} = \left(\tau(\tilde{p}_{ij})\right)_{n\times n}$ is a consistent IMPR. Then, we have

$$h^-_{ij}h^+_{ji} = 1, \quad h^-_{ik}h^+_{ik}h^-_{kj}h^+_{kj} = h^-_{ij}h^+_{ij}, \quad i, j, k = 1, 2, \ldots, n. \tag{14}$$

Denoting $h_{ij} = h^-_{ij} = h^+_{ij} = \frac{\mu_{ij}}{1-\mu_{ij}}=\frac{1-\nu_{ij}}{\nu_{ij}}$, from Eq. (14), one can obtain

$$h_{ij}h_{ji} = 1, \quad h_{ik}h_{kj} = h_{ij}, \quad i, j, k = 1, 2, \ldots, n.$$

Consequently, $\tilde{H} = \left(\tau(\tilde{p}_{ij})\right)_{n\times n}$ degenerates to consistent MPR $H = (h_{ij})_{n\times n}$, where $h_{ij} = \tau(\tilde{p}_{ij})$. According to Eq. (2), the priority weight vector of $\tilde{H} = \left(\tau(\tilde{p}_{ij})\right)_{n\times n}$ satisfies

$$h_{ij} = \tau(\tilde{p}_{ij}) = w_i \big/ w_j, \quad i, j = 1, 2, \ldots, n. \tag{15}$$

From Eq. (15), Model (12) can be rewritten as

$$\begin{aligned} &\underset{\mu,\phi}{\text{Max}}\, \phi_k \\ &s.t. \begin{cases} \sum_{i=1}^{n} \mu_i w_i / w_k \geq \phi_k, & j = 1, 2, \ldots, n, \\ \sum_{i=1}^{n} \mu_i \leq 1, \\ \mu_i \geq 0, \quad i = 1, 2, \ldots, n. \end{cases} \end{aligned} \tag{16}$$

Without loss of generality, it is assumed that $w_1 \leq w_2 \leq \cdots \leq w_n$. The objective function ϕ_k is maximal if and only if the constraint $\sum_{i=1}^{n} \mu_i \leq 1$ reduces to equality, i.e., $\sum_{i=1}^{n} \mu_i = 1$.

For any $k \in \{1, 2, \cdots, n\}$, by $\mu_k = 1 - \mu_1 - \cdots - \mu_{k-1} - \mu_{k+1} - \cdots - \mu_n$, the first constraint of Model (16) becomes

$$1 + \sum_{i\neq k, i=1}^{n} \mu_i\left(w_i \big/ w_k - 1\right) \geq \phi_k. \tag{17}$$

Because $w_1 \big/ w_k - 1 \leq w_2 \big/ w_k - 1 \leq \cdots \leq w_n \big/ w_k - 1$, it follows that ϕ_k is maximal if and only if $\mu_1 = \mu_2 = \cdots = \mu_{n-1} = 0$, $\mu_n = 1$. Thus, the optimal objective function value of Model (12) is $\phi^*_k = w_n \big/ w_k$.

Furthermore, since $w_i > 0$, $i = 1, 2, \ldots, n$, and $\sum_{i=1}^{n} w_i = 1$, we have $w_k = (\phi^*_k)^{-1} \big/ \sum_{l=1}^{n} (\phi^*_l)^{-1}$, $k = 1, 2, \ldots, n$. □

In fact, Theorem 1 provides a method of deriving the priority weight vector of consistent IFPR using the proposed DEA model. However, when $\mu_{ij}+\nu_{ij} \neq 1$, this method is not applicable. In order to solving this problem, based on interval aggregation function ρ, we propose the following CCR DEA model:

$$\underset{\mu,\phi}{\text{Max}}\ \phi_k$$
$$s.t.\begin{cases}\sum_{i=1}^{n}\mu_i\rho\big(\tau(\tilde{p}_{ij})\big)\geq\phi_k\rho\big(\tau(\tilde{p}_{kj})\big), & j=1,2,\ldots,n,\\ \sum_{i=1}^{n}\mu_i\rho(\tau(\tilde{p}_0))\leq\rho(\tau(\tilde{p}_0)),\\ \mu_i\geq 0, \quad i=1,2,\ldots,n.\end{cases} \tag{18}$$

where $\tau(\tilde{p}_{ij})=[h_{ij}^-,h_{ij}^+]=[\frac{\mu_{ij}}{1-\mu_{ij}},\frac{1-\nu_{ij}}{\nu_{ij}}]$ and $\rho\big(\tau(\tilde{p}_{ij})\big)=\sqrt{h_{ij}^-\cdot h_{ij}^+}=\sqrt{\frac{\mu_{ij}(1-\nu_{ij})}{\nu_{ij}(1-\mu_{ij})}}$.

In case of $\tilde{P}=(\tilde{p}_{ij})_{n\times n}=\big(\langle\mu_{ij},\nu_{ij}\rangle\big)_{n\times n}$ being a multiplicative consistent IFPR, according to Theorem 1, it is evident that $\tilde{H}=\Big([h_{ij}^-,h_{ij}^+]\Big)_{n\times n}=\big(\tau(\tilde{p}_{ij})\big)_{n\times n}$ is a consistent IMPR. Moreover, by Eq. (4), we have $\rho(\tau(\tilde{p}_{ik}))\cdot\rho(\tau(\tilde{p}_{kj}))=\rho(\tau(\tilde{p}_{ij}))$, $i,j,k=1,2,\ldots,n$, which implies that $\hat{H}=\big(\rho(\tau(\tilde{p}_{ij}))\big)_{n\times n}$ is a consistent MPR. According to the proof of Theorem 2, we have the following proposition.

Proposition 1 *Let $\tilde{P}=(\tilde{p}_{ij})_{n\times n}=\big(\langle\mu_{ij},\nu_{ij}\rangle\big)_{n\times n}$ be a multiplicative consistent IFPR with $\mu_{ij},\nu_{ij}>0$ and ϕ_k^* be the optimal objective function value of Model (18). Then, the priority weight vector of $\hat{H}=\big(\rho(\tau(\tilde{p}_{ij}))\big)_{n\times n}$ can be derived by*

$$w_k=(\phi_k^*)^{-1}\Bigg/\sum_{l=1}^{n}(\phi_l^*)^{-1},\quad k=1,2,\ldots,n. \tag{19}$$

For any multiplicative consistent IFPR $\tilde{P}=(\tilde{p}_{ij})_{n\times n}=\big(\langle\mu_{ij},\nu_{ij}\rangle\big)_{n\times n}$ over the alternative set $X=\{x_1,x_2,\ldots,x_n\}$, the inverse of optimal objective function value of model (18), $(\phi_k^*)^{-1}$, is the efficiency score of alternative x_k, $k=1,2,\ldots,n$. Moreover, the corresponding priority weight vector of $\tilde{P}$ can be derived by Proposition 1.

3.3 Deriving the intuitionistic fuzzy weights based on DEA cross-efficiency

It is worth noting that deriving priority weight vector based Model (18) is not applicable for inconsistent IFPR. To resolve this problem, we will develop a new method of deriving the normalized intuitionistic fuzzy priority weight vector from an IFPR based on DEA cross-efficiency. The cross-efficiency DEA uses the DMUs to evaluate each other, and the final efficiency evaluation value of the DMU is obtained from the self-evaluation and the peer-evaluation (Liang et al. 2008).

Based on Table 1 and interval aggregation function ρ, the input-oriented CCR DEA model is formulated as the following linear programming:

$$\text{Max}\quad \alpha_{kk}=\sum_{r=1}^{n}\omega_{rk}\rho(\tau(\tilde{p}_{kr}))$$
$$s.t.\begin{cases}\sum_{r=1}^{n}\omega_{rk}\rho\big(\tau(\tilde{p}_{jr})\big)-\eta_k\rho(\tau(\tilde{p}_0))\leq 0, & j=1,2,\ldots,n,\\ \eta_k\rho(\tau(\tilde{p}_0))=1,\\ \omega_{rk},\eta_k\geq 0, \quad k=1,2,\ldots,n.\end{cases} \tag{20}$$

For each alternative x_k $(k = 1, 2, \ldots, n)$ under evaluation, we can obtain the optimal objective value (efficiency value) α^*_{kk} and a set of optimal weights $\eta^*_k, \omega^*_{1k}, \omega^*_{2k}, \ldots, \omega^*_{nk}$ associated with inputs and outputs.

