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Stackelberg game models between two competitive retailers in fuzzy decision environment

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Abstract In this paper, we study the pricing problem in a fuzzy supply chain that consists of a manufacturer and two competitive retailers. There is a single product produced by a manufacturer and then sold by two competitive retailers to the consumers. The manufacturer acting as a leader determines the wholesale price, and the retailers acting as the followers set their sale prices independently. Both the manufacturing cost and the demand for product are characterized as fuzzy variables, we analyze how the manufacturer and the retailers make their pricing decisions with the duopolistic retailers' different behaviors: competition strategy and collusion strategy, and develop the expected value models in this paper. Finally, numerical examples illustrate the effectiveness of the proposed two-echelon models using fuzzy set theory.

Keywords Supply chain · Stackelberg game · Collusion · Competition · Fuzzy variable · Pricing decisions

1 Introduction

Modern supply chains are complex networks consisting of many firms working together, each firm's primary objective is its own profit, as a result, an important issue is the coordination of disparate members to achieve optimal supply chain performance. A considerable supply chain system in this issue is modeled as Stackerberg

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structure, in which a manufacturer acts as a Stackelberg leader, and a retailer/buyer acts as a follower, the demand in the market is assumed that it varies with the product's price based on some deterministic "demand functions", the manufacturer who maximizes his profit by receiving orders from the retailers; While the retailer hopes her order to match the demands of the customers, to minimize her inventory cost and to maximize her profit, see (Jeuland and Shugan 1983, 1988; Shugan 1985) for more details. Opposite to the manufacturer who acts as a leader, the retailers are often much larger than the manufacturer in channel (Walmart, B&Q). Another Stackelberg structure is that the retailer acts as a Stackelberg leader, and the manufacturer acts as the followers, see (Choi 1996; Ertek and Griffin 2003) for more details.

Other related two-echelon Stackelberg models focus on not only a single retailer but multiple retailers, commonly, two duopolistic retailers. Choi (1991) investigated price competition in a channel structure, in which the retailer acts as a supply chain leader in a Stackelberg game. Ingene and Parry (1995) explored the impact of two retailers' Cournot behavior in the channel under the assumption that demand functions are linear. Yang and Zhou (2006) analyzed the effects of the two duopolistic retailers' different competitive behaviors, Cournut, collusion, and Stackelberg, on the optimal decisions of the manufacturer and the duopolistic retailers themselves. Wang et al. (2012) extended Yang and Zhou (2006)'s work, and researched the duopolistic retailer behavior and the non-linear demand function problem. Li and Huo (2010) investigated the pricing and coordination decisions under a Stackelberg structure, in which the manufacturer acting as a leader declares her wholesale price and implements a common-replenishment epochs (CRE) schedule to competitive retailers. Raul et al. (2011) proposed a simulation method to transform competitive supply networks into collaborative supply networks.

All studies mentioned above assume that the customer demand is uncertain, which involves a type of probability distribution. This assumption seems to be restrictive. Firstly, in real world situations, there is a lack of statistical data to forecast the demand. For example, for iphone 5, a new digital product produced by apple Inc., there is no historical data available to the decision maker. Secondly, perhaps due to recent changes, *"probability distribution may simply not be available, or may not be easily or accurately estimated"* (Xie et al. 2006), such as strike by employees, logistics out of control. In addition, Handfield et al. (2009) pointed out that because of time constraints or other reasons, it is impossible to collect data on the random variables of interests in some cases. For example, the parameters a_i and θ, i.e., the measure of sensitivity of retailer's sales to the change of retailer's price, whose accurate values are time consuming and expensive to be determined.

Compared with the stochastic model, it is more time efficient to estimate the uncertainty parameters by the experts or the senior managers with fuzzy linguistic form. Fuzzy set theory proposed by Zadeh (1965) is a suitable tool to characterize the experts' or the senior managers' judges, which has many applications on supply chain management. Petrovic et al. (1998, 1999) firstly used fuzzy set theory in supply chain management. Later, fuzzy set theory was applied to Supplier selection (Kumar et al. 2004; Chen et al. 2006), supply chain network design (Xu et al. 2009), supply chain planning (Torabi and Hassini 2008; Selim et al. 2008), and supply chain coordination (Ryu and Yücesan 2010; Sinha and Sarmah 2008), etc.

In recent supply chain two-echelon coordination work, some researchers have already adopted fuzzy set theory to depict uncertainty of demand, cost, etc. Chen et al. (2006) formulated a game framework to investigate the behavior of supply chain partners based on fuzzy multi-objective programming. Ryu and Yücesan (2010) proposed a fuzzy newsvendor approach for supply chain coordination. Yu and Jin (2011) studied the return policy model with fuzzy demand and asymmetric information. Chen and Ho (2011) proposed an analysis method for single period inventory problem with fuzzy demands and incremental quantity discounts. Sinha and Sarmah (2008) designed a coordination mechanism through quantity discount policy under uncertain costs and fuzzy demands.

Thus, so far, to our knowledge, there have been seldom studies on pricing problems considering retailer competitive behaviors in fuzzy environment. Liang et al. (2008) developed a new optimum output quantity decision analysis of a duopoly market under a fuzzy decision environment. Zhao et al. (2012a,b) studied the pricing problem of substitutable products in a supply chain with one manufacturer and two competitive retailers, they developed one centralized pricing model and three decentralized pricing models using game theoretic approach. However, how the supply chain members make decisions with duopolistic retailers' different behaviors under fuzzy environment is an interesting issue, and as far as we know, no study has been done on it by now.

