Fuzzy Optim Decis Making (2013) 12:323–356
DOI 10.1007/s10700-013-9158-9

An interactive method for multiple criteria group decision analysis based on interval type-2 fuzzy sets and its application to medical decision making

Ting-Yu Chen

Published online: 9 March 2013

Abstract The theory of interval type-2 fuzzy sets provides an intuitive and computationally feasible way of addressing uncertain and ambiguous information in decision-making fields. The aim of this paper is to develop an interactive method for handling multiple criteria group decision-making problems, in which information about criterion weights is incompletely (imprecisely or partially) known and the criterion values are expressed as interval type-2 trapezoidal fuzzy numbers. With respect to the relative importance of multiple decision-makers and group consensus of fuzzy opinions, a hybrid averaging approach combining weighted averages and ordered weighted averages was employed to construct the collective decision matrix. An integrated programming model was then established based on the concept of signed distance-based closeness coefficients to determine the importance weights of criteria and the priority ranking of alternatives. Subsequently, an interactive procedure was proposed to modify the model according to the decision-makers' feedback on the degree of satisfaction toward undesirable solution results for the sake of gradually improving the integrated model. The feasibility and applicability of the proposed methods are illustrated with a medical decision-making problem of patient-centered medicine concerning basilar artery occlusion. A comparative analysis with other approaches was performed to validate the effectiveness of the proposed methodology.

Keywords Interval type-2 fuzzy set · Interactive method · Multiple criteria group decision-making · Interval type-2 trapezoidal fuzzy number · Medical decision-making

T.-Y. Chen (✉)
Department of Industrial and Business Management, Graduate Institute of Business and Management, College of Management, Chang Gung University, 259, Wen-Hwa 1st Road, Kwei-Shan, Taoyuan 333, Taiwan
e-mail: tychen@mail.cgu.edu.tw

1 Introduction

Decision-making information provided by group decision-makers is often imprecise and uncertain because of a lack of data, time pressure, or the decision-makers' limited attention and information-processing capabilities (Xu 2010). Accordingly, research pertaining to multiple criteria group decision-making (MCGDM) problems has often been performed within a fuzzy environment (Park et al. 2011). The concept of type-2 fuzzy sets (T2FSs) (Zadeh 1975) is an extension of type-1 fuzzy sets (T1FSs). T2FSs are superior to T1FSs because they can model second-order uncertainties (Greenfield et al. 2009). Interval type-2 fuzzy sets (IT2FSs), also known as interval-valued fuzzy sets (Sambuc 1975; Zadeh 1975), are the most widely used type of T2FSs because of their relative simplicity. IT2FSs are valuable for both modeling imprecision and their ability to easily reflect the ambiguous nature of subjective judgments. Therefore, many valuable methods have been developed to solve various decision-making problems (Chen 2012a,b; Chen et al. 2012; Wang et al. 2012). However, the research related to the interactive group decision-making approach within the IT2FS environment has been relatively less discussed. Many new interactive methods have been developed and discussed in multiple criteria decision analysis (Katagiri and Sakawa 2011; Kaliszewski et al. 2012). Nevertheless, very few studies focus on interactive MCGDM methods in the context of an IT2FS framework.

The purpose of this paper is to develop a new interactive method to handle interval type-2 fuzzy MCGDM problems with incomplete preference information. Based on the IT2FS framework, this paper employs a popular fuzzy number with the trapezoidal form called an interval type-2 trapezoidal fuzzy number (IT2TrFN) (Chen 2012b) to establish a group decision-making method and an interactive procedure using a signed distance-based closeness coefficient approach. In this paper, we extend the concept of closeness coefficients in the technique for order preference by similarity to ideal solution (TOPSIS) (Hwang and Yoon 1981) to propose the signed distance-based closeness coefficient as the core of our interactive MCGDM method.

This paper differs from the current literature in the following ways. First, we fuse multiple IT2TrFN ratings to build a collective decision environment using a hybrid averaging (HA) approach by combining weighted averaging (WA) and ordered weighted averaging (OWA) operations. Next, an integrated programming model based on the signed distance-based closeness coefficients is constructed to estimate the importance weights of criteria from incomplete preference information. Third, an interactive procedure is employed to gradually improve the integrated model according to the group decision-makers' requirements for the satisfactory solutions. The interactive procedure can actualize the process of interactive group decision making by providing the decision-makers with the solution results and modifying the integrated model according to their responses based on degrees of satisfaction. To facilitate such a procedure, we use a signed distance-based representation method for the degree of satisfaction (defined by the signed distance-based closeness coefficient). The concept of signed distances makes it easy to compare the degrees of satisfaction for the solution result and the group member's requirements. In the process of interaction, the decision-makers provide and modify their preference information such that the unsatisfactory results can be adjusted gradually until the most preferred alternative is obtained.

To demonstrate the feasibility and effectiveness of the proposed method, we investigated an MCGDM problem of patient-centered medicine. Given the increasing awareness of health rights, the rights of patients have drawn increasing attention. Thus, the focus of providing healthcare has shifted from the perspective of medical personnel to a patient-centered approach. Diseases are sources of stress for the patients, their family members, and medical personnel. When a patient is subjected to an emergent and life-threatening disease, selecting the most appropriate treatment is a difficult and complex process. This process involves very complex group decision making by the medical personnel, the patients, and their family members. Although the physician usually provides a limited number of treatment protocols to the patient (because there are only several treatments that have significant therapeutic effects), the decision-making process still involves numerous, complex, and possibly contradictory assessment criteria (in addition to the fuzziness, imprecision, and uncertainty of the issues). Therefore, we studied the proposed interactive MCGDM method for handling patient-centered group decision-making problems to validate its applicability.

The article is organized in the following manner. Section 2 briefly reviews the concepts of IT2TrFNs and signed distances. Section 3 formulates an MCGDM problem with IT2TrFNs data and constructs the collective decision matrix using hybrid averages. Section 4 develops an integrated programming model using the concept of signed distance-based closeness coefficients under incomplete preference information. This section also provides an interactive procedure for acquiring a satisfactory solution. Section 5 demonstrates the feasibility and applicability of the proposed methodology by applying it to a patient-centered medical problem and conducting a comparative analysis with the widely used TOPSIS method. Section 6 presents our conclusions.

2 Preliminaries

Select relevant definitions and operations of IT2FSs and IT2TrFNs are briefly reviewed in this section. The concept of signed distances in the context of IT2TrFNs is described as well.

2.1 The concept of IT2TrFNs

Definition 1 Let X be an ordinary finite nonempty set. Let Int([0, 1]) denote a set of all closed subintervals of [0, 1]. The mapping A: $X \rightarrow$Int([0, 1]) is known as an IT2FS on X. All IT2FSs on X are denoted by IT2FS(X).

Definition 2 If $A \in$ IT2FS(X), let $A(x) = [A^L(x), A^U(x)]$, where $x \in X$ and $0 \leq A^L(x) \leq A^U(x) \leq 1$. The two T1FSs $A^L : X \rightarrow [0, 1]$ and $A^U : X \rightarrow [0, 1]$ are known as the lower and upper fuzzy sets, respectively, with respect to A. If $A(x)$ is convex and defined on a closed and bounded interval, then A is known as "an interval type-2 fuzzy number (IT2FN) on X". All IT2FNs on X are denoted by IT2FN(X).

Definition 3 Let $A^L(= (a_1^L, a_2^L, a_3^L, a_4^L; h_A^L))$ and $A^U(= (a_1^U, a_2^U, a_3^U, a_4^U; h_A^U))$ be the lower and upper trapezoidal fuzzy numbers defined on the universe of discourse X, where $a_1^L \le a_2^L \le a_3^L \le a_4^L$, $a_1^U \le a_2^U \le a_3^U \le a_4^U$, $0 \le h_A^L \le h_A^U \le 1$, $a_1^U \le a_1^L$, $a_4^L \le a_4^U$, and $A^L \subset A^U$. Let $\xi \in \{L, U\}$. The membership function of A^ξ for each ξ is expressed as the following:

$$A^\xi(x) = \begin{cases} h_A^\xi \left(x - a_1^\xi\right) \Big/ \left(a_2^\xi - a_1^\xi\right) & \text{for } a_1^\xi \le x \le a_2^\xi, \\ h_A^\xi & \text{for } a_2^\xi \le x \le a_3^\xi, \\ h_A^\xi \left(a_4^\xi - x\right) \Big/ \left(a_4^\xi - a_3^\xi\right) & \text{for } a_3^\xi \le x \le a_4^\xi, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

An IT2TrFN A on X is represented by the following:

$$A = [A^L, A^U] = \left[(a_1^L, a_2^L, a_3^L, a_4^L; h_A^L), (a_1^U, a_2^U, a_3^U, a_4^U; h_A^U)\right]. \tag{2}$$

The extension principle (Zadeh 1975) can be employed to develop fuzzy arithmetic defined as T2FSs (Aisbett et al. 2010; Gilan et al. 2012). Let $\oplus$ denote the addition operation and let A and B denote IT2TrFNs. By using Zadeh's extension principle, we define an IT2TrFN for a set of all real numbers $A \oplus B$ with the following equation:

$$(A \oplus B)(z) = \sup_{z=x+y} \min[A(x), B(y)], \tag{3}$$

where sup is the supremum. Based on interval-valued arithmetic, standard arithmetic operations on trapezoidal-shape fuzzy numbers can be extended to IT2TrFNs.

Definition 4 Let A and B be two non-negative IT2TrFNs. $A = [(a_1^L, a_2^L, a_3^L, a_4^L; h_A^L), (a_1^U, a_2^U, a_3^U, a_4^U; h_A^U)]$ and $B = [(b_1^L, b_2^L, b_3^L, b_4^L; h_B^L), (b_1^U, b_2^U, b_3^U, b_4^U; h_B^U)]$ on X. The arithmetic operations on A and B are defined as follows (Wei and Chen 2009):

(i) Addition operation:

$$A \oplus B = \left[\left(a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L; \min\left\{h_A^L, h_B^L\right\}\right),\right.$$
$$\left.\left(a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_4^U; \min\left\{h_A^U, h_B^U\right\}\right)\right]. \tag{4}$$

(ii) Multiplication by an ordinary number:

$$q \cdot A = A \cdot q$$
$$= \begin{cases} \left[(q \times a_1^L, q \times a_2^L, q \times a_3^L, q \times a_4^L; h_A^L), (q \times a_1^U, q \times a_2^U, q \times a_3^U, q \times a_4^U; h_A^U)\right] & \text{if } q \ge 0, \\ \left[(q \times a_4^L, q \times a_3^L, q \times a_2^L, q \times a_1^L; h_A^L), (q \times a_4^U, q \times a_3^U, q \times a_2^U, q \times a_1^U; h_A^U)\right] & \text{if } q \le 0. \end{cases} \tag{5}$$

(iii) Division by an ordinary number (q is a nonzero number):

$$A/q = \begin{cases} \left[\left(\frac{a_1^L}{q}, \frac{a_2^L}{q}, \frac{a_3^L}{q}, \frac{a_4^L}{q}; h_A^L\right), \left(\frac{a_1^U}{q}, \frac{a_2^U}{q}, \frac{a_3^U}{q}, \frac{a_4^U}{q}; h_A^U\right)\right] & \text{if } q > 0, \\ \left[\left(\frac{a_4^L}{q}, \frac{a_3^L}{q}, \frac{a_2^L}{q}, \frac{a_1^L}{q}; h_A^L\right), \left(\frac{a_4^U}{q}, \frac{a_3^U}{q}, \frac{a_2^U}{q}, \frac{a_1^U}{q}; h_A^U\right)\right] & \text{if } q < 0. \end{cases} \quad (6)$$

2.2 The concept of signed distances

In this paper, we use a simple procedure based on signed distances to define ordering, which means we can use both positive and negative values. The concept of signed distances, also referred to as oriented distances or directed distances, can be used to determine the rankings of IT2TrFN values (Chen 2011, 2012a). In general, the ranking of fuzzy numbers can be performed using many methods, such as the coefficient of variation (i.e., CV index) and distance metric between fuzzy sets. However, some limitations have been found when ranking fuzzy numbers with the CV index, distance between fuzzy sets, centroid point and original point, and weighted mean value (Yao and Wu 2000; Abbasbandy and Asady 2006). Conversely, the ranking method that employs signed distances can effectively rank various fuzzy numbers and their images (Yao and Wu 2000). Additionally, calculating the signed distance method is far simpler than calculating other approaches (Abbasbandy and Asady 2006). Notice that the signed distance method can use both positive and negative values to define the ordering of fuzzy numbers. This approach is considerably different from the ordinary distance measures (Yao and Wu 2000) because this study employed a signed distance-based approach for the comparison with IT2TrFN values. For the proof of the following proposition and properties, see Chen (2011, 2012a).

Proposition 1 *Let A be an IT2TrFN defined in the universe of discourse X and $A = [A^L, A^U] = \left[(a_1^L, a_2^L, a_3^L, a_4^L; h_A^L), (a_1^U, a_2^U, a_3^U, a_4^U; h_A^U)\right]$, where $0 < h_A^L \leq h_A^U \leq 1$. Let the level 1 fuzzy numbers $\tilde{0}_1$ and $\tilde{1}_1$ map onto the y-axis at $x = 0$ and at $x = 1$, respectively. The signed distances from A to $\tilde{0}_1$ (y-axis at $x = 0$) or to $\tilde{1}_1$ (y-axis at $x = 1$) are the following:*

$$d(A, \tilde{0}_1) = \frac{1}{8}\left(a_1^L + a_2^L + a_3^L + a_4^L + 4a_1^U + 2a_2^U + 2a_3^U + 4a_4^U + 3(a_2^U + a_3^U - a_1^U - a_4^U)\frac{h_A^L}{h_A^U}\right), \quad (7)$$

$$d(A, \tilde{1}_1) = \frac{1}{8}\left(a_1^L + a_2^L + a_3^L + a_4^L + 4a_1^U + 2a_2^U + 2a_3^U + 4a_4^U + 3(a_2^U + a_3^U - a_1^U - a_4^U)\frac{h_A^L}{h_A^U} - 16\right). \quad (8)$$

Property 1 Let A be an IT2TrFN defined on X and $A = \left[(a_1^L, a_2^L, a_3^L, a_4^L; h_A^L), (a_1^U, a_2^U, a_3^U, a_4^U; h_A^U)\right]$.

(i) $d(A, \tilde{0}_1) - d(A, \tilde{1}_1) = 2$.
(ii) A is located at $\tilde{1}_1$ (i.e., $a_1^L = a_2^L = a_3^L = a_4^L = a_1^U = a_2^U = a_3^U = a_4^U = 1$) if and only if $d(A, \tilde{1}_1) = 0$ and $d(A, \tilde{0}_1) = 2$.
(iii) A is located at $\tilde{0}_1$ (i.e., $a_1^L = a_2^L = a_3^L = a_4^L = a_1^U = a_2^U = a_3^U = a_4^U = 0$) if and only if $d(A, \tilde{0}_1) = 0$ and $d(A, \tilde{1}_1) = -2$.

Property 2 Let A, B, C, and D be four IT2TrFNs defined in the universe of discourse X, where $A = [(a_1^L, a_2^L, a_3^L, a_4^L; h_A^L), (a_1^U, a_2^U, a_3^U, a_4^U; h_A^U)]$, $B = [(b_1^L, b_2^L, b_3^L, b_4^L; h_B^L), (b_1^U, b_2^U, b_3^U, b_4^U; h_B^U)]$, $C = [(1, 1, 1, 1; 1), (1, 1, 1, 1; 1)]$, and $D = [(0, 0, 0, 0; 1), (0, 0, 0, 0; 1)]$.

(i) A is closer to C than B if and only if $d(A, \tilde{1}_1) > d(B, \tilde{1}_1)$.
(ii) A is farther from D than B if and only if $d(A, \tilde{0}_1) > d(B, \tilde{0}_1)$.