Note that the optimal weights obtained from Model (20) may not be unique. Inspired by the benevolent DEA cross-efficiency model and aggressive DEA cross-efficiency model (Doyle and Green 1994), we propose two new DEA cross-efficiency models for IFPR. Since $\tau(\tilde{p}_{ij}) = [h^-_{ij}, h^+_{ij}] = [\frac{\mu_{ij}}{1-\mu_{ij}}, \frac{1-\nu_{ij}}{\nu_{ij}}]$ and $\tau(\tilde{p}_0) = 1$, we construct the following two mathematical programming model for each alternative x_k $(k = 1, 2, \cdots, n)$.

$$\text{Max} \quad \sum_{j=1}^{n}\sum_{r=1}^{n} u_{rk}h^-_{jr}$$

$$s.t. \begin{cases} v_k \cdot n = 1, \\ \sum_{r=1}^{n} u_{rk}h^+_{kr} - \alpha^*_{kk}v_k = 0, \\ \sum_{r=1}^{n} u_{rk}h^+_{kr} - v_k \leq 0, \quad j = 1, 2, \ldots, n; j \neq k, \\ v_k, u_{rk} \geq 0, \quad k = 1, 2, \ldots, n. \end{cases} \tag{21}$$

$$\text{Min} \quad \sum_{j=1}^{n}\sum_{r=1}^{n} u_{rk}h^-_{jr}$$

$$s.t. \begin{cases} v_k \cdot n = 1, \\ \sum_{r=1}^{n} u_{rk}h^+_{kr} - \alpha^*_{kk}v_k = 0, \\ \sum_{r=1}^{n} u_{rk}h^+_{kr} - v_k \leq 0, \quad j = 1, 2, \ldots, n; j \neq k, \\ v_k, u_{rk} \geq 0, \quad k = 1, 2, \ldots, n. \end{cases} \tag{22}$$

In Models (21) and (22), α^*_{kk} is the target efficiency of alternative x_k, which is obtained from Model (20). As can be seen, Model (21) aims to maximize the lower bound of overall cross-efficiency values of all alternatives and Model (22) aims to minimize the lower bound of overall cross-efficiency values of all alternatives. The second equality constraint of Models (21) and (22), $\sum_{r=1}^{n} u_{rk}h^+_{kr} - \alpha^*_{kk}v_k = 0$, is used to ensure that the upper bound efficiency value of alternative x_k is equal to its optimal efficiency α^*_{kk}, i.e., $\sum_{r=1}^{n} u_{rk}h^+_{kr} \big/ v_k = \alpha^*_{kk}$.

By solving Models (21) and (22), we can obtain the optimal weights $u^* = (u^*_{1k}, u^*_{2k}, \ldots, u^*_{nk})^T$ and v^*_k, which are the associated output weights and input weight. Thus, the cross-efficiency of alternative x_j $(j = 1, 2, \ldots, n)$ using the optimal weights of alternative $x_k (k = 1, 2, \ldots, n)$ can be calculated as in Eq. (23),

$$\theta^-_{jk} = \sum_{r=1}^{n} u^*_{rk}h^-_{jr} \Big/ v^*_k \quad \text{and} \quad \theta^+_{jk} = \sum_{r=1}^{n} u^*_{rk}h^+_{jr} \Big/ v^*_k, \quad j = 1, 2, \ldots, n, \tag{23}$$

where θ^-_{jk} and θ^+_{jk} are the lower bound and upper bound of the cross-efficiency, respectively. Therefore, we get an interval cross-efficiency value of alternative x_j, i.e., $\tilde{\theta}_{jk} = [\theta^-_{jk}, \theta^+_{jk}]$, which represents the peer-evaluation of alternative x_k to alternative x_j.

In Models (21) and (22), while alternative x_k $(k = 1, 2, \ldots, n)$ is changed, one can get the interval cross-efficiency matrix, namely,

$$\Theta = (\tilde{\theta}_{jk})_{n\times n} = \begin{pmatrix} [\theta_{11}^-, \theta_{11}^+] & [\theta_{12}^-, \theta_{12}^+] & \cdots & [\theta_{1n}^-, \theta_{1n}^+] \\ [\theta_{21}^-, \theta_{21}^+] & [\theta_{22}^-, \theta_{22}^+] & \cdots & [\theta_{2n}^-, \theta_{2n}^+] \\ \cdots & \cdots & \cdots & \cdots \\ [\theta_{n1}^-, \theta_{n1}^+] & [\theta_{n2}^-, \theta_{n2}^+] & \cdots & [\theta_{nn}^-, \theta_{nn}^+] \end{pmatrix}.$$

Furthermore, we aggregate the cross-efficiency values of alternative x_j $(j = 1, 2, \ldots, n)$ based on Eq. (24), and obtain the average cross-efficiency value $\tilde{\theta}_j$.

$$\tilde{\theta}_j = \left[\theta_j^-, \theta_j^+\right] = \left[\frac{1}{n}\sum_{k=1}^{n}\theta_{jk}^-, \frac{1}{n}\sum_{k=1}^{n}\theta_{jk}^+\right], \quad j = 1, 2, \ldots, n. \tag{24}$$

Based on the average cross-efficiency values of all alternatives, we can calculate the intuitionistic fuzzy priority weight vector $\tilde{w} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)^T$ with $\tilde{w}_j = \left\langle w_j^\mu, w_j^\nu \right\rangle (j = 1, 2, \ldots, n)$, based on the following formulas:

$$w_j^\mu = \frac{\theta_j^-}{\psi}, \quad w_j^\nu = \frac{1}{\psi}\sum_{r=1, r\neq j}^{n}\theta_r^+, \quad j = 1, 2, \ldots, n, \tag{25}$$

where $\psi = \sum_{r=1}^{n}\theta_r^+ + \frac{1}{n-2}\max\limits_k\left\{\theta_k^+ - \theta_k^-\right\}$.

It is worth noting that the intuitionistic fuzzy priority weight vector obtained from Eq. (25) satisfies the following theorem.

Theorem 3 *For any IFPR $\tilde{P} = (\tilde{p}_{ij})_{n\times n}$, its intuitionistic fuzzy priority weight vector $\tilde{w} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)^T$, which is obtained from Eq. (25), is normalized.*

Proof According to Formula (25), since $0 \le \theta_j^- \le \theta_j^+ \le 1$, it follows that w_j^μ, $w_i^\nu \in [0, 1]$ and

$$w_j^\mu + w_j^\nu = \frac{\theta_j^- + \sum_{r=1, r\neq j}^{n}\theta_r^+}{\sum_{r=1}^{n}\theta_r^+ + \frac{1}{n-2}\max\limits_k\left\{\theta_k^+ - \theta_k^-\right\}} \le \frac{\theta_j^- + \sum_{r=1, r\neq j}^{n}\theta_r^+}{\sum_{r=1}^{n}\theta_r^+} \le 1.$$

Note that $\frac{\sum_{i=1, i\neq j}^{n}\theta_i^-}{\sum_{r=1}^{n}\theta_r^+ + \frac{1}{n-2}\max\limits_k\{\theta_k^+ - \theta_k^-\}} \le \frac{\sum_{r=1, r\neq j}^{n}\theta_r^+}{\sum_{r=1}^{n}\theta_r^+ + \frac{1}{n-2}\max\limits_k\{\theta_k^+ - \theta_k^-\}}$, we have $\sum_{i=1, i\neq j}^{n} w_i^\mu \le w_j^\nu$.