This paper extends Yang and Zhou (2006) work by characterizing the manufacturing cost and the customer demand with fuzzy variables, proposes two-echelon supply chain models under fuzzy environment, and solves them using credibility theory proposed by Nahmias (1978) and Liu (2002). The rest of the paper is organized as follows: Sect. 2 introduces some preliminaries. The two expected value models are proposed under retailers' different behaviors in Sect. 3, and the corresponding propositions are given under different scenarios. Numerical examples are given to illustrate the proposed models in Sect. 4. Section 5 discusses the contributions and limitations of this paper, and points out some further research directions.

2 Preliminaries

A possibility space is defined as a triplet, $(\Theta, p(\Theta), Pos)$, where Θ is a nonempty set, $p(\Theta)$ is the power of Θ, and Pos is a possibility measure. Each element in $p(\Theta)$ is called a fuzzy event. For each event A, $Pos(A)$ indicates the possibility that A will occur. Nahmias (1978) and Liu (2002) gave the following four axioms:

Axiom 1 $Pos(\Theta) = 1$.

Axiom 2 $Pos(\phi) = 1$, where ϕ denotes an empty set.

Axiom 3 $Pos\left(\bigcup_{i=1}^{m} A_i\right) = \sup_{1 \le i \le m} Pos(A_i)$, for any collection of events $A_i (i = 1, 2, \ldots, m)$ in $p(\Theta)$.

Axiom 4 Let Θ_i be a nonempty set, on which $Pos_i(\Theta_i)(i = 1, 2, \ldots, n)$ are the possibility measures satisfying the first three axioms, and $\Theta = \prod_{i=1}^{n} \Theta_i$. Then

$$Pos(A) = sup_{(\theta_1, \theta_2, \ldots, \theta_n) \in A} \{Pos_1(\theta_1) \wedge Pos_2(\theta_2) \wedge \cdots \wedge Pos_n(\theta_n)\}$$

for each $A \in p(\Theta)$. In that case we denote $Pos = \wedge_{i=1}^{n} Pos_i$.

Lemma 1 (Liu 2002) *Suppose that* $(\Theta_i, p(\Theta_i), Pos_i)$ $(i = 1, 2, \ldots n)$ *are a collection of possibility spaces. By Axiom 4,* $\left(\prod_{i=1}^{n} \Theta_i, p\left(\prod_{i=1}^{n} \Theta_i\right), \wedge_{i=1}^{n} Pos_i\right)$ *is also a possibility space, which is called a product possibility space.*

Definition 1 (Nahmias 1978) A fuzzy variable is defined as a function from the possibility space $(\Theta, p(\Theta), Pos)$ to the set of real numbers and its membership function is derived from

$$\mu_{\xi}(x) = Pos(\{\theta \in \Theta \,|\, \xi(\theta) = x\}), \quad \forall x \in R$$

Definition 2 (Liu 2002) A fuzzy variable ξ is nonnegative (or positive) if

$$Pos(\{\xi < 0\}) = 0 \ \ (\text{or } Pos(\{\xi \le 0\}) = 0)$$

Definition 3 (Liu 2002) Let $f : R^n \to R$ be a function, and $\xi_i (i = 1, 2, \ldots n)$ be a collection of fuzzy variables defined on the possibility spaces $(\Theta_i, p(\Theta_i), Pos_i)$ $(i = 1, 2, \ldots n)$, respectively. Then $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$ is a fuzzy variable defined on the product possibility space $\left(\prod_{i=1}^{n} \Theta_i, p\left(\prod_{i=1}^{n} \Theta_i\right), \wedge_{i=1}^{n} Pos_i\right)$.

The independence of fuzzy variable has been discussed by several researchers, such as (Liu 2002; Nahmias 1978), and (Zadeh 1978).

Definition 4 (Liu 2002) The fuzzy variables $\xi_i (i = 1, 2, \ldots, n)$ are independent if for any sets $B_i (i = 1, 2, \ldots, n)$ of R:

$$Pos(\xi_i \in B_i, i = 1, 2, \ldots n) = \min_{1 \le i \le n} Pos(\{\xi_i \in B_i\})$$

Lemma 2 (Liu 2002) *Let* $\xi_i (i = 1, 2, \ldots, n)$ *be a collection of independent fuzzy variables, and* $f_i : R \to R$ $(i = 1, 2, \ldots, n)$ *be a collection of functions. Then* $f_i(\xi_i)$ $(i = 1, 2, \ldots, n)$ *are also the independent fuzzy variables.*

Definition 5 (Liu 2002) Let ξ be a fuzzy variable on the possibility space $(\Theta, p(\Theta), Pos)$ and $\alpha \in (0, 1]$. Then

$$\xi_{\alpha}^{L} = \inf\left[r \,|Pos([\xi \le r]) \ge \alpha\right], \ \xi_{\alpha}^{U} = \sup\left[r \,|Pos([\xi \ge r]) \ge \alpha\right]$$

are called the α—pessimistic value and the α —optimistic value of ξ, respectively.

Example 1 The triangular fuzzy variable $\xi = (a_1, a_2, a_3)$ has its α—pessimistic value and the α—optimistic value respectively:

$$\xi_{\alpha}^{L} = a_2\alpha + a_1(1 - \alpha), \ \xi_{\alpha}^{U} = a_2\alpha + a_3(1 - \alpha)$$

Lemma 3 (Wang et al. 2007) *Let* $\xi_i (i = 1, 2, \ldots, n)$ *be a collection of independent fuzzy variables defined on the possibility space* $(\Theta_i, p(\Theta_i), Pos_i)$*, and* $f : X \subset R^n \to R$ *be a measurable function. If* $f(x_1, x_2, \ldots, x_n)$ *is monotonic with respect to* x_i*, respectively, then*

