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A possibilistic approach to the modeling and resolution of uncertain closed-loop logistics

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Abstract Closed-loop logistics planning is an important tactic for the achievement of sustainable development. However, the correlation among the demand, recovery, and landfilling makes the estimation of their rates uncertain and difficult. Although the fuzzy numbers can present such kinds of overlapping phenomena, the conventional method of defuzzification using level-cut methods could result in the loss of information. To retain complete information, the possibilistic approach is adopted to obtain the possibilistic mean and mean square imprecision index (*MSII*) of the shortage and surplus for uncertain factors. By applying the possibilistic approach, a multi-objective, closed-loop logistics model considering shortage and surplus is formulated. The two objectives are to reduce both the total cost and the root *MSII*. Then, a non-dominated solution can be obtained to support decisions with lower perturbation and cost. Also, the information on prediction interval can be obtained from the possibilistic mean and root *MSII* to support the decisions in the uncertain environment. This problem is non-deterministic polynomial-time hard, so a new algorithm based on the spanning tree-based genetic algorithm has been developed. Numerical experiments have shown that the proposed algorithm can yield comparatively efficient and accurate results.

Keywords Closed-loop logistics · Fuzzy number · Possibilistic mean · Genetic algorithms · Shortage and surplus

1 Introduction

For the sustainability of the earth, green supply chain (GSC) management has been considered an effective approach because it is implemented on closed-loop logistics,

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including forward and reverse logistics, so that the requirements of reproduction, reuse, and redesign for sustainability can be facilitated. A number of studies, such as those of Lu et al. (2000), Baumgarten et al. (2003), and Schultmann et al. (2006), significantly contributed to the development of GSC because of its importance.

However, based on the studies of Kongar (2004), Listes (2007), Salema et al. (2007), and Wang and Hsu (2010b), not only the uncertainty embedded in reverse logistics, but also the uncertain factors of the reverse supply chain, which are more complex than those of the forward supply chain, present a challenge for GSC managers. To cope with such complicated uncertainty, Wang and Hsu (2010b) emphasized that fuzzy numbers are appropriate for describing uncertain factors and that the statistical approach toward possibility has a sound foundation to synthesize fuzzy information because fuzzy presentation is capable of processing uncertain patterns in GSC.

Among the difficulties in management that result from uncertain factors, the shortage and surplus of demands are the most serious. Resolving such uncertainty by simply providing uncertain parameters is not always sufficient. The closed-loop logistics problem concerns the recycling of goods through the methods for reusing, and the recovery and landfilling rates are the factors that determine the end of reusable materials in the reverse logistics, so the insufficiency or surplus of these factors will directly affect plans for reproduction. Therefore, studies should consider the logistics problems on shortage and surplus to cope with unstable environments. The possibilistic mean and mean square imprecision index (*MSII*) of shortage and surplus are thus developed in the present study, and the proposed mathematical model is applied with these properties to build a multi-objective model. The objectives are to reduce both the cost and the root mean square imprecision index (*RMSII*), which directly affects the risk level.

To resolve these problems, three issues should be considered: the model, its applicability, and the efficiency of the solution.

Although the fuzzy numbers are useful and efficient tools for investigating imprecise and overlapping factors, the determination of the expected amounts of shortage and surplus for demand, landfilling, and recovery from these numbers remains a problem. Several objectives should be achieved, so a multi-objective model should be formulated, and non-dominated solutions should be obtained to yield lower cost and perturbation. In addition, the location-allocation transportation problem is a non-deterministic polynomial-time (NP) problem; hence, an efficient algorithm is necessary.

After reviewing the literature on the analysis of GSC in uncertain environments using defuzzification procedures and genetic algorithms (GAs), as shown in Sect. 2, the possibilistic means of shortage and surplus are introduced in Sect. 3. A multi-objective mathematical programming model for GSC logistics is proposed in Sect. 4. The solution process for the revised spanning tree-based genetic algorithm is then presented in detail in Sect. 5. A numerical example of an uncertain GSC with shortage and surplus is presented and discussed in Sect. 6. Finally, in Sect. 7, the conclusion is drawn.

2 Literature review

This section focuses on the resolution of uncertain issues related to GSC, as discussed in the literature. After covering the basic knowledge on uncertain GSC, the spanning tree-based GAs are also discussed.

2.1 Uncertain green logistics and methods for resolving uncertainty

After Fleischmann et al. (2000) and Kongar (2004) showed the difficulties associated with the high level of uncertainty embedded in a reverse supply chain, Biehl et al. (2007) used an experimental design for the reverse supply chain to confirm the high degrees of uncertainty in the collection and landfill rates. The factors of demand, landfill, and recovery rates have overlapping patterns, so Wang and Hsu (2010b) resolved their uncertainties using the fuzzy set theory.

However, during the resolution process of the fuzzy approach, "defuzzification" is an issue when generating final solutions for decision making. In the literature, a number of studies addressed this defuzzification issue using different approaches. For instance, López-González et al. (2000) established the fuzzy objective function based on the Hamming distance. In recent studies, the credibility measure was also used. Peng and Liu (2004), Zheng and Liu (2006), Ke and Liu (2007), Yang et al. (2007), and Wen and Iwamura (2008) were among those who used the credibility measure. However, the credibility measure does not consider the complete information of fuzzy numbers at any one time, which therefore, remains an issue.

To overcome such limitations, Dubois and Prade (1987) proposed an alternative measure of the possibilistic mean to retain the complete information of fuzzy numbers. The idea is to define an interval-valued expectation of a fuzzy number by viewing it as a consonant random set. This idea was extended by Carlsson and Fullér (2001) into the interval-valued possibilistic mean of a continuous possibility distribution, such that both the extension principle (Zadeh 1975) and the well-known definition of expectation in the probability theory can be integrated. Wang and Hsu (2010b) then applied the possibilistic mean value method to retain the information of fuzzy numbers, which was then successfully used for GSC logistics.

In terms of the resolution, the issue on the location selection of conventional logistics planning has been extensively discussed as an NP-hard problem (Gen and Cheng 1997; Syarif et al. 2002). The definition of the fitness function for developing the algorithms is important because of this uncertainty. In the literature, both the Hamming distance and the credibility measure are used to define the fitness function (López-González et al. 2000; Yang et al. 2007; Wen and Iwamura 2008). The method to adequately define the fitness function to reflect the possible shortage and surplus is the first issue addressed in the present study.

GAs are powerful algorithms for solving engineering design and optimization problems. The fuzzy methods with GAs have been used in many areas of uncertainty. López-González et al. (2000) discussed transportation problems with fuzzy application based on the Hamming distance and used GAs to solve the problem. Yang et al. (2007) adopted a chance-constrained method with the credibility measure of the fuzzy parameters and applied the hybrid algorithms of Tabu Search and GA to solve logistics distribution center location problems. Wen and Iwamura (2008) also used the credibility measure and GAs to solve the facility location-allocation problem using optimistic and pessimistic criteria. Other applications of uncertainty with credibility measure and GAs have been used in numerous studies, such as those by Peng and Liu (2004), Zheng and Liu (2006), and Ke and Liu (2007). For multi-objective problems, Duenas and Petrovic (2008) also applied GAs for a single-machine scheduling problem under fuzziness. Among the different issues discussed in different studies, one common and important issue for GAs is to determine a suitable coding type. Therefore, the second aim of the present study is to find specific coding methods to represent the multi-stage logistics transformation with location-allocation under uncertainty.

2.2 Spanning tree-based genetic algorithm

For the basic concepts of GAs, the study by Gen and Cheng (1997) can be used as reference. The concept of applying a spanning tree to supply chain network problems was first proposed by Syarif et al. (2002). The multi-stage logistics transformation with location-allocation is a fixed charge transportation problem (FCTP). Syarif et al. (2002) and Jo et al. (2007) have successfully adopted spanning tree-based GAs with Prüfer encoding to solve FCTPs. However, empirical investigations have shown that Prüfer encoding is an unsatisfactory method for evolutionary algorithms and should therefore be avoided (Yeh 2005). However, the spanning tree-based GAs have the advantage of being capable of using the least number of arcs between two stages to solve logistics problems, thereby saving memory and time (Yao and Hsu 2009).

Abuali et al. (1995) proposed the determinant encoding and proved that this code is better than that of Prüfer. Both the works of Chou et al. (2001) and Yao and Hsu (2009) on logistic networks further confirmed this conclusion. Wang and Hsu (2010a) proposed a revised spanning tree-based genetic algorithm with determinant encoding that was applied successfully to the closed-loop logistics model. Therefore, determinant encoding is employed in the present work.

Although the spanning tree-based GAs with determinant encoding is capable of solving the transportation location-allocation problem, overcoming uncertainty still remains difficult. Real-number encoding should be integrated into the spanning tree-based GAs because of the properties of uncertainty. The primary challenge for real number encoding is the use of a pair of real-parameter decision variable vectors to create a new pair of offspring vectors or perturbing a decision variable vector into a mutated vector in a meaningful manner (Chang 2006; Kumar and Naresh 2007).

In previous studies on real-number encoding, Arumugam et al. (2005) proposed the arithmetic crossover (AMXO) and average convex crossover methods (ACXO) with dynamic mutation (DM). Note that ACXO is a special case of AMXO. Deep and Thakur (2007) suggested that the heuristic crossover (HX) and power mutation (PM) operators have the best performance and that the PM is better than DM; however, they did not compare HX with AMXO or ACXO. AMXO is the most common and efficient operator, whereas PM dominates other methods; hence, one of the feasible combinations of operators is the AMXO and PM method.