For any $j = 1, 2, \ldots, n$, we have

$$w_j^\mu + n - 2 = \frac{\theta_j^- + (n-2)\sum_{r=1}^{n}\theta_r^+ + \max\limits_k\left\{\theta_k^+ - \theta_k^-\right\}}{\sum_{r=1}^{n}\theta_r^+ + \frac{1}{n-2}\max\limits_k\left\{\theta_k^+ - \theta_k^-\right\}},$$

and

$$\sum_{i=1,i\neq j}^{n} w_i^{\nu} = \frac{\sum_{i=1,i\neq j}^{n}\sum_{r=1,r\neq i}^{n}\theta_r^{+}}{\sum_{r=1}^{n}\theta_r^{+} + \frac{1}{n-2}\max\limits_k\left\{\theta_k^{+}-\theta_k^{-}\right\}} = \frac{(n-1)\sum_{r=1}^{n}\theta_r^{+} - \sum_{i\neq j}^{n}\theta_i^{+}}{\sum_{r=1}^{n}\theta_r^{+} + \frac{1}{n-2}\max\limits_k\left\{\theta_k^{+}-\theta_k^{-}\right\}}.$$

Thus, on can get

$$w_j^{\mu} + n - 2 - \sum_{i=1,i\neq j}^{n} w_i^{\nu} = \frac{\max\limits_k\left\{\theta_k^{+}-\theta_k^{-}\right\} - (\theta_j^{+}-\theta_j^{-})}{\sum_{r=1}^{n}\theta_r^{+} + \frac{1}{n-2}\max\limits_k\left\{\theta_k^{+}-\theta_k^{-}\right\}} \geq 0.$$

The Proof is completed. □

Based on the afore-mentioned discussion, we have the ranking method of IFPR $\tilde{P} = (\tilde{p}_{ij})_{n\times n} = \left(\langle \mu_{ij}, \nu_{ij}\rangle\right)_{n\times n}$, which involves the following steps:

Step 1 For IFPR $\tilde{P} = (\tilde{p}_{ij})_{n\times n} = \left(\langle \mu_{ij}, \nu_{ij}\rangle\right)_{n\times n}$, use Model (20) to obtain α_{kk}^{*}, $k = 1, 2, \cdots, n$.
Step 2 Set α_{kk}^{*} in the DEA cross-efficiency Models (21) or (22), and solve the Model (21) or Model (22) to obtain the optimal solutions $u_{1k}^{*}, u_{2k}^{*}, \ldots, u_{nk}^{*}, v_k^{*}$, $k = 1, 2, \ldots, n$.
Step 3 Use Formula (23) to compute the lower bound and upper bound of the cross-efficiency, then get the interval cross-efficiency matrix $\Theta = (\tilde{\theta}_{jk})_{n\times n}$, where $\tilde{\theta}_{jk} = [\theta_{jk}^{-}, \theta_{jk}^{+}]$, $j, k = 1, 2, \ldots, n$.
Step 4 Calculate the average cross-efficiency value $\tilde{\theta}_j = [\theta_j^{-}, \theta_j^{+}]$ of alternative x_j based on formula (24), $j = 1, 2, \ldots, n$.
Step 5 Calculate the normalized intuitionistic fuzzy priority weight vector $\tilde{w} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)^T$ by using formula (25), where $\tilde{w}_i = \langle w_i^{\mu}, w_i^{\nu}\rangle$.
Step 6 Rank the normalized intuitionistic fuzzy priority weights $\tilde{w}_j$ in descending order, $j = 1, 2, \ldots, n$.
Step 7 Rank the alternatives x_j $(j = 1, 2, \ldots, n)$ according to the ranking of $\tilde{w}_j$, $j = 1, 2, \cdots, n$.
Step 8 End.

Example 1 Consider the alternative set $X = \{x_1, x_2, x_3, x_4\}$, and the IFPR $\tilde{P}$ given by the decision maker is as follows.

$$\tilde{P} = \begin{pmatrix} \langle 0.5, 0.5\rangle & \langle 0.6, 0.2\rangle & \langle 0.8, 0.2\rangle & \langle 0.6, 0.3\rangle \\ \langle 0.2, 0.6\rangle & \langle 0.5, 0.5\rangle & \langle 0.5, 0.3\rangle & \langle 0.3, 0.5\rangle \\ \langle 0.2, 0.8\rangle & \langle 0.3, 0.5\rangle & \langle 0.5, 0.5\rangle & \langle 0.3, 0.6\rangle \\ \langle 0.3, 0.6\rangle & \langle 0.5, 0.3\rangle & \langle 0.6, 0.3\rangle & \langle 0.5, 0.5\rangle \end{pmatrix}$$

Step 1 According to Model (20), we can get the relative efficiency value of x_k, $k = 1, 2, 3, 4$, i.e.,

$$\alpha_{11}^{*} = 1, \quad \alpha_{22}^{*} = 0.4082, \quad \alpha_{33}^{*} = 0.2857, \quad \alpha_{44}^{*} = 0.6236.$$

Step 2 Set α_{kk}^* in the cross-efficiency DEA Model (21), and solve the Model (21) to obtain the optimal solutions.

$$\begin{array}{lllll} u_{11}^* = 0.2500, & u_{21}^* = 0.0000, & u_{31}^* = 0.0000, & u_{41}^* = 0.0000, & v_1^* = 0.2500; \\ u_{12}^* = 0.0000, & u_{22}^* = 0.0328, & u_{32}^* = 0.0297, & u_{42}^* = 0.0000, & v_2^* = 0.2500; \\ u_{13}^* = 0.0000, & u_{23}^* = 0.0000, & u_{33}^* = 0.0000, & u_{43}^* = 0.1071, & v_3^* = 0.2500; \\ u_{14}^* = 0.1208, & u_{24}^* = 0.0000, & u_{34}^* = 0.0323, & u_{44}^* = 0.0000, & v_4^* = 0.2500. \end{array}$$

Step 3 Based on formula (23), we get the interval cross-efficiency matrix

$$\Theta = \begin{pmatrix} [1.0000, 1.0000] & [0.6717, 1.0000] & [0.6429, 1.0000] & [1.0000, 1.0000] \\ [0.2500, 0.6667] & [0.2500, 0.4082] & [0.1837, 0.4286] & [0.2500, 0.6236] \\ [0.2500, 0.2500] & [0.1750, 0.2500] & [0.1837, 0.2857] & [0.2500, 0.2500] \\ [0.4286, 0.6667] & [0.3093, 0.5833] & [0.4286, 0.4286] & [0.4009, 0.6236] \end{pmatrix}.$$

Step 4 From Θ, we can get the average cross-efficiency value of each alternative respectively based on Formula (24), i.e.,

$$\tilde{\theta}_1 = [0.8286, 1.0000], \quad \tilde{\theta}_2 = [0.2334, 0.5318],$$
$$\tilde{\theta}_3 = [0.2147, 0.2589], \quad \tilde{\theta}_4 = [0.3918, 0.5755].$$

Step 5 According to Formula (25), we get the normalized intuitionistic fuzzy priority weight vector, i.e.,

$$\tilde{w} = (\langle 0.3294, 0.5431\rangle, \langle 0.0928, 0.7293\rangle, \langle 0.0853, 0.8378\rangle, \langle 0.1558, 0.7119\rangle)^T.$$

Step 6 Based on Definition 6, we can get the ranking of normalized intuitionistic fuzzy priority weight vector, which is as follows:

$$\tilde{w}_1 > \tilde{w}_4 > \tilde{w}_2 > \tilde{w}_3.$$

Step 7 Thus, the rank of the alternatives x_j according to the ranking of $\tilde{w}_j$ $(j = 1, 2, 3, 4)$ is as follows:

$$x_1 > x_4 > x_2 > x_3.$$

Similarly, if we use Model (22) instead of Model (21) in Step 2, we can get

$$\Theta = \begin{pmatrix} [0.3750, 1.0000] & [0.6124, 0.6124] & [0.6429, 1.0000] & [0.6771, 1.0000] \\ [0.2500, 0.2500] & [0.1531, 0.4082] & [0.1837, 0.4286] & [0.2500, 0.4514] \\ [0.1071, 0.2500] & [0.1531, 0.1531] & [0.1837, 0.2857] & [0.1762, 0.2500] \\ [0.2500, 0.5833] & [0.2624, 0.4082] & [0.4286, 0.4286] & [0.3363, 0.6236] \end{pmatrix},$$

Moreover, we have the average cross-efficiency value of four alternatives

$$\tilde{\theta}_1 = [0.5768, 0.9031], \quad \tilde{\theta}_2 = [0.2092, 0.3846],$$
$$\tilde{\theta}_3 = [0.1550, 0.2347], \quad \tilde{\theta}_4 = [0.3193, 0.5109].$$

Further, we can get

$$\tilde{w} = (\langle 0.2626, 0.5146\rangle, \langle 0.0952, 0.7506\rangle, \langle 0.0706, 0.8189\rangle, \langle 0.1454, 0.6931\rangle)^T.$$

It can be seen that we have the same ranking result of the derived intuitionistic fuzzy priority weights, which is $\tilde{w}_1 > \tilde{w}_4 > \tilde{w}_2 > \tilde{w}_3$.