(a) $f_\alpha^U(\xi) = f(\xi_{1\alpha}^V, \xi_{2\alpha}^V, \ldots, \xi_{n\alpha}^V)$, *where* $\xi_{i\alpha}^V = \xi_{i\alpha}^U$, *if* $f(x_1, x_2, \ldots, x_n)$ *is non-decreasing with respect to* x_i, *then* $\xi_{i\alpha}^V = \xi_{i\alpha}^L$; *Otherwise,*

(b) $f_\alpha^L(\xi) = f(\xi_{1\alpha}^{\bar V}, \xi_{2\alpha}^{\bar V}, \ldots, \xi_{n\alpha}^{\bar V})$, *where* $\xi_{i\alpha}^{\bar V} = \xi_{i\alpha}^L$, *if* $f(x_1, x_2, \ldots, x_n)$ *is non-decreasing with respect to* x_i, *then* $\xi_{i\alpha}^{\bar V} = \xi_{i\alpha}^U$, *where* $f_\alpha^U(\xi)$ *and* $f_\alpha^L(\xi)$ *denote the α—optimistic value and the α—pessimistic value of the fuzzy variable* $f(\xi)$, *respectively.*

Definition 6 (Liu and Liu 2002) Let $(\Theta, p(\Theta), Pos)$ be a possibility space and A be a set in $p(\Theta)$. The credibility measure of A is defined as:

$$Cr(A) = \frac{1}{2}\left(1 + Pos(A) - Pos(A^c)\right)$$

where A^c denotes the complement of A.

Definition 7 (Liu and Liu 2002) Let ξ be a fuzzy variable, the expected value of ξ is defined as:

$$E[\xi] = \int_0^{+\infty} Cr(\xi \geq x)dx - \int_{-\infty}^0 Cr(\xi \leq x)dx$$

provided that at least one of the two integrals is finite.

Example 2 The triangular fuzzy variable $\xi = (a_1, a_2, a_3)$ has an expected value:

$$E(\xi) = \frac{a_1 + 2a_2 + a_3}{4}$$

Definition 8 (Liu and Liu 2002) Let f be a function on $R \to R$ and ξ be a fuzzy variable, then the expected value of ξ is

$$E[\xi] = \int_0^{+\infty} Cr(f(\xi) \geq x)dx - \int_{-\infty}^0 Cr(f(\xi) \leq x)dx$$

provided that at least one of the two integrals is finite.

Lemma 4 (Liu and Liu 2002) *Let* ξ *be a fuzzy variable with a finite expected value. Then*

$$E[\xi] = \int_0^1 \left(\xi_\alpha^L + \xi_\alpha^U\right)d\alpha$$

Lemma 5 (Liu and Liu 2002) *Let* ξ *and* η *be two independent fuzzy variables with the finite expected values. Then for any numbers a and b, we have*

$$E\left[a\xi + b\eta\right] = aE\left[\xi\right] + bE\left[\eta\right]$$

3 Two-echelon supply chain models

This paper focuses on the Stackelberg game models between a manufacturer and two competitive retailers in a supply chain. The manufacturer, indexed by m, produces the product and wholesales it to the two retailers i $(i = 1, 2)$. The manufacturer and two retailers make their own pricing decisions to maximize their own expected profits, respectively. For the sake of clarity, the following notations are used to formulate the fuzzy supply chain models:

p_i : The sale price charged to customers by the retailer i, $p_i > 0$, $i = 1, 2$;
$\tilde{c}_i$: Unit cost incurred by the manufacturer, $i = 1, 2$;
$\tilde{Q}_i$: The ith retailer's quantity order, $i = 1, 2$;
w : Unit wholesale price that the manufacturer charges the retailers;
π_{ri} : The ith retailer's profit, $i = 1, 2$;
π_m : The manufacturer's profit;

Similar to (Mcguire and Staelin 1984), the ith retailer's demand function is defined as a linear form of the two retailers' prices, which is downward sloping in its own price, increasing with respect to the competitor's price. The demand for the retailer i can be expressed as:

$$\tilde{Q}_i = \tilde{D}_i - \tilde{a}_i p_i + \tilde{\theta} p_j, \quad i = 1, 2, \ j = 3 - i \tag{1}$$

The parameters $\tilde{\theta}$, $\tilde{D}_i$, $\tilde{a}_i (i = 1, 2)$ are fuzzy variables, where $\tilde{D}_i$ represents the market base of product, $\tilde{a}_i$ and $\tilde{\theta}$ denote the measures of the responsiveness of product's demand to its own price and to its competitor's price, respectively.

Because the expected demand for a product should be more sensitive to the changes in its own price than to the changes in the price of the other product, the makeup and the consumers' demand are nonnegative in the practical applications, thus,

$$Pos\left(\{w - \tilde{c}\}\right) = 0, \ Pos\left(\left\{\tilde{D}_i - \tilde{a}_i p_i + \tilde{\theta} p_j < 0\right\}\right) = 0, \quad i = 1, 2, \ j = 3 - i$$

We make the following assumptions:

A1. All activities occur within a single period.
A2. The fuzzy parameters $\tilde{\theta}$, $\tilde{D}_i$, $\tilde{a}_i$ $(i = 1, 2)$ are independent and nonnegative, the parameters $\tilde{a}_i$ and $\tilde{\theta}$ satisfy $E[\tilde{a}_i] > E[\tilde{\theta}]$, which means that the expected demand for product should be more sensitive to change in its retailer's price than to change in the competitor's price.
A3. The manufacturer and the two retailers have the prefect information of demand and cost structures of other members.
A4. The logistic cost components of manufactures and retailers, e.g., transportation cost and inventory, etc., are not considered for convenience.