The concept of linguistic variables provides a means for approximate characterization of phenomena that are complex or ill-defined to be amenable to description in conventional quantitative terms (Zadeh 1975; Yuen 2012). In many decision situations, ratings cannot be measured precisely as decision-makers may express their assessments or judgments using linguistic terms (Li et al. 2012). Thus, linguistic variables that can be represented by membership functions are suitable for most decision models (Yuen 2012). Several studies have provided the linguistic rating system that contains a nine-point (Wei and Chen 2009; Chen 2012a,b) or seven-point (Chen et al. 2012; Wang et al. 2012) linguistic rating scale and the corresponding IT2TrFNs to measure alternative ratings. Furthermore, most of the IT2TrFNs associated with linguistic terms are bounded within the interval [0, 1]. Let A and B be two IT2TrFN associated with linguistic terms. By applying Property 1, it follows that the signed distances are $d(A, \tilde{0}_1) \in [0, 2]$ and $d(A, \tilde{1}_1) \in [-2, 0]$. In addition to the boundary conditions, the signed distances also satisfy the law of trichotomy. Because $d(A, \tilde{0}_1)$ and $d(B, \tilde{0}_1)$ are real numbers, they satisfy linear ordering. That is, one of the following three conditions must be true: $d(A, \tilde{0}_1) > d(B, \tilde{0}_1)$, $d(A, \tilde{0}_1) = d(B, \tilde{0}_1)$, or $d(A, \tilde{0}_1) < d(B, \tilde{0}_1)$. Similarly, $d(A, \tilde{1}_1)$ and $d(B, \tilde{1}_1)$ also satisfy linear ordering. Accordingly, the signed distance-based procedure can be employed to rank the IT2TrFN values.

3 Collective decision-making context with incomplete information

Consider an MCGDM problem. Assume that $E = \{E_1, E_2, \ldots, E_K\}$ is the set of decision-makers involved in the decision process. Let $\pi = (\pi_1, \pi_2, \ldots, \pi_K)$ be the weight vector of the decision-makers, where $\pi_k \geq 0$ for $k = 1, 2, \ldots, K$ and $\sum_{k=1}^{K} \pi_k = 1$. An alternative set $A = \{A_1, A_2, \ldots, A_m\}$ consists of m non-inferior decision alternatives and a criterion set $X = \{x_1, x_2, \ldots, x_n\}$. The criterion set X can be generally divided into two sets, X_b and X_c, where X_b denotes a collection of benefit criteria, X_c denotes a collection of cost criteria, $X_b \cap X_c = \varnothing$, and $X_b \cup X_c = X$. In the following, we present an HA approach to aggregate the IT2TrFN data for constructing a collective decision-making context in the MCGDM analysis.

3.1 Collective decision environment

Based on a WA operation and a signed distance-based OWA operation, we employ the HA operation to aggregate IT2TrFN information and to build a collective decision matrix. The operation of hybrid averages can reflect the importance degrees of each decision-maker and the agreement of individual opinions via the WA and OWA operations, respectively.

Let an IT2TrFN $A_{ij}^{k} = \left[A_{ij}^{kL}, A_{ij}^{kU}\right]$ be a criterion value of alternative $A_i \in A$ with respect to criterion $x_j \in X$ provided by the kth decision-maker, where $k = 1, 2, \ldots, K$. In addition, $A_{ij}^{kL} = \left(a_{1ij}^{kL}, a_{2ij}^{kL}, a_{3ij}^{kL}, a_{4ij}^{kL}; h_{A_{ij}}^{kL}\right)$, $A_{ij}^{kU} = \left(a_{1ij}^{kU}, a_{2ij}^{kU}, a_{3ij}^{kU}, a_{4ij}^{kU}; h_{A_{ij}}^{kU}\right)$, and $A_{ij}^{kL} \subset A_{ij}^{kU}$. We consider the relative importance of each decision-maker and incorporate the WA operation into the HA operation. Additionally, it is essential to obtain group consensus ratings. Thus, the HA operation aggregates individual weighted ratings to form a common rating using the signed distance-based OWA operation. The OWA operation requires reordering all of the given arguments in descending order and then weighting these ordered arguments (Chen 2012a). Because the signed distance based on IT2TrFNs satisfies the law of trichotomy, in this paper, a comparison of the IT2TrFN values has been drawn via the signed distance from $\tilde{0}_1$.

Many methods can be used to determine the OWA weights (Xu 2005; Xu and Yager 2006). Specifically, Xu (2005) developed a normal distribution-based method, which is defined as follows:

$$\tau_k = \frac{e^{-\frac{(k-u_K)^2}{2\cdot v_K^2}}}{\sum_{\hbar=1}^{K} e^{-\frac{(\hbar-u_K)^2}{2\cdot v_K^2}}}, \quad k = 1, 2, \ldots K, \tag{9}$$

where u_K is the mean of the collection of $1, 2, \ldots, K$, and v_K ($v_K > 0$) is the standard deviation of the collection of $1, 2, \ldots, K$. That is:

$$u_K = \frac{1}{K} \cdot \frac{K(1+K)}{2} = \frac{1+K}{2}, \tag{10}$$

$$v_K = \sqrt{\frac{1}{K} \sum_{k=1}^{K} (k - u_K)^2}. \tag{11}$$

The HA operation $\text{HA}\left(A_{ij}^{1}, A_{ij}^{2}, \ldots, A_{ij}^{K}\right)$ with the associated OWA weight vector $\tau = (\tau_1, \tau_2, \ldots, \tau_K)$, where $\tau_k \in [0, 1]$ and $\sum_{k=1}^{K} \tau_k = 1$, is obtained by $(\tau_1 \dot{A}_{ij}^{\sigma(1)}) \oplus (\tau_2 \cdot \dot{A}_{ij}^{\sigma(2)}) \oplus \cdots \oplus (\tau_K \cdot \dot{A}_{ij}^{\sigma(K)})$, as indicated in the following definition.

Definition 5 Let an IT2TrFN $A_{ij}^k = \left[\left(a_{1ij}^{kL}, a_{2ij}^{kL}, a_{3ij}^{kL}, a_{4ij}^{kL}; h_{A_{ij}}^{kL}\right), \left(a_{1ij}^{kU}, a_{2ij}^{kU}, a_{3ij}^{kU}, a_{4ij}^{kU}; h_{A_{ij}}^{kU}\right)\right]$ denote the rating of alternative $A_i \in A$ with respect to criterion $x_j \in X$ provided by the decision-maker $E_k \in E$. Let $\dot{A}_{ij}^k = \pi_k \cdot A_{ij}^k$ for all $k = 1, 2, \ldots, K$ as follows:

$$\begin{aligned}\dot{A}_{ij}^k &= \left[\left(\dot{a}_{1ij}^{kL}, \dot{a}_{2ij}^{kL}, \dot{a}_{3ij}^{kL}, \dot{a}_{4ij}^{kL}; h_{\dot{A}_{ij}}^{kL}\right), \left(\dot{a}_{1ij}^{kU}, \dot{a}_{2ij}^{kU}, \dot{a}_{3ij}^{kU}, \dot{a}_{4ij}^{kU}; h_{\dot{A}_{ij}}^{kU}\right)\right] \\ &= \left[\left(\pi_k \times a_{1ij}^{kL}, \pi_k \times a_{2ij}^{kL}, \pi_k \times a_{3ij}^{kL}, \pi_k \times a_{4ij}^{kL}; h_{A_{ij}}^{kL}\right),\right. \\ &\qquad \left.\left(\pi_k \times a_{1ij}^{kU}, \pi_k \times a_{2ij}^{kU}, \pi_k \times a_{3ij}^{kU}, \pi_k \times a_{4ij}^{kU}; h_{A_{ij}}^{kU}\right)\right]. \end{aligned} \tag{12}$$

Referring to the OWA weight vector $\tau = (\tau_1, \tau_2, \ldots, \tau_K)$, the hybrid average evaluation of alternative A_i with respect to criterion x_j is defined by:

$$\begin{aligned}\hat{A}_{ij} &= \left[\hat{A}_{ij}^L, \hat{A}_{ij}^U\right] = \left[\left(\hat{a}_{1ij}^L, \hat{a}_{2ij}^L, \hat{a}_{3ij}^L, \hat{a}_{4ij}^L; h_{\hat{A}_{ij}}^L\right), \left(\hat{a}_{1ij}^U, \hat{a}_{2ij}^U, \hat{a}_{3ij}^U, \hat{a}_{4ij}^U; h_{\hat{A}_{ij}}^U\right)\right] \\ &= \left[\left(\sum_{k=1}^K \left(\tau_k \times \dot{a}_{1ij}^{\sigma(k)L}\right), \sum_{k=1}^K \left(\tau_k \times \dot{a}_{2ij}^{\sigma(k)L}\right), \sum_{k=1}^K \left(\tau_k \times \dot{a}_{3ij}^{\sigma(k)L}\right),\right.\right. \\ &\qquad \left.\sum_{k=1}^K \left(\tau_k \times \dot{a}_{4ij}^{\sigma(k)L}\right); \min_k h_{\dot{A}_{ij}}^{\sigma(k)L}\right), \\ &\qquad \left(\sum_{k=1}^K \left(\tau_k \times \dot{a}_{1ij}^{\sigma(k)U}\right), \sum_{k=1}^K \left(\tau_k \times \dot{a}_{2ij}^{\sigma(k)U}\right), \sum_{k=1}^K \left(\tau_k \times \dot{a}_{3ij}^{\sigma(k)U}\right),\right. \\ &\qquad \left.\left.\sum_{k=1}^K \left(\tau_k \times \dot{a}_{4ij}^{\sigma(k)U}\right); \min_k h_{\dot{A}_{ij}}^{\sigma(k)U}\right)\right], \end{aligned} \tag{13}$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(K))$ is a permutation of $(1, 2, \ldots, K)$ such that $d(\dot{A}_{ij}^{\sigma(k)}, \tilde{0}_1) \geq d(\dot{A}_{ij}^{\sigma(k-1)}, \tilde{0}_1)$ for all k. Additionally, $0 \leq \hat{a}_{1ij}^L \leq \hat{a}_{2ij}^L \leq \hat{a}_{3ij}^L \leq \hat{a}_{4ij}^L \leq 1$, $0 \leq \hat{a}_{1ij}^U \leq \hat{a}_{2ij}^U \leq \hat{a}_{3ij}^U \leq \hat{a}_{4ij}^U \leq 1$, $0 \leq h_{\hat{A}_{ij}}^L \leq h_{\hat{A}_{ij}}^U \leq 1$, $\hat{a}_{1ij}^U \leq \hat{a}_{1ij}^L$, $\hat{a}_{4ij}^L \leq \hat{a}_{4ij}^U$, and $\hat{A}_{ij}^L \subset \hat{A}_{ij}^U$.

The collective decision matrix $\hat{D}$ is expressed in the following way:

$$\hat{D} = \begin{matrix} & x_1 & x_2 & \cdots & x_n \\ A_1 & [\hat{A}_{11}^L, \hat{A}_{11}^U] & [\hat{A}_{12}^L, \hat{A}_{12}^U] & \cdots & [\hat{A}_{1n}^L, \hat{A}_{1n}^U] \\ A_2 & [\hat{A}_{21}^L, \hat{A}_{21}^U] & [\hat{A}_{22}^L, \hat{A}_{22}^U] & \cdots & [\hat{A}_{2n}^L, \hat{A}_{2n}^U] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & [\hat{A}_{m1}^L, \hat{A}_{m1}^U] & [\hat{A}_{m2}^L, \hat{A}_{m2}^U] & \cdots & [\hat{A}_{mn}^L, \hat{A}_{mn}^U] \end{matrix}. \tag{14}$$

The collective data with respect to each criterion are normalized to the maximum criterion values for benefit criteria and the minimum criterion values for cost criteria.

Let $\hat{a}_j^+ = \max_{i_1} \hat{a}_{4i_1j}^U$ (for $x_j \in X_b$) and $\hat{a}_j^- = \min_{i_1} \hat{a}_{1i_1j}^U$ (for $x_j \in X_c$). The transformed outcome of $\hat{A}_{ij}$ is obtained by the following:

$$A_{ij} = \left[A_{ij}^L, A_{ij}^U\right] = \left[\left(a_{1ij}^L, a_{2ij}^L, a_{3ij}^L, a_{4ij}^L; h_{A_{ij}}^L\right), \left(a_{1ij}^U, a_{2ij}^U, a_{3ij}^U, a_{4ij}^U; h_{A_{ij}}^U\right)\right]$$
$$= \begin{cases} \left[\left(\frac{\hat{a}_{1ij}^L}{\hat{a}_j^+}, \frac{\hat{a}_{2ij}^L}{\hat{a}_j^+}, \frac{\hat{a}_{3ij}^L}{\hat{a}_j^+}, \frac{\hat{a}_{4ij}^L}{\hat{a}_j^+}; h_{\hat{A}_{ij}}^L\right), \left(\frac{\hat{a}_{1ij}^U}{\hat{a}_j^+}, \frac{\hat{a}_{2ij}^U}{\hat{a}_j^+}, \frac{\hat{a}_{3ij}^U}{\hat{a}_j^+}, \frac{\hat{a}_{4ij}^U}{\hat{a}_j^+}; h_{\hat{A}_{ij}}^U\right)\right] & \text{if } x_j \in X_b, \\ \left[\left(\frac{\hat{a}_j^-}{\hat{a}_{4ij}^L}, \frac{\hat{a}_j^-}{\hat{a}_{3ij}^L}, \frac{\hat{a}_j^-}{\hat{a}_{2ij}^L}, \frac{\hat{a}_j^-}{\hat{a}_{1ij}^L}; h_{\hat{A}_{ij}}^L\right), \left(\frac{\hat{a}_j^-}{\hat{a}_{4ij}^U}, \frac{\hat{a}_j^-}{\hat{a}_{3ij}^U}, \frac{\hat{a}_j^-}{\hat{a}_{2ij}^U}, \frac{\hat{a}_j^-}{\hat{a}_{1ij}^U}; h_{\hat{A}_{ij}}^U\right)\right] & \text{if } x_j \in X_c. \end{cases} \tag{15}$$

The normalized collective decision matrix D is constructed as follows:

$$D = \begin{matrix} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} \begin{matrix} x_1 & x_2 & \cdots & x_n \\ \left[\begin{matrix} [A_{11}^L, A_{11}^U] & [A_{12}^L, A_{12}^U] & \cdots & [A_{1n}^L, A_{1n}^U] \\ [A_{21}^L, A_{21}^U] & [A_{22}^L, A_{22}^U] & \cdots & [A_{2n}^L, A_{2n}^U] \\ \vdots & \vdots & \ddots & \vdots \\ [A_{m1}^L, A_{m1}^U] & [A_{m2}^L, A_{m2}^U] & \cdots & [A_{mn}^L, A_{mn}^U] \end{matrix}\right] \end{matrix}. \tag{16}$$

Additionally, the characteristics of alternative A_i can be represented by the IT2TrFN in the following way:

$$A_i = \left\{\left\langle x_j, [A_{ij}^L, A_{ij}^U]\right\rangle \middle| x_j \in X\right\}. \tag{17}$$

3.2 Incomplete preference structure

In the MCGDM process, decision-makers may express some preference relations on weights of criteria according to their knowledge, past experience, and subjective judgments. Usually such information of criterion weights is incomplete (Li 2011). The incomplete information about criterion weights can be generally constructed by several basic ranking forms (Xu 2010; Wei et al. 2011; Li 2011).

Definition 6 Let w_j be the weight of criterion $x_j \in X$, which satisfies the normalization conditions: $w_j \in [0, 1]$ $(j = 1, 2, \ldots, n)$ and $\sum_{j=1}^n w_j = 1$. Let Γ_0 denote a set of all weight vectors, and

$$\Gamma_0 = \left\{(w_1, w_2, \ldots, w_n) \middle| w_j \geq 0 (j = 1, 2, \ldots, n), \quad \sum_{j=1}^n w_j = 1\right\}. \tag{18}$$

The five basic ranking forms of incomplete weight information are as follows:

(i) A weak ranking:

$$\Gamma_1 = \left\{(w_1, w_2, \ldots, w_n) \in \Gamma_0 | \, w_{j_1} \geq w_{j_2} \text{ for all } j_1 \in \Upsilon_1 \text{ and } j_2 \in \Lambda_1\right\}, \tag{19}$$

where Υ_1 and Λ_1 are two disjointed subsets of the subscript index set $N = \{1, 2, \ldots, n\}$ for all criteria.

(ii) A strict ranking:

$$\Gamma_2 = \left\{ (w_1, w_2, \ldots, w_n) \in \Gamma_0 | \, \beta_{j_1 j_2} \geq w_{j_1} - w_{j_2} \geq \delta_{j_1 j_2} \text{ for all } j_1 \in \Upsilon_2 \text{ and } j_2 \in \Lambda_2 \right\}, \tag{20}$$

where $\beta_{j_1 j_2} > 0$ and $\delta_{j_1 j_2} > 0$ are constants, satisfying $\beta_{j_1 j_2} > \delta_{j_1 j_2}$; Υ_2 and Λ_2 are two disjointed subsets of N.

(iii) A ranking of differences (or strength of preference):

$$\Gamma_3 = \left\{ (w_1, w_2, \ldots, w_n) \in \Gamma_0 | \, w_{j_1} - w_{j_2} \geq w_{j_2} - w_{j_3} \text{ for all } j_1 \in \Upsilon_3, j_2 \in \Lambda_3, \text{ and } j_3 \in \Omega_3 \right\}, \tag{21}$$

where Υ_3, Λ_3, and Ω_3 are three disjointed subsets of N.