2.3 Summary

Uncertainty is one of the primary concerns of GSC management, and the demand, recovery, and landfilling rates are the three main factors of uncertainty. Although the use of fuzzy numbers is appropriate for presenting uncertainty, the literature has shown that defuzzification procedures based on level cuts always result in information loss. A realistic model capable of coping with the uncertainty of shortage and surplus in closed-loop logistics remains lacking. Therefore, the possibilistic approach will be adopted to obtain the possibilistic mean and *MSII* of shortage and surplus. In addition, the issues of uncertainty for this problem can be summarized into two, namely, the realistic modeling with shortage and surplus and the efficient algorithm, because closed-loop logistics is an NP-hard problem.

3 Possibilistic approach for shortage and surplus

Uncertainty is one of the primary issues in GSC logistics. The uncertain factors are assumed to be fuzzy numbers described by fuzzy membership functions because of the uncertainty embedded in the customer demand, recycling rate, and landfill rate. To describe the exact situation of the uncertain environment, the resultant shortage and surplus are discussed in this section under a possibilistic approach to retain the complete information of fuzzy numbers using the possibilistic mean and *MSII* of shortage and surplus for demand, recovery, and landfilling amounts.

3.1 Possibilistic mean of insufficient and surplus amounts

In this section, the process of obtaining the possibilistic mean of insufficient and surplus amounts is shown. The case of uncertain demand is used as an example.

Possibilistic mean of insufficient amounts

In an interval number of uncertain demand with lower bound d_L and upper bound d_U, each value has the same possibility between d_L and d_U. Let z be the decision variable that refers to the assigned transportation amount, with w as the actual amount, then the total expected shortage of the interval number can be described as follows:

$$E(shortage) = \begin{cases} \int\limits_{w=d_L}^{d_U} \frac{w-z}{d_U-d_L} dw = \frac{d_L+d_U}{2} - z, & z \leq d_L \\ \int\limits_{w=z}^{d_U} \frac{w-z}{d_U-d_L} dw = \frac{(d_U-z)^2}{2(d_U-d_L)}, & d_L < z < d_U \\ 0, & z \geq d_U \end{cases} \tag{1}$$

For example, when $d_L < z < d_U$, the expected shortage amounts of the interval number is derived by:

$$E(shortage) = \int\limits_{w=z}^{d_U} \frac{w-z}{d_U-d_L} dw = \frac{d_U^2 - z^2}{2(d_U-d_L)} - \frac{(d_U-z)z}{d_U-d_L} = \frac{(d_U-z)^2}{2(d_U-d_L)}.$$

By defining a fuzzy set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | \mu_{\tilde{A}}(x) \in [0, 1], \forall x \in R\}$, where $\mu_{\tilde{A}}(x)$ is the membership function or degree of truth of x in A, a crisp set of elements belonging to fuzzy set $\tilde{A}$, at least to a degree of γ, is called a γ-level set of $\tilde{A}$ and is defined by $A_\gamma = \{x \in R | \mu_{\tilde{A}}(x) \geq \gamma, 0 \leq \gamma \leq 1\}$. If $\tilde{A}$ is a fuzzy number, then for each γ level, A_γ is a closed interval that can be defined by its lower and upper bounds as $[a_L(\gamma), a_U(\gamma)]$ (Zadeh 1975), and the modes of fuzzy number $\tilde{A}$ can be defined as the values with the maximal degree, $\{x \in R | \mu_{\tilde{A}}(x) = 1\}$.

One method for collecting the information is to integrate all levels, which can be attributed to Carlsson and Fullér (2001) based on the concept of mean in the probability theory. The fuzzy numbers can be ranked using the method of Goetschel and Voxman (1986), which compares the possibilistic means of the fuzzy numbers, as defined by the arithmetic means of all γ-level sets, as shown below.

$$\begin{aligned} \overline{M}(\tilde{A}) &= \int\limits_0^1 \gamma[a_L(\gamma)+a_U(\gamma)]d\gamma = \frac{\int_0^1 \gamma \left[\frac{a_L(\gamma)+a_U(\gamma)}{2}\right] d\gamma}{\int_0^1 \gamma d\gamma} \\ &= \frac{1}{2}\left(\frac{\int_0^1 \gamma a_L(\gamma)d\gamma}{\int_0^1 \gamma d\gamma} + \frac{\int_0^1 \gamma a_U(\gamma)d\gamma}{\int_0^1 \gamma d\gamma}\right) \end{aligned} \tag{2}$$

For example, if $\tilde{A} = (a, S_L, S_U)$ is a triangular fuzzy number with center a, left spread from a, S_L and right spread from a, S_U, then

$$\begin{aligned} \overline{M}(\tilde{A}) &= \int\limits_0^1 \gamma \left[a - (1-\gamma)S_L + a + (1-\gamma)S_U\right] d\gamma = \frac{\int_0^1 \gamma \left[\frac{2a+(1-\gamma)(S_U-S_L)}{2}\right] d\gamma}{\int_0^1 \gamma d\gamma} \\ &= a + \frac{S_U - S_L}{6} \end{aligned}$$

Therefore, the method of Goetschel and Voxman (1986) for ranking fuzzy numbers is actually the level-weight average of mean for each level cut.

Generally, the aim of a transportation problem is to satisfy the demand and to avoid a surplus. For a given fuzzy number, the values beyond the lowest and highest possible values are assumed impossible with membership degree 0, therefore, the possible demands only fall between the lower and upper bounds of the fuzzy number, which are the only values that should be considered and satisfied.

Each level cut of the fuzzy number is a closed interval with lower and upper bounds, so Eq. (1) can be used to derive the expected shortage in the form of an interval.

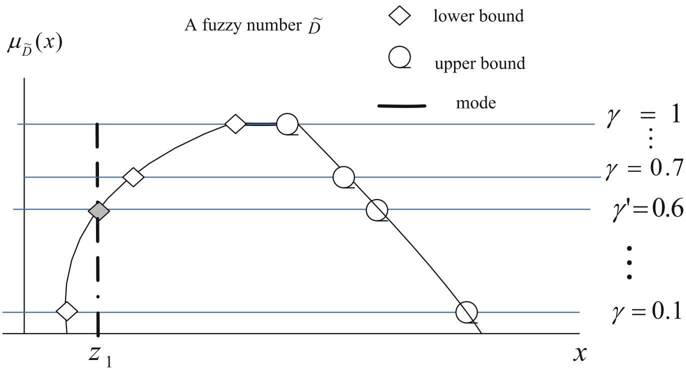

Fig. 1 The visualized example when z is smaller than the mode

Therefore, applying the possibility theory, the possibilistic mean of shortage of a fuzzy demand with lower and upper bounds, $[d_L(\gamma), d_U(\gamma)]$, of which the mode defined at $\gamma = 1$ is the interval $[m_1, m_2] = [d_L(1), d_U(1)]$, can be calculated.

If $d_L(0) \leq z < m_1$, there are two cases for the closed interval of γ-level sets, namely, $d_L(\gamma) < z < d_U(\gamma)$ and $z \leq d_L(\gamma)$.

$$\overline{M_d^-} = \int_0^{\gamma'} \gamma \left[\frac{(d_U(\gamma) - z)^2}{d_U(\gamma) - d_L(\gamma)} \right] d\gamma + \int_{\gamma'}^{1} \gamma [d_L(\gamma) + d_U(\gamma) - 2z] d\gamma \tag{3}$$

where γ' is the point of the level cut making $z \leq d_L(\gamma)$. The visualized figure is similar to that shown in Fig. 1.

Figure 1 is a visualized example where z_1 is smaller than the mode. $\tilde{D}$ is a fuzzy number, and γ-level sets of $\tilde{D}$ are the closed intervals with lower and upper bounds, $[d_L(\gamma), d_U(\gamma)]$. When $\gamma = 0.1$ in Fig. 1, z_1 is in the interval of $[d_L(0.1), d_U(0.1)]$, but when $\gamma = 0.7$, z_1 is out of the interval and smaller than $d_L(0.7)$. In Fig. 1, $z_1 = d_L(0.6)$, indicating that if $\gamma > 0.6$, the lower bound is higher and z_1 is out of the range. The definition of γ' is the point of the level cut making $z \leq d_L(\gamma)$, so $\gamma' = 0.6$. If γ' can be determined, Eq. (3) can be derived from Eq. (1) with different cases. Equations (4) and (5) are similar.

If $m_1 \leq z \leq m_2$, it is necessary that $d_L(\gamma) < z < d_U(\gamma)$ for the closed interval of γ-level sets.

$$\overline{M_d^-} = \int_0^{1} \gamma \left[\frac{(d_U(\gamma) - z)^2}{d_U(\gamma) - d_L(\gamma)} \right] d\gamma \tag{4}$$

If $m_2 < z \leq d_U(0)$, there are two cases for the closed interval of γ-level sets, namely, $d_L(\gamma) < z < d_U(\gamma)$ and $z \geq d_U(\gamma)$.

$$\overline{M_d^-} = \int_0^{\gamma'} \gamma \left[\frac{(d_U(\gamma) - z)^2}{d_U(\gamma) - d_L(\gamma)} \right] d\gamma \tag{5}$$

where γ' is the point of level cut making $z \geq d_U(\gamma)$.