Therefore, the retailer's profit functions can be expressed as:

$$\pi_{ri} = (p_i - c)\tilde{Q}_i, \quad i = 1, 2 \tag{2}$$

The manufacturer's profit function can be expressed as:

$$\pi_m = (w - c)\sum_{i=1}^{2} \tilde{Q}_i, \quad i = 1, 2 \tag{3}$$

3.1 Two-echelon model with two retailers' Cournot behaviors

In this two-echelon Stackelberg game case, the manufacturer is a leader, and two retailers are the followers. The two retailers adopt the Cournot behaviors, then the following game model can be formulated:

$$\begin{cases} \max_w E\left[\pi_m\left(w, p_i^*\right)\right] \\ \text{s.t. } Pos\left(\{w - \tilde{c}\}\right) = 0 \\ \quad p_i^*(w), i = 1, 2 \text{ are derived from solving the problem :} \\ \quad \max_{p_i} E\left[\pi_{ri}\left(p_1, p_2\right)\right], \quad i = 1, 2 \\ \quad \text{s.t. } Pos\left(\left\{\tilde{D}_i - \tilde{a}_i p_i + \tilde{\theta} p_j < 0\right\}\right) = 0, \quad i = 1, 2 \\ \quad\quad p_i > w, \quad i = 1, 2 \end{cases} \tag{4}$$

Proposition 1 *If the retailers adopt the Cournot behaviors, given the wholesale prices w made earlier by the manufacturer, then two retailers' optimal retailer prices are*

$$p_1^* = A_1 w + B_1 \tag{5}$$

$$p_2^* = A_2 w + B_2 \tag{6}$$

under the conditions that

$$Pos\left(\left\{\tilde{D}_1 - \tilde{a}_1\left(A_1 w + B_1\right) + \tilde{\theta}\left(A_2 w + B_2\right) < 0\right\}\right) = 0$$

$$Pos\left(\left\{\tilde{D}_2 - \tilde{a}_2\left(A_2 w + B_2\right) + \tilde{\theta}\left(A_1 w + B_1\right) < 0\right\}\right) = 0$$

where

$$A_1 = \frac{2E[\tilde{a}_1]E[\tilde{a}_2] + E[\tilde{\theta}]E[\tilde{a}_2]}{4E[\tilde{a}_1]E[\tilde{a}_2] - (E(\tilde{\theta}))^2}, \quad B_1 = \frac{2E[\tilde{a}_2]E[\tilde{D}_1] + E[\tilde{\theta}]E[\tilde{D}_2]}{4E[\tilde{a}_1]E[\tilde{a}_2] - (E(\tilde{\theta}))^2}$$

$$A_2 = \frac{2E[\tilde{a}_1]E[\tilde{a}_2] + E[\tilde{\theta}]E[\tilde{a}_1]}{4E[\tilde{a}_1]E[\tilde{a}_2] - (E(\tilde{\theta}))^2}, \quad B_2 = \frac{2E[\tilde{a}_1]E[\tilde{D}_2] + E[\tilde{\theta}]E[\tilde{D}_1]}{4E[\tilde{a}_1]E[\tilde{a}_2] - (E(\tilde{\theta}))^2}$$

Proof Note that the fuzzy variables $\tilde{\theta}, \tilde{D}_i, \tilde{a}_i \ (i = 1, 2)$ are all independent and nonnegative, then by Lemma 3, the expected profit is

$$
\begin{aligned}
E(\pi_{ri}) &= E((p_i - w)\tilde{Q}_i) \\
&= \frac{1}{2}\int_0^1 \left\{(p_i - w)\left(\tilde{D}^L_{i\alpha} - \tilde{\alpha}^U_{i\alpha} p_i + \tilde{\theta}^L p_j\right)\right. \\
&\quad \left. + (p_i - w)\left(\tilde{D}^U_{i\alpha} - \tilde{\alpha}^L_{i\alpha} p_i + \tilde{\theta}^U p_j\right)\right\} d\alpha \\
&= (p_i - w)(E(\tilde{D}_i) - E[\tilde{a}_i]p_i + E(\tilde{\theta})p_j), \quad i = 1, 2, j = 3 - i \qquad (7)
\end{aligned}
$$

and the first-order and second-order partial derivatives of $E[\pi_{ri}(p_i, p_j)]$ with respect to (p_1, p_2) are shown as:

$$
\frac{dE\left[\pi_{r1}\left(p_1, p_2\right)\right]}{dp_1} = E[\tilde{D}_1] - 2E[\tilde{a}_1]p_1 + E[\tilde{\theta}]p_2 + wE[\tilde{a}_1] \qquad (8)
$$

$$
\frac{dE\left[\pi_{r2}\left(p_i, p_j\right)\right]}{dp_2} = E[\tilde{D}_2] - 2E[\tilde{a}_2]p_2 + E[\tilde{\theta}]p_1 + wE[\tilde{a}_2] \qquad (9)
$$

$$
\frac{d^2E[\pi_{ri}(p_i, p_j)]}{dp_i^2} = -2E[\tilde{a}_i] < 0 \qquad (10)
$$

It follows from Eq. (10) that the expected profit $E[\pi_{ri}(p_i, p_j)]$ of the retailer i is a concave function of p_i. Let Eqs. (8) and (9) be equal to zero, then we get the first conditions:

$$
E[\tilde{D}_1] - 2E[\tilde{a}_1]p_1 + E[\tilde{\theta}]p_2 + wE[\tilde{a}_1] = 0 \qquad (11)
$$

$$
E[\tilde{D}_2] - 2E[\tilde{a}_2]p_2 + E[\tilde{\theta}]p_1 + wE[\tilde{a}_2] = 0 \qquad (12)
$$

solving Eqs. (11) and (12), we can easily have Eqs. (5) and (6). □

Knowing the two retailers' reaction functions, the manufacturer sets the optimal wholesale price to maximize his expected profit $E\left[\pi_m\left(w, p_1^*(w), p_2^*(w)\right)\right]$.