(iv) An interval bound:

$$\Gamma_4 = \left\{ (w_1, w_2, \ldots, w_n) \in \Gamma_0 | \, \delta_{j_1} + \varepsilon_{j_1} \geq w_{j_1} \geq \delta_{j_1} \text{ for all } j_1 \in \Upsilon_4 \right\}, \tag{22}$$

where $\delta_{j_1} \geq 0$ and $\varepsilon_{j_1} \geq 0$ are constants, satisfying $0 \leq \delta_{j_1} \leq \delta_{j_1} + \varepsilon_{j_1} \leq 1$; Υ_4 is a subsets of N.

(v) A ratio bound (or ranking with multiples):

$$\Gamma_5 = \left\{ (w_1, w_2, \ldots, w_n) \in \Gamma_0 | \, w_{j_1} \geq \delta_{j_1 j_2} \ldots w_{j_2} \text{for all } j_1 \in \Upsilon_5 \text{ and } j_2 \in \Lambda_5 \right\}, \tag{23}$$

where $\delta_{j_{12}}$ is a constant, satisfying $0 \leq \delta_{j_{12}} \leq 1$; Υ_5 and Λ_5 are two disjointed subsets of N. Finally, the known information structure Γ consists of the above five sets $\Gamma_1, \Gamma_2, \ldots$, and Γ_5 as follows:

$$\Gamma = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \cup \Gamma_4 \cup \Gamma_5. \tag{24}$$

The multiple decision-makers may provide different, even conflicting, opinions on the preference information of the criteria. Thus, some conditions in Γ do not simultaneously hold because of inconsistent information. In this case, there may be no feasible solutions for the criterion weights. To overcome this difficulty, several deviation variables were introduced to mitigate the inconsistent weight information in this study. These deviation variables could relax the constraints of criterion weights in Γ to solve the feasible weights under the situation of preference conflict in criterion importance. For $j_1 \neq j_2 \neq j_3$, the conditions in Γ are relaxed to Γ' by introducing the non-negative deviation variables $e^-_{(\mathrm{i}) j_1 j_2}$, $e^-_{(\mathrm{ii}) j_1 j_2}$, $e^+_{(\mathrm{ii}) j_1 j_2}$, $e^-_{(\mathrm{iii}) j_1 j_2 j_3}$, $e^-_{(\mathrm{iv}) j_1}$, $e^+_{(\mathrm{iv}) j_1}$, and $e^-_{(\mathrm{v}) j_1 j_2}$, which are defined as follows:

(i) A relaxed weak ranking:

$$\Gamma_1' = \left\{ (w_1, w_2, \ldots, w_n) \in \Gamma_0 | \, w_{j_1} + e^-_{(\mathrm{i})j_1 j_2} \geq w_{j_2} \text{ for all } j_1 \in \Upsilon_1 \text{ and } j_2 \in \Lambda_1 \right\}. \tag{25}$$

(ii) A relaxed strict ranking:

$$\Gamma_2' = \left\{ (w_1, w_2, \ldots, w_n) \in \Gamma_0 \left| w_{j_1} - w_{j_2} + e^-_{(\mathrm{ii})j_1 j_2} \geq \delta_{j_1 j_2}, w_{j_1} - w_{j_2} - e^+_{(\mathrm{ii})j_1 j_2} \leq \beta_{j_1 j_2} \right. \right. \\ \left. \text{for all } j_1 \in \Upsilon_2 \text{ and } j_2 \in \Lambda_2 \right\}. \tag{26}$$

(iii) A relaxed ranking of differences:

$$\Gamma_3' = \left\{ (w_1, w_2, \ldots, w_n) \in \Gamma_0 | \, w_{j_1} - 2w_{j_2} + w_{j_3} + e^-_{(\mathrm{iii})j_1 j_2 j_3} \geq 0 \text{ for all } j_1 \in \Upsilon_3, \right. \\ \left. j_2 \in \Lambda_3, \text{ and } j_3 \in \Omega_3 \right\}. \tag{27}$$

(iv) A relaxed interval bound:

$$\Gamma_4' = \left\{ (w_1, w_2, \ldots, w_n) \in \Gamma_0 | \, w_{j_1} + e^-_{(\mathrm{iv})j_1} \geq \delta_{j_1}, w_{j_1} - e^+_{(\mathrm{iv})j_1} \leq \delta_{j_1} + \varepsilon_{j_1} \text{ for all } j_1 \in \Upsilon_4 \right\}. \tag{28}$$

(v) A relaxed ratio bound:

$$\Gamma_5' = \left\{ (w_1, w_2, \ldots, w_n) \in \Gamma_0 \left| \frac{w_{j_1}}{w_{j_2}} + e^-_{(\mathrm{v})j_1 j_2} \geq \delta_{j_1 j_2} \text{ for all } j_1 \in \Upsilon_5 \text{ and } j_2 \in \Lambda_5 \right. \right\}. \tag{29}$$

Finally, the set Γ' of the relaxed conditions about criterion weights is given by:

$$\Gamma' = \Gamma_1' \cup \Gamma_2' \cup \Gamma_3' \cup \Gamma_4' \cup \Gamma_5'. \tag{30}$$

4 Interactive decision-making method on IT2TrFNs

This section presents an interactive method for solving an MCGDM problem with IT2TrFN data. An integrated nonlinear programming model based on the concept of signed distance-based closeness coefficients was constructed to estimate criterion weights under the incomplete preference structure. Furthermore, according to the decision-makers' feedback on the acceptable levels of satisfaction toward the solution results, an interactive procedure was developed to achieve an optimum satisfactory solution among the decision-makers.

4.1 The integrated programming model

In the IT2TrFN framework, we employed the concept of signed distances to derive the separations for each alternative from the positive-ideal and negative-ideal solutions independently and then to determine the signed distance-based closeness coefficient. Because the values of the normalized ratings are between zero and one, the specifications of the positive-ideal solution, denoted as A^+, and the negative-ideal solution, denoted as A^-, are as follows:

$$A^+ = \left\{ \langle x_j, [(1,1,1,1;1),(1,1,1,1;1)] \rangle \,\middle|\, x_j \in X \right\}, \tag{31}$$

$$A^- = \left\{ \langle x_j, [(0,0,0,0;1),(0,0,0,0;1)] \rangle \,\middle|\, x_j \in X \right\}. \tag{32}$$

Let $A_j^+ = [(1,1,1,1;1),(1,1,1,1;1)]$ and $A_j^- = [(0,0,0,0;1),(0,0,0,0;1)]$ for all $x_j \in X$. According to Property 1, we know that $d(A_j^+, \tilde{1}_1) = 0$, $d(A_j^+, \tilde{0}_1) = 2$, $d(A_j^-, \tilde{1}_1) = -2$, and $d(A_j^-, \tilde{0}_1) = 0$. For each A_i, it is obvious that the signed distances from A_{ij} to A_j^+ and A_{ij} to A_j^- can be calculated with $d(A_{ij}, \tilde{1}_1)$ and $d(A_{ij}, \tilde{0}_1)$, respectively, because the positions of the individual ratings in the positive-ideal and negative-ideal solutions are placed on the y-axis at x=1 and at $x = 0$, respectively. It follows that:

$$d(A_{ij}, A_j^+) = d(A_{ij}, \tilde{1}_1), \tag{33}$$

$$d(A_{ij}, A_j^-) = d(A_{ij}, \tilde{0}_1). \tag{34}$$

Let $S(A_{ij}, A_j^+)$ and $S(A_{ij}, A_j^-)$ denote the weighted signed distances from A_{ij} to A_j^+ and A_{ij} to A_j^-, respectively. For each set, $A_i \in A$ and $x_j \in X$,

$$\begin{aligned} S(A_{ij}, A_j^+) &= w_j \cdot d(A_{ij}, A_j^+) \\ &= \frac{w_j}{8} \Big(a_{1ij}^L + a_{2ij}^L + a_{3ij}^L + a_{4ij}^L + 4a_{1ij}^U + 2a_{2ij}^U + 2a_{3ij}^U + 4a_{4ij}^U \\ &\quad +3(a_{2ij}^U + a_{3ij}^U - a_{1ij}^U - a_{4ij}^U) \cdot \left(h_{A_{ij}}^L \middle/ h_{A_{ij}}^U \right) - 16 \Big), \end{aligned} \tag{35}$$

$$\begin{aligned} S(A_{ij}, A_j^-) &= w_j \cdot d(A_{ij}, A_j^-) \\ &= \frac{w_j}{8} \Big(a_{1ij}^L + a_{2ij}^L + a_{3ij}^L + a_{4ij}^L + 4a_{1ij}^U + 2a_{2ij}^U + 2a_{3ij}^U + 4a_{4ij}^U \\ &\quad +3(a_{2ij}^U + a_{3ij}^U - a_{1ij}^U - a_{4ij}^U) \cdot \left(h_{A_{ij}}^L \middle/ h_{A_{ij}}^U \right) \Big). \end{aligned} \tag{36}$$

The closeness of the alternative A_i and the ideal solutions was determined based on the weighted signed distances. More specifically, the average signed distances $(1/n) \cdot \sum_{j=1}^n S(A_{ij}, A_j^+)$ and $(1/n) \cdot \sum_{j=1}^n S(A_{ij}, A_j^-)$ were calculated to identify the signed distance-based closeness coefficient of each A_i. For $i = 1, 2, \ldots, m$, let CC_i denote the signed distance-based closeness coefficient of A_i as follows:

$$CC_i = \frac{\frac{1}{n}\sum_{j=1}^n S(A_{ij}, A_j^-)}{\frac{1}{n}\sum_{j=1}^n S(A_{ij}, A_j^-) - \frac{1}{n}\sum_{j=1}^n S(A_{ij}, A_j^+)}. \tag{37}$$

Recall that $d(A, \tilde{0}_1) - d(A, \tilde{1}_1) = 2$ in Property 1. For the denominator in the right-hand side, we obtain:

$$\frac{1}{n}\sum_{j=1}^{n} S(A_{ij}, A_j^-) - \frac{1}{n}\sum_{j=1}^{n} S(A_{ij}, A_j^+) = \frac{1}{n}\cdot\sum_{j=1}^{n}\left[w_j\left(d(A_{ij}, \tilde{0}_1) - d(A_{ij}, \tilde{1}_1)\right)\right]$$
$$= \frac{2}{n}\cdot\sum_{j=1}^{n} w_j = \frac{2}{n}.$$

Therefore, CC_i can be expressed as follows:

$$\begin{aligned} CC_i &= \frac{1}{2}\sum_{j=1}^{n} S(A_{ij}, A_j^-) \\ &= \frac{1}{16}\sum_{j=1}^{n}\Big[w_j\cdot\Big(a_{1ij}^L + a_{2ij}^L + a_{3ij}^L + a_{4ij}^L + 4a_{1ij}^U + 2a_{2ij}^U + 2a_{3ij}^U + 4a_{4ij}^U \\ &\quad + 3(a_{2ij}^U + a_{3ij}^U - a_{1ij}^U - a_{4ij}^U)\cdot\left(h_{A_{ij}}^L \Big/ h_{A_{ij}}^U\right)\Big)\Big] \end{aligned} \tag{38}$$

It is clear that $0 \le CC_i \le 1$. In addition, CC_i=1 if $A_i = A_j^+$, and $CC_i = 0$ if $A_i = A_j^-$. The alternative A_i is closer to A_j^+ and farther from A_j^- as CC_i approaches 1. The alternative with the largest signed distance-based closeness coefficient is the one prescribed to the decision-makers.

However, the crisp values of criterion weights are unknown, and we only have some imprecise or partial information about the importance weights. Thus, an integrated programming model based on signed distance-based closeness coefficients was established to estimate the importance weights of criteria from incomplete and inconsistent information. For each alternative A_i, we can obtain the largest signed distance-based closeness coefficient by maximizing CC_i under the relaxed conditions Γ' about criterion weights. Conversely, for smaller values of the deviation variables, the criterion weights w_j are closer to the constraints in Γ. Furthermore, if the deviation variables are close to zero, then there is no gross violation of the conditions in Γ. Therefore, we have a second objective to minimize $\sum_{j_1,j_2,j_3}(e_{(\text{i})j_1j_2}^- + e_{(\text{ii})j_1j_2}^- + e_{(\text{ii})j_1j_2}^+ + e_{(\text{iii})j_1j_2j_3}^- + e_{(\text{iv})j_1}^- + e_{(\text{iv})j_1}^+ + e_{(\text{v})j_1j_2}^-)$. With consideration of the two objectives of maximal signed distance-based closeness coefficient and minimal deviation variables under the relaxed conditions Γ', a bi-objective programming model was established as follows:

$$\max\{CC_i\}$$
$$\min\left\{\sum_{j_1,j_2,j_3}\left(e_{(\text{i})j_1j_2}^- + e_{(\text{ii})j_1j_2}^- + e_{(\text{ii})j_1j_2}^+ + e_{(\text{iii})j_1j_2j_3}^- + e_{(\text{iv})j_1}^- + e_{(\text{iv})j_1}^+ + e_{(\text{v})j_1j_2}^-\right)\right\}$$

$$[\mathrm{M1}]\mathrm{s.t.}\begin{cases}(w_1,w_2,\ldots,w_n)\in\Gamma' & \\ e^-_{(\mathrm{i})j_1j_2}\geq 0 & j_1\in\Upsilon_1 \text{ and } j_2\in\Lambda_1,\\ e^-_{(\mathrm{ii})j_1j_2}\geq 0, e^+_{(\mathrm{ii})j_1j_2}\geq 0 & j_1\in\Upsilon_2 \text{ and } j_2\in\Lambda_2,\\ e^-_{(\mathrm{iii})j_1j_2j_3}\geq 0 & j_1\in\Upsilon_3, j_2\in\Lambda_3, \text{ and } j_3\in\Omega_3,\\ e^-_{(\mathrm{iv})j_1}\geq 0, e^+_{(\mathrm{iv})j_1}\geq 0 & j_1\in\Upsilon_4,\\ e^-_{(\mathrm{v})j_1j_2}\geq 0 & j_1\in\Upsilon_5 \text{ and } j_2\in\Lambda_5,\end{cases} \tag{39}$$

for each $A_i \in A$.