If a specific fuzzy number is known, the closed form can be derived. For example, if $\tilde{D} = (d, S_L, S_U)$ is a triangular fuzzy number with center d, left spread from d, S_L and right spread from d, S_U, then

If $d - S_L \leq z < d$, then $\gamma' = 1 - \frac{d-z}{S_L}$. There are two cases, $d - (1-\gamma)S_L < z < d + (1-\gamma)S_U$ if $\gamma < \gamma'$, and $z \leq d - (1-\gamma)S_L$ if $\gamma \geq \gamma'$, for the closed interval of γ-level sets.

$$\begin{aligned}\overline{M_d^-} &= \int_0^{\gamma'} \gamma \left[\frac{[(d-z) + (1-\gamma)S_U]^2}{(1-\gamma)(S_L+S_U)} \right] d\gamma + \int_{\gamma'}^{1} \gamma [2d + (1-\gamma)(S_U - S_L) - 2z] d\gamma \\ &= \left. \frac{3\gamma^2(2S_U(S_U - z + d) - S_U^2) - 2\gamma^3 S_U^2 - 6(d-z)^2(\ln|1-\gamma| + \gamma)}{6(S_L+S_U)} \right|_0^{\gamma'} \\ &\quad + \gamma^2 \left. \left[\frac{(S_U - S_L)(3-2\gamma)}{6} + (d-z) \right] \right|_{\gamma'}^{1}\end{aligned}$$

If $z = d$, it is necessary that $d - (1-\gamma)S_L < z < d + (1-\gamma)S_U$ for the closed interval of γ-level sets.

$$\overline{M_d^-} = \int_0^{1} \gamma \left[\frac{[(d-z) + (1-\gamma)S_U]^2}{(1-\gamma)(S_L+S_U)} \right] d\gamma = \int_0^1 \frac{\gamma(1-\gamma)S_U^2}{(S_L+S_U)} d\gamma = \frac{S_U^2}{6(S_L+S_U)}$$

If $d < z \leq d + S_U$, then $\gamma' = 1 - \frac{z-d}{S_U}$. There are two cases, $d - (1-\gamma)S_L < z < d + (1-\gamma)S_U$ and $z \geq d + (1-\gamma)S_U$, for the closed interval of γ-level sets.

$$\begin{aligned}\overline{M_d^-} &= \int_0^{\gamma'} \gamma \left[\frac{[(d-z) + (1-\gamma)S_U]^2}{(1-\gamma)(S_L+S_U)} \right] d\gamma \\ &= \left. \frac{3\gamma^2(2S_U(S_U - z + d) - S_U^2) - 2\gamma^3 S_U^2 - 6(d-z)^2(\ln|1-\gamma| + \gamma)}{6(S_L+S_U)} \right|_0^{\gamma'}\end{aligned}$$

The process to drive the possibilistic mean of surplus amounts is similar to that of shortage, as summarized in the following section.

Possibilistic mean of surplus amounts

The process is similar to that previously discussed. The expected surplus amounts of the interval number can be described as follows:

$$E(surplus) = \begin{cases} 0, & z \le d_L \\ \int\limits_{w=d_L}^{z} \dfrac{z-w}{d_U-d_L} dw = \dfrac{(z-d_L)^2}{2(d_U-d_L)}, & d_L < z < d_U \\ \int\limits_{w=d_L}^{d_U} \dfrac{z-w}{d_U-d_L} dw = z - \dfrac{d_L+d_U}{2}, & z \ge d_U \end{cases} \tag{6}$$

The possibility theory can be applied to calculate the possibilistic mean of surplus of a fuzzy demand with lower and upper bounds, $[d_L(\gamma), d_U(\gamma)]$.

If $d_L(0) \le z < m_1$, then

$$\overline{M}_d^+ = \int\limits_0^{\gamma'} \gamma \left[\frac{(z-d_L(\gamma))^2}{d_U(\gamma)-d_L(\gamma)}\right] d\gamma \tag{7}$$

where γ' is the point of level cut making $z \le d_L(\gamma)$.

If $m_1 \le z \le m_2$, then

$$\overline{M}_d^+ = \int\limits_0^{1} \gamma \left[\frac{(z-d_L(\gamma))^2}{d_U(\gamma)-d_L(\gamma)}\right] d\gamma \tag{8}$$

If $m_2 < z \le d_U(0)$, then

$$\overline{M}_d^+ = \int\limits_0^{\gamma'} \gamma \left[\frac{(z-d_L(\gamma))^2}{d_U(\gamma)-d_L(\gamma)}\right] d\gamma + \int\limits_{\gamma'}^{1} \gamma \left[2z - d_U(\gamma) - d_L(\gamma)\right] d\gamma \tag{9}$$

where γ' is the point of level cut making $z \ge d_U(\gamma)$.

If a specific fuzzy number known, the closed form can be derived. For example, if $\tilde{D} = (d, S_L, S_U)$ is a triangular fuzzy number with center d, left spread from d, S_L and right spread from d, S_U, then

If $d - S_L \le z < d$, then $\gamma' = 1 - \frac{d-z}{S_L}$

$$\overline{M_d^+} = \int_0^{\gamma'} \gamma \left[\frac{[(z-d)+(1-\gamma)S_L]^2}{(1-\gamma)(S_L+S_U)} \right] d\gamma$$
$$= \left. \frac{3\gamma^2(2S_L(z+S_L-d)-S_L^2)-2\gamma^3 S_L^2-6(d-z)^2(\ln|1-\gamma|-\gamma)}{6(S_L+S_U)} \right|_0^{\gamma'}$$

If $z = d$, then

$$\overline{M_d^+} = \int_0^1 \gamma \left[\frac{[(z-d)+(1-\gamma)S_L]^2}{(1-\gamma)(S_L+S_U)} \right] d\gamma = \int_0^1 \frac{\gamma(1-\gamma)S_L^2}{(S_L+S_U)} d\gamma = \frac{S_L^2}{6(S_L+S_U)}$$

If $d < z \leq d + S_U$, then $\gamma' = 1 - \frac{z-d}{S_U}$

$$\overline{M_d^+} = \int_0^{\gamma'} \gamma \left[\frac{[(z-d)+(1-\gamma)S_L]^2}{(1-\gamma)(S_L+S_U)} \right] d\gamma + \int_{\gamma'}^1 \gamma[2z-2d-(1-\gamma)(S_U-S_L)]d\gamma$$
$$= \left. \frac{3\gamma^2(2S_L(z+S_L-d)-S_L^2)-2\gamma^3 S_L^2-6(d-z)^2(\ln|1-\gamma|-\gamma)}{6(S_L+S_U)} \right|_0^{\gamma'}$$
$$+\gamma^2 \left. \left[\frac{(S_U-S_L)(2\gamma-3)}{6} - d + z) \right] \right|_{\gamma'}^1$$

The case of the possibilistic mean for uncertain demand is similar to that previously discussed. The possibilistic means of shortage and surplus for other fuzzy numbers, as well as recovery and landfill rates, can be derived using similar methods. After deriving the possibilistic mean of shortage and surplus, the *MSII* will be derived in the next section to help decision makers understand the uncertainty for certain decisions.

3.2 MSII for insufficient and surplus amounts

In this section, the process of obtaining the *MSII* based on the possibilistic theory for insufficient and surplus amounts is shown. Also, the case of uncertain demand is used as an example.

MSII of insufficient amounts

In an interval number of uncertain demand with lower bound, d_L, and upper bound, d_U, each value has the same possibility between d_L and d_U. Let z be the decision variable that refers to the assigned transportation amount, with w as the actual amount, then, all of the possible *MSII* of shortage amounts for the interval number can be described as follows:

$$MSII(shortage) = \begin{cases} \int\limits_{w=d_L}^{d_U} \frac{(w-z)^2}{d_U-d_L}dw - \left[\int\limits_{w=d_L}^{d_U} \frac{(w-z)}{d_U-d_L}dw\right]^2 = \frac{(d_U-d_L)^2}{12} & z \le d_L \\ \int\limits_{w=z}^{d_U} \frac{(w-z)^2}{d_U-d_L}dw - \left[\int\limits_{w=z}^{d_U} \frac{(w-z)}{d_U-d_L}dw\right]^2 = \frac{(d_U-z)^3(d_U-4d_L+3z)}{12(d_U-d_L)^2}, & d_L < z < d_U \\ 0, & z \ge d_U \end{cases} \tag{10}$$

For example, when $d_L < z < d_U$, the *MSII* for shortage amounts of the interval number is derived by

$$\begin{aligned} MSII(shortage) &= \int\limits_{w=z}^{d_U} \frac{(w-z)^2}{d_U-d_L}dw - \left[\int\limits_{w=z}^{d_U} \frac{(w-z)}{d_U-d_L}dw\right]^2 \\ &= \frac{4(d_U-z)^3(d_U-d_L)-3(d_U-z)^4}{12(d_U-d_L)^2} \\ &= \frac{(d_U-z)^3[4(d_U-d_L)-3(d_U-z)]}{12(d_U-d_L)^2} \\ &= \frac{(d_U-z)^3(d_U-4d_L+3z)}{12(d_U-d_L)^2} \end{aligned}$$

Carlsson and Fullér (2001) proposed a variance based on the end points of a fuzzy number in relation to the possibilistic mean, and as previously explained, the information of end points are insufficient for the determination of shortage and surplus. Therefore, the *MSII* of a fuzzy number is given below using the arithmetic means of γ-level sets based on all continuous points:

$$MSII(\tilde{A}) = \frac{\int_0^1 \gamma \left(\frac{[a_U(\gamma)-a_L(\gamma)]^2}{12}\right) d\gamma}{\int_0^1 \gamma d\gamma} = \frac{1}{6}\int\limits_0^1 \gamma \left(\left[a_U(\gamma)-a_L(\gamma)\right]^2\right) d\gamma \tag{11}$$

For example, if $\tilde{A} = (a, S_L, S_U)$ is a triangular fuzzy number with center a; left spread from a, S_L, and right spread from a, S_U, then

$$MSII(\tilde{A}) = \frac{1}{6}\int\limits_0^1 \gamma(a+S_U(1-\gamma)-(a-S_L(1-\gamma)))^2 d\gamma = \frac{(S_L+S_U)^2}{72}$$

Each level cut of the fuzzy number is a closed interval with lower and upper bounds. Equation (10) can be used to determine the *MSII* of the interval numbers. Therefore, applying the possibility theory, the possibilistic *MSII* of shortage of a fuzzy demand with lower and upper bounds, $[d_L(\gamma), d_U(\gamma)]$, can be calculated.