Proposition 2 *With the two retailers' Cournot behaviors, the manufacturer's optimal wholesale price is*

$$
w^* = \frac{A_1E[(\tilde{\theta}-\tilde{a}_1)\tilde{c}] + A_2E[(\tilde{\theta}-\tilde{a}_2)\tilde{c}] - (E[\tilde{\theta}] - E[\tilde{a}_1])B_1 - (E[\tilde{\theta}] - E[\tilde{a}_2])B_2 - E[\tilde{D}_1] - E[\tilde{D}_2]}{2\{(E[\tilde{\theta}] - E[\tilde{a}_2])A_1 + (E[\tilde{\theta}] - E[\tilde{a}_2])A_2\}} \qquad (13)
$$

under the condition that

$$
Pos\left(\left\{\begin{array}{c}
\dfrac{A_1E[(\tilde{\theta}-\tilde{a}_1)\tilde{c}] + A_2E[(\tilde{\theta}-\tilde{a}_2t)\tilde{c}] - (E[\tilde{\theta}] - E[\tilde{a}_1])B_1}{2\{(E[\tilde{\theta}] - E[\tilde{a}_2])A_1 + (E[\tilde{\theta}] - E[\tilde{a}_2])A_2\}} \\
-\dfrac{-(E[\tilde{\theta}] - E[\tilde{a}_2])B_2 - E[\tilde{D}_1] - E[\tilde{D}_2]}{2\{(E[\tilde{\theta}] - E[\tilde{a}_2])A_1 + (E[\tilde{\theta}] - E[\tilde{a}_2])A_2\}} - \tilde{c} < 0
\end{array}\right\}\right) = 0
$$

Proof By Lemma 3, we can get the manufacturer's expected profit as follows:

$$\begin{aligned} E[\pi_m(w)] &= E[(w-\tilde{c})(\tilde{Q}_1+\tilde{Q}_2)] \\ &= -\frac{1}{2}\int_0^1 \left(\tilde{\theta}_\alpha^L - \tilde{a}_{1\alpha}^U\right)\tilde{c}_\alpha^U + \left(\tilde{\theta}_\alpha^U - a_{1\alpha}^L\right)\tilde{c}_\alpha^L d\alpha\,(A_1 w + B_1) \\ &\quad -\frac{1}{2}\int_0^1 \left(\tilde{\theta}_\alpha^L - \tilde{a}_{2\alpha}^U\right)\tilde{c}_\alpha^U + \left(\tilde{\theta}_\alpha^U - a_{2\alpha}^L\right)\tilde{c}_\alpha^L d\alpha\,(A_2 w + B_2) \\ &\quad + \{(E[\tilde{\theta}] - E[\tilde{a}_1])A_1 + (E[\tilde{\theta}] - E[\tilde{a}_2])A_2\}w^2 \\ &\quad + (E[\tilde{D}_1] + E[\tilde{D}_2])w + \{(E[\tilde{\theta}] - E[\tilde{a}_1])B_1 \\ &\quad + (E[\tilde{\theta}] - E[\tilde{a}_2])B_2\}w \\ &\quad -\frac{1}{2}\left(\int_0^1 (\tilde{D}_{1\alpha}^L + \tilde{D}_{2\alpha}^L)\tilde{c}_\alpha^U d\alpha - \int_0^1 (\tilde{D}_{1\alpha}^U + \tilde{D}_{2\alpha}^U)\tilde{c}_\alpha^L d\alpha\right) \end{aligned} \tag{14}$$

The first-order and second-order partial derivatives of $E[\pi_m(w, p_1^*(w), p_2^*(w))]$ with respect to w can be shown as:

$$\begin{aligned} \frac{\partial E[\pi_m(w)]}{\partial w} &= 2\{(E[\tilde{\theta}] - E[\tilde{a}_1])A_1 + (E[\tilde{\theta}] - E[\tilde{a}_2])A_2\}w \\ &\quad + (E[\tilde{D}_1] + E[\tilde{D}_2]) + (E[\tilde{\theta}] - E[\tilde{a}_1])B_1 \\ &\quad + (E[\tilde{\theta}] - E[\tilde{a}_2])B_2 - \frac{A_1}{2}\int_0^1 \left(\tilde{\theta}_\alpha^L - \tilde{a}_{1\alpha}^U\right)\tilde{c}_\alpha^U \\ &\quad + \left(\tilde{\theta}_\alpha^U - a_{1\alpha}^L\right)\tilde{c}_\alpha^L d\alpha - \frac{A_2}{2}\int_0^1 \left(\tilde{\theta}_\alpha^L - \tilde{a}_{2\alpha}^U\right)\tilde{c}_\alpha^U + \left(\tilde{\theta}_\alpha^U - a_{2\alpha}^L\right)\tilde{c}_\alpha^L d\alpha \end{aligned} \tag{15}$$

$$\frac{\partial^2 E[\pi_m(w)]}{\partial^2 w} = 2\{(E[\tilde{\theta}] - E[\tilde{a}_1])A_1 + (E[\tilde{\theta}] - E[\tilde{a}_2])A_2\} \tag{16}$$

According to the assumption A2, $E[\tilde{\theta}] < E[\tilde{a}_1]$, and $E[\tilde{\theta}] < E[\tilde{a}_2]$, we can get $\frac{\partial^2 E[\pi_m(w)]}{\partial^2 w} < 0$. It follows from Eq. (16) that the expected profit $E\left[\pi_m\left(w, p_1^*(w), p_2^*(w)\right)\right]$ of manufacturer is a concave function of w. Let Eq. (14) be equal to zero, similar to Proposition 1, the wholesale price w^* can easily be solved as Eq. (13). □