There are m alternatives in the set A; thus, a total of m bi-objective programming models need to be solved to produce m optimal solutions of criterion weights. Although the optimal weight vector for each alternative can be determined, these optimal weights may be different among the m models in general. As a result, the corresponding signed distance-based closeness coefficients for the m alternatives cannot be compared on the equity basis. Accordingly, an integrated programming model needs to be constructed to determine common weight vectors for a consistent comparative basis. In view of the fact that the decision-makers cannot easily or evidently judge the preference relations among all of the non-inferior alternatives, it is reasonable to assume that all non-inferior alternatives are of equal importance. In addition, the models in [M1] for all $A_i \in A$ have the same constraints, and the m models can be combined to formulate a single multiple objective programming model:

$$\max\{CC_1, CC_2,\ldots,CC_m\}$$

$$\min\left\{\sum_{j_1,j_2,j_3}\left(e^-_{(\mathrm{i})j_1j_2}+e^-_{(\mathrm{ii})j_1j_2}+e^+_{(\mathrm{ii})j_1j_2}+e^-_{(\mathrm{iii})j_1j_2j_3}+e^-_{(\mathrm{iv})j_1}+e^+_{(\mathrm{iv})j_1}+e^-_{(\mathrm{v})j_1j_2}\right)\right\}$$

$$[\mathrm{M2}]\mathrm{s.t.}\begin{cases}(w_1,w_2,\ldots,w_n)\in\Gamma' & \\ e^-_{(\mathrm{i})j_1j_2}\geq 0 & j_1\in\Upsilon_1 \text{ and } j_2\in\Lambda_1,\\ e^-_{(\mathrm{ii})j_1j_2}\geq 0, e^+_{(\mathrm{ii})j_1j_2}\geq 0 & j_1\in\Upsilon_2 \text{ and } j_2\in\Lambda_2,\\ e^-_{(\mathrm{iii})j_1j_2j_3}\geq 0 & j_1\in\Upsilon_3, j_2\in\Lambda_3, \text{ and } j_3\in\Omega_3,\\ e^-_{(\mathrm{iv})j_1}\geq 0, e^+_{(iv)j_1}\geq 0 & j_1\in\Upsilon_4,\\ e^-_{(\mathrm{v})j_1j_2}\geq 0 & j_1\in\Upsilon_5 \text{ and } j_2\in\Lambda_5.\end{cases} \tag{40}$$

The second objective function in [M2] is equivalent to the following objective function: $\max\left\{-\sum_{j_1,j_2,j_3}(e^-_{(\mathrm{i})j_1j_2}+e^-_{(\mathrm{ii})j_1j_2}+e^+_{(\mathrm{ii})j_1j_2}+e^-_{(\mathrm{iii})j_1j_2j_3}+e^-_{(\mathrm{iv})j_1}+e^+_{(\mathrm{iv})j_1}+e^-_{(\mathrm{v})j_1j_2})\right\}$. Let λ be a real number. By utilizing the max-min operator, the model in [M2] can be integrated with the following single-objective nonlinear programming model:

$$\max\lambda$$

$$[\mathrm{M3}]\,\mathrm{s.t.}\begin{cases}CC_i\geq\lambda & i=1,2,\ldots,m,\\ -\sum_{j_1,j_2,j_3\in N}\left(e^-_{(\mathrm{i})j_1j_2}+e^-_{(\mathrm{ii})j_1j_2}+e^+_{(\mathrm{ii})j_1j_2}+e^-_{(\mathrm{iii})j_1j_2j_3}+e^-_{(\mathrm{iv})j_1}+e^+_{(\mathrm{iv})j_1}+e^-_{(\mathrm{v})j_1j_2}\right)\geq\lambda, & \\ (w_1,w_2,\ldots,w_n)\in\Gamma', & \\ e^-_{(\mathrm{i})j_1j_2}\geq 0 & j_1\in\Upsilon_1 \text{ and } j_2\in\Lambda_1,\\ e^-_{(\mathrm{ii})j_1j_2}\geq 0, e^+_{(\mathrm{ii})j_1j_2}\geq 0 & j_1\in\Upsilon_2 \text{ and } j_2\in\Lambda_2,\\ e^-_{(\mathrm{iii})j_1j_2j_3}\geq 0 & j_1\in\Upsilon_3, j_2\in\Lambda_3, \text{ and } j_3\in\Omega_3,\\ e^-_{(\mathrm{iv})j_1}\geq 0, e^+_{(\mathrm{iv})j_1}\geq 0 & j_1\in\Upsilon_4,\\ e^-_{(\mathrm{v})j_1j_2}\geq 0 & j_1\in\Upsilon_5 \text{ and } j_2\in\Lambda_5.\end{cases} \tag{41}$$

The optimal weight vector $\bar{w} = (\bar{w}_1, \bar{w}_2, \ldots, \bar{w}_n)$ and the optimal deviation values $\bar{e}^-_{(\mathrm{i})j_1j_2}$, $\bar{e}^-_{(\mathrm{ii})j_1j_2}$, $\bar{e}^+_{(\mathrm{ii})j_1j_2}$, $\bar{e}^-_{(\mathrm{iii})j_1j_2j_3}$, $\bar{e}^-_{(\mathrm{iv})j_1}$, $\bar{e}^+_{(\mathrm{iv})j_1}$, and $\bar{e}^-_{(\mathrm{v})j_1j_2}$ $(j_1, j_2, j_3 \in N)$ can be obtained by solving the programming problem in [M3]. If all values of $\bar{e}^-_{(\mathrm{i})j_1j_2}$, $\bar{e}^-_{(\mathrm{ii})j_1j_2}$, $\bar{e}^+_{(\mathrm{ii})j_1j_2}$, $\bar{e}^-_{(\mathrm{iii})j_1j_2j_3}$, $\bar{e}^-_{(\mathrm{iv})j_1}$, $\bar{e}^+_{(\mathrm{iv})j_1}$, and $\bar{e}^-_{(\mathrm{v})j_1j_2}$ are equal to zero, then the optimal weight vector $\bar{w}$ satisfies all conditions contained in the incomplete preference information provided by the decision-makers. Otherwise, the resulting weight values are in conflict with some constraints in Γ. Finally, we apply $(\bar{w}_1, \bar{w}_2, \ldots, \bar{w}_n)$ to calculate the corresponding signed distance-based closeness coefficients $\overline{CC}_i$ for all $A_i \in A$. The ranking of all alternatives can be determined along the decreasing order of the $\overline{CC}_i$ values.

4.2 The interactive method

To reach a consensus and acquire a satisfactory solution, an interactive procedure was developed to facilitate the decision-makers to modify or complete their requirements for satisfactory solutions with ease and compared the reference solution yielded by the integrated programming model in [M3]. In doing this, the unsatisfied decision-makers must provide an acceptable satisfaction level (i.e., the lower bounds of the degree of satisfaction) for the undesirable results. Next, we incorporated a set of relevant conditions into the model [M3] to represent additional bounds on signed distance-based closeness coefficients.

A high signed distance-based closeness coefficient CC_i of alternative A_i reflects a higher the degree of satisfaction of A_i. Thus, the CC_i values for each $A_i \in A$ can be employed to identify the degree of satisfaction of each alternative. Let $Z(A_i) = Z\left(\left\{\left\langle x_j, [A^L_{ij}, A^U_{ij}]\right\rangle \middle| x_j \in X\right\}\right)$ denote a satisfactory function regarding the overall performance of the alternative A_i. The function of signed distance-based closeness coefficients can be employed to define a satisfactory function, but a satisfactory function is not necessarily a function of closeness coefficients. Accordingly, the discussed satisfactory function fulfills the following property:

$$CC_{i_1} \geq CC_{i_2} \Rightarrow Z(A_{i_1}) \geq Z(A_{i_2}). \tag{42}$$

Possible maximal and minimal values of CC_i are 1 and 0, respectively. For brevity, it was assumed that the satisfactory function takes the form of a signed distance-based closeness coefficient in this paper; that is:

$$Z(A_i) = CC_i. \tag{43}$$

The more complicated forms can be expected to define the satisfactory function in future research.

Assume that there are κ decision-makers dissatisfied with the solution results, where $\kappa \leq K$. Let $\overset{\frown}{Z}_k(A_i)$ $(i = 1, 2, \ldots, m; k = 1, 2, \ldots, \kappa)$ denote the lower acceptable bound for the degree of satisfaction for alternative A_i provided by the kth decision-maker according to the $\overline{Z}(A_i)\,(= \overline{CC}_i)$ values yielded by [M3]. Considering the individual weights of the κ decision-makers, the aggregated lower bound of the degree of satisfaction for alternative A_i can be obtained by fusing the unsatisfied decision-makers' opinions in the following:

$$\widehat{Z}(A_i) = \frac{\sum_{k=1}^{\kappa} \pi_k \cdot \widehat{Z}_k(A_i)}{\sum_{k=1}^{\kappa} \pi_k}. \tag{44}$$

Let ε_i denote an indifference threshold of the aggregated degree of satisfaction $\widehat{Z}\ (A_i)$ for each alternative A_i, where $\varepsilon_i \geq 0$. That is, the difference in $\widehat{Z}\ (A_i)$ of ε_i or less is assumed to be insignificant. Let the unsatisfactory set Ψ denote the index set for the subset of all alternatives for which $\overline{Z}(A_i) < \widehat{Z}\ (A_i) - \varepsilon_i$:

$$\begin{aligned} \Psi &= \left\{ i \left| \widehat{Z}(A_i) - \overline{Z}(A_i) > \varepsilon_i \right., i = 1, 2, \ldots, m \right\} \\ &= \left\{ i \left| \widehat{Z}\ (A_i) - \overline{CC}_i > \varepsilon_i \right., i = 1, 2, \ldots, m \right\}. \end{aligned} \tag{45}$$

If $\Psi = \varnothing$, then the signed distance-based closeness coefficient $\overline{CC}_i$ of each alternative A_i fulfills the decision-makers' requirements in some sense, and the solution results achieve an acceptable level of satisfaction for common preferences. Conversely, if $\Psi \neq \varnothing$, the analyst should incorporate additional constraints of $CC_i \geq \widehat{Z}(A_i) - \varepsilon_i$ for $i \in \Psi$ into the integrated programming model with the signed distance-based closeness coefficient approach based on IT2TrFNs to ensure that the solution results can gradually satisfy the decision-makers' preferences in the course of decision making.

Recall that solving the model in [M3] would produce the initial optimal weight vector $\bar{w} = (\bar{w}_1, \bar{w}_2, \ldots, \bar{w}_n)$. Accordingly, the corresponding signed distance-based closeness coefficient $\overline{CC}_i$, which represents the satisfactory degree $\overline{Z}(A_i)$, could be obtained for all $A_i \in A$. In the interactive process, the decision-makers are requested to refer $\overline{Z}(A_i)$ values. If some decision-makers are dissatisfied with the solution results, they are requested to give the acceptable lower bound of the degree of satisfaction for those undesirable results. After aggregating the lower bound opinions to acquire $\widehat{Z}(A_i)$ of $A_i \in A$, we establish the following nonlinear programming model:

$$[\text{M4}] \quad \begin{aligned} & \max \lambda \\ & \text{s.t.} \begin{cases} CC_i \geq \lambda & i = 1, 2, \ldots, m, \\ -\sum_{j_1, j_2, j_3 \in N} \left(e^-_{(\text{i}) j_1 j_2} + e^-_{(\text{ii}) j_1 j_2} + e^+_{(\text{ii}) j_1 j_2} + e^-_{(\text{iii}) j_1 j_2 j_3} + e^-_{(\text{iv}) j_1} + e^+_{(\text{iv}) j_1} + e^-_{(\text{v}) j_1 j_2} \right) \geq \lambda, & \\ CC_i \geq \widehat{Z}\ (A_i) - \varepsilon_i & i \in \Psi, \\ (w_1, w_2, \ldots, w_n) \in \Gamma', & \\ e^-_{(\text{i}) j_1 j_2} \geq 0 & j_1 \in \Upsilon_1 \ \text{ and } \ j_2 \in \Lambda_1, \\ e^-_{(\text{ii}) j_1 j_2} \geq 0, e^+_{(\text{ii}) j_1 j_2} \geq 0 & j_1 \in \Upsilon_2 \ \text{ and } \ j_2 \in \Lambda_2, \\ e^-_{(\text{iii}) j_1 j_2 j_3} \geq 0 & j_1 \in \Upsilon_3, j_2 \in \Lambda_3, \ \text{ and } \ j_3 \in \Omega_3, \\ e^-_{(\text{iv}) j_1} \geq 0, e^+_{(\text{iv}) j_1} \geq 0 & j_1 \in \Upsilon_4, \\ e^-_{(\text{v}) j_1 j_2} \geq 0 & j_1 \in \Upsilon_5 \ \text{ and } \ j_2 \in \Lambda_5. \end{cases} \end{aligned} \tag{46}$$

Solving the model in [M4], we can obtain the optimal weight vector $\bar{\bar{w}} = (\bar{\bar{w}}_1, \bar{\bar{w}}_2, \ldots, \bar{\bar{w}}_n)$ and the optimal deviation values $\bar{\bar{e}}^-_{(\text{i}) j_1 j_2}, \bar{\bar{e}}^-_{(\text{ii}) j_1 j_2}, \bar{\bar{e}}^+_{(\text{ii}) j_1 j_2}, \bar{\bar{e}}^-_{(\text{iii}) j_1 j_2 j_3}$,

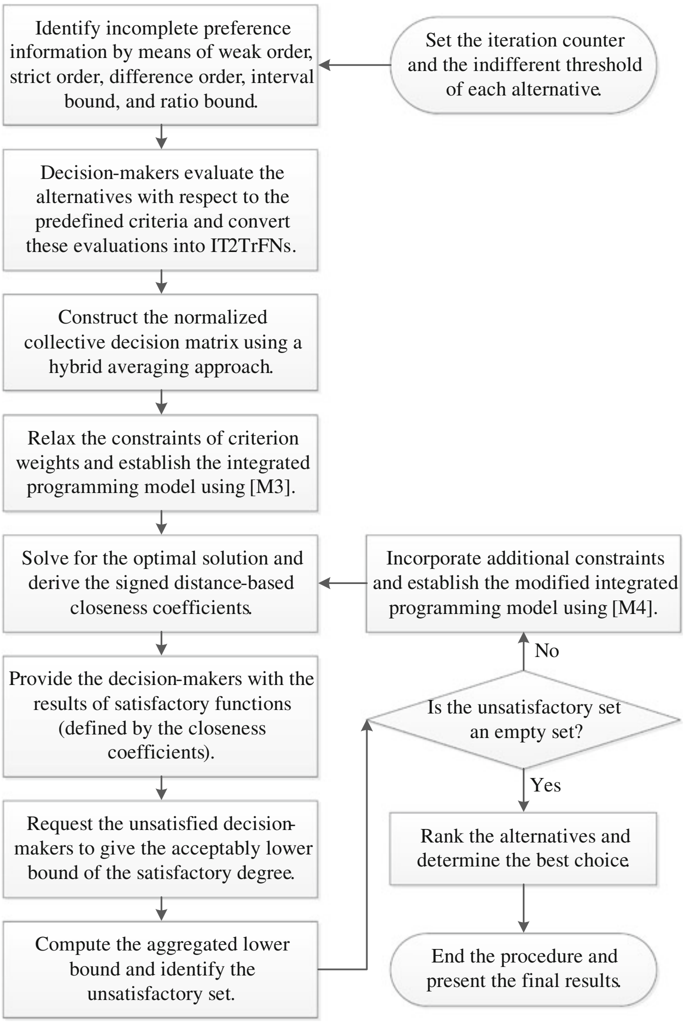

Fig. 1 Flowchart of the proposed method

$\bar{\bar{e}}^{-}_{(\text{iv})j_1}$, $\bar{\bar{e}}^{+}_{(\text{iv})j_1}$, and $\bar{\bar{e}}^{-}_{(\text{v})j_1 j_2}$ $(j_1, j_2, j_3 \in N)$. Next, we apply $(\bar{\bar{w}}_1, \bar{\bar{w}}_2, \ldots, \bar{\bar{w}}_n)$ to calculate the corresponding signed distance-based closeness coefficient $\overline{\overline{CC}}_i$ for all $A_i \in A$. Show the updated degree of satisfaction $\bar{\bar{Z}}(A_i)(= \overline{\overline{CC}}_i)$ to the decision-makers and proceed the interactive method until a satisfactory solution with group consensus is achieved. If there are no feasible solutions in the model of [M4], the decision-makers need to reconsider and readjust the lower bound $\widehat{Z}_k(A_i)(k = 1, 2, \ldots, \kappa)$ regarding the degree of satisfaction for alternative $A_i (i \in \Psi)$ until a feasible solution is obtained. Furthermore, the condition of $\Psi = \varnothing$ can be used as a stopping rule in the process of interactions. While the stopping condition is satisfied, the resulting ranking order of m alternatives can be obtained according to the decreasing order of the $\overline{\overline{CC}}_i$ values.

4.3 The proposed algorithm

For an MCGDM problem, form a committee of decision-makers ($E = \{E_1, E_2, \ldots, E_K\}$) and identify the relative importance weights ($\pi = (\pi_1, \pi_2, \ldots, \pi_K)$) of these decision-makers. Specify the evaluation criteria ($X = \{x_1, x_2, \ldots, x_n\}$) and generate feasible alternatives ($A = \{A_1, A_2, \ldots, A_m\}$). Figure 1 shows the flowchart of the developed method.

For an IT2TrFN environment with incomplete/inconsistent preference information, the interactive group decision-making method using an integrated programming model based on the signed distance-based closeness coefficient approach is given by the following series of successive steps:

- Step 0: Each iteration in this algorithm will be labeled ζ, where $\zeta = 0, 1, 2, \ldots$. Set the iteration counter: $\zeta = 0$. Set the value of the indifferent threshold ε_i for each $A_i \in A$, where $\varepsilon_i \geq 0$.
- Step 1: Request the decision-makers to provide their preference over all criteria by means of weak order, strict order, difference order, interval bound, and ratio bound, as depicted in (19)–(23), respectively, and further construct the set Γ with the known information using (24).
- Step 2: Request the decision-makers to evaluate the alternatives using each criterion and then, convert these evaluations into IT2TrFNs.
- Step 3: Construct the normalized collective decision matrix D and acquire the signed distance-based closeness coefficient CC_i for each $A_i \in A$.
  - 3.1: Apply (12) to calculate the weighted ratings $\dot{A}_{ij}^k$ of individual IT2TrFN data by multiplying the weights of the decision-makers.
  - 3.2: Derive the weighting vector ($\tau = (\tau_1, \tau_2, \ldots, \tau_K)$) in the OWA operation according to the normal distribution based method in (9)–(11).
  - 3.3: Calculate the signed distance $d(\dot{A}_{ij}^k, \tilde{0}_1)$ from $\dot{A}_{ij}^k$ to $\tilde{0}_1$ using Proposition 1 and reorder all of the weighted ratings in descending order of signed distances.
  - 3.4: Compute the hybrid average $\hat{A}_{ij}$ for the group consensus opinion using the HA operation in (13) for each $A_i \in A$ and $x_j \in X$.
  - 3.5: Construct the collective decision matrix $\hat{D}$ in (14).
  - 3.6: Apply (15) to establish the normalized collective decision matrix D in (16). In addition, calculate the signed distance $d(A_{ij}, \tilde{0}_1)$ for each $A_i \in A$ and $x_j \in X$.
  - 3.7: For each alternative characteristic of A_i in (17), determine the signed distance-based closeness coefficient CC_i using (38).
- Step 4: Relax the constraints of criterion weights by introducing the deviation variables, as depicted in (25)–(29). Next, construct the set Γ' of the relaxed conditions using (30).
- Step 5: Establish the integrated programming model using [M3] and solve for the optimal weight vector $\bar{w}$. Apply $\bar{w}$ to calculate the corresponding signed distance-based closeness coefficients $\overline{CC}_i$ for all $A_i \in A$.
- Step 6: Identify the unsatisfactory set Ψ^{ζ}.