If $d_L(0) \le z < m_1$, there are two cases for the closed interval of γ-level sets, namely, $d_L(\gamma) < z < d_U(\gamma)$ and $z \le d_L(\gamma)$.

$$MSII_d^- = \int_0^{\gamma'} \gamma \left[\frac{(d_U(\gamma) - z)^3 (d_U(\gamma) - 4d_L(\gamma) + 3z)}{6(d_U(\gamma) - d_L(\gamma))^2} \right] d\gamma + \int_{\gamma'}^{1} \gamma \left[\frac{(d_U(\gamma) - d_L(\gamma))^2}{6} \right] d\gamma \tag{12}$$

where γ' is the point of level cut making $z \le d_L(\gamma)$.

If $m_1 \le z \le m_2$, it is necessary that $d_L(\gamma) < z < d_U(\gamma)$ for the closed interval of γ-level sets.

$$MSII_d^- = \int_0^{1} \gamma \left[\frac{(d_U(\gamma) - z)^3 (d_U(\gamma) - 4d_L(\gamma) + 3z)}{6(d_U(\gamma) - d_L(\gamma))^2} \right] d\gamma \tag{13}$$

If $m_2 < z \le d_U(0)$, there are two cases for the closed interval of γ-level sets, namely, $d_L(\gamma) < z < d_U(\gamma)$ and $z \ge d_U(\gamma)$.

$$MSII_d^- = \int_0^{\gamma'} \gamma \left[\frac{(d_U(\gamma) - z)^3 (d_U(\gamma) - 4d_L(\gamma) + 3z)}{6(d_U(\gamma) - d_L(\gamma))^2} \right] d\gamma \tag{14}$$

where γ' is the point of level cut that makes $z \ge d_U(\gamma)$

Again, if a specific fuzzy number is known, the closed form can be derived. For example, if $\tilde{D} = (d, S_L, S_U)$ is a triangular fuzzy number with center d, left spread from d, S_L and right spread from d, S_U, then

If $d - S_L \le z < d$, then $\gamma' = 1 - \frac{d-z}{S_L}$

$$\begin{aligned} MSII_d^- &= \int_0^{\gamma'} \gamma \left[\frac{[(d-z) + (1-\gamma)S_U]^3 [(1-\gamma)(S_U + 4S_L) + 3(z-d)]}{6(1-\gamma)^2 (S_L + S_U)^2} \right] d\gamma \\ &\quad + \int_{\gamma'}^{1} \gamma \left[\frac{(1-\gamma)^2 (S_L + S_U)^2}{6} \right] d\gamma \\ &= \frac{1}{6(S_L + S_U)^2} \left[\begin{array}{l} \frac{S_U^3 \gamma^4 (S_U + 4S_L)}{4} - \frac{2S_U^2 \gamma^3 (S_U(S_U + 4S_L) + 6S_L(d-z))}{3} \\ \quad + \frac{S_U \gamma^2 (S_U^3 + 4S_L(S_U^2 + 3(d-z)^2) - 6S_U(d-z)(d-z-2S_L))}{2} \\ \quad -4\gamma(d-z)^3 (S_L - 2S_U) - \ln|1-\gamma| \\ \quad \times \left[(d-z)^3 (3(d-z) - 4(2S_U - S_L)) \right] - \frac{3(d-z)^4}{1-\gamma} \end{array} \right]_0^{\gamma'} \\ &\quad + \left. \frac{\gamma^2 (S_L + S_U)^2 (3\gamma^2 - 8\gamma + 6)}{72} \right|_{\gamma'}^{1} \end{aligned}$$

If $z = d$, then

$$MSII = \int_0^1 \gamma \left[\frac{[(d-z)+(1-\gamma)S_U]^3[(1-\gamma)(S_U+4S_L)+3(z-d)]}{6(1-\gamma)^2(S_L+S_U)^2} \right] d\gamma$$

$$= \frac{1}{6(S_L+S_U)^2} \left[\begin{array}{l} \frac{S_U^3\gamma^4(S_U+4S_L)}{4} - \frac{2S_U^2\gamma^3(S_U(S_U+4S_L)+6S_L(d-z))}{3} \\ + \frac{S_U\gamma^2(S_U^3+4S_L(S_U^2+3(d-z)^2)-6S_U(d-z)(d-z-2S_L))}{2} \\ -4\gamma(d-z)^3(S_L-2S_U) - \ln|1-\gamma| \\ \times \left[(d-z)^3(3(d-z)-4(2S_U-S_L))\right] - \frac{3(d-z)^4}{1-\gamma} \end{array} \right] \Bigg|_0^1$$

If $d < z \leq d + S_U$, then $\gamma' = 1 - \frac{z-d}{S_U}$

$$MSII_d^- = \int_0^{\gamma'} \gamma \left[\frac{[(d-z)+(1-\gamma)S_U]^3[(1-\gamma)(S_U+4S_L)+3(z-d)]}{6(1-\gamma)^2(S_L+S_U)^2} \right] d\gamma$$

$$= \frac{1}{6(S_L+S_U)^2} \left[\begin{array}{l} \frac{S_U^3\gamma^4(S_U+4S_L)}{4} - \frac{2S_U^2\gamma^3(S_U(S_U+4S_L)+6S_L(d-z))}{3} \\ + \frac{S_U\gamma^2(S_U^3+4S_L(S_U^2+3(d-z)^2)-6S_U(d-z)(d-z-2S_L))}{2} \\ -4\gamma(d-z)^3(S_L-2S_U) - \ln|1-\gamma| \\ \times \left[(d-z)^3(3(d-z)-4(2S_U-S_L))\right] - \frac{3(d-z)^4}{1-\gamma} \end{array} \right] \Bigg|_0^{\gamma'}$$

The process to drive the *MSII* of surplus amounts is similar to that of shortage and is briefly described as follows:

MSII of surplus amounts

The process is similar to that previously discussed. The *MSII* of shortage amounts for the interval number can be described as follows:

$$MSII(surplus) = \begin{cases} 0, & z \leq d_L \\ \int\limits_{w=d_L}^{z} \frac{(z-w)^2}{d_U-d_L} dw - \left[\int\limits_{w=d_L}^{z} \frac{(z-w)}{d_U-d_L} dw \right]^2 = \frac{(d_L-z)^3(d_L-4d_U+3z)}{12(d_U-d_L)^2}, & d_L < z < d_U \\ \int\limits_{w=d_L}^{d_U} \frac{(z-w)^2}{d_U-d_L} dw - \left[\int\limits_{w=d_L}^{d_U} \frac{(z-w)}{d_U-d_L} dw \right]^2 = \frac{(d_U-d_L)^2}{12}, & z \geq d_U \end{cases} \tag{15}$$

The possibility theory can be applied to calculate the *MSII* of surplus of a fuzzy demand with lower and upper bounds, $[d_L(\gamma), d_U(\gamma)]$.

If $d_L(0) \le z < m_1$, then

$$MSII_d^+ = \int_0^{\gamma'} \gamma \left[\frac{(d_L(\gamma) - z)^3 (d_L(\gamma) - 4d_U(\gamma) + 3z)}{6(d_U(\gamma) - d_L(\gamma))^2} \right] d\gamma \tag{16}$$

where γ' is the point of level cut that makes $z \le d_L(\gamma)$.

If $m_1 \le z \le m_2$, then

$$MSII_d^+ = \int_0^{1} \gamma \left[\frac{(d_L(\gamma) - z)^3 (d_L(\gamma) - 4d_U(\gamma) + 3z)}{6(d_U(\gamma) - d_L(\gamma))^2} \right] d\gamma \tag{17}$$

If $m_2 < z \le d_U(0)$, then

$$\begin{aligned} MSII_d^+ = & \int_0^{\gamma'} \gamma \left[\frac{(d_L(\gamma) - z)^3 (d_L(\gamma) - 4d_U(\gamma) + 3z)}{6(d_U(\gamma) - d_L(\gamma))^2} \right] d\gamma \\ & + \int_{\gamma'}^{1} \gamma \left[\frac{(d_U(\gamma) - d_L(\gamma))^2}{6} \right] d\gamma \end{aligned} \tag{18}$$

where γ' is the point of level cut that makes $z \ge d_U(\gamma)$.

Again, taking $\tilde{D} = (d, S_L, S_U)$, which a triangular fuzzy number with center d, left spread from d, S_L, and right spread from d, S_U, as an example, the closed form can be derived as follows:

If $d - S_L \le z < d$, then $\gamma' = 1 - \frac{d-z}{S_L}$

$$\begin{aligned} MSII_d^+ &= \int_0^{\gamma'} \gamma \left[\frac{[(d - z) - (1 - \gamma)S_L]^3 [3(z - d) - (1 - \gamma)(S_L + 4S_U)]}{6(1 - \gamma)^2 (S_L + S_U)^2} \right] d\gamma \\ &= \frac{1}{6(S_L + S_U)^2} \left[\begin{array}{l} \frac{S_L^3 \gamma^4 (S_L + 4S_U)}{4} - \frac{2S_L^2 \gamma^3 (S_L(S_L + 4S_U) - 6S_U(d+z))}{3} \\ + \frac{S_L \gamma^2 (S_L^3 + 4S_U(S_L^2 + 3(d-z)^2) - 6S_L^2(d-z)(d-z+2S_U))}{2} \\ -4\gamma(d - z)^3 (2S_L - S_U) - \ln|1 - \gamma| \\ \times \left[(d - z)^3 (3(d - z) + 4(2S_L - S_U))\right] - \frac{3(d-z)^4}{1-\gamma} \end{array} \right] \Bigg|_0^{\gamma'} \end{aligned}$$

If $z = d$, then

$$MSII_d^+ = \int_0^1 \gamma \left[\frac{[(d-z)-(1-\gamma)S_L]^3[3(z-d)-(1-\gamma)(S_L+4S_U)]}{6(1-\gamma)^2(S_L+S_U)^2} \right] d\gamma$$

$$= \frac{1}{6(S_L+S_U)^2} \left[\begin{array}{l} \frac{S_L^3\gamma^4(S_L+4S_U)}{4} - \frac{2S_L^2\gamma^3(S_L(S_L+4S_U)-6S_U(d+z))}{3} \\ + \frac{S_L\gamma^2(S_L^3+4S_U(S_L^2+3(d-z)^2)-6S_L^2(d-z)(d-z+2S_U))}{2} \\ -4\gamma(d-z)^3(2S_L-S_U) - \ln|1-\gamma| \\ \times \left[(d-z)^3(3(d-z)+4(2S_L-S_U))\right] - \frac{3(d-z)^4}{1-\gamma} \end{array} \right] \Bigg|_0^1$$