3.2 Two-echelon model with two retailers' Collusion behaviors

In this two-echelon stackelberg game case, the manufacturer is a leader, and the two retailers are the followers. The two retailers adopt the collusion behaviors, and then the following game model can be formulated:

$$\begin{aligned}
&\max_w E\left[\pi_m\left(w, p_i^*\right)\right] \\
&\text{ s.t. } Pos\left(\{w-\tilde{c}\}\right)=0 \\
&\qquad p_i^*\left(w\right), i=1,2 \text{ are derived from solving the problem:} \\
&\quad \max_{p_i} \sum_{i=1}^{2} E\left[\pi_{ri}\left(p_1, p_2\right)\right], \quad i=1,2 \\
&\quad \text{s.t. } Pos\left(\left\{\tilde{D}_i-\tilde{a}_i p_i+\tilde{\theta} p_j<0\right\}\right)=0, \quad i=1,2 \\
&\qquad p_i>w, \quad i=1,2.
\end{aligned} \tag{17}$$

Proposition 3 *If the retailers adopt collusion behaviors, given the wholesale prices* w *made earlier by the manufacturer, then two retailers' optimal retailer prices are*

$$p_1^{**}=0.5w+B_3 \tag{18}$$

$$p_2^{**}=0.5w+B_4 \tag{19}$$

under the conditions that

$$\begin{aligned}
&Pos\left(\left\{\tilde{D}_1-\tilde{a}_1\left(0.5w+B_3\right)+\tilde{\theta}\left(0.5w+B_4\right)<0\right\}\right)=0 \\
&Pos\left(\left\{\tilde{D}_2-\tilde{a}_2\left(0.5w+B_4\right)+\tilde{\theta}\left(0.5w+B_3\right)<0\right\}\right)=0
\end{aligned}$$

where

$$B_3=\frac{E[\tilde{a}_2]E[\tilde{D}_1]+E[\tilde{\theta}]E[\tilde{D}_2]}{2(E[\tilde{a}_1]E[\tilde{a}_2]-(E(\tilde{\theta}))^2}, \quad B_4=\frac{E[\tilde{a}_1]E[\tilde{D}_2]+E[\tilde{\theta}]E[\tilde{D}_1]}{2(E[\tilde{a}_1]E[\tilde{a}_2]-(E(\tilde{\theta}))^2)}$$

Proof Note that the fuzzy variables $\tilde{\theta}, \tilde{D}_i, \tilde{a}_i \ (i=1,2)$ are all independent and nonnegative, then by Lemma 3, the expected profit is

$$\begin{aligned}
\sum_{i=1}^{2} E(\pi_{ri}(p_i, p_j)) &= (p_1-w)(E[\tilde{D}_1]-E[\tilde{a}_1]p_1+E[\tilde{\theta}]p_2) \\
&\quad +(p_2-w)(E[\tilde{D}_2]-E[\tilde{a}_2]p_2+E[\tilde{\theta}]p_1)
\end{aligned} \tag{20}$$

The first-order and partial derivatives $\sum_{i=1}^{2} E(\pi_{ri}(p_i, p_j))$ with respect to (p_1, p_2) can be shown as:

$$\frac{\partial \sum_{i=1}^{2} E[\pi_{ri}(p_1, p_2)]}{\partial p_1} = E[\tilde{D}_1] - 2E[\tilde{a}_1]p_1 + E[\tilde{\theta}]p_2 + w(E[\tilde{a}_1] - E[\tilde{\theta}]) \tag{21}$$

$$\frac{\partial \sum_{i=1}^{2} E[\pi_{r1}(p_i, p_j)]}{\partial p_2} = E[\tilde{D}_2] - 2E[\tilde{a}_2]p_2 + E[\tilde{\theta}]p_1 + w(E[\tilde{a}_2] - E[\tilde{\theta}]) \tag{22}$$

The second-order partial derivatives of $\sum_{i=1}^{2} E(\pi_{ri}(p_i, p_j))$ with respect to (p_1, p_2) can be shown as:

$$\frac{\partial^2 \sum_{i=1}^{2} E[\pi_{ri}(p_i, p_j)]}{\partial p_1^2} = -2E[\tilde{a}_1] \tag{23}$$

$$\frac{\partial^2 \sum_{i=1}^{2} E[\pi_{ri}(p_i, p_j)]}{\partial p_2^2} = -2E[\tilde{a}_2] \tag{24}$$

$$\frac{\partial^2 \sum_{i=1}^{2} E[\pi_{ri}(p_i, p_j)]}{\partial p_1 \partial p_2} = -2E[\tilde{\theta}] \tag{25}$$

then the hessian matrix

$$H_2 = \begin{bmatrix} -2E[\tilde{a}_1] & 2E[\tilde{\theta}] \\ 2E[\tilde{\theta}] & -2E[\tilde{a}_2] \end{bmatrix}$$

is negative definite, so the expected profit $\sum_{i=1}^{2} E[\pi_{ri}(p_i, p_j)]$ is a concave function with respect to (p_1, p_2). Let Eqs. (21) and (22) be equal to zero, then we can get the first conditions:

$$E[\tilde{D}_1] - 2E[\tilde{a}_1]p_1 + E[\tilde{\theta}]p_2 + w(E[\tilde{a}_1] - E[\tilde{\theta}]) = 0 \tag{26}$$

$$E[\tilde{D}_2] - 2E[\tilde{a}_2]p_2 + E[\tilde{\theta}]p_1 + w(E[\tilde{a}_2] - E[\tilde{\theta}]) = 0 \tag{27}$$

solving Eqs. (26) and (27), we can easily have Eqs. (18) and (19). □

Knowing the two retailers' reaction functions, the manufacturer sets the optimal wholesale price to maximize his expected profit $E\left[\pi_m\left(w, p_1^{**}(w), p_2^{**}(w)\right)\right]$.