In a comparison of our method with some existing ranking methods of IFPR, our proposed method has the following advantages.

First, the given IFPRs need an inconsistency repairing process to yield the acceptable consistent IFPRs in Xu and Xia (2014), Liao et al. (2015) and Meng et al. (2017). The inconsistency repairing have changed the original evaluation information given by the decision maker, which can distort the given decision making information and make the decision result less reliable. As to our ranking method, it is based entirely on the original preference information given by decision maker and does not require the consistency adjustment, so the result is more credible.

Second, in Wu et al. (2018), Liao and Xu (2014) and Wang (2013), it is assumed that there exist specific relationships between the priority weight vector and the consistent IFPRs. Then, the optimization models are established based on these assumptions to get the priority weight vector. Since these assumptions are based on consistent IFPR, the obtained priority weight vectors are always unreasonable for some inconsistent IFPRs. Besides, these assumptions are not unique. By contrast, our approach does not need to make any assumptions in advance, so it can avoid the risk of assumption dependent.

Third, in our ranking method, the DEA cross-efficiency models combine the results of self-evaluation and peer-evaluation to get the final efficiency value of each alternative, which makes the ranking result fairer.

4 Group decision making with intuitionistic fuzzy preference relations

Let $D = \{d_1, d_2, \ldots, d_m\}$ be the set of decision makers and $X = \{x_1, x_2, \ldots, x_n\}$ be the set of alternatives. Suppose that d_l provides his/her preference on $X = \{x_1, x_2, \ldots, x_n\}$ using IFPR $\tilde{P}^{(l)} = (\tilde{p}_{ij}^{(l)})_{n\times n} = \left(\left\langle \mu_{ij}^{(l)}, \nu_{ij}^{(l)}\right\rangle\right)_{n\times n}$, $l = 1, 2, \ldots, m$, and $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)^T$ is the weight vector of decision makers satisfying $\lambda_l \geq 0$ and $\sum_{l=1}^{m} \lambda_l = 1$, in which λ_l can reflect the importance degree of d_l.

In order to measure the similarity between two IFPRs, the definition of compatibility degree was introduced by Xu (2013) as follows.

Definition 11 Let $\tilde{P}^{(l)} = \left(\left\langle \mu_{ij}^{(l)}, \nu_{ij}^{(l)} \right\rangle\right)_{n\times n}$ and $\tilde{P}^{(k)} = \left(\left\langle \mu_{ij}^{(k)}, \nu_{ij}^{(k)} \right\rangle\right)_{n\times n}$ be two IFPRs, given by two decision makers d_l and d_k, $k, l = 1, 2, \ldots, m$. Then, we call

$$c(\tilde{P}^{(k)}, \tilde{P}^{(l)}) = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\mu_{ij}^{(k)}\mu_{ij}^{(l)} + \upsilon_{ij}^{(k)}\upsilon_{ij}^{(l)}\right)}{\max\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}[(\mu_{ij}^{(k)})^2 + (\nu_{ij}^{(k)})^2], \sum_{i=1}^{n}\sum_{j=1}^{n}[(\mu_{ij}^{(l)})^2 + (\nu_{ij}^{(l)})^2]\right\}}$$

the compatibility degree of $\tilde{P}^{(l)}$ and $\tilde{P}^{(k)}$.

It can be seen easily that the greater the value of $c(\tilde{P}^{(k)}, \tilde{P}^{(l)})$, the nearer the two IFPRs $\tilde{P}^{(k)}$ and $\tilde{P}^{(l)}$ will be. Moreover, we have $0 \leq c(\tilde{P}^{(k)}, \tilde{P}^{(l)}) \leq 1$, and $c(\tilde{P}^{(k)}, \tilde{P}^{(l)}) = 1$ if and only if $\tilde{P}^{(k)} = \tilde{P}^{(l)}$, which means that $\tilde{P}^{(l)}$ and $\tilde{P}^{(k)}$ are perfectly compatible.

Besides, based on the proposed interval transform function τ, $\tilde{P}^{(l)} = (\tilde{p}_{ij}^{(l)})_{n\times n} = \left(\left\langle \mu_{ij}^{(l)}, \nu_{ij}^{(l)} \right\rangle\right)_{n\times n}$ can be transformed into IMPR $\tilde{H}^{(l)} = \left([h_{ij}^{(l),-}, h_{ij}^{(l),+}]\right)_{n\times n}$, where

$$\tau(\tilde{p}_{ij}^{(l)}) = \left[h_{ij}^{(l),-}, h_{ij}^{(l),+}\right] = \left[\frac{\mu_{ij}^{(l)}}{1-\mu_{ij}^{(l)}}, \frac{1-\nu_{ij}^{(l)}}{\nu_{ij}^{(l)}}\right], \quad l = 1, 2, \ldots, m, \quad i, j = 1, 2, \ldots, n. \tag{26}$$

Then, the IFPRs provided by m decision makers can be aggregated into a synthetic interval preference relation $H^s = \left([h_{ij}^{s,-}, h_{ij}^{s,+}]\right)_{n\times n}$, where

$$h_{ij}^{s,-} = \prod_{l=1}^{m}\left(h_{ij}^{(l),-}\right)^{\lambda_l} \quad \text{and} \quad h_{ij}^{s,+} = \prod_{l=1}^{m}\left(h_{ij}^{(l),+}\right)^{\lambda_l}, \quad i, j = 1, 2, \ldots, n. \tag{27}$$

It is evident that H^s is an IMPR. Note that the closer H^s and each of $\tilde{H}^{(l)}$ $(l = 1, 2, \ldots, m)$, the more representative the synthetic preference relation H^s is. Thus, we develop the following goal programming to obtain the weight vector of decision makers.

$$\text{Min} \quad J = \sum_{i=1}^{n}\sum_{j=i+1}^{n}\sum_{l=1}^{m}\left[\varepsilon_{ij}^{(l),+} + \varepsilon_{ij}^{(l),-} + \gamma_{ij}^{(l),+} + \gamma_{ij}^{(l),-}\right]$$

$$s.t. \begin{cases} \ln h_{ij}^{(l),-} - \sum_{l=1}^{m}\lambda_l \ln h_{ij}^{(l),-} - \varepsilon_{ij}^{(l),+} + \varepsilon_{ij}^{(l),-} = 0, \quad i < j,\ i, j = 1, 2, \ldots n, \quad l = 1, 2, \ldots, m, \\ \ln h_{ij}^{(l),+} - \sum_{l=1}^{m}\lambda_l \ln h_{ij}^{(l),+} - \gamma_{ij}^{(l),+} + \gamma_{ij}^{(l),-} = 0, \quad i < j,\ i, j = 1, 2, \ldots n, \quad l = 1, 2, \ldots, m, \\ \sum_{l=1}^{m}\lambda_l = 1, \\ \lambda_l, \varepsilon_{ij}^{(l),+}, \varepsilon_{ij}^{(l),-}, \gamma_{ij}^{(l),+}, \gamma_{ij}^{(l),-} \geq 0, \quad i < j,\ i, j = 1, 2, \ldots n, \quad l = 1, 2, \ldots, m. \end{cases} \tag{28}$$