If $d < z \leq d + S_U$, then $\gamma' = 1 - \frac{z-d}{S_U}$

$$MSII_d^+ = \int_0^{\gamma'} \gamma \left[\frac{[(d-z)-(1-\gamma)S_L]^3[3(z-d)-(1-\gamma)(S_L+4S_U)]}{6(1-\gamma)^2(S_L+S_U)^2} \right] d\gamma$$

$$+ \int_{\gamma'}^1 \gamma \left[\frac{(1-\gamma)^2(S_L+S_U)^2}{6} \right] d\gamma$$

$$= \frac{1}{6(S_L+S_U)^2} \left[\begin{array}{l} \frac{S_L^3\gamma^4(S_L+4S_U)}{4} - \frac{2S_L^2\gamma^3(S_L(S_L+4S_U)-6S_U(d+z))}{3} \\ + \frac{S_L\gamma^2(S_L^3+4S_U(S_L^2+3(d-z)^2)-6S_L^2(d-z)(d-z+2S_U))}{2} \\ -4\gamma(d-z)^3(2S_L-S_U) - \ln|1-\gamma| \\ \times \left[(d-z)^3(3(d-z)+4(2S_L-S_U))\right] - \frac{3(d-z)^4}{1-\gamma} \end{array} \right] \Bigg|_0^{\gamma'}$$

$$+ \frac{\gamma^2(S_L+S_U)^2(3\gamma^2-8\gamma+6)}{72} \Bigg|_{\gamma'}^1$$

The case of *MSII* for uncertain demand is shown above. The *MSII* of shortage and surplus for other fuzzy numbers, recovery, and landfill rates can be derived using similar methods.

RMSII is the root of *MSII*. *RMSII* is an index to test the perturbation of solutions. Therefore, the goals are to reduce both the *RMSII* and cost as much as possible, so the model should be formulated as a multi-objective problem. If all the uncertain parameters are described by the symmetric triangular fuzzy numbers, the prediction interval with the known possibilistic mean and *RMSII* are relatively easy to obtain.

A prediction interval is an estimate of an interval in which future observations will fall under a certain probability, and the probability can be derived by calculating the area using the possibility theory (De Groot 2007). The prediction interval of a normal probability distribution with known mean and variance between $\mu + 2\sigma$ and $\mu - 2\sigma$ is 95.44%. Adapting this to a symmetric triangular possibility distribution with known

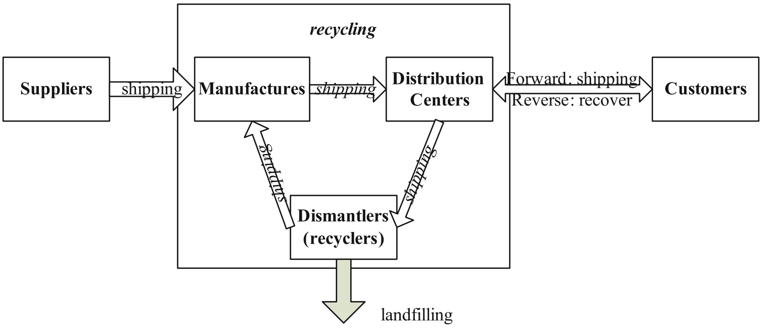

Fig. 2 Framework of GSC logistics

mean and variance, the prediction interval $[\overline{M}_d^- - 2RMSII, \overline{M}_d^- + 2RMSII]$ for the shortage of customer demand is approximated to be 96.63%. For example, for an uncertain demand with symmetric triangular fuzzy number, $\tilde{D} = (400, 100, 100)$, the possibilistic mean is 400 from Eq. (2), and the *MSII* is 1666.667 from Eq. (11). The prediction interval $[\overline{M}_d^- - 2RMSII, \overline{M}_d^- + 2RMSII]$ is [318.3503, 381.6497]. The total area of the symmetric triangular fuzzy number is 100, and the area between 318.3503 and 381.6497 is 96.63266. The probability that future observations will fall in the interval [318.3503, 381.6497] is $\frac{96.63266}{100} = 96.63\%$.

Extending to nonsymmetric cases, the area of the range can also be used to calculate the probability. For example, if the goal is to obtain 95% of the prediction interval for mean, 2.5% can be excluded from both the left and right areas to determine the range. This information can help decision makers clearly understand the uncertainty under probability.

4 The mathematical model for uncertainty

Wang and Hsu (2010a) mentioned that past studies have not shown a high degree of correlation between forward and reverse chains and that their solutions can be obtained separately. The operations of transportation are often subcontracted separately to a third-party logistics service provider. To reduce operational cost, Hsu and Wang (2009) developed a model to determine the relationship between the forward and reverse chains to facilitate delivery and pick-up operations conducted by the same fleet of vehicles. The framework is presented in Fig. 2.

Based on the report by Hsu and Wang, the model can be extended to an uncertain model with insufficient and surplus amounts. Uncertain parameters are described in terms of fuzzy numbers. The possibilistic mean of the shortage and surplus amounts for all uncertain parameters are applied to obtain the possibilistic mean objection function of the model proposed by Hsu and Wang. Based on the literature, demand, landfill, and recovery rates are the basic factors that contribute to the uncertainty in GSC logistics (Biehl et al. 2007; Zikopoulos and Tagaras 2007; Kongar 2004). The notations of indices are listed below.

Indices:

I: The number of suppliers with $i = 1, 2, \ldots, I$
J: The number of manufactories with $j = 1, 2, \ldots, J$
K: The number of distribution centers (DCs) with $k = 1, 2, \ldots, K$
L: The number of customers with $l = 1, 2, \ldots, L$
M: The number of dismantlers with $m = 1, 2, \ldots, M$

The insufficient costs or shortage costs are C_d^-, C_r^-, and C_l^- for the uncertain demand, recovery rate, and landfill rate, respectively. The surplus costs are C_d^+, C_r^+, and C_l^+, for the uncertain demand, recovery rate, and landfill rate, respectively.

The total cost of uncertainty TUC would be

$$\begin{aligned} TUC = {} & C_d^- \sum_l \overline{M}_{d,l}^- + C_r^- \sum_l \overline{M}_{r,l}^- + C_l^- \sum_m \overline{M}_{l,m}^- + C_d^+ \sum_l \overline{M}_{d,l}^+ \\ & + C_r^+ \sum_l \overline{M}_{r,l}^+ + C_l^+ \sum_m \overline{M}_{l,m}^+ \end{aligned} \tag{19}$$

where $\overline{M}_{d,l}^-$, $\overline{M}_{r,l}^-$, and $\overline{M}_{l,m}^-$ are the possibilistic means of the insufficient amounts for the lth customer with demand and recovery, and mth is the dismantler with landfilling rate. Here, $\overline{M}_{d,l}^+$, $\overline{M}_{r,l}^+$, and $\overline{M}_{l,m}^+$ are the possibilistic means of the surplus amounts for lth customer with demand and recovery, and mth is the dismantler with landfilling rate.

The relationship between insufficiency and surplus cost can then be discussed. For costs of the uncertain demands, $C_d^- > C_d^+$ because managers always aim to satisfy all the needs of the customers. The shortage cost may include the loss of expected benefits and of business goodwill. In some situations, $C_d^- < C_d^+$ because these kinds of goods have a short lifecycle, and the benefits will be lost immediately.

For costs of the uncertain recovery rates, $C_r^- > C_r^+$ because managers always aim to deliver all the recovered items. If the recovery cannot be delivered to the dismantler, the high recovery rate will be insignificant. If the product has a short lifecycle, the situation will be sufficient, that is, $C_r^- \gg C_r^+$. Here, $C_r^- < C_r^+$ if and only if the recovery materials are inexpensive. However, if that is the case, the materials may not be recovered. For the costs of uncertain landfilling rates, $C_l^- > C_l^+$ because of environmental protection. If the actual landfilling amounts cannot be addressed, environmental problems may emerge.

The costs of the uncertain factors are presented as Eq. (19). A possibilistic-based mathematical programming model with shortage and surplus is proposed accordingly. The notations of this programming model, as well as the possibilistic mean-based green closed-loop logistics (GCLL) model, are defined in the following.

Parameters:

a_i: Capacity of supplier i

b_j: Capacity of manufactory j

Sc_k: Capacity of the DC k

pc_l^L: The lower bound of uncertain recovery percentage of customer l

pc_l^U: The upper bound of uncertain recovery percentage of customer l

pl_m^L: The lower bound of uncertain landfilling rate for dismantler m

pl_m^U: The upper bound of uncertain landfilling rate for dismantler m

d_l^L: The lower bound of uncertain demand of the customer l

d_l^U: The upper bound of uncertain demand of the customer l

e_m: Capacity of dismantler m

s_{ij}: Unit cost of production in manufactory j using materials from supplier i

t_{jk}: Unit cost of transportation from each manufactory j to each DC k

u_{kl}: Unit cost of transportation from DC k to customer l

v_{km}: Unit cost of transportation from DC k to dismantler m

w_{mj}: Unit cost of transportation from dismantler m to manufactory j

Ru_{lk}: Unit cost of recovery in DC k from customer l

f_j: Fixed cost for operating Manufactory j

g_k: Fixed cost for operating DC k

h_m: Fixed cost for operating dismantler m

LC: Fixed cost for landfilling per unit.