Proposition 4 *With the two retailers' collusion behaviors, the manufacturer's optimal wholesale price is*

$$w^* = \frac{0.5(E[(\tilde{\theta} - \tilde{a}_1)\tilde{c}] + E[(\tilde{\theta} - \tilde{a}_2)\tilde{c}]) - (E[\tilde{\theta}] - E[\tilde{a}_1])B_3 - (E[\tilde{\theta}] - E[\tilde{a}_2])B_4 - E[\tilde{D}_1] - E[\tilde{D}_2]}{2E[\tilde{\theta}] - E[\tilde{a}_1] - E[\tilde{a}_2]} \tag{28}$$

under the condition that

$$Pos\left(\left\{\begin{array}{l}\dfrac{0.5(E[(\tilde{\theta}-\tilde{a}_1)\tilde{c}]+E[(\tilde{\theta}-\tilde{a}_2)\tilde{c}])-(E[\tilde{\theta}]-E[\tilde{a}_1])B_1}{2E[\tilde{\theta}]-E[\tilde{a}_1]-E[\tilde{a}_2]}\\ \quad -\dfrac{-(E[\tilde{\theta}]-E[\tilde{a}_2])B_2-E[\tilde{D}_1]-E[\tilde{D}_2]}{2E[\tilde{\theta}]-E[\tilde{a}_1]-E[\tilde{a}_2]}-\tilde{c}<0\end{array}\right\}\right)=0$$

Proof Similar to the proof of Proposition 2, the wholesale price w^* can easily be solved as Eq. (28). □

4 Numerical examples

In this section, we examine a manufacturer that produces a certain new precision instrument face to the aged, has two retailers working for customers in east China, the manufacturer has no historical statistic data to make wholesale pricing decision, so the manufacturer calls for experts and senior managers as a team, and the relationship between the linguistic expressions and the triangular fuzzy variables for the manufacturing cost, market bases, and price elastic coefficients are determined by the experts' experiences (see Table 1).

There are many types of fuzzy membership functions, such as the triangular membership function, the trapezoidal fuzzy function, and the Gaussian fuzzy function, etc. The triangular distribution is one of the most commonly used in fuzzy field, *"Because of their simplicity, trigonometric shaped, and simplified versions, are widely used"* (Pedrycz 1994). Based on the above analysis, we assume that the demands and the costs obey the triangular distributions.

Table 1 Relations between linguistic expressions and triangular fuzzy variables

Scenarios	Linguistic expressions	Triangular fuzzy variables
Manufacturing cost $\tilde{c}$	Low (about 3)	(2, 3, 4)
	Medium (about 4.5)	(3, 4, 5)
	High (about 5)	(4, 5, 6)
Market base $\tilde{D}_1$	Large (about 400)	(360, 400, 440)
	Small (about 300)	(240, 300, 360)
Market base $\tilde{D}_2$	Large (about 360)	(320, 360, 400)
	Small (about 200)	(160, 200, 240)
Price elastic coefficient $\tilde{a}_1$	Very sensitive (about 20)	(18, 20, 22)
	Sensitive (about 15)	(13, 15, 17)
Price elastic coefficient $\tilde{a}_2$	Very sensitive (about 18)	(16, 18, 20)
	Sensitive (about 12)	(10, 12, 14)
Price elastic coefficient $\tilde{\theta}$	Very sensitive (about 15)	(10, 15, 20)
	Sensitive (about 10)	(8, 10, 12)

Table 2 The optimal value of price and the expected profits with fuzzy variables

Scenarios	w^*	p_1^*	p_2^*	$E[\pi_m^*]$	$E[\pi_{r1}^*]$	$E[\pi_{r2}^*]$
Cournot	23.5505	29.2493	29.9011	4,104	649.8041	725.7044
Collusion	23.5370	32.5377	33.3070	3,003	740.9016	838.7643

Consider the case that the manufacturing cost $\tilde{c}$ in the higher level is about 5, the market base $\tilde{D}_1$ and $\tilde{D}_2$ are in large level, ($\tilde{D}_1$ is about 400, $\tilde{D}_2$ is about 360), the price elastic coefficients $\tilde{a}_1, \tilde{a}_2, \tilde{\theta}$ are in sensitive level, ($\tilde{a}_1$ is about 15, $\tilde{a}_2$ is about 12, $\tilde{\theta}$ is about 10), then through Table 1, the corresponding triangular fuzzy variables are $\tilde{c} = (4, 5, 6)$, $\tilde{D}_1 = (360, 400, 440)$, $\tilde{D}_2 = (320, 360, 400)$, $\tilde{a}_1 = (18, 20, 22)$, $\tilde{a}_2 = (16, 18, 20)$, and $\tilde{\theta} = (8, 10, 12)$, respectively.

Moreover, the α—pessimistic values and the α—optimistic values of $\tilde{c}$, $\tilde{\theta}$, $\tilde{D}_i$, and $\tilde{a}_i$ $(i = 1, 2)$ are $\tilde{c}^L_\alpha = 4 + \alpha$, $\tilde{c}^U_\alpha = 6 - \alpha$, $\tilde{D}^L_{1\alpha} = 360 + 40\alpha$, $\tilde{D}^U_{1\alpha} = 440 - 40\alpha$, $\tilde{D}^L_{2\alpha} = 320 + 40\alpha$, $\tilde{D}^U_{2\alpha} = 400 - 40\alpha$, $\tilde{a}^L_{1\alpha} = 18 + 2\alpha$, $\tilde{a}^U_{1\alpha} = 22 - 2\alpha$, $\tilde{a}^L_{2\alpha} = 16 + 2\alpha$, $\tilde{a}^U_{2\alpha} = 20 - 2\alpha$, $\tilde{\theta}^L = 8 + 2\alpha$, and $\tilde{\theta}^U = 12 - 2\alpha$. The expected values of parameters are

$$E[\tilde{c}] = \frac{4 + 2 \times 5 + 6}{4} = 5, \ E[\tilde{D}_1] = \frac{360 + 2 \times 400 + 440}{4} = 400$$
$$E[\tilde{a}_2] = \frac{16 + 2 \times 18 + 20}{4} = 18, \ E[\tilde{D}_2] = \frac{320 + 2 \times 360 + 400}{4} = 360$$
$$E[\tilde{a}_1] = \frac{18 + 2 \times 20 + 22}{4} = 20, \ E[\tilde{\theta}] = \frac{8 + 2 \times 10 + 12}{4} = 10$$