6.1: Provide the decision-makers with the results of the satisfactory function $\overline{Z}(A_i)(=\overline{CC}_i)$ for all $A_i \in A$. Request the unsatisfied function to give the acceptable lower bound $\widehat{Z}_k\ (A_i)$ for each undesirable result.
6.2: Apply the individual weights of the unsatisfied decision-makers to compute the aggregated lower bound $\widehat{Z}\ (A_i)$ of the degree of satisfaction using (44).
6.3: Use the condition of $\widehat{Z}\ (A_i) - \overline{Z}(A_i) > \varepsilon_i$ (i.e., $\widehat{Z}\ (A_i) - \overline{CC}_i > \varepsilon_i$) in (45) to identify the unsatisfactory set Ψ^{ζ}.

Step 7: While the stopping condition is false, do Steps 8-11.

Step 8: Establish the modified integrated programming model using [M4] and solve for the optimal weight vector $\bar{\bar{\mathrm{w}}}^{\zeta}$.

Step 9: Apply $\bar{\bar{\mathrm{w}}}^{\zeta}$ to calculate the signed distance-based closeness coefficient $\overline{\overline{CC}}_i^{\zeta}$ for all $A_i \in A$. Let the updated satisfactory degree $\bar{\bar{Z}}(A_i)^{\zeta}$ be $\overline{\overline{CC}}_i^{\zeta}$.

Step 10: Update the unsatisfactory set Ψ^{ζ}.
10.1: Show the updated satisfactory function $\bar{\bar{Z}}(A_i)^{\zeta}$ of each $A_i \in A$ and request the unsatisfied decision-makers to give the acceptable lower bound $\widehat{Z}_k\ (A_i)^{\zeta}$ for each undesirable result.
10.2: Compute the aggregated lower bound $\widehat{Z}\ (A_i)^{\zeta}$ of the satisfactory degree using (44).
10.3: Use the condition of $\widehat{Z}\ (A_i)^{\zeta} - \bar{\bar{Z}}(A_i)^{\zeta} > \varepsilon_i$ (i.e., $\widehat{Z}\ (A_i)^{\zeta} - \overline{\overline{CC}}_i^{\zeta} > \varepsilon_i$) in (45) to identify the unsatisfactory set Ψ^{ζ}.

Step 11: Test for the stopping condition and update the iteration counter. If $\Psi^{\zeta} = \varnothing$, then go to Step 12; otherwise, reset the ζ value ($\zeta^{(new)} = \zeta^{(old)} + 1$) and continue.

Step 12: Rank the m alternatives in decreasing order of the $\overline{\overline{CC}}_i^{\zeta}$ values and determine the best choice.

5 A case study of patient-centered medicine

The decision-making environment of a patient-centered healthcare system is more complex than the decision-making process of an individual due to the involvement of multiple decision-makers, including healthcare personnel, patients, and their families. The attending physician cannot independently perform many crucial healthcare decisions for the patients. Compared with the decision making by individuals, the patient-centered healthcare system adopts a group decision-making method that considers the professional judgments of the entire medical team, the patients' inclinations, and the opinions of the family. The viewpoints and subjective opinions of multiple decision-makers are integrated to comprehensively consider the assessed issue. The following practical example involves a patient-centered medical problem of basilar artery occlusion (BAO) to illustrate the implementation of the proposed interactive signed distance-based closeness coefficient method in the IT2TrFN framework.

5.1 Problem description

The case study was from the Department of Neurology, Chang Gung Memorial Hospital in Taiwan. The patient was an 82-year-old widowed male with a history of hypertension. Because of complaints of physical discomfort, the patient was brought to the hospital by his eldest son and daughter-in-law, who live in the same residence. The physician made an initial diagnosis of acute cerebrovascular disease and arranged an immediate examination. The patient was a retired government employee with two sons and one daughter, who were all married with children. He lived a frugal life with his eldest son and considered physical examinations and treatments to be a waste of money due to his old age and probability of dying at any time (which would allow him to join his wife). In addition to his concerns about expensive medical bills, he disliked the feeling of staying in a hospital and hoped to return home as soon as possible. While waiting for his examination results and diagnosis, his condition deteriorated, and he fell into a coma when the definitive diagnosis was made. His younger son and daughter rushed to the hospital upon being informed by their older brother.

Because the patient was unconscious, the attending physician explained the diagnosis of BAO to his family members. BAO is an acute cerebrovascular disease caused by a complete or partial occlusion of the basilar artery. BAO is characterized by a gradual disturbance in consciousness, rapid progression, and a critical and poor prognosis. The attending physician assessed the patient's medical history and current physical conditions and provided the following treatment options: intravenous thrombolysis, intra-arterial thrombolysis, antiplatelet treatment, and heparinization. To allow the patient's family members to fully understand the advantages and disadvantages of each treatment, the physician described the four treatment methods using several criteria, as summarized in Table 1.

Each treatment has its own advantages and disadvantages. The physician wanted the family members to thoroughly discuss these options and select the treatment method with a group decision. The family members' opinions diverged during the discussion. The patient lived with his eldest son and family, and his eldest son and daughter-in-law were providing his daily care. Therefore, the eldest son thought that he did not need to pay for a portion of the cost of the surgery, while his wife was more concerned about the prognosis of her father-in-law's self-care capacity, worrying about whether she would need to exert more efforts when caring for her father-in-law. The younger son felt that the cost of the surgery did not matter, but he proposed that each of the three siblings pay an equal fraction of the cost and take turns caring for their father during the hospitalization. The daughter was the youngest of the three siblings and received the most attention from her father since childhood. Thus, she was the closest to her father. She wanted the physician to utilize the most effective treatment, such that her father would recover completely. She also worried about the development of complications, which may pose a significant physical burden on her elderly father.

The patient's three children expressed their preferences with respect to some of the assessment criteria. Because the responsibilities of daily care usually fell to the eldest son's wife, she was also asked to express her opinion. Although the patient had already lost consciousness, when he was awake, he indicated that he did not want to pay high medical expenses for his treatment due to his old age and wanted to return to

Table 1 Descriptions of the treatment methods using the assessment criteria

A_1 *(Intravenous thrombolysis)*
1. A very high survival rate
2. The possibility of allergic reaction. These side effects are not expected to have a high severity
3. The possibility of an intracerebral hemorrhage[a] as a complication
4. A 60 % probability of a cure if recanalization is achieved
5. No pain/discomfort during treatment
6. A greater out-of-pocket expense, even though the procedure is covered by the patient's health benefits
7. A shorter hospitalization if recanalization is achieved
8. The probability of recurrence is less than average
9. The prognosis for the patient's self-care capacity is less than average
A_2 *(Intra-arterial thrombolysis)*
1. A very high survival rate
2. The possibility of a blood pressure drop and local contusions. These side effects are not expected to have a high severity
3. The possibility of an intracerebral hemorrhage as a complication. The probability is a little higher than that of A_1
4. A very high probability of a cure
5. An intravenous catheter is used during the treatment, which generates more discomfort than an intravenous injection
6. The procedure is not covered by health insurance and is expensive (about NT$200K) compared with the out-of-pocket expenses under health benefits
7. A short hospitalization
8. A very low probability of a recurrence
9. A moderate prognosis for the patient's self-care capacity
A_3 *(Antiplatelet treatment)*
1. A moderate survival rate
2. The possibility of a low-severity contusion or allergic reaction
3. The possibility of progressive stroke[b] as a complication, which may aggravate and prolong the disease course
4. A very low or near-zero probability of a cure
5. No pain/discomfort during treatment
6. Health insurance covers most of the expenses, with a very low out-of-pocket expense
7. A very long hospitalization
8. The highest probability of a recurrence due to ineffective therapeutic effects
9. The worst prognosis for the patient's self-care capacity
A_4 *(Heparinization)*
1. A high survival rate
2. The possibility of contusions, allergic reactions, and thrombocytopenia as side effects. The severity is relatively high
3. The possibility of an intracerebral hemorrhage as a complication, but with a lower severity compared with A_1 and A_2
4. A relatively low probability of a cure
5. No pain/discomfort during the treatment
6. Low coverage by the patient's health insurance and moderately higher out-of-pocket expenses

Table 1 continued

7. A slightly longer hospitalization
8. A significantly high probability of a recurrence
9. A poor prognosis for the patient's self-care ability

[a] Refers to the large hemorrhages caused by blood vessel ruptures in the cerebral parenchyma. Of these, 80 % occur in the cerebral hemispheres (primarily in the basal ganglia), and the other 20 % occur in the brain stem and cerebellum

[b] Refers to the mild symptoms upon disease onset and the gradually aggravated conditions that develop with the prolonging of the disease course until a fully developed stroke occurs. The condition may progress from either mild to complete paralysis or from mild to severe paresis

home as soon as possible. Using the format of five basic preferences, the incomplete preference information provided by the patient and his family was presented in a mathematical format. In addition, based on the physician's description of the four treatment options, the three siblings were asked to provide an assessment using a nine-point linguistic scale. To help the family reach a consensus, an initial solution was obtained using the method developed in this study. Next, an interactive procedure was used to identify a satisfaction solution, and the treatment method was determined based on this satisfaction solution.

5.2 An illustrative application of the algorithm

The patient's three children, eldest son, younger son, and daughter, are the three decision-makers E_1, E_2, and E_3, respectively. The three decision-makers considered various criteria, including survival rate (x_1), severity of the side effects (x_2), severity of the complications (x_3), probability of a cure (x_4), discomfort index of the treatment (x_5), cost (x_6), number of days of hospitalization (x_7), probability of a recurrence (x_8), and self-care capacity (x_9). Here, x_1, x_4, and x_9 are benefit criteria, whereas the remaining criteria are cost ones. The set of evaluation criteria is denoted as $X = \{x_1, x_2, \ldots, x_9\}$, with $X_b = \{x_1, x_4, x_9\}$ and $X_c = \{x_2, x_3, x_5, x_6, x_7, x_8\}$. The weight vector of the three decision-makers is given by $\pi = (\pi_1, \pi_2, \pi_3) = (0.40, 0.35, 0.25)$. There are four treatment options available, including intravenous thrombolysis (A_1), intra-arterial thrombolysis (A_2), antiplatelet treatment (A_3), and heparinization (A_4). The set of all alternatives is denoted by $A = \{A_1, A_2, A_3, A_4\}$. In the following, we illustrate the implementation process of the proposed interactive group decision-making method in the patient-centered medicine problem step by step:

In Step 0, let the initial iteration counter ζ be zero. In addition, let the indifferent threshold $\varepsilon_i = 0.05$ for each $A_i \in A$ for simplicity. In Step 1, let $\Gamma_0 = \left\{(w_1, w_2, \ldots, w_9) \middle| w_j \geq 0\, (j = 1, 2, \ldots, 9), \sum_{j=1}^{9} w_j = 1\right\}$. The preference relationships over all criteria, provided by the three decision-makers and the two influencers (the patient and his daughter-in-law) are given by:

E_1: $0.12 \leq w_1 \leq 0.25, \quad w_2 \geq 0.9 \cdot w_6;$
E_2: $0.07 \leq w_4 - w_6 \leq 0.15, \quad w_7 - w_3 \geq w_3 - w_8;$
E_3: $w_1 \geq w_2, \quad w_4 - w_8 \geq w_8 - w_7, \quad w_3 \geq 1.1 \cdot w_9;$
The patient: $w_6 \geq w_4, \quad 0.1 \leq w_7 \leq 0.2;$

The patient's daughter-in-law: $0.05 \leq w_9 - w_5 \leq 0.12$.

The following set Γ of known information about criterion weights given by the three decision-makers and the two influencers can be described as:

$$\Gamma = \{(w_1, w_2, \ldots, w_9) \in \Gamma_0 \,|\, w_1 \geq w_2, \quad w_6 \geq w_4, \quad 0.07 \leq w_4 - w_6 \leq 0.15, \\ 0.05 \leq w_9 - w_5 \leq 0.12, \quad w_4 - w_8 \geq w_8 - w_7, \quad w_7 - w_3 \geq w_3 - w_8, \\ 0.12 \leq w_1 \leq 0.25, \quad 0.1 \leq w_7 \leq 0.2, \quad w_2 \geq 0.9 \cdot w_6, \quad w_3 \geq 1.1 \cdot w_9\}.$$

The information in Γ is incomplete and partially inconsistent. For example, the ranking priority of w_4 and w_6 given by E_2 does not agree with the patient's opinion.

In Step 2, the decision-makers applied the nine-point linguistic rating scales to evaluate the four treatment options based on the nine criteria. The rating results are presented in Table 2. Applying the transformation standards proposed in author's previous research (Chen 2012a,b), the linguistic evaluations were easily converted into IT2TrFNs.

In Step 3.1, the weighted ratings $\dot{A}_{ij}^k$ were computed for all $k = 1, 2, 3$. Take A_{27} as an example. The linguistic ratings provided by the three decision-makers are L, ML, and M, and their corresponding IT2TrFNs are [(0.0875, 0.12, 0.16, 0.1825; 0.8), (0.04, 0.1, 0.18, 0.23; 1)], [(0.2325, 0.255, 0.325, 0.3575; 0.8), (0.17, 0.22, 0.36, 0.42; 1)], and [(0.4025, 0.4525, 0.5375, 0.5675; 0.8), (0.32, 0.41, 0.58, 0.65; 1)], respectively, according to Table 1 in Chen (2012a,b). Considering $\pi = (0.40, 0.35, 0.25)$, the weighted ratings $\dot{A}_{27}^1$, $\dot{A}_{27}^2$, and $\dot{A}_{27}^3$ are calculated as follows:

$$\dot{A}_{27}^1 = [(0.0350, 0.0480, 0.0640, 0.0730; 0.8), (0.0160, 0.0400, 0.0720, 0.0920; 1)],$$
$$\dot{A}_{27}^2 = [(0.0814, 0.0893, 0.1138, 0.1251; 0.8), (0.0595, 0.0770, 0.1260, 0.1470; 1)],$$
$$\dot{A}_{27}^3 = [(0.1006, 0.1131, 0.1344, 0.1419; 0.8), (0.0800, 0.1025, 0.1450, 0.1625; 1)].$$

In Step 3.2, the OWA weight vector $\tau = (\tau_1, \tau_2, \tau_3) = (0.2429, 0.5142, 0.2429)$. In Step 3.3, we compute the signed distance $d(\dot{A}_{ij}^k, \tilde{0}_1)$ from $\dot{A}_{ij}^k$ to $\tilde{0}_1$. For example, we use $d(\dot{A}_{27}^1, \tilde{1}_1) = 0.1107$, $d(\dot{A}_{27}^2, \tilde{1}_1) = 0.2042$, and $d(\dot{A}_{27}^3, \tilde{1}_1) = 0.2459$ because $d(\dot{A}_{27}^3, \tilde{1}_1) > d(\dot{A}_{27}^2, \tilde{1}_1) > d(\dot{A}_{27}^1, \tilde{1}_1)$, $\sigma(1) = 3$, $\sigma(2) = 2$, and $\sigma(3) = 1$; moreover, $\dot{A}_{27}^{\sigma(1)} = \dot{A}_{27}^3$, $\dot{A}_{27}^{\sigma(2)} = \dot{A}_{27}^2$, and $\dot{A}_{27}^{\sigma(3)} = \dot{A}_{27}^1$.

In Step 3.4, the group consensus opinions on the ratings of the alternatives with respect to each criterion were derived by utilizing the HA operation. Table 3 summarizes the aggregated rating $\hat{A}_{ij}$ of alternative A_i on criterion x_j. Then, we can establish the collective decision matrix $\hat{D}$ in Step 3.5.