Decision Variables:

x_{ij}: Quantity produced at manufactory j using raw materials from supply i

y_{jk}: Amount shipped from manufactory j to DC k

z_{kl}: Amount shipped from DC k to customer l

o_{km}: Amount shipped from DC k to dismantler m

Rd_{mj}: Amount shipped from dismantler m to manufactory j

Rz_{lk}: Quantity recovered at DC k from customer l

ppc_l: Percentage in range (pc_l^L, pc_l^U) of recovery rate for customer l

ppl_m: Percentage in range (pl_m^L, pl_m^U) of landfilling rate for dismantler m

pd_l: Percentage in range (d_l^L, d_l^U) of demand for customer l.

$$\alpha_j = \begin{cases} 1, & \text{if production takes place at manufactory } j \\ 0, & \text{otherwise} \end{cases}$$

$$\beta_k = \begin{cases} 1, & \text{if DC} k \text{ is opened} \\ 0, & \text{otherwise} \end{cases}$$

$$\delta_m = \begin{cases} 1, & \text{if dismantler} m \text{ is opened} \\ 0, & \text{otherwise} \end{cases}$$

Possibilistic-GCLL model

Object function:

$$\min TC = \sum_i \sum_j s_{ij} x_{ij} + \sum_j \sum_k t_{jk} y_{jk} + \sum_k \sum_l u_{kl} z_{kl} + \sum_k \sum_m v_{km} o_{km}$$
$$+ \sum_m \sum_j w_{mj} Rd_{mj} + \sum_l \sum_k Ru_{lk} Rz_{lk} + \sum_j f_i \alpha_j + \sum_k g_k \beta_k$$
$$+ \sum_m h_m \delta_m + LC \sum_m \left[pl_m^L + ppl_m (pl_m^U - pl_m^L) \right] \sum_k o_{km} + TUC \tag{20}$$

Subject to:

$$\sum_j x_{ij} \le a_i, \quad \forall i \tag{21}$$

$$\sum_k y_{jk} - b_j \alpha_j \le 0, \quad \forall j \tag{22}$$

$$\sum_i x_{ij} + \sum_m Rd_{mj} - \sum_k y_{jk} = 0, \quad \forall j \tag{23}$$

$$\sum_l z_{kl} + \sum_m o_{km} - Sc_k \beta_k \le 0, \quad \forall k \tag{24}$$

$$\sum_j y_{jk} - \sum_l z_{kl} = 0, \quad \forall k \tag{25}$$

$$\sum_l Rz_{lk} - \sum_m o_{km} = 0, \quad \forall k \tag{26}$$

$$\sum_k Rz_{lk} - \left[pc_l^L + ppc_l (pc_l^U - pc_l^L) \right] \sum_k z_{kl} \ge 0, \quad \forall l \tag{27}$$

$$\sum_k z_{kl} \ge d_l^L + pd_l (d_l^U - d_l^L), \quad \forall l \tag{28}$$

$$\sum_j Rd_{mj} + \left[pl_m^L + ppl_m (pl_m^U - pl_m^L) \right] \sum_k o_{km} - e_m \delta_m \le 0, \quad \forall m \tag{29}$$

$$\sum_k o_{km} - \sum_j Rd_{mj} - \left[pl_m^L + ppl_m (pl_m^U - pl_m^L) \right] \sum_k o_{km} = 0, \quad \forall m \tag{30}$$

$$\alpha_j, \beta_k, \delta_m \in \{0, 1\}, \quad \forall j, k, m \tag{31}$$

$$0 \le ppc_l, ppl_m, pd_l \le 1, \quad \forall l, m \tag{32}$$

$$x_{ij}, y_{jk}, z_{kl}, o_{km}, Rd_{mj}, Rz_{lk} \in \mathrm{N} \cup \{0\} \quad \forall i, j, k, l, m \tag{33}$$

$pl_m^L + ppl_m (pl_m^U - pl_m^L)$, $pc_l^L + ppc_l (pc_l^U - pc_l^L)$ and $d_l^L + pd_l (d_l^U - d_l^L)$ in the objective function and constraints (27), (28), (29), (30) are used to make the decisions between the range of uncertain landfilling rates, recovery rates, and demands. ppl_m, ppc_l, and pd_l are the decision variables between 0 and 1, and ppl_m, ppc_l, and pd_l percentages of the uncertain ranges ($pl_m^U - pl_m^L$, $pc_l^U - pc_l^L$, and $d_l^U - d_l^L$) are added from the lower bounds (pl_m^L, pc_l^L, and d_l^L) of these uncertain factors. For example, if an uncertain demand with symmetric triangular fuzzy number is $\tilde{D} = (400, 100, 100)$, and $pd_l = 0.5$ is a feasible solution, the decision of demand equal to 400 ($300 + 0.5(500 - 300) = 400$) is made. Then, the possibilistic mean and *MSII* of shortage can be calculated under this triangular fuzzy number (i.e., possibilistic mean of shortage from Eq. (4) equal to $\frac{S_U^2}{6(S_L+S_U)} = \frac{100^2}{6\times 200} = 8.3333$). The details of the objective and constraints are explained below.

The objective is to minimize the possible total cost (*TC*) of transportation, operations, and uncertain possible loss. The constraints are categorized into two, namely, limited capacities and the law of flow conservation. Constraints (21) and (22) represent the possible amounts that could be provided by the suppliers and manufacturers in forward logistics, respectively. Constraint (24) is the joint-capacity limit between the forward and reverse in DCs. Constraint (29) is the uncertain reverse capacity limit of the dismantlers. Constraint (27) describes the possible recovery amount of the customers from an uncertain recovery rate. Constraints (23), (25), (26), and (30) ensure the law of flow conservation through the possible in/out flows. Constraint (28) refers to the customer demands that need to be satisfied. Constraint (31) denotes the binary decision variables, and Constraint (33) is the non-negative, integral condition of the proposed model.

In this model, Constraint (32) plays an important role in making decisions among the ranges of uncertain parameters. The total cost, *TC*, with the cost of uncertainty, *TUC*, is used to obtain the best estimation of uncertainty with minimal expected cost. The problem is a logistics problem with transportation; therefore, the reduction of total cost is the primary objective. Except for total cost, a stable solution is necessary for reducing the risk in an uncertain environment. Therefore, after obtaining a possible good solution, the second objective is to minimize the *RMSII*.

$$\min RMSII(TC) = RMSII(TUC) \tag{34}$$

Formula (34) describes the perturbation for the solution obtained from Formula (20).

5 Revised spanning tree-based genetic algorithm

A closed-loop logistics problem is a kind of capacitated location-allocation issue and can be viewed as a multiple-choice knapsack problem. This issue is known to be an NP-hard problem (Gen and Cheng 1997; Jo et al. 2007; Wang and Hsu 2010a). Although this model has an issue of uncertainty, the problem is visibly difficult in nature. Therefore, an efficient algorithm should be developed to solve the model.

In a spanning-tree network with n nodes, $n-1$ arcs are present. Yao and Hsu (2009) showed that no more than $(n-1)$ links exist when optimality is reached in two consecutive stages in a logistics problem. Wang and Hsu (2010a) emphasized that the rules of spanning tree properties can be relaxed and a possibility of simultaneously improving the solution exists. The primary details and processes of the spanning tree-based GA

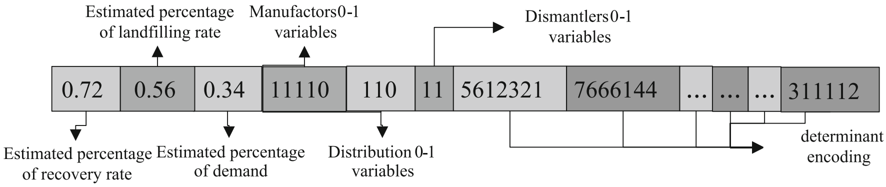

Fig. 3 Chromosome in revised spanning tree-based GA

for a closed-loop logistics problem are based on the model of Wang and Hsu (2010a). The main steps and the different revisions for the uncertain environment are described in the following subsection.

5.1 Revised determinant encoding

Determinate encoding

Encoding:
Assume there are N nodes,

Step 1: Generate an $N-1$ length of determinate encoding;
Step 2: Use the random or heuristic method to set the codes.

Decoding:
The decoding algorithm addresses each allele of the gene corresponding to its position in the chromosome, and the position represents its direct connecting node. The first gene is decoded as fixed-position 2, the second as fixed-position 3, and so on.

The procedure for the determinate decoding process is as follows:

Step 1: Let C be the given determination string and l be its length. If $\mathrm{C}(j)$ is the j-th allele in the chromosome and $1 \leq j \leq l$, the number of the nodes in the given graph G is $l+1$, where a node is denoted as node (x) and $1 \leq x \leq l+1$.
Step 2: Set $j = 1$, if $0 < j < l+1$, go to Step 3; otherwise, stop.
Step 3: Connect node $(j+1)$ with node $\mathrm{C}(j)$. Set $j = j+1$. Go back to Step 2.

Three "illegal" situations are present in the determinant encoding, namely, cycling, reflexivity, and missing node 1. Only the situation of the missing node 1 may occur in this problem. However, solving the problem while simultaneously improving the solution is easy. The repairing process tests the nodes at the fixed positions with the costs connecting to Node 1, and the node with the minimum cost will be replaced by Node 1. Thus, the missing node 1 problem is resolved.

Figure 3 shows an example of chromosome presentation.

The chromosome has three parts. The first part is the real-number encoding to address the uncertainty, which is used to design the estimated amount of uncertain parameters. The second is binary encoding to design the places, whether open or otherwise. The third is the evaluation of the determinant encoding for establishing the flows.

A heuristic of the revised determinant encoding was used to establish the flows in the problem. To ensure the feasibility of a substring in determinant encoding, the range of the encoding value should be restricted. The units are restricted to flow only between different stages of the network. The cases of "reflexivity" and "cycling" will never occur, provided that the range of the encoding value is restricted to certain values.