From Lemmas 3 and 4, we get

$$E[(\tilde{\theta} - \tilde{a}_1)\tilde{c}_1] = -48.6667, \ E[(\tilde{\theta} - \tilde{a}_2)\tilde{c}_2] = -38.6667$$

The optimal prices and the expected profits are shown in Table 2.

In the above example, we select the fuzzy linguistic expressions:

$$\tilde{c} = (4, 5, 6), \ \tilde{D}_1 = (360, 400, 440), \ \tilde{D}_2 = (320, 360, 400)$$
$$\tilde{a}_1 = (18, 20, 22), \ \tilde{a}_2 = (16, 18, 20), \ \tilde{\theta} = (8, 10, 12)$$

If we take $c = 5$, $D_1 = 400$, $D_2 = 360$, $a_1 = 20$, $a_2 = 18$, and $\theta = 10$, then our model can be simplified as that of Yang and Zhou (2006), and then the corresponding results according to Yang and Zhou (2006)'s method, are shown in Table 3.

We compare our results with those of Yang and Zhou (2006), and find that our wholesale prices are lower, sale prices are higher, the profits of the manufacture are lower, and the profits of the retailers are higher.

It's very interesting to compare our results with those of Zhao et al. (2012a), which also use fuzzy variables to characterize the parameters in the pricing problem for substitutable products. We find that our results are completely reversed. In the model

Table 3 The optimal value of price and profits with Yang and Zhou (2006)'s model

Scenarios	w^*	p_1^*	p_2^*	$E[\pi_m^*]$	$E[\pi_{r1}^*]$	$E[\pi_{r2}^*]$
Cournot	23.6491	29.3166	29.9680	4,235	642.4020	718.7218
Collusion	23.6111	32.5748	33.3440	3,177	734.5234	832.7041

Table 4 The change of optimal solutions with the fuzzy degree of the parameter $\tilde{a}_1$

Scenarios	$\tilde{a}_1$	w^*	p_1^*	p_2^*	$E[\pi_m^*]$	$E[\pi_{r1}^*]$	$E[\pi_{r2}^*]$
Cournot	(18, 20, 22)	23.551	29.2493	29.9011	4,104	649.804	725.704
	(19, 29, 21)	23.560	29.256	29.908	4,139.2	649.093	725.024
	(19.5, 20, 20.5)	23.565	29.259	29.911	4,144.1	648.750	724.696
Collusion	(18, 20, 22)	23.537	32.538	33.307	3,003	740.907	838.764
	(19, 29, 21)	23.546	32.542	33.312	3,013.7	740.101	838.002
	(19.5, 20, 20.5)	23.551	32.545	33.314	3,019.1	739.702	837.623

Table 5 The change of optimal solutions with the fuzzy degree of the parameter $\tilde{\theta}$

Scenarios	$\tilde{\theta}$	w^*	p_1^*	p_2^*	$E[\pi_m^*]$	$E[\pi_{r1}^*]$	$E[\pi_{r2}^*]$
Cournot	(8, 10, 12)	23.551	29.249	29.901	4,104	649.804	725.704
	(9, 10, 11)	23.569	29.262	29.914	4,149.2	648.392	724.353
	(9.5, 10, 10.5)	23.565	29.269	29.921	4,156.1	647.630	724.623
Collusion	(8, 10, 12)	23.537	32.538	33.307	3,003	740.902	838.764
	(9, 10, 11)	23.556	32.547	33.307	3,003	739.304	837.243
	(9.5, 10, 10.5)	23.565	32.552	33.321	3,036	738.507	836.485

of Zhao et al. (2012a), the manufacturers' profits are higher than those with crisp parameters, and the retailer's profits are lower than those with crisp parameters.

We have also examined the fuzzy degrees of the parameters $\tilde{a}_1$ and $\tilde{\theta}$ with the change of pricing and the expected profits, the results are shown in Tables 4, 5. As we can see, as the lower limits of fuzzy degrees of the parameters $\tilde{a}_1$ and $\tilde{\theta}$, the manufacturer's profits are increasing, and the retailers' profits are decreasing. Thus, the manufacturer should pursue as low fuzzy degrees of the parameters as possible.

5 Conclusions

The main contribution of this paper is to characterize the pricing decisions under duopolistic retailers' different behaviors in a fuzzy environment. Two different Stackelberg optimization models have been developed with fuzzy demands and fuzzy costs, which extend Yang and Zhou (2006)'s deterministic model. By mean of fuzzy set theory and Stackelberg game theoretic approach, the equilibrium solutions of two-echelon fuzzy optimization problems have been deduced. Some analyses about the results have been examined, which can provide some insights for supply chain incentive mechanism design.

Our models are developed for a single product between a manufacturer and two retailers for analytical convenience, and the demand function is linear. Topics for future studies include the use of other fuzzy membership functions, such as S-curve membership function and Gaussian membership function to model the customer demands and the manufacturing costs. Beside, the pricing problem addressed in this paper can be extended to the situations with multiple manufacturers, multiple retailers and multiple periods. Fuzzy set theory and fuzzy logic, as a flexible tool, can be effectively applied to tackle complicated supply chain problems.

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