As can be seen, the objective function J, which reflects group consensus, can measure the agreement between the individual preference relations and the synthetic preference relation. Especially, when all $\tilde{P}^{(l)}$ $(1, 2, \ldots, m)$ are perfectly compatible, the following result holds:

Theorem 4 *For any $l, k \in \{1, 2, \ldots, m\}$, $c(\tilde{P}^{(l)}, \tilde{P}^{(k)}) = 1$ if and only if $J^* = 0$, where J^* is the optimal objective function value of Model (28).*

Proof (Sufficiency) If $c(\tilde{P}^{(l)}, \tilde{P}^{(k)}) = 1$ for any $l, k \in \{1, 2, \ldots, m\}$, then $\tilde{P}^{(1)} = \tilde{P}^{(2)} = \cdots = \tilde{P}^{(m)}$. According to Eq. (26), we have $h_{ij}^{(1),-} = h_{ij}^{(2),-} = \cdots = h_{ij}^{(m),-}$ and $h_{ij}^{(1),+} = h_{ij}^{(2),+} = \cdots = h_{ij}^{(m),+}$, $i, j = 1, 2, \ldots n$. Thus, all deviation variables $\varepsilon_{ij}^{(l),+}, \varepsilon_{ij}^{(l),-}, \gamma_{ij}^{(l),+}$ and $\gamma_{ij}^{(l),-}$ are equal to 0 in Model (28), and therefore $J^* = 0$.

(Necessity) If $J^* = 0$, then it is evident that $h_{ij}^{(1),-} = h_{ij}^{(2),-} = \cdots = h_{ij}^{(m),-}$ and $h_{ij}^{(1),+} = h_{ij}^{(2),+} = \cdots = h_{ij}^{(m),+}$, $i, j = 1, 2, \cdots n$. Based on Eq. (26), we have $\tilde{P}^{(1)} = \tilde{P}^{(2)} = \cdots = \tilde{P}^{(m)}$. Thus $c(\tilde{P}^{(l)}, \tilde{P}^{(k)}) = 1$ holds for any $l, k \in \{1, 2, \ldots, m\}$. □

In conclusion, the process of group decision making method based on DEA cross-efficiency with intuitionistic fuzzy preference relations can be summarized as follows:

Step 1 Use interval transform function τ to translate $\tilde{P}^{(l)} = (\tilde{p}_{ij}^{(l)})_{n\times n} = \left(\left\langle \mu_{ij}^{(l)}, \nu_{ij}^{(l)} \right\rangle\right)_{n\times n}$ into $\tilde{H}^{(l)} = \left([h_{ij}^{(l),-}, h_{ij}^{(l),+}]\right)_{n\times n}$ based on Eq. (26), $l = 1, 2, \ldots, m$;
Step 2 Solve Model (28) to get the optimal weight vector $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)^T$ of decision makers;
Step 3 Aggregate $\tilde{H}^{(l)} = \left([h_{ij}^{(l),-}, h_{ij}^{(l),+}]\right)_{n\times n}$ into the synthetic preference relation $H^s = \left([h_{ij}^{s,-}, h_{ij}^{s,+}]\right)_{n\times n}$ based on Eq. (27);
Step 4 For multiplicative preference relation $H^s = \left([h_{ij}^{s,-}, h_{ij}^{s,+}]\right)$, obtain the normalized intuitionistic fuzzy priority weight vector $\tilde{\varpi} = (\tilde{\varpi}_1, \tilde{\varpi}_2, \ldots, \tilde{\varpi}_n)^T$ based on the ranking method of IFPR in Sect. 3.3, where $\tilde{\varpi}_i = \left\langle \varpi_i^{\mu}, \varpi_i^{\nu} \right\rangle$, $i = 1, 2, \ldots, n$;
Step 5 Rank the normalized intuitionistic fuzzy priority weights $\tilde{\varpi}_i$ in the descending order based on Definition 6, $i = 1, 2, \ldots, n$.
Step 6 Rank the alternatives x_i $(i = 1, 2, \ldots, n)$ according to the ranking of $\tilde{\varpi}_i$, $i = 1, 2, \cdots, n$.
Step 7 End.

Let $\tilde{P}^{(l)} = (\tilde{p}_{ij}^{(l)})_{n\times n} = \left(\left\langle \mu_{ij}^{(l)}, \nu_{ij}^{(l)} \right\rangle\right)_{n\times n}$ $(l = 1, 2, \ldots, m)$ be m IFPRs, which are given by m decision makers respectively, over the alternative set $X = \{x_1, x_2, \ldots, x_n\}$, where $\mu_{ij}, \nu_{ij} > 0$. Generally speaking, when all the decision makers think that alternative x_i is better than x_j, for any $k \in \{1, 2, \ldots, n\}$, then $\mu_{ik}^{(l)} \geq \mu_{jk}^{(l)}$ and $\nu_{ik}^{(l)} \leq \nu_{jk}^{(l)}$, $l = 1, 2, \ldots, m$. Then, we have the following theorem.

Theorem 5 *Let $\tilde{P}^{(l)} = (\tilde{p}_{ij}^{(l)})_{n\times n} = \left(\left\langle \mu_{ij}^{(l)}, \nu_{ij}^{(l)} \right\rangle\right)_{n\times n}$ $(l = 1, 2, \ldots, m)$ be m IFPRs, which are given by m decision makers respectively. If for any $k \in \{1, 2, \ldots, n\}$ and $l \in \{1, 2, \ldots, m\}$, $\mu_{ik}^{(l)} \geq \mu_{jk}^{(l)}$ and $\nu_{ik}^{(l)} \leq \nu_{jk}^{(l)}$ hold, then $\tilde{\varpi}_i \geq \tilde{\varpi}_j$, $i, j = 1, 2, \ldots, n$.*

Proof Let $\tilde{P}^{(l)}=(\tilde{p}_{ij}^{(l)})_{n\times n}=\left(\left\langle\mu_{ij}^{(l)},\nu_{ij}^{(l)}\right\rangle\right)_{n\times n}$ $(l=1,2,\ldots,m)$ be m IFPRs. From Eqs. (26) and (27), we have $\tau(\tilde{p}_{ik}^{(l)})=[h_{ik}^{(l),-},h_{ik}^{(l),+}]=[\frac{\mu_{ik}^{(l)}}{1-\mu_{ik}^{(l)}},\frac{1-\nu_{ik}^{(l)}}{\nu_{ik}^{(l)}}]$, $\tau(\tilde{p}_{jk}^{(l)})=[h_{jk}^{(l),-},h_{jk}^{(l),+}]=[\frac{\mu_{jk}^{(l)}}{1-\mu_{jk}^{(l)}},\frac{1-\nu_{jk}^{(l)}}{\nu_{jk}^{(l)}}]$, $h_{ik}^{s,-}=\prod_{l=1}^{m}\left(h_{ik}^{(l),-}\right)^{\lambda_l}$ and $h_{jk}^{s,+}=\prod_{l=1}^{m}\left(h_{jk}^{(l),+}\right)^{\lambda_l}$.