Initialization procedure:

Two kinds of algorithms for establishing determinate encoding are used, namely, random and heuristic encoding. Let $\rho\%$ of the population be randomly generated, and $(100-\rho)\%$ of the population be heuristically generated.

The process of heuristic generation is as follows: (Let $q=2$)

Step1: The heuristic method begins from the qth fixed position in the determinant encoding, wherein the gene with the minimum cost is allocated to the fixed position. If more than one of the minimum points exists, any one will be arbitrarily selected.

Step 2: Set $q=q+1+$. If $q \le I+J$ (If it is in the first stage of I suppliers and J manufacturers), then go to Step 1; otherwise, stop.

The ratio for the use of the heuristic and random settings is 9:1 ($\rho\% = 10\%$) in the initial population. The heuristic setting can effectively help in the determination of a good solution, and the random setting is used to avoid the optimum generated from a local population.

5.2 Fitness function

Flows and cost of determinate encoding

A random number stream is generated to determine the order of flows, and the details are described using the example between suppliers and manufacturers with a stream established by a random number between 1 and $i+j-1$.

Step 1: Use the random stream to choose from among the fixed positions. Then, select the smaller capacity to be the flow between the fixed positions and the corresponding gene in the chromosome. This means that assigning the available amount of units to $x_{ij} = \min\{a_i, b_j\}$ is needed.

Step 2: Update the availability $a_i = a_i - x_{ij}$ and $b_j = b_j - x_{ij}$.

Step 3: If the number of units to assign is not available, stop. If a remaining supply of node r and demand of node s are present, then add edge (r, s) to the tree, and assign the available amount of units $x_{rs} = \min\{a_r, b_s\}$ to the edge.

After assigning the delivered amounts for each variable using determinate encoding, Eq. (20) can be used to calculate the total cost to be minimized. The information on assigned amounts also allows for the obtainment the *RMSII* of the total cost using Eq. (34) with the possibilistic approach. In the first five iterations of the algorithm, the GA is conducted with only the first priority of the objective, which is the fitness function of total cost. After the cost converges to a better solution set, the second priority of the objective is added, that is, the *RMSII* of total cost, as one of the fitness functions. Then, 80% of the population follows the total cost as a fitness function to select the next generation, whereas 20% of the population follows the *RMSII* of the total cost. This process helps in the determination of a solution with a lower *RMSII* or risk, and the best solution in each iteration should be simultaneously compared with cost and *RMSII*. If both the cost and *RMSII* for the solution are better than the current best solution, the best solution is then updated. In other words, the solution is non-dominated.

In a closed-loop supply chain, there are two kinds of systems, namely, push for reverse and pull for forward. In the proposed GA, the push system is first employed to determine the reverse amounts, and then how much the suppliers should assign to the manufacturers can be determined using the pull system.

5.3 Genetic operations

Based on the reports by Chou et al. (2001), Yao and Hsu (2009), and Wang and Hsu (2010a), the proper methods for the crossover and mutation of spanning-tree problems are the two-point crossover and exchange mutation. For these methods, the heuristic is applied to restrict the connection of the same stage and to improve the solution. The details can be found in the study by Wang and Hsu (2010a).

To address the uncertain property, real-number encoding is used to pre-assign a percentage for the range of uncertainty. This percentage provides the decision maker a suitable estimation for the uncertainty. The operator of the real code is shown as follows:

Operator of real-number coded GA

AMXO:

The basic concept of this method is borrowed from the convex set theory. Simple arithmetic operators are defined as the combination of two vectors (chromosomes):

$$x_1' = \lambda_1 x_1 + \lambda_2 x_2$$
$$x_2' = \lambda_2 x_2 + \lambda_1 x_1$$

If the multipliers are restricted as

$$\lambda_1 + \lambda_2 = 1, \quad \lambda_1 > 0, \quad \lambda_2 > 0$$

where λ_1 is a uniformly distributed random variable between 0 and 1, then a convex crossover occurs.

The convex crossover is the most commonly used method for the operator of a real-number code. When restricting $\lambda_1 = \lambda_2 = 0.5$, a special case is generated, which is called the ACXO or intermediate crossover.

Power mutation (PM) operator:

The proposed mutation operator is based on power distribution, and its distribution function is given by

$$f(x) = px^{p-1}, \quad 0 \leq x \leq 1$$

and the density function is given by

$$F(x) = x^p, \quad 0 \leq x \leq 1$$

where p is the index of the distribution. PM is used to create a solution y in the vicinity of a parent solution $\bar{x}$ in the following manner: First, a uniform random number x between 0 and 1 is created, and a random number s is created following the above-mentioned distribution. The following formula is then used to create the muted solution:

$$y = \begin{cases} \bar{x} - s(\bar{x} - x^l), & \text{if } t < r \\ \bar{x} + s(x^u - \bar{x}), & \text{if } t \geq r \end{cases}$$

where $t = \frac{\bar{x}-x^l}{x^u-x^l}$ and x^l and x^u are lower and upper bounds of the decision variable, and r is a uniformly distributed random number between 0 and 1. The strength of the mutation is governed by the index of the mutation (p). For small values of p, less perturbation in the solution is expected. For large values of p, more diversity is achieved. In the present research, p is set as 2.

The selection approach adopts the $(\mu + \lambda)$ method suggested by Chou et al. (2001), where the μ parents and λ offsprings compete for survival, and the μ best solutions are selected for the next generation.

Several termination conditions are established based on the number of generations, computing time, and fitness convergence. Fitness convergence occurs when all the chromosomes in the population have the same fitness value. In the present work, fitness convergence is selected as the termination criterion. The evolutionary process in GA is stopped when the best chromosome on hand did not improve in the last 15 generations ($Max\_bestonhand$). The iteration number of the best solution on hand is denoted as $Num\_bestonhand$. Simultaneously, if the number of generations is greater than 750 ($Max\_iteration$), the algorithms are also stopped. These termination parameters result in the development of the dynamic adjustable crossover and mutation rate described below.

Dynamic adjustable crossover and mutation rate

In the proposed GA, a dynamic adjustable crossover and mutation policy are applied to improve search efficiency. The initial crossover rate (p_c) and mutation rate (p_m) are equal to 0.8 and 0.2, respectively, according to the following rules:

Step 1. If the best solution in the current iteration does not improve, go to Step 2. Otherwise, go to Step 3.

Step 2. The crossover rate will increase and the mutation rate will decrease as the following function to raise the possible improvement of the best solution:

$$p_c = p_c + (1 - p_c)/(Max\_iteration + 1 - current\_iteration)$$
$$p_m = p_m - p_m/(Max\_iteration + 1 - current\_iteration)$$

Step 3. The crossover rate will decrease and the mutation rate will increase as the following function to raise the possible global search.

$$p_c = p_c - (p_c - p_m)/2(Max\_bestonhand + 1 - Num\_bestonhand)$$
$$p_m = p_m + (p_c - p_m)/2(Max\_bestonhand + 1 - Num\_bestonhand)$$

5.4 Summary of the proposed algorithm

Based on the above-mentioned description, real-number, binary, and revised determinate encoding methods are used with the heuristic initial population to build up the chromosome in a spanning tree-based GA. The fitness functions with cost and *RMSII* are considered in searching for a solution with lower perturbation. The final result is a non-dominated solution for a decision maker. By applying AMXO and PM as the operators for a real-number code, uncertain parameters can be addressed. The dynamic adjustable crossover and mutation rate methods are proposed to improve efficiency. The above-mentioned process is repeated until the termination condition is satisfied.

6 Numerical experiments and the evaluation

Accuracy and efficiency are the basic concerns in developing an algorithm. No appropriate commercial software is available for comparison because of the nonlinearity of the objective function. Therefore, the results developed by Wang and Hsu (2010a) will be adopted for the crisp closed-loop logistics problems.

To test for accuracy, the uncertain parameters were transformed into crisp forms by giving a zero spread of fuzzy numbers. Then, the commercial software LINGO was used to find the solutions for comparison. To test for efficiency, different sizes of the test problems were generated by doubling the numbers of the nodes at each stage, and the averaged time was compared by running each problem thirty times. These experiments were conducted using a PC with Intel^R Pentium^R M processor 1.86 GHz and 1.0 G RAM.

6.1 Test the accuracy

The test is based on the same problem used for the illustration, containing three suppliers (I), five manufactories (J), three distributors (K), four customers (L), and two dismantlers (M) with a \$5 unit landfilling cost (LC). By consulting the most possible value and its range from a DM with different scenarios, the input data is similar to that of Wang and Hsu (2010a) and Hsu and Wang (2009) with fuzzified parameters

Table 1 Input scale and data for the example

Suppliers	Manufactories	DCs	Customers	Dismantlers	$\tilde{pc}_l$	$\tilde{pl}_m$	LC
3	5	3	4	2	(0.1, 0.1)	(0.1, .0.1)	5

Table 2 Input fuzzy demand $\tilde{d}_l$ (mode, spread)

$\tilde{d}_1$	$\tilde{d}_2$	$\tilde{d}_3$	$\tilde{d}_4$
(500, 100)	(300, 100)	(400, 100)	(300, 100)

Table 3 Results of GA for different crisp problems (30 instances)

Population size	I=3, J=5, K=3, L=4, M=2 (A1)			I=6, J=10, K=6, L=8, M=4 (A2)		
	Minimum cost	Average cost	Average time (s)	Minimum cost	Average cost	Average time (s)
100	***29815****	29927.83	2.7730	***58110****	58792.53	12.3386
50	***29815****	29975.83	2.1681	58350	59261	5.9002
20	***29815****	30007.5	1.4205	58480	59591.77	2.5608
LINGO	***29815****		6	***58110****		13

* The global optimal solution
The bold italicized numbers mean the best values of different size problems

$\tilde{pc}_l$, demand $\tilde{d}_l$ for each customer l, and $\tilde{pl}_m$ for dismantler m, as listed in Tables 1 and 2, wherein the triangular membership functions are assumed to have similar left and right spreads.