In Step 3.6, from the data in Table 3, it is known that $a_1^+ = 0.3379$, $a_2^- = 0.0024$, $a_3^- = 0.0402$, $a_4^+ = 0.3379$, $a_5^- = 0.0024$, $a_6^- = 0.0039$, $a_7^- = 0.0539$, $a_8^- = 0.0039$, and $a_9^+ = 0.2028$. The normalized decision matrix D was constructed, and Table 4 summarizes the normalized rating A_{ij} of alternative A_i on criterion x_j. In addition, the computation results of the signed distance $d(A_{ij}, \tilde{0}_1)$ for each $A_i \in A$ and $x_j \in X$ are also shown in Table 4.

Table 2 The therapeutic ratings evaluated by the decision-makers

Criteria	Treatment options	Decision-makers		
		E_1 (eldest son)	E_2 (younger son)	E_3 (daughter)
x_1 (Survival rate)	A_1	AH	VH	H
	A_2	VH	VH	AH
	A_3	MH	M	MH
	A_4	MH	H	H
x_2 (Severity of the side effects)	A_1	VL	VL	L
	A_2	VL	L	VL
	A_3	L	VL	VL
	A_4	ML	L	ML
x_3 (Severity of the complications)	A_1	ML	M	L
	A_2	MH	H	MH
	A_3	ML	ML	M
	A_4	L	ML	ML
x_4 (Probability of a cure)	A_1	MH	MH	H
	A_2	AH	VH	VH
	A_3	AL	VL	VL
	A_4	ML	M	L
x_5 (Discomfort index of the treatment)	A_1	VL	L	AL
	A_2	L	ML	L
	A_3	VL	AL	L
	A_4	VL	VL	L
x_6 (Cost)	A_1	ML	ML	L
	A_2	AH	AH	H
	A_3	L	VL	VL
	A_4	M	MH	ML
x_7 (Number of days of hospitalization)	A_1	MH	M	M
	A_2	L	ML	M
	A_3	AH	AH	VH
	A_4	MH	H	MH
x_8 (Probability of a recurrence)	A_1	ML	L	M
	A_2	L	VL	AL
	A_3	AH	AH	AH
	A_4	AH	VH	VH
x_9 (Self-care capacity)	A_1	L	ML	ML
	A_2	MH	ML	M
	A_3	AL	VL	VL
	A_4	VL	ML	L

AL absolutely low, *VL* very low, *L* low, *ML*, medium low, *M* medium, *MH* medium high, *H* high, *VH* very high, *AH* absolutely high

Table 3 The aggregated ratings of treatment options in $\hat{D}$

	$\hat{A}_{ij}^L$					$\hat{A}_{ij}^U$				
	$\hat{a}_{1ij}^L$	$\hat{a}_{2ij}^L$	$\hat{a}_{3ij}^L$	$\hat{a}_{4ij}^L$	$h_{\hat{A}_{ij}}^L$	$\hat{a}_{1ij}^U$	$\hat{a}_{2ij}^U$	$\hat{a}_{3ij}^U$	$\hat{a}_{4ij}^U$	$h_{\hat{A}_{ij}}^U$
$\hat{A}_{11}$	0.3152	0.3240	0.3295	0.3309	0.8000	0.3083	0.3209	0.3330	0.3360	1.0000
$\hat{A}_{12}$	0.0075	0.0095	0.0141	0.0263	0.8000	0.0024	0.0061	0.0167	0.0343	1.0000
$\hat{A}_{13}$	0.0874	0.0982	0.1223	0.1328	0.8000	0.0646	0.0862	0.1343	0.1556	1.0000
$\hat{A}_{14}$	0.2276	0.2359	0.2637	0.2740	0.8000	0.2045	0.2220	0.2776	0.2972	1.0000
$\hat{A}_{15}$	0.0090	0.0117	0.0167	0.0263	0.8000	0.0034	0.0085	0.0194	0.0340	1.0000
$\hat{A}_{16}$	0.0698	0.0780	0.0998	0.1101	0.8000	0.0495	0.0670	0.1107	0.1304	1.0000
$\hat{A}_{17}$	0.1600	0.1743	0.2030	0.2133	0.8000	0.1334	0.1599	0.2173	0.2400	1.0000
$\hat{A}_{18}$	0.0797	0.0901	0.1131	0.1235	0.8000	0.0578	0.0786	0.1246	0.1454	1.0000
$\hat{A}_{19}$	0.0581	0.0662	0.0850	0.0941	0.8000	0.0402	0.0567	0.0944	0.1120	1.0000
$\hat{A}_{21}$	0.3233	0.3337	0.3358	0.3358	0.8000	0.3185	0.3323	0.3379	0.3379	1.0000
$\hat{A}_{22}$	0.0094	0.0122	0.0176	0.0295	0.8000	0.0034	0.0085	0.0206	0.0382	1.0000
$\hat{A}_{23}$	0.2397	0.2485	0.2771	0.2876	0.8000	0.2157	0.2341	0.2913	0.3116	1.0000
$\hat{A}_{24}$	0.3252	0.3343	0.3361	0.3361	0.8000	0.3210	0.3330	0.3379	0.3379	1.0000
$\hat{A}_{25}$	0.0431	0.0537	0.0703	0.0790	0.8000	0.0251	0.0453	0.0786	0.0970	1.0000
$\hat{A}_{26}$	0.3246	0.3266	0.3309	0.3322	0.8000	0.3209	0.3245	0.3330	0.3360	1.0000
$\hat{A}_{27}$	0.0748	0.0850	0.1067	0.1165	0.8000	0.0539	0.0742	0.1175	0.1374	1.0000
$\hat{A}_{28}$	0.0098	0.0130	0.0183	0.0272	0.8000	0.0039	0.0097	0.0211	0.0349	1.0000
$\hat{A}_{29}$	0.1347	0.1452	0.1703	0.1801	0.8000	0.1119	0.1326	0.1829	0.2028	1.0000
$\hat{A}_{31}$	0.1809	0.1903	0.2167	0.2266	0.8000	0.1581	0.1771	0.2299	0.2494	1.0000
$\hat{A}_{32}$	0.0103	0.0135	0.0192	0.0304	0.8000	0.0039	0.0097	0.0223	0.0392	1.0000
$\hat{A}_{33}$	0.0920	0.1016	0.1271	0.1384	0.8000	0.0689	0.0889	0.1399	0.1616	1.0000
$\hat{A}_{34}$	0.0016	0.0016	0.0032	0.0112	0.8000	0.0000	0.0000	0.0043	0.0149	1.0000
$\hat{A}_{35}$	0.0069	0.0088	0.0128	0.0219	0.8000	0.0024	0.0061	0.0150	0.0284	1.0000
$\hat{A}_{36}$	0.0103	0.0135	0.0192	0.0304	0.8000	0.0039	0.0097	0.0223	0.0392	1.0000
$\hat{A}_{37}$	0.3347	0.3370	0.3374	0.3374	0.8000	0.3336	0.3366	0.3379	0.3379	1.0000
$\hat{A}_{38}$	0.3379	0.3379	0.3379	0.3379	1.0000	0.3379	0.3379	0.3379	0.3379	1.0000
$\hat{A}_{39}$	0.0016	0.0016	0.0032	0.0112	0.8000	0.0000	0.0000	0.0043	0.0149	1.0000
$\hat{A}_{41}$	0.2477	0.2571	0.2848	0.2947	0.8000	0.2242	0.2433	0.2986	0.3183	1.0000
$\hat{A}_{42}$	0.0599	0.0678	0.0870	0.0962	0.8000	0.0418	0.0582	0.0966	0.1144	1.0000
$\hat{A}_{43}$	0.0581	0.0662	0.0850	0.0941	0.8000	0.0402	0.0567	0.0944	0.1120	1.0000
$\hat{A}_{44}$	0.0874	0.0982	0.1223	0.1328	0.8000	0.0646	0.0862	0.1343	0.1556	1.0000
$\hat{A}_{45}$	0.0075	0.0095	0.0141	0.0263	0.8000	0.0024	0.0061	0.0167	0.0343	1.0000
$\hat{A}_{46}$	0.1522	0.1657	0.1947	0.2056	0.8000	0.1254	0.1512	0.2092	0.2323	1.0000
$\hat{A}_{47}$	0.2397	0.2485	0.2771	0.2876	0.8000	0.2157	0.2341	0.2913	0.3116	1.0000
$\hat{A}_{48}$	0.3252	0.3343	0.3361	0.3361	0.8000	0.3210	0.3330	0.3379	0.3379	1.0000
$\hat{A}_{49}$	0.0318	0.0378	0.0497	0.0589	0.8000	0.0196	0.0316	0.0557	0.0721	1.0000

Table 4 The normalized collective decision matrix D

	A_{ij}^L					A_{ij}^U					$d(A_{ij}, \tilde{0}_1)$
	a_{1ij}^L	a_{2ij}^L	a_{3ij}^L	a_{4ij}^L	$h_{A_{ij}}^L$	a_{1ij}^U	a_{2ij}^U	a_{3ij}^U	a_{4ij}^U	$h_{A_{ij}}^U$	
A_{11}	0.9328	0.9589	0.9751	0.9793	0.8000	0.9124	0.9497	0.9855	0.9944	1.0000	1.9265
A_{12}	0.0913	0.1702	0.2526	0.3200	0.8000	0.0700	0.1437	0.3934	1.0000	1.0000	0.6137
A_{13}	0.3027	0.3287	0.4094	0.4600	0.8000	0.2584	0.2993	0.4664	0.6223	1.0000	0.7849
A_{14}	0.6736	0.6981	0.7804	0.8109	0.8000	0.6052	0.6570	0.8215	0.8796	1.0000	1.4805
A_{15}	0.0913	0.1437	0.2051	0.2667	0.8000	0.0706	0.1237	0.2824	0.7059	1.0000	0.4670
A_{16}	0.0354	0.0391	0.0500	0.0559	0.8000	0.0299	0.0352	0.0582	0.0788	1.0000	0.0957
A_{17}	0.2527	0.2655	0.3092	0.3369	0.8000	0.2246	0.2480	0.3371	0.4040	1.0000	0.5931
A_{18}	0.0316	0.0345	0.0433	0.0489	0.8000	0.0268	0.0313	0.0496	0.0675	1.0000	0.0831
A_{19}	0.2865	0.3264	0.4191	0.4640	0.8000	0.1982	0.2796	0.4655	0.5523	1.0000	0.7469
A_{21}	0.9568	0.9876	0.9938	0.9938	0.8000	0.9426	0.9834	1.0000	1.0000	1.0000	1.9709
A_{22}	0.0814	0.1364	0.1967	0.2553	0.8000	0.0628	0.1165	0.2824	0.7059	1.0000	0.4569
A_{23}	0.1398	0.1451	0.1618	0.1677	0.8000	0.1290	0.1380	0.1717	0.1864	1.0000	0.3102
A_{24}	0.9624	0.9893	0.9947	0.9947	0.8000	0.9500	0.9855	1.0000	1.0000	1.0000	1.9747
A_{25}	0.0304	0.0341	0.0447	0.0557	0.8000	0.0247	0.0305	0.0530	0.0956	1.0000	0.0906
A_{26}	0.0117	0.0118	0.0119	0.0120	0.8000	0.0116	0.0117	0.0120	0.0122	1.0000	0.0237
A_{27}	0.4627	0.5052	0.6341	0.7206	0.8000	0.3923	0.4587	0.7264	1.0000	1.0000	1.2206
A_{28}	0.1434	0.2131	0.3000	0.3980	0.8000	0.1117	0.1848	0.4021	1.0000	1.0000	0.6769
A_{29}	0.6642	0.7160	0.8397	0.8881	0.8000	0.5518	0.6538	0.9019	1.0000	1.0000	1.5545
A_{31}	0.5354	0.5632	0.6413	0.6706	0.8000	0.4679	0.5241	0.6804	0.7381	1.0000	1.2050
A_{32}	0.0789	0.1250	0.1778	0.2330	0.8000	0.0612	0.1076	0.2474	0.6154	1.0000	0.4074
A_{33}	0.2905	0.3163	0.3957	0.4370	0.8000	0.2488	0.2873	0.4522	0.5835	1.0000	0.7531
A_{34}	0.0047	0.0047	0.0095	0.0331	0.8000	0.0000	0.0000	0.0127	0.0441	1.0000	0.0223
A_{35}	0.1096	0.1875	0.2727	0.3478	0.8000	0.0845	0.1600	0.3934	1.0000	1.0000	0.6360
A_{36}	0.1283	0.2031	0.2889	0.3786	0.8000	0.0995	0.1749	0.4021	1.0000	1.0000	0.6621
A_{37}	0.1598	0.1598	0.1599	0.1610	0.8000	0.1595	0.1595	0.1601	0.1616	1.0000	0.3201
A_{38}	0.0115	0.0115	0.0115	0.0115	0.8000	0.0115	0.0115	0.0115	0.0115	1.0000	0.0230
A_{39}	0.0079	0.0079	0.0158	0.0552	0.8000	0.0000	0.0000	0.0212	0.0735	1.0000	0.0372
A_{41}	0.7331	0.7609	0.8429	0.8722	0.8000	0.6635	0.7200	0.8837	0.9420	1.0000	1.6043
A_{42}	0.0249	0.0276	0.0354	0.0401	0.8000	0.0210	0.0248	0.0412	0.0574	1.0000	0.0680
A_{43}	0.4272	0.4729	0.6073	0.6919	0.8000	0.3589	0.4258	0.7090	1.0000	1.0000	1.1708
A_{44}	0.2587	0.2906	0.3619	0.3930	0.8000	0.1912	0.2551	0.3975	0.4605	1.0000	0.6523
A_{45}	0.0913	0.1702	0.2526	0.3200	0.8000	0.0700	0.1437	0.3934	1.0000	1.0000	0.6137
A_{46}	0.0190	0.0200	0.0235	0.0256	0.8000	0.0168	0.0186	0.0258	0.0311	1.0000	0.0450
A_{47}	0.1874	0.1945	0.2169	0.2249	0.8000	0.1730	0.1850	0.2302	0.2499	1.0000	0.4159
A_{48}	0.0116	0.0116	0.0117	0.0120	0.8000	0.0115	0.0115	0.0117	0.0121	1.0000	0.0233
A_{49}	0.1568	0.1864	0.2451	0.2904	0.8000	0.0966	0.1558	0.2747	0.3555	1.0000	0.4370

In Step 3.7, we acquired the signed distance-based closeness coefficient CC_i for each characteristics of alternative A_i. For example:

$$CC_1 = \frac{1}{2}\left(1.9265w_1 + 0.6137w_2 + 0.7849w_3 + 1.4805w_4 + 0.4670w_5 + 0.0957w_6 + 0.5931w_7 + 0.0831w_8 + 0.7469w_9\right).$$

In Step 4, several non-negative deviation variables were introduced to construct the set Γ' of the relaxed conditions as follows:

$$\Gamma' = \Big\{(w_1, w_2, \ldots, w_9) \in \Gamma_0 \Big| w_1 + e^-_{(i)12} \geq w_2,\ w_6 + e^-_{(i)64} \geq w_4,\ w_4 - w_6 + e^-_{(ii)46} \geq 0.07, w_4 - w_6 - e^+_{(ii)46} \leq 0.15,\ w_9 - w_5 + e^-_{(ii)95} \geq 0.05,\ w_9 - w_5 - e^+_{(ii)95} \leq 0.12, w_4 - 2w_8 + w_7 + e^-_{(iii)487} \geq 0,\ w_7 - 2w_3 + w_8 + e^-_{(iii)738} \geq 0,\ w_1 + e^-_{(iv)1} \geq 0.12, w_1 - e^+_{(iv)1} \leq 0.25,\quad w_7 + e^-_{(iv)7} \geq 0.1,\quad w_7 - e^+_{(iv)7} \leq 0.2,\quad \frac{w_2}{w_6} + e^-_{(v)26} \geq 0.9,\ \frac{w_3}{w_9} + e^-_{(v)39} \geq 1.1\Big\}.$$