If $\mu_{ik}^{(l)}\geq\mu_{jk}^{(l)}$ and $\nu_{ik}^{(l)}\leq\nu_{jk}^{(l)}$ for all $k\in\{1,2,\ldots,n\}$ and $l\in\{1,2,\ldots,m\}$, it follows that $h_{ik}^{s,-}\geq h_{jk}^{s,-}$ and $h_{ik}^{s,+}\geq h_{jk}^{s,+}$. Assume that $u^*=(u_{1k}^*,u_{2k}^*,\ldots,u_{nk}^*)^T$ and v_k^* are the optimal weights obtained by Model (21) or (22), since

$$\theta_{jk}^-=\sum_{r=1}^{n}u_{rk}^*h_{ik}^{s,-}\Big/v_k^*,\quad\text{and}\quad\theta_{jk}^+=\sum_{r=1}^{n}u_{rk}^*h_{ik}^{s,+}\Big/v_k^*,\quad j=1,2,\ldots,n,$$

we have $\theta_{ik}^-\geq\theta_{jk}^-$ and $\theta_{ik}^+\geq\theta_{jk}^+$. According to Eq. (24), it is evident that $\theta_i^-\geq\theta_j^-$ and $\theta_i^+\geq\theta_j^+$. From Eq. (25), we have $\varpi_i^{\mu}=\frac{\theta_i^-}{\psi}$, $\varpi_j^{\mu}=\frac{\theta_j^-}{\psi}$, $\varpi_i^{\nu}=\frac{1}{\psi}\sum_{r=1,r\neq i}^{n}\theta_r^+$ and $\varpi_j^{\nu}=\frac{1}{\psi}\sum_{r=1,r\neq j}^{n}\theta_r^+$, where $\psi=\sum_{r=1}^{n}\theta_r^++\frac{1}{n-2}\max_k\{\theta_k^+-\theta_k^-\}$. Thus, the score function of $\tilde{\varpi}_i$ and $\tilde{\varpi}_j$ are $S(\tilde{\varpi}_i)=\frac{1}{\psi}(\theta_i^--\sum_{r\neq i}^{n}\theta_r^+)$ and $S(\tilde{\varpi}_j)=\frac{1}{\psi}(\theta_j^--\sum_{r\neq j}^{n}\theta_r^+)$, respectively. Consequently, we have

$$S(\tilde{\varpi}_i)-S(\tilde{\varpi}_j)=\frac{1}{\psi}\left[(\theta_i^--\theta_j^-)+(\theta_i^+-\theta_j^+)\right]\geq 0,$$

which implies that $\tilde{\varpi}_i\geq\tilde{\varpi}_j$. □

Theorem 5 shows that the obtained priority vector from our method is in accord with the decision makers' preferences. In other words, the ranking result can reflect the preference of all the decision makers.

5 Numerical examples and comparison analysis

5.1 Numerical example

In the background of knowledge economy, the competition in the market is not just the competition between enterprises, but the competition between supply chain alliances. The supply chain is a complex system built by interdependent organizations and processes, and the evaluation criteria of the supply chain involve customer service level, financial status, innovation level, production flexibility, etc. The evaluation process is somewhat subjective as the experts may not be very familiar with all the situation of the supply chains they evaluated. Thus, it is suitable for the decision makers to use the IFPRs to express their evaluation information. There are five decision makers d_1, d_2, d_3, d_4 and d_5 who provide their IFPRs on four supply chains $\{A_1,A_2,A_3,A_4\}$. After doing some pairwise comparisons, the decision makers offer five IFPRs as follows:

$$\tilde{P}^{(1)} = \begin{pmatrix} \langle 0.50, 0.50\rangle & \langle 0.50, 0.20\rangle & \langle 0.70, 0.10\rangle & \langle 0.50, 0.30\rangle \\ \langle 0.20, 0.50\rangle & \langle 0.50, 0.50\rangle & \langle 0.60, 0.20\rangle & \langle 0.30, 0.60\rangle \\ \langle 0.10, 0.70\rangle & \langle 0.20, 0.60\rangle & \langle 0.50, 0.50\rangle & \langle 0.30, 0.60\rangle \\ \langle 0.30, 0.50\rangle & \langle 0.60, 0.30\rangle & \langle 0.60, 0.30\rangle & \langle 0.50, 0.50\rangle \end{pmatrix},$$

$$\tilde{P}^{(2)} = \begin{pmatrix} \langle 0.50, 0.50\rangle & \langle 0.60, 0.20\rangle & \langle 0.60, 0.20\rangle & \langle 0.60, 0.30\rangle \\ \langle 0.20, 0.60\rangle & \langle 0.50, 0.50\rangle & \langle 0.50, 0.20\rangle & \langle 0.30, 0.50\rangle \\ \langle 0.20, 0.60\rangle & \langle 0.20, 0.50\rangle & \langle 0.50, 0.50\rangle & \langle 0.20, 0.60\rangle \\ \langle 0.30, 0.60\rangle & \langle 0.50, 0.30\rangle & \langle 0.60, 0.20\rangle & \langle 0.50, 0.50\rangle \end{pmatrix},$$

$$\tilde{P}^{(3)} = \begin{pmatrix} \langle 0.50, 0.50\rangle & \langle 0.55, 0.25\rangle & \langle 0.65, 0.20\rangle & \langle 0.35, 0.55\rangle \\ \langle 0.25, 0.55\rangle & \langle 0.50, 0.50\rangle & \langle 0.40, 0.25\rangle & \langle 0.55, 0.30\rangle \\ \langle 0.20, 0.65\rangle & \langle 0.25, 0.40\rangle & \langle 0.50, 0.50\rangle & \langle 0.60, 0.20\rangle \\ \langle 0.55, 0.35\rangle & \langle 0.30, 0.55\rangle & \langle 0.20, 0.60\rangle & \langle 0.50, 0.50\rangle \end{pmatrix},$$

$$\tilde{P}^{(4)} = \begin{pmatrix} \langle 0.50, 0.50\rangle & \langle 0.60, 0.30\rangle & \langle 0.70, 0.20\rangle & \langle 0.60, 0.30\rangle \\ \langle 0.30, 0.60\rangle & \langle 0.50, 0.50\rangle & \langle 0.50, 0.40\rangle & \langle 0.30, 0.40\rangle \\ \langle 0.20, 0.70\rangle & \langle 0.40, 0.50\rangle & \langle 0.50, 0.50\rangle & \langle 0.30, 0.60\rangle \\ \langle 0.30, 0.60\rangle & \langle 0.40, 0.30\rangle & \langle 0.60, 0.30\rangle & \langle 0.50, 0.50\rangle \end{pmatrix},$$

$$\tilde{P}^{(5)} = \begin{pmatrix} \langle 0.50, 0.50\rangle & \langle 0.60, 0.30\rangle & \langle 0.75, 0.15\rangle & \langle 0.60, 0.20\rangle \\ \langle 0.30, 0.60\rangle & \langle 0.50, 0.50\rangle & \langle 0.45, 0.20\rangle & \langle 0.60, 0.20\rangle \\ \langle 0.15, 0.75\rangle & \langle 0.20, 0.45\rangle & \langle 0.50, 0.50\rangle & \langle 0.40, 0.40\rangle \\ \langle 0.20, 0.60\rangle & \langle 0.20, 0.60\rangle & \langle 0.40, 0.40\rangle & \langle 0.50, 0.50\rangle \end{pmatrix}.$$

Step 1 Based on Eq. (26), $\tilde{P}^{(1)}$, $\tilde{P}^{(2)}$, $\tilde{P}^{(3)}$, $\tilde{P}^{(4)}$ and $\tilde{P}^{(5)}$ are transformed into $\tilde{H}^{(l)}$ $(l = 1, 2, 3, 4, 5)$ respectively, where

$$\tilde{H}^{(1)} = \begin{pmatrix} [1.0000, 1.0000] & [1.0000, 4.0000] & [2.3333, 9.0000] & [1.0000, 2.3333] \\ [0.2500, 1.0000] & [1.0000, 1.0000] & [1.5000, 4.0000] & [0.4286, 0.6667] \\ [0.1111, 0.4286] & [0.2500, 0.6667] & [1.0000, 1.0000] & [0.4286, 0.6667] \\ [0.4286, 1.0000] & [1.5000, 2.3333] & [1.5000, 2.3333] & [1.0000, 1.0000] \end{pmatrix},$$

$$\tilde{H}^{(2)} = \begin{pmatrix} [1.0000, 1.0000] & [1.5000, 4.0000] & [1.5000, 4.0000] & [1.5000, 2.3333] \\ [0.2500, 0.6667] & [1.0000, 1.0000] & [1.0000, 4.0000] & [0.4286, 1.0000] \\ [0.2500, 0.6667] & [0.2500, 1.0000] & [1.0000, 1.0000] & [0.2500, 0.6667] \\ [0.4286, 0.6667] & [1.0000, 2.3333] & [1.5000, 4.0000] & [1.0000, 1.0000] \end{pmatrix},$$