If same data are run without the spread, the results with different population sizes are shown in Table 3.

The population size affects the time to converge for a solution and the quality. The results for the small-sized problem (A1) in Table 3 show that even for a small population size, the algorithm can achieve the global optimal solution. Therefore, less time is needed to reach the optimal. When the problems have been doubled in size to $I = 6$, $J = 10$, $K = 6$, $L = 8$, and $M = 4$ (A2), the optimal solution still can be obtained. The commercial software cannot reach the optimal solution after doubling the data from A2 because of memory limitations. Therefore, the results cannot be compared. Similar situations can be correlated with the work of Wang and Hsu (2010a).

6.2 Different types of shortage and surplus costs

The cost structure may affect the speed of search. Although the relationship between shortage and surplus cost has been discussed in Sect. 4, a comparison for the cost should still be conducted because of its importance in this setting. Table 4 shows two different types of cost relations. Problem 1 shows that the shortage cost is higher than the surplus cost. Problem 2 shows that the two costs are the same.

Table 4 Different relation types for the cost of the uncertain factors

Problem	C_d^-	C_r^-	C_l^-	C_d^+	C_r^+	C_l^+
1	35	40	30	15	10	8
2	15	10	8	15	10	8

Table 5 Results of different cost types for shortage and surplus (30 instances each)

Problem	Population size		Minimum cost	Minimum SD	Average	Average time (s)
1	20	Cost	33071.67	34020.72	34687.05	***1.8444***
		RMSII	1093.50	725.59	1153.58	
		C.V.	3.3065%	2.1328%	3.3257%	
	50	Cost	33157.52	34919.63	34040.35	5.9075
		RMSII	1160.11	***570.06***	997.927	
		C.V.	3.4988%	***1.6325%***	2.9316%	
	100	Cost	***32938.86***	33352.21	33646.15	10.9981
		RMSII	1129.76	804.84	1085.97	
		C.V.	3.4299%	2.4132%	3.2567%	
2	20	Cost	28833.07	29706.68	30427.84	***3.3461***
		RMSII	752.17	***402.56***	666.65	
		C.V.	2.6087%	***1.3551%***	2.1909%	
	50	Cost	29092.74	30187.46	29852.59	9.4802
		RMSII	730.32	421.04	711.92	
		C.V.	2.5103%	1.3948%	2.3848%	
	100	Cost	***28777.93***	28945.85	29354.07	18.6659
		RMSII	764.54	483.87	727.28	
		C.V.	2.6567%	1.6716%	2.4776%	

The bold italicized numbers mean the best values of different indices in different problems

The results of different population sizes with the information on *RMSII* are shown in Table 5. The coefficient of variance (C.V.) in the table is derived when *RMSII* is divided by the cost.

As shown in Table 5, when the shortage and surplus costs are similar to those in Problem 2, the algorithm requires more time to obtain a good solution. The whole cost of Problem 2 is smaller than that of the others. Therefore, the C.V. is also smaller than that of the others, indicating that the risk is smaller in Problem 2 because of the lower cost of uncertainty. However, as discussed in Sect. 4, the cost structure in Problem 2 is not realistic. Therefore, in the following section, further analysis on efficiency will be based on the cost structure of Problem 1.

6.3 Testing for efficiency

To test for efficiency, different sizes of test problems are used by doubling the number of nodes at each stage, as shown in Table 6, and each problem is run thirty times.

Table 6 Different size structures of the test problems

Size	Suppliers	Manufactories	DCs	Customers	Dismantlers
1	3	5	3	4	2
2	6	10	6	8	4
3	12	20	12	16	8
4	24	40	24	32	16

Table 7 Results for different-sized problems with population size = 100

Problem size		Minimum cost	Minimum SD	Average	Average time (s)
1	Cost	32938.86	33352.21	33646.15	10.9881
	RMSII	1129.76	804.84	1085.97	
	C.V.	3.4299%	2.4132%	3.2567%	
2	Cost	67116.02	69372.28	68822.94	18.2443
	RMSII	1634.23	772.53	1540.34	
	C.V.	2.4349%	1.1136%	2.2381%	
3	Cost	112264.64	115672.4	119843.26	58.7679
	RMSII	1442.96	194.66	1226.43	
	C.V.	1.2853%	0.1683%	1.0234%	
4	Cost	230358.67	257891.95	249003.94	147.0322
	RMSII	2019.07	865.95	994.71	
	C.V.	0.8765%	0.3385%	0.3995%	

* C.V. = RMSII/cost

Table 7 shows the results of different-sized problems with a population size equal to 100. The average run time increases significantly with the problem size. Problem size 1 is the same problem used in the crisp test discussed in Sect. 5.1, but with an extended spread of uncertainty. When the population size is equal to 100, the average run time for the crisp test without the spread of uncertainty in Table 3 is 2.773 (s), and the time with the spread of uncertainty in Table 7 is 10.9881 (s). The difference is 8.2151 (s), indicating that the problem with uncertainty makes obtaining an acceptable solution more difficult, and the uncertain problem is more complex than crisp.

As can be observed in Table 7, when the problem size increases, the C.V. decreases. The number of uncertain factors for demand, recovery, and landfill rates increases as problem size increases, whereas the value of C.V. decreases significantly, implying that the algorithm successfully controls the *RMSII* in a stable lower value and that when the problem size is larger, the relative risk of uncertainty is lower. When the problem is large, the tolerance for the estimation of uncertainty is higher, so the relative risk is lower. Also, the possibilistic mean of cost and *RMSII* in Table 7 can be used to obtain the prediction interval. For example, the minimum cost of problem 1 is 32938.86, and the 96.63% prediction interval is between [30679.34, 35198.38], which is the possibilistic mean of cost minus and plus two *RMSII*. In other words, if the decision maker chooses 32938.86 as his expected cost in a certain decision, there

Table 8 Average run time compared with that of Wang and Hsu (2010a)

Problem size	Current algorithm for uncertain environment	Time increased percentage	Crisp problem Wang and Hsu (2010a)	Time increased percentage
1	10.9881 (s)	–	2.04 (s)	–
2	18.2443 (s)	0.6604	6.84 (s)	2.3529
3	58.7679 (s)	2.2212	22.49 (s)	2.2880
4	147.0322 (s)	1.5019	72.74 (s)	2.2343

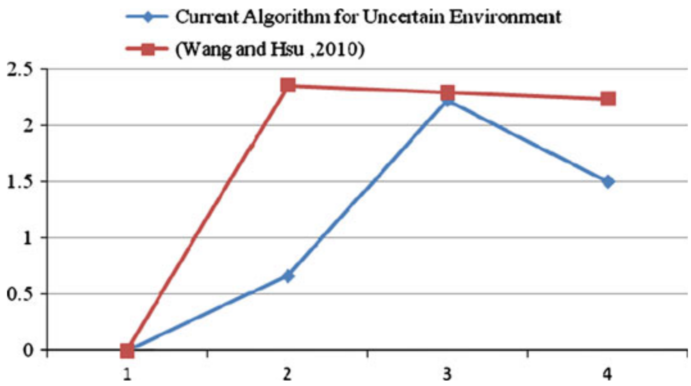

Fig. 4 Time increase percentage for the current algorithm and for that of Wang and Hsu (2010a)

is a 96.63% probability that the cost will not be worse than 35198.38 and better than 30679.34. This probability is important for a decision maker to be able to realize the uncertainty.

Proving the efficiency of the algorithm remains difficult, especially when faced with an uncertain problem with a nonlinear objective function. No commercial software is available for use to obtain a solution and for comparison. Based on the results, the average run time was compared with the spanning tree-based GA for the crisp closed-loop logistics proposed by Wang and Hsu (2010a), which has been proven accurate and effective.

Table 8 shows the comparison of the run time between the current algorithm for an uncertain environment and that of Wang and Hsu (2010a). The percentage of time increase is defined as the difference between the current and last problem size divided by the last problem size. Figure 4 shows the graph comparing the time increase percentage between the current algorithm and that of Wang and Hsu (2010a).

Figure 4 shows that although the problem of uncertainty is more complex than the crisp issue, the time increase rate is still slower than that of Wang and Hsu (2010a). This finding indicates that the current algorithm can efficiently address large problems compared with that proposed by Wang and Hsu (2010a).

6.4 Summary

After comparing the accuracy of the crisp problem and the efficiency with the model of Wang and Hsu (2010a), the current spanning tree-based genetic algorithm for uncertain environments can be confirmed as accurate and efficient. The non-dominated solution can provide the decision marker an acceptable solution with lower perturbation. In addition, the situation is more significant when the problem size is larger. *RMSII* and C.V. are the reference values for risk and can be obtained from the possibilistic approach. Also, the information on mean and *RMSII* can be used to determine the prediction interval and to provide a solution under a certain probability in an uncertain environment.

7 Conclusions

An uncertain environment has its own properties, including shortage and surplus. In the present study, the focus was on the uncertain closed-loop logistics problem with shortage and surplus for demand, recovery, and landfill amounts. To reflect a realistic situation, the uncertainty was described using fuzzy sets, and the possibilistic approach was used to derive the possibilistic mean and *MSII* of shortage and surplus and to retain the complete information of fuzzy numbers. This approach resulted in the proposal of a possibilistic-based closed-loop logistics model with shortage and surplus. The objectives of this model are to reduce both the cost and *RMSII* so that decision makers can understand the risk level through *RMSII* under certain conditions. The prediction interval with a certain significance level can be obtained from the possibilistic mean and *RMSII*, which also provide the information to realize the uncertainty.

The problem is NP-hard, so a spanning tree-based GA was developed and revised to facilitate adjustable crossover and mutation rates. The accuracy and efficiency of the algorithm were tested, and the proposed model was shown to have promising results. Therefore, the current research provides decision makers with an acceptable non-dominated solution with low perturbation for low-risk results.

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