In Step 5, the following integrated programming model using [M3] was constructed to estimate the importance of the criterion weights:

$$\begin{aligned}
\max\ & \lambda \\
\text{s.t.}\ & \frac{1}{2}\left(1.9265w_1 + 0.6137w_2 + 0.7849w_3 + 1.4805w_4 + 0.4670w_5 + 0.0957w_6 + 0.5931w_7 + 0.0831w_8 + 0.7469w_9\right) \geq \lambda, \\
& \frac{1}{2}\left(1.9709w_1 + 0.4569w_2 + 0.3102w_3 + 1.9747w_4 + 0.0906w_5 + 0.0237w_6 + 1.2206w_7 + 0.6769w_8 + 1.5545w_9\right) \geq \lambda, \\
& \frac{1}{2}\left(1.2050w_1 + 0.4074w_2 + 0.7531w_3 + 0.0223w_4 + 0.6360w_5 + 0.6621w_6 + 0.3201w_7 + 0.0230w_8 + 0.0372w_9\right) \geq \lambda, \\
& \frac{1}{2}\left(1.6043w_1 + 0.0680w_2 + 1.1708w_3 + 0.6523w_4 + 0.6137w_5 + 0.0450w_6 + 0.4159w_7 + 0.0233w_8 + 0.4370w_9\right) \geq \lambda, \\
& -\Big(e^-_{(i)12} + e^-_{(i)64} + e^-_{(ii)46} + e^+_{(ii)46} + e^-_{(ii)95} + e^+_{(ii)95} + e^-_{(iii)487} + e^-_{(iii)738} + e^-_{(iv)1} + e^+_{(iv)1} + e^-_{(iv)7} + e^+_{(iv)7} + e^-_{(v)26} + e^-_{(v)39}\Big) \geq \lambda,\ (w_1, w_2, \ldots, w_9) \in \Gamma', \\
& e^-_{(i)12} \geq 0,\quad e^-_{(i)64} \geq 0,\quad e^-_{(ii)46} \geq 0,\quad e^+_{(ii)46} \geq 0,\quad e^-_{(ii)95} \geq 0,\quad e^+_{(ii)95} \geq 0, \\
& e^-_{(iii)487} \geq 0,\quad e^-_{(iii)738} \geq 0,\quad e^-_{(iv)1} \geq 0,\quad e^+_{(iv)1} \geq 0,\quad e^-_{(iv)7} \geq 0,\quad e^+_{(iv)7} \geq 0, \\
& e^-_{(v)26} \geq 0,\quad e^-_{(v)39} \geq 0.
\end{aligned} \tag{47}$$

The model in (47) was solved to obtain the optimal weight vector, $\bar{w} = (\bar{w}_1, \bar{w}_2, \ldots, \bar{w}_9) = (0.1490, 0.0557, 0.1352, 0.1318, 0.0730, 0.0618, 0.1923, 0.0782, 0.1230)$; the optimal deviation values, $\bar{e}^-_{(i)12} = \bar{e}^-_{(ii)46} = \bar{e}^+_{(ii)46} = \bar{e}^-_{(ii)95} =$

$\bar{e}^{+}_{\text{(ii)}95} = \bar{e}^{-}_{\text{(iii)}487} = \bar{e}^{-}_{\text{(iii)}738} = \bar{e}^{-}_{\text{(iv)}1} = \bar{e}^{+}_{\text{(iv)}1} = \bar{e}^{-}_{\text{(iv)}7} = \bar{e}^{+}_{\text{(iv)}7} = \bar{e}^{-}_{\text{(v)}26} = \bar{e}^{-}_{\text{(v)}39} = 0$, and $\bar{e}^{-}_{\text{(i)}64} = 0.07$; and the optimal objective value, $\bar{\lambda} = -0.07$. The signed distance-based closeness coefficients were calculated as follows: $\overline{CC}_1 = 0.4374$, $\overline{CC}_2 = 0.5541$, $\overline{CC}_3 = 0.2311$, and $\overline{CC}_4 = 0.3351$. The corresponding ranking of the treatments is: $A_2 \succ A_1 \succ A_4 \succ A_3$.

In Step 6.1, the analyst showed the solution results to the three decision-makers, including the resulting satisfactory functions of $\overline{Z}(A_1) = 0.4374$, $\overline{Z}(A_2) = 0.5541$, $\overline{Z}(A_3) = 0.2311$, and $\overline{Z}(A_4) = 0.3351$. Then, the decision-makers spoke with the analyst, revealing that the patient's two sons were not satisfied with the results of $\overline{Z}(A_3)$. They provided their acceptable lower bounds of the satisfaction function for the alternative A_3 as follows: $\widehat{Z}_1\ (A_3) = 0.28$ and $\widehat{Z}_2\ (A_3) = 0.33$. In Step 6.2, considering the individual weights of the patient's sons, the aggregated lower bound of the satisfaction degree was obtained as follows:

$$\widehat{Z}(A_3) = \frac{0.4 \times 0.28 + 0.35 \times 0.33}{0.4 + 0.35} = 0.3033.$$

In Step 6.3, because $\widehat{Z}(A_3) - \overline{Z}(A_3) = 0.3033 - 0.2311 = 0.0722 > \varepsilon_3(= 0.05)$, we identified that the unsatisfactory set $\Psi^0 = \{3\}$.

In Step 7, the solution result regarding A_3 in this iteration does not satisfy the first two decision-makers' requirements. Because $\Psi^0 \neq \varnothing$, the stopping condition is false. Let $\zeta^{(new)} = \zeta^{(old)} + 1 = 0 + 1 = 1$ and continue the algorithm.

In Step 8, we added the constraint of $CC_3 \geq \widehat{Z}\ (A_3) - \varepsilon_3(= 0.3033 - 0.05 = 0.2533)$ into the model in (47) and constructed the following modified integrated programming model using [M4]:

$$\begin{aligned}
\max\ & \lambda \\
\text{s.t.}\ & \frac{1}{2}\,(1.9265w_1 + 0.6137w_2 + 0.7849w_3 + 1.4805w_4 + 0.4670w_5 + 0.0957w_6 \\
& +0.5931w_7 + 0.0831w_8 + 0.7469w_9) \geq \lambda, \\
& \frac{1}{2}\,(1.9709w_1 + 0.4569w_2 + 0.3102w_3 + 1.9747w_4 + 0.0906w_5 + 0.0237w_6 \\
& +1.2206w_7 + 0.6769w_8 + 1.5545w_9) \geq \lambda, \\
& \frac{1}{2}\,(1.2050w_1 + 0.4074w_2 + 0.7531w_3 + 0.0223w_4 + 0.6360w_5 + 0.6621w_6 \\
& +0.3201w_7 + 0.0230w_8 + 0.0372w_9) \geq \lambda, \\
& \frac{1}{2}\,(1.6043w_1 + 0.0680w_2 + 1.1708w_3 + 0.6523w_4 + 0.6137w_5 + 0.0450w_6 \\
& +0.4159w_7 + 0.0233w_8 + 0.4370w_9) \geq \lambda, \\
& -\Big(e^{-}_{\text{(i)}12} + e^{-}_{\text{(i)}64} + e^{-}_{\text{(ii)}46} + e^{+}_{\text{(ii)}46} + e^{-}_{\text{(ii)}95} + e^{+}_{\text{(ii)}95} + e^{-}_{\text{(iii)}487} + e^{-}_{\text{(iii)}738} + e^{-}_{\text{(iv)}1} + e^{+}_{\text{(iv)}1} \\
& \quad + e^{-}_{\text{(iv)}7} + e^{+}_{\text{(iv)}7} + e^{-}_{\text{(v)}26} + e^{-}_{\text{(v)}39}\Big) \geq \lambda,
\end{aligned}$$

$$\frac{1}{2}\left(1.2050w_1 + 0.4074w_2 + 0.7531w_3 + 0.0223w_4 + 0.6360w_5 + 0.6621w_6 + 0.3201w_7 + 0.0230w_8 + 0.0372w_9\right) \geq 0.2533, (w_1, w_2, \ldots, w_9) \in \Gamma',$$
$$e^-_{(\mathrm{i})12} \geq 0, e^-_{(\mathrm{i})64} \geq 0, e^-_{(\mathrm{ii})46} \geq 0, e^+_{(\mathrm{ii})46} \geq 0, e^-_{(\mathrm{ii})95} \geq 0, e^+_{(\mathrm{ii})95} \geq 0, e^-_{(\mathrm{iii})487} \geq 0,$$
$$e^-_{(\mathrm{iii})738} \geq 0, e^-_{(\mathrm{iv})1} \geq 0, e^+_{(\mathrm{iv})1} \geq 0, e^-_{(\mathrm{iv})7} \geq 0, e^+_{(\mathrm{iv})7} \geq 0, e^-_{(\mathrm{v})26} \geq 0, e^-_{(\mathrm{v})39} \geq 0. \quad (48)$$

The above model was solved to obtain the optimal weight vector, $\bar{\bar{w}}^1 = (\bar{\bar{w}}_1^1, \bar{\bar{w}}_2^1, \ldots, \bar{\bar{w}}_9^1) = (0.2218, 0.2057, 0.0859, 0.1017, 0.0000, 0.0678, 0.1489, 0.0901, 0.0781)$; the optimal deviation values, $\bar{\bar{e}}^{-,1}_{(\mathrm{i})12} = \bar{\bar{e}}^{+,1}_{(\mathrm{ii})46} = \bar{\bar{e}}^{-,1}_{(\mathrm{ii})95} = \bar{\bar{e}}^{+,1}_{(\mathrm{ii})95} = \bar{\bar{e}}^{-,1}_{(\mathrm{iii})487} = \bar{\bar{e}}^{-,1}_{(\mathrm{iii})738} = \bar{\bar{e}}^{-,1}_{(\mathrm{iv})1} = \bar{\bar{e}}^{+,1}_{(\mathrm{iv})1} = \bar{\bar{e}}^{-,1}_{(\mathrm{iv})7} = \bar{\bar{e}}^{+,1}_{(\mathrm{iv})7} = \bar{\bar{e}}^{-,1}_{(\mathrm{v})26} = \bar{\bar{e}}^{-,1}_{(\mathrm{v})39} = 0$, $\bar{\bar{e}}^{-,1}_{(\mathrm{i})64} = 0.0339$, and $\bar{\bar{e}}^{-,1}_{(\mathrm{ii})46} = 0.0361$; and the optimal objective value, $\bar{\bar{\lambda}}^1 = -0.07$.

In Step 9, applying $\bar{\bar{w}}^1$, the signed distance-based closeness coefficients were calculated to be: $\overline{\overline{CC}}_1^1 = 0.4661$, $\overline{\overline{CC}}_2^1 = 0.5622$, $\overline{\overline{CC}}_3^1 = 0.2578$, and $\overline{\overline{CC}}_4^1 = 0.3190$ with the updated satisfaction degree $\bar{\bar{Z}}(A_i)^1 = \overline{\overline{CC}}_i^1$ for each $A_i \in A$.

In Step 10.1, the analyst showed the updated satisfactory function $\bar{\bar{Z}}(A_i)^1$ of each A_i to the three decision-makers. In this time, only the patient's daughter was dissatisfied with the results of $\bar{\bar{Z}}(A_4)^1$. She offered the acceptable lower bound $\widehat{Z}_3\ (A_4)^1 = 0.32$. In Step 10.2, the aggregated lower bound $\widehat{Z}(A_4)^1 = (0.25 \times 0.32)/0.25 = 0.32$. In Step 10.3, the unsatisfactory set $\Psi^1 = \varnothing$ because $\widehat{Z}(A_4)^1 - \bar{\bar{Z}}(A_4)^1 = 0.32 - 0.319 = 0.001 < \varepsilon_4 (= 0.05)$.

In Step 11, the solution results of the model in (48) satisfy the stopping condition; thus, go to Step 12. In Step 12, the optimal ranking of the four treatment options was determined according to the decreasing order of the $\overline{\overline{CC}}_i^1$ values: $A_2 \succ A_1 \succ A_4 \succ A_3$. Therefore, A_2 is the most appropriate treatment option for the patient. Finally, the patient's family decided to adopt intra-arterial thrombolysis (A_2) as the treatment. The patient became conscious after the treatment and is still undergoing rehabilitation.

5.3 Comparative analysis

A comparative study was conducted to validate the results of the proposed method with those of other approaches. We based the analysis on the same input data that are presented in Sect. 5.1 and chose a well-known and widely used method, the TOPSIS approach, to facilitate the comparison analysis.

The basic concept of the TOPSIS method is that the chosen alternative should have the shortest distance from the positive-ideal solution and the farthest distance from the negative-ideal solution (Hwang and Yoon 1981). Considering $(w_1, w_2, \ldots, w_n) \in \Gamma'$, the weighted IT2TrFN value A_i^w of A_i is acquired by the following:

$$A_i^w = \left\{ \left\langle x_j, \left[\left(w_j a^L_{1ij}, w_j a^L_{2ij}, w_j a^L_{3ij}, w_j a^L_{4ij}; h^L_{A_{ij}} \right), \left(w_j a^U_{1ij}, w_j a^U_{2ij}, w_j a^U_{3ij}, w_j a^U_{4ij}; h^U_{A_{ij}} \right) \right] \right\rangle \middle| x_j \in X \right\}. \quad (49)$$

The weighted positive-ideal and negative-ideal solutions are obtained as follows:

$$A^{w+} = \left\{ \left\langle x_j, \left[(w_j, w_j, w_j, w_j; 1), (w_j, w_j, w_j, w_j; 1) \right] \right\rangle \middle| x_j \in X \right\}, \quad (50)$$

$$A^{w-} = \left\{ \left\langle x_j, [(0, 0, 0, 0; 1), (0, 0, 0, 0; 1)] \right\rangle \middle| x_j \in X \right\}. \quad (51)$$

The Euclidean distances, $d^E(A_i^w, A^{w+})$ and $d^E(A_i^w, A^{w-})$, of each weighted IT2TrFN value A_i^w from the weighted positive-ideal and negative-ideal solutions, respectively, are derived from:

$$d^E(A_i^w, A^{w+}) = \left[\frac{1}{8} \sum_{j=1}^{n} w_j^2 \left(\left(1 - a_{1ij}^L\right)^2 + \left(1 - a_{2ij}^L\right)^2 + \left(1 - a_{3ij}^L\right)^2 + \left(1 - a_{4ij}^L\right)^2 + \left(1 - a_{1ij}^U\right)^2 + \left(1 - a_{2ij}^U\right)^2 + \left(1 - a_{3ij}^U\right)^2 + \left(1 - a_{4ij}^U\right)^2 \right) \right]^{\frac{1}{2}}. \quad (52)$$

$$d^E(A_i^w, A^{w-}) = \left[\frac{1}{8} \sum_{j=1}^{n} w_j^2 \left(\left(a_{1ij}^L\right)^2 + \left(a_{2ij}^L\right)^2 + \left(a_{3ij}^L\right)^2 + \left(a_{4ij}^L\right)^2 + \left(a_{1ij}^U\right)^2 + \left(a_{2ij}^U\right)^2 + \left(a_{3ij}^U\right)^2 + \left(a_{4ij}^U\right)^2 \right) \right]^{\frac{1}{2}}. \quad (53)$$

Then, the closeness coefficient CC_i^E $(0 \leq CC_i^E \leq 1)$ of the alternative A_i is defined with the following formula:

$$CC_i^E = \frac{d^E(A_i^w, A^{w-})}{d^E(A_i^w, A^{w+}) + d^E(A_i^w, A^{w-})}. \quad (54)$$

A single-objective nonlinear programming model can be constructed with incomplete information as follows:

$$[\text{M5}] \quad \max \lambda' \quad \text{s.t.} \begin{cases} CC_i^E \geq \lambda' & i = 1, 2, \ldots, m, \\ -\sum_{j_1, j_2, j_3 \in N} \left(e_{(\text{i}) j_1 j_2}^- + e_{(\text{ii}) j_1 j_2}^- + e_{(\text{ii}) j_1 j_2}^+ + e_{(\text{iii}) j_1 j_2 j_3}^- + e_{(\text{iv}) j_1}^- + e_{(\text{iv}) j_1}^+ + e_{(\text{v}) j_1 j_2}^- \right) \geq \lambda', & \\ (w_1, w_2, \ldots, w_n) \in \Gamma', & \\ e_{(\text{i}) j_1 j_2}^- \geq 0 & j_1 \in \Upsilon_1 \text{ and } j_2 \in \Lambda_1, \\ e_{(\text{ii}) j_1 j_2}^- \geq 0, e_{(\text{ii}) j_1 j_2}^+ \geq 0 & j_1 \in \Upsilon_2 \text{ and } j_2 \in \Lambda_2, \\ e_{(\text{iii}) j_1 j_2 j_3}^- \geq 0 & j_1 \in \Upsilon_3, j_2 \in \Lambda_3, \text{ and } j_3 \in \Omega_3, \\ e_{(\text{iv}) j_1}^- \geq 0, e_{(\text{iv}) j_1}^+ \geq 0 & j_1 \in \Upsilon_4, \\ e_{(\text{v}) j_1 j_2}^- \geq 0 & j_1 \in \Upsilon_5 \text{ and } j_2 \in \Lambda_5. \end{cases} \quad (55)$$