$$\tilde{H}^{(3)} = \begin{pmatrix} [1.0000, 1.0000] & [1.2222, 3.0000] & [1.8571, 4.0000] & [0.5385, 0.8182] \\ [0.3333, 0.8182] & [1.0000, 1.0000] & [0.6667, 3.0000] & [1.2222, 2.3333] \\ [0.2500, 0.5385] & [0.3333, 1.5000] & [1.0000, 1.0000] & [1.5000, 4.0000] \\ [1.2222, 1.8571] & [0.4286, 0.8182] & [0.2500, 0.6667] & [1.0000, 1.0000] \end{pmatrix},$$

$$\tilde{H}^{(4)} = \begin{pmatrix} [1.0000, 1.0000] & [1.5000, 2.3333] & [2.3333, 4.0000] & [1.5000, 2.3333] \\ [0.4286, 0.6667] & [1.0000, 1.0000] & [1.0000, 1.5000] & [0.4286, 1.5000] \\ [0.2500, 0.4286] & [0.6667, 1.0000] & [1.0000, 1.0000] & [0.4286, 0.6667] \\ [0.4286, 0.6667] & [0.6667, 2.3333] & [1.5000, 2.3333] & [1.0000, 1.0000] \end{pmatrix},$$

$$\tilde{H}^{(5)} = \begin{pmatrix} [1.0000, 1.0000] & [1.5000, 2.3333] & [3.0000, 5.6667] & [1.5000, 4.0000] \\ [0.4286, 0.6667] & [1.0000, 1.0000] & [0.8182, 4.0000] & [1.5000, 4.0000] \\ [0.1765, 0.3333] & [0.2500, 1.2222] & [1.0000, 1.0000] & [0.6667, 1.5000] \\ [0.2500, 0.6667] & [0.2500, 0.6667] & [0.6667, 1.5000] & [1.0000, 1.0000] \end{pmatrix}.$$

Step 2 By solving Model (28), we get the optimal weight vector of five decision makers, i.e., $\lambda = (0.0883,\ 0.3689,\ 0.0941,\ 0.2657,\ 0.1830)^T$;
Step 3 Using Eq. (27), $\tilde{H}^{(l)}$ $(l = 1, 2, 3, 4, 5)$ are aggregated into the synthetic preference relation

$$H^s = \begin{pmatrix} [1.0000, 1.0000] & [1.4196, 3.0567] & [2.0316, 4.5796] & [1.3142, 2.3333] \\ [0.3271, 0.7044] & [1.0000, 1.0000] & [0.9616, 3.0000] & [0.5949, 1.5000] \\ [0.2184, 0.4922] & [0.3333, 1.0399] & [1.0000, 1.0000] & [0.4286, 0.9154] \\ [0.4286, 0.7609] & [0.6667, 1.6809] & [1.0924, 2.3333] & [1.0000, 1.0000] \end{pmatrix}.$$

Step 4 According to the ranking method for IFPR in Sect. 3.3 with Model (21), we obtain the normalized intuitionistic fuzzy priority weight vector

$$\tilde{\varpi} = (\langle 0.2798, 0.5975\rangle, \langle 0.1036, 0.7164\rangle, \langle 0.0694, 0.7929\rangle, \langle 0.1318, 0.7133\rangle)^T.$$

Meanwhile, if we use Model (22) instead of Model (21), we have

$$\varpi' = (\langle 0.1866, 0.5155\rangle, \langle 0.1043, 0.7059\rangle, \langle 0.0646, 0.7889\rangle, \langle 0.0923, 0.6918\rangle)^T.$$

Step 5 Based on Definition 6, the ranking of the normalized intuitionistic fuzzy priority weight vector $\tilde{\varpi}$ in the descending order is as follows:

$$\tilde{\varpi}_1 = \langle 0.2798, 0.5975\rangle > \tilde{\varpi}_4 = \langle 0.1318, 0.7133\rangle > \tilde{\varpi}_2' = \langle 0.1036, 0.7164\rangle > \tilde{\varpi}_3 = \langle 0.1318, 0.7133\rangle$$

Additionally, the ranking of ϖ' is

$$\tilde{\varpi}_1' = \langle 0.1866, 0.5155\rangle > \tilde{\varpi}_4' = \langle 0.0923, 0.6918\rangle > \tilde{\varpi}_2' = \langle 0.1043, 0.7059\rangle > \tilde{\varpi}_3' = \langle 0.0646, 0.7889\rangle$$

As can be seen, $\tilde{\varpi}$ and ϖ' have the same ranking results.
Step 6 The final ranking of the four alternative supply chains is $A_1 > A_4 > A_2 > A_3$.
Step 7 Thus, A_1 is the best supply chain.

5.2 Comparison analysis

This subsection further compares our method with several existing representative methods, which were developed in Wang (2013), Meng et al. (2017), Xu (2012) and Wu et al. (2018). First of all, we use these methods to solve the above problem respectively.

(1) Using Wang (2013)'s Method. Using the linear goal programming model (4.18) in Wang (2013), we obtain the normalized intuitionistic fuzzy weight vector:

$$\tilde{w} = (\langle 0.4701, 0.4709\rangle, \langle 0.0749, 0.6250\rangle, \langle 0, 0.9034\rangle, \langle 0.1549, 0.6626\rangle)^T.$$

Moreover, the score functions of the four weights are obtained as follows:

$$S(\tilde{w}_1) = -0.0008,\ S(\tilde{w}_2) = -0.5501,\ S(\tilde{w}_3) = -0.9034,\ S(\tilde{w}_4) = -0.5077.$$

So, the final ranking is $A_1 > A_4 > A_2 > A_3$, which is the same as that derived by our approach.

(2) Using Meng et al. (2017)'s Method. The intuitionistic fuzzy priority weight vector is derived using Meng et al. (2017)'s method, i.e., $\tilde{w} = (\langle 0.6146, 0.2560\rangle, \langle 0.3686, 0.3861\rangle, \langle 0.2469, 0.5385\rangle, \langle 0.4080, 0.4286\rangle)$. From Zhang and Xu (2012) for IFVs, we have

$$L(\tilde{w}_1) = 0.6588,\ L(\tilde{w}_2) = 0.4930,\ L(\tilde{w}_3) = 0.3800,\ L(\tilde{w}_4) = 0.4911.$$

Hence, the ranking is $A_1 > A_2 > A_4 > A_3$, which is different from the results derived by our approach, as well as the Wang (2013)'s method as the positions of the supply chain A_2 and A_4 are changed.

Furthermore, we will use the method in Wu et al. (2018) and Xu (2012) to obtain the priority vector of synthetic IFPR, which is yielded by aggregate the five IFPRs, i.e., $\tilde{P}^{(l)}, l = 1, 2, \ldots, 5$.

(3) Using Xu (2012)'s Method. We can get the priority weight vector,

$$\tilde{w} = ([0.2845, 0.3059], [0.2290, 0.2597], [0.1971, 0.2251], [0.2369, 0.2617])^T.$$

The following possibility degree matrix is construct to rank the priority weights.

$$P = \begin{pmatrix} 0.5 & 1 & 1 & 1 \\ 0 & 0.5 & 1 & 0.4111 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0.5889 & 1 & 0.5 \end{pmatrix}.$$

By summing all elements in each line of P, we have $p_1 = 3.5000$, $p_2 = 1.9111$, $p_3 = 0.5000$, $p_4 = 2.0889$. Hence, the ranking of the four alternative supply chains is $A_1 > A_4 > A_2 > A_3$, which is the same as that derived by our approach.

(4) Using Wu et al. (2018)'s Method. The crisp priority vector is obtained by using the arithmetic mean operator:

$$w_1 = 0.8750, w_2 = 0.4430, w_3 = 0.1250, w_4 = 0.5570.$$

And thus the final ranking of the four alternative supply chains is $A_1 > A_4 > A_2 > A_3$, which is the same as that derived by our approach.