Incorporating the data of the patient-centered medical problem into [M5], we constructed the following integrated programming model:

$$\begin{aligned}
\max \quad & \lambda' \\
\text{s.t.} \quad & \left[\frac{1}{8}\left(7.3939w_1^2 + 1.3838w_2^2 + 1.3400w_3^2 + 4.4541w_4^2 + 0.7405w_5^2 + 0.0201w_6^2\right.\right. \\
& \left.\left. + 0.7323w_7^2 + 0.0152w_8^2 + 1.2187w_9^2\right)\right]^{0.5} \Big/ \left\{\left[\frac{1}{8}\left(0.0177w_1^2 + 4.5014w_2^2\right.\right.\right. \\
& \left.\left. + 3.0456w_3^2 + 0.6015w_4^2 + 4.9617w_5^2 + 7.2551w_6^2 + 3.9763w_7^2 + 7.3482w_8^2\right.\right. \\
& \left.\left. + 3.2355w_9^2\right)\right]^{0.5} + \left[\frac{1}{8}\left(7.3939w_1^2 + 1.3838w_2^2 + 1.3400w_3^2 + 4.4541w_4^2\right.\right. \\
& \left.\left.\left. + 0.7405w_5^2 + 0.0201w_6^2 + 0.7323w_7^2 + 0.0152w_8^2 + 1.2187w_9^2\right)\right]^{0.5}\right\} \geq \lambda', \\
& \left[\frac{1}{8}\left(7.7217w_1^2 + 0.7247w_2^2 + 0.1948w_3^2 + 7.7575w_4^2 + 0.0207w_5^2 + 0.0011w_6^2\right.\right. \\
& \left.\left. + 3.2826w_7^2 + 1.5227w_8^2 + 4.9930w_9^2\right)\right]^{0.5} \Big/ \left\{\left[\frac{1}{8}\left(0.0057w_1^2 + 5.0499w_2^2\right.\right.\right. \\
& \left.\left. + 5.7158w_3^2 + 0.0043w_4^2 + 7.2833w_5^2 + 7.8113w_6^2 + 1.4826w_7^2 + 4.0165w_8^2\right.\right. \\
& \left.\left. + 0.5620w_9^2\right)\right]^{0.5} + \left[\frac{1}{8}\left(7.7217w_1^2 + 0.7247w_2^2 + 0.1948w_3^2 + 7.7575w_4^2\right.\right. \\
& \left.\left.\left. + 0.0207w_5^2 + 0.0011w_6^2 + 3.2826w_7^2 + 1.5227w_8^2 + 4.9930w_9^2\right)\right]^{0.5}\right\} \geq \lambda', \\
& \left[\frac{1}{8}\left(2.9662w_1^2 + 0.5630w_2^2 + 1.2214w_3^2 + 0.0033w_4^2 + 1.4300w_5^2 + 1.4867w_6^2\right.\right. \\
& \left.\left. + 0.2052w_7^2 + 0.0011w_8^2 + 0.0093w_9^2\right)\right]^{0.5} \Big/ \left\{\left[\frac{1}{8}\left(1.3242w_1^2 + 5.2704w_2^2\right.\right.\right. \\
& \left.\left. + 3.1988w_3^2 + 7.7857w_4^2 + 4.3190w_5^2 + 4.1359w_6^2 + 5.6428w_7^2 + 7.8171w_8^2\right.\right. \\
& \left.\left. + 7.6463w_9^2\right)\right]^{0.5} + \left[\frac{1}{8}\left(2.9662w_1^2 + 0.5630w_2^2 + 1.2214w_3^2 + 0.0033w_4^2\right.\right. \\
& \left.\left.\left. + 1.4300w_5^2 + 1.4867w_6^2 + 0.2052w_7^2 + 0.0011w_8^2 + 0.0093w_9^2\right)\right]^{0.5}\right\} \geq \lambda', \\
& \left[\frac{1}{8}\left(5.2145w_1^2 + 0.0103w_2^2 + 3.0665w_3^2 + 0.9085w_4^2 + 1.3838w_5^2 + 0.0042w_6^2\right.\right. \\
& \left.\left. + 0.3502w_7^2 + 0.0011w_8^2 + 0.4392w_9^2\right)\right]^{0.5} \Big/ \left\{\left[\frac{1}{8}\left(0.3779w_1^2 + 7.4655w_2^2\right.\right.\right. \\
& \left.\left. + 1.6805w_3^2 + 3.6915w_4^2 + 4.5014w_5^2 + 7.6434w_6^2 + 5.0266w_7^2 + 7.8137w_8^2\right.\right. \\
& \left.\left. + 4.9166w_9^2\right)\right]^{0.5} + \left[\frac{1}{8}\left(5.2145w_1^2 + 0.0103w_2^2 + 3.0665w_3^2 + 0.9085w_4^2\right.\right. \\
& \left.\left.\left. + 1.3838w_5^2 + 0.0042w_6^2 + 0.3502w_7^2 + 0.0011w_8^2 + 0.4392w_9^2\right)\right]^{0.5}\right\} \geq \lambda', \\
& -\Big(e^-_{(\mathrm{i})12} + e^-_{(\mathrm{i})64} + e^-_{(\mathrm{ii})46} + e^+_{(\mathrm{ii})46} + e^-_{(\mathrm{ii})95} + e^+_{(\mathrm{ii})95} + e^-_{(\mathrm{iii})487} + e^-_{(\mathrm{iii})738} + e^-_{(\mathrm{iv})1} + e^+_{(\mathrm{iv})1} \\
& + e^-_{(\mathrm{iv})7} + e^+_{(\mathrm{iv})7} + e^-_{(\mathrm{v})26} + e^-_{(\mathrm{v})39}\Big) \geq \lambda', (w_1, w_2, \ldots, w_9) \in \Gamma', \\
& e^-_{(\mathrm{i})12} \geq 0, e^-_{(\mathrm{i})64} \geq 0, e^-_{(\mathrm{ii})46} \geq 0, e^+_{(\mathrm{ii})46} \geq 0, e^-_{(\mathrm{ii})95} \geq 0, e^+_{(\mathrm{ii})95} \geq 0, e^-_{(\mathrm{iii})487} \geq 0, \\
& e^-_{(\mathrm{iii})738} \geq 0, e^-_{(\mathrm{iv})1} \geq 0, e^+_{(\mathrm{iv})1} \geq 0, e^-_{(\mathrm{iv})7} \geq 0, e^+_{(\mathrm{iv})7} \geq 0, e^-_{(\mathrm{v})26} \geq 0, e^-_{(\mathrm{v})39} \geq 0. \quad (56)
\end{aligned}$$

Solving the model in (56), we obtained $(\bar{w}_1, \bar{w}_2, \ldots, \bar{w}_9)$=(0.1200, 0.0746, 0.1471, 0.1157, 0.0428, 0.0829, 0.1576, 0.1367, 0.1226). The closeness coefficients were calculated as follows: $\overline{CC}_1^E = 0.3803$, $\overline{CC}_2^E = 0.7488$, $\overline{CC}_3^E = 0.0337$, and $\overline{CC}_4^E = 0.2301$. The corresponding ranking of the treatments is: $A_2 \succ A_1 \succ A_4 \succ A_3$.

The models in (47) and (56) yielded a similar distribution of criterion weights and the same ranking results of the treatment options. However, the formulation of the model in (56) is much heavier and more complicated than that of the model in (47). To determine the separations of each alternative from the positive-ideal and negative-ideal solutions, our proposed method employed the signed distances between IT2TrFNs to identify the closeness coefficient, whereas the Euclidean distances were used in the classical TOPSIS methodology. According to the comparative results, the employment of the signed distance-based closeness coefficients can significantly reduce the complexity in modeling and simplify the solution efforts.

Furthermore, our proposed method provided more reasonable results than the TOPSIS method even though the two methods yielded the same ranking orders of the four treatment options. The TOPSIS method acquired the following closeness coefficients: 0.3803, 0.7488, 0.0337, and 0.2301 for intravenous thrombolysis, intra-arterial thrombolysis, antiplatelet treatment, and heparinization, respectively. Nevertheless, these results are very strange and unreasonable. Note that there is a noteworthy difference between the closeness coefficients of intra-arterial thrombolysis (0.7488) and antiplatelet treatment (0.0337). In the real-life situations, most of the physicians recognize that too many treatment choices would render it difficult for the patients and their family members to make a choice. If the antiplatelet treatment does not have an evident therapeutic effect during the cure of the disease, it is impossible for the physician to provide this treatment protocols for the patient. Furthermore, the resulting closeness coefficient of intra-arterial thrombolysis is markedly larger than that of other treatments. This implies that a dominant alternative exists in the patient-centered medicine problem of the BAO disease. However, it is not true in the discussed case. With respect to the solution results produced by the model in (47), the obtained signed distance-based closeness coefficients are 0.4374, 0.5541, 0.2311, and 0.3351 for the four treatment options. These results seem much more reasonable and meaningful than the TOPSIS results. Therefore, despite minimizing the tedious computation requirements, the proposed method can generate credible solution results in the patient-centered medical problem. The comparative analysis demonstrates the potential of the proposed method in practical applications.

6 Conclusions

In this paper, we developed an interactive decision-making model for solving MCGDM problems within the IT2TrFN environment. This paper makes several important contributions to the existing literature on the topic of interactive decision-making methodologies. First, we fused individual IT2TrFN ratings to construct a collective decision matrix using the operation of hybrid averages, which reflect the importance degrees of each decision-maker and the agreement of individual opinions. Second, we utilized the concept of signed distances between IT2TrFNs to determine the closeness coefficient involving the separations of each alternative from the positive-ideal and negative-ideal solutions. This approach is completely different from the traditional definition of the closeness coefficient in TOPSIS methodology. Third, we established the integrated programming model with the signed distance-based closeness coefficient approach to estimate the importance weights of criteria from incomplete and inconsistent information and to determine the priority ranking of the alternatives. Fourth, we developed an interactive method to facilitate the decision-makers to modify or complete their requirements for satisfactory solutions with ease compared to the reference solution obtained by the integrated programming model. Fifth, we applied the developed methodology to the field of patient-centered medicine. The comparative analysis also validated the feasibility and effectiveness of the proposed methods. Briefly, we conducted a study to construct a new interactive decision-making model involving the signed distance-based HA operation and closeness coefficient approach for solving interval type-2 fuzzy MCGDM problems with incomplete preference information.

A possible limitation of the developed interactive MCGDM method based on IT2TrFNs is the data collection of incomplete preference information. The proposed method is useful because of its flexibility with regard to incomplete information. Incomplete information about criterion weights provided by the decision makers can be generally constructed using the five basic ranking forms, including a weak ranking, strict ranking, ranking of differences, interval bound, and ratio bound. However, it might be difficult to obtain some ranking forms, such as ranking of differences and ratio bound. According to the application experience of the patient-centered medical problem, it is arduous to collect the patient's and his family's opinions to complete the basic ranking relations of criterion importance. In addition, an effective method is lacking for justifying the evidence base for these subjective opinions. If we have difficulty in collecting incomplete preference information, we can adopt another approach to acquire criterion importance by using linguistic variables, in a manner similar to that of alternative ratings via the nine-point linguistic scales.

We assumed that the satisfactory function had taken the form of the signed distance-based closeness coefficient in our interactive procedure. In future studies, we can consider a more complicated form to redefine the satisfactory function. Nevertheless, this paper has demonstrated the applicability of the proposed method in a real life patient-centered medical problem. Future research will extend the proposed interactive MCGDM model and method such that it is suitable for a patient-centered healthcare system. It is anticipated that the developed method can be a useful support model for the future promotion of integrated decision making to assist the medical personnel in the humanized medical care and improve the quality of medical care.

Acknowledgments The author is very grateful to the respected editor and the anonymous referees for their insightful and constructive comments, which helped to improve the overall quality of the paper. Special thanks are due to Dr. Chien-Hung Chang (Stroke Center and Stroke Section, Department of Neurology, Linkou Medical Center, Chang Gung Memorial Hospital) for providing me with the sample case. This research is financially supported by the National Science Council of Taiwan (Grant Nos. NSC 99-2410-H-182-022-MY3 and NSC 100-2632-H-182- 001-MY2).

References

Abbasbandy, S., & Asady, B. (2006). Ranking of fuzzy numbers by sign distance. *Information Sciences*, *176*(16), 2405–2416.

Aisbett, J., Rickard, J. T., & Morgenthaler, D. G. (2010). Type-2 fuzzy sets as functions on spaces. *IEEE Transactions on Fuzzy Systems*, *18*(4), 841–844.

Chen, T.-Y. (2011). Signed distanced-based TOPSIS method for multiple criteria decision analysis based on generalized interval-valued fuzzy numbers. *International Journal of Information Technology & Decision Making*, *10*(6), 1131–1159.

Chen, S.-M., Yang, M.-W., Lee, L.-W., & Yang, S.-W. (2012). Fuzzy multiple attributes group decision-making based on ranking interval type-2 fuzzy sets. *Expert Systems with Applications*, *39*(5), 5295–5308.

Chen, T.-Y. (2012a). Multiple criteria group decision-making with generalized interval-valued fuzzy numbers based on signed distances and incomplete weights. *Applied Mathematical Modelling*, *36*(7), 3029–3052.

Chen, T.-Y. (2012b). A signed-distance-based approach to importance assessment and multi-criteria group decision analysis based on interval type-2 fuzzy set. *Knowledge and Information Systems*. doi: 10.1007/s10115-012-0497-6.

Gilan, S. S., Sebt, M. H., & Shahhosseini, V. (2012). Computing with words for hierarchical competency based selection of personnel in construction companies. *Applied Soft Computing*, *12*(2), 860–871.

Greenfield, S., Chiclana, F., Coupland, S., & John, R. (2009). The collapsing method of defuzzification for discretised interval type-2 fuzzy sets. *Information Sciences*, *179*(13), 2055–2069.

Hwang, C. L., & Yoon, K. (1981). *Multiple attribute decision making: Methods and applications*. Berlin: Springer.

Kaliszewski, I., Miroforidis, J., & Podkopaev, D. (2012). Interactive multiple criteria decision making based on preference driven evolutionary multiobjective optimization with controllable accuracy. *European Journal of Operational Research*, *216*(1), 188–199.

Katagiri, H., & Sakawa, M. (2011). Interactive multiobjective fuzzy random programming through the level set-based probability model. *Information Sciences*, *181*(9), 1641–1650.

Li, D.-F. (2011). Closeness coefficient based nonlinear programming method for interval-valued intuitionistic fuzzy multiattribute decision making with incomplete preference information. *Applied Soft Computing*, *11*(4), 3402–3418.

Li, Y., Liu, X., & Chen, Y. (2012). Supplier selection using axiomatic fuzzy set and TOPSIS methodology in supply chain management. *Fuzzy Optimization and Decision Making*, *11*(2), 147–176.

Park, J. H., Cho, H. J., & Kwun, Y. C. (2011). Extension of the VIKOR method for group decision making with interval-valued intuitionistic fuzzy information. *Fuzzy Optimization and Decision Making*, *10*(3), 233–253.

Sambuc, R. (1975). *Fonctions ϕ-floues, application a l'aide au diagnostic en pathologie thyroidienne*. These de Doctorat en Medicine: University of Marseille, France.

Wang, W., Liu, X., & Qin, Y. (2012). Multi-attribute group decision making models under interval type-2 fuzzy environment. *Knowledge-Based Systems*, *30*, 121–128.

Wei, G.-W., Wang, H.-J., & Lin, R. (2011). Application of correlation coefficient to interval-valued intuitionistic fuzzy multiple attribute decision-making with incomplete weight information. *Knowledge and Information Systems*, *26*(2), 337–349.

Wei, S.-H., & Chen, S.-M. (2009). Fuzzy risk analysis based on interval-valued fuzzy numbers. *Expert Systems with Applications*, *36*(2–1), 2285–2299.

Xu, Z. S. (2005). An overview of methods for determining OWA weights. *International Journal of Intelligent Systems*, *20*(8), 843–865.

Xu, Z. S. (2010). An integrated model-based interactive approach to FMAGDM with incomplete preference information. *Fuzzy Optimization and Decision Making*, *9*(3), 333–357.

Xu, Z. S., & Yager, R. R. (2006). Some geometric aggregation operators based on intuitionistic fuzzy sets. *International Journal of General Systems*, *35*(4), 417–433.

Yao, J. S., & Wu, K. (2000). Ranking fuzzy numbers based on decomposition principle and signed distance. *Fuzzy Sets and Systems*, *116*(2), 275–288.

Yuen, K. K. F. (2012). Membership maximization prioritization methods for fuzzy analytic hierarchy process. *Fuzzy Optimization and Decision Making*, *11*(2), 113–133.

Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning-I. *Information Sciences*, *8*(3), 199–249.