The above numerical example shows that our approach produces the same ranking order, i.e., $A_1 > A_4 > A_2 > A_3$, with Xu's, Wu et al.'s and Wang's methods, while Meng et al.'s method derives a slightly different order in which the positions of A_2 and A_4 are changed, which is a further proof of the credibility and reliability of our approach.

We further illustrate the advantages of our method by the Example 2 compared with the four existing representative methods.

Example 2 Let $\tilde{P}$ be an IFPR on alternative set $X = \{x_1, x_2, x_3, x_4\}$, and

$$\tilde{P} = \begin{pmatrix} \langle 0.50, 0.50 \rangle & \langle 0.35, 0.55 \rangle & \langle 0.40, 0.40 \rangle & \langle 0.35, 0.55 \rangle \\ \langle 0.55, 0.35 \rangle & \langle 0.50, 0.50 \rangle & \langle 0.70, 0.10 \rangle & \langle 0.60, 0.20 \rangle \\ \langle 0.40, 0.40 \rangle & \langle 0.10, 0.70 \rangle & \langle 0.50, 0.50 \rangle & \langle 0.70, 0.20 \rangle \\ \langle 0.55, 0.35 \rangle & \langle 0.20, 0.60 \rangle & \langle 0.20, 0.70 \rangle & \langle 0.50, 0.50 \rangle \end{pmatrix}.$$

Using our approach, we have

$$\tilde{w} = (\langle 0.1145, 0.7653 \rangle, \langle 0.2034, 0.5490 \rangle, \langle 0.1106, 0.6983 \rangle, \langle 0.1101, 0.7397 \rangle)^T,$$

then the ranking order is $x_2 > x_3 > x_4 > x_1$.

Using Wang (2013)'s Method, we have

$$S(\tilde{w}_1) = -0.6237, S(\tilde{w}_2) = 0.1239, S(\tilde{w}_3) = -0.6237, S(\tilde{w}_4) = -0.6761,$$

by which the ranking order is $x_2 > x_1 = x_3 > x_4$.

Using Xu (2012)'s method, we have

$$p_1 = 1.5309, p_2 = 3.5000, p_3 = 2.2753, p_4 = 0.6938,$$

which means that the ranking order is $x_2 > x_3 > x_1 > x_4$.

Using Wu et al. (2018)'s Method, we have

$$w_1 = 0.2250, w_2 = 0.8750, w_3 = 0.2750, w_4 = 0.6250,$$

by which the ranking order is $x_2 > x_4 > x_3 > x_1$.

Using Meng et al. (2017)'s Method, since $\tilde{P}$ has poor consistency, and it can be adjusted to a multiplicative consistent IFPR $\tilde{T}$.

$$\tilde{T}=\begin{pmatrix} \langle 0.5000, 0.5000\rangle & \langle 0.2799, 0.7066\rangle & \langle 0.3747, 0.4243\rangle & \langle 0.4372, 0.3443\rangle \\ \langle 0.7066, 0.2799\rangle & \langle 0.5000, 0.5000\rangle & \langle 0.5906, 0.2227\rangle & \langle 0.6517, 0.1695\rangle \\ \langle 0.4243, 0.3747\rangle & \langle 0.2227, 0.5906\rangle & \langle 0.5000, 0.5000\rangle & \langle 0.5646, 0.4161\rangle \\ \langle 0.3443, 0.4372\rangle & \langle 0.1695, 0.6517\rangle & \langle 0.4161, 0.5646\rangle & \langle 0.5000, 0.5000\rangle \end{pmatrix}.$$

Then, from $\tilde{T}$, we have

$$L(\tilde{w}_1)=0.4526,\ L(\tilde{w}_2)=0.6891,\ L(\tilde{w}_3)=0.4680,\ L(\tilde{w}_4)=0.3910.$$

Thus, the ranking order is $x_2 > x_3 > x_1 > x_4$.

Comparing the ranking of $x_i (i = 1, 2, \ldots, 4)$ derived by our approach with those of Wang (2013), Xu (2012), Wu et al. (2018) and Meng et al. (2017), we find that these methods generate a different ranking, especially on x_1 and x_4. From the IFPR matrix $\tilde{P} = (\tilde{p}_{ij})_{4\times4}$, we have $\tilde{p}_{41} = \langle 0.55, 0.35\rangle$, which means the certainty degree that x_4 is preferred to x_1 is 0.55, while the certainty degree of x_4 is non-preferred to x_1 is 0.35. It is obvious that x_1 should not be superior to x_4. Therefore, the ranking result of our method is more reasonable than that of Xu (2012), Wu et al. (2018), Wang (2013) and Meng et al. (2017) in this case. What's more, from the multiplicative consistent IFPR $\tilde{T} = (\tilde{t}_{ij})_{4\times4}$ adjusted from $\tilde{P}$, we have $\tilde{t}_{41} = \langle 0.3443, 0.4372\rangle$, which is quite different from $\tilde{p}_{41} = \langle 0.55, 0.35\rangle$. The distortion of the given decision making information makes the decision result less reliable.

As can be seen, compared with Xu (2012), Wu et al. (2018), Wang (2013) and Meng et al. (2017), our proposed method has the following advantages:

(1) In Meng et al. (2017)'s method, the inconsistent IFPRs need to be repaired to yield the acceptable consistent IFPRs, which can distort the given decision making information and make the decision result less reliable. Besides, the inconsistency repairing process is complex and tedious. As to our method, the decision making process is based entirely on the original preference information given by decision makers and does not require consistent improving. Thus, our method can avoid information distortion and the result is more credible.
(2) In Wu et al. (2018)'s method and Xu (2012)'s method, the priority vector of IFPR is given in the form of interval, which is not normalized. From Theorem 3, we find that our method can yield the normalized intuitionistic fuzzy weight vector.
(3) In Wu et al. (2018)'s method and Wang (2013)'s method, it is assumed that there exist some relationships between priority weight vector and consistent IFPRs, then the optimization models are established based on these assumptions to get the priority weight vector. Since these assumptions are based on consistent IFPR, the obtained priority weight vectors are always unreasonable for some inconsistent IFPRs. Besides, these assumptions are not unique. By contrast, our approach does not need to make any assumptions in advance, so it can avoid the risk of assumption dependent.

In summary, our method does not need consistency adjustment, making any assumptions and can yield a normalized intuitionistic fuzzy weight vector. In particular, when the IFPR given by the decision maker has a poor consistency, our method has better applicability and can derive more reasonable ranking result than some known methods.

6 Conclusions

This paper has been studying group decision making based on DEA cross-efficiency with intuitionistic fuzzy preference relations.

Firstly, the defined interval transform function has established the substantial relationship between IFPRs into IMPRs, and it has been proved that the transform function can ensure the consistency of the relationship. Then, an interval transform function based DEA model is developed to obtain the ranking vector of consistent IFPR, in which each of the alternatives is viewed as a DMU. Moreover, for any IFPR, we proposed two DEA cross-efficiency models to get the peer-evaluation cross-efficiency values of all alternatives, and we can calculate the normalized intuitionistic fuzzy weight vector of the IFPR based on the cross-efficiency values. Because the proposed normalized intuitionistic fuzzy weight vector derivation method from IFPR does not need repairing the given IFPR, it can avoid information distortion and obtain more credible decision making results. A goal programming model is investigated to derive the weights of decision makers. Meanwhile, a step-by-step procedure for group decision making approach based on DEA cross-efficiency with intuitionistic fuzzy preference relations is presented. Finally, the proposed approach is verified by numerical examples and the comparison analysis, by which one can find the advantages of the new method.

To the best of our knowledge, employing the theory of DEA cross-efficiency to the group decision making with IFPRs are completely new to the literature and have not been studied elsewhere before. Probably, we also can use the DEA cross-efficiency model to any other decision making environments, such as triangular fuzzy preference relations, trapezoidal fuzzy preference relation.

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