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Supplier selection using axiomatic fuzzy set and TOPSIS methodology in supply chain management

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Abstract The current paper presents a comprehensive methodology for supplier selection. In the first stage, the linguistic values expressed as trapezoidal fuzzy numbers are used to assess the weights of the criteria. The Axiomatic Fuzzy Set clustering (AFS) method, which handles ambiguity and fuzziness in the supplier selection problem effectively, is applied to cluster the suppliers and evaluate each potential supplier that aims at obtaining initial supplier ranking. In the second stage, the Fuzzy Analytic Hierarchy Process (FAHP) model is constructed to determine the weight of various quantitative and qualitative criteria. To address multiple decision criteria in supplier ranking, the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is employed to select the final suppliers. A numerical example composed of 30 suppliers and 6 criteria is studied, and the experimental results show that the proposed evaluation framework is suitable for supplier selection decisions even with the dependent criteria/attributes.

Keywords Fuzzy analytic hierarchy process · Qualitative and qualitative criteria · Technique for order preference by similarity to ideal solution · Axiomatic fuzzy set · Supplier selection

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1 Introduction

The contemporary supply management aims to maintain long-term partnership with suppliers; it uses few but reliable suppliers. Therefore, choosing the right suppliers involves more than scanning a series of price lists, and the choices depend on a wide range of both quantitative and qualitative factors. Extensive multi-criteria decision-making approaches have been proposed for supplier selection, such as the analytic hierarchy process (AHP), technique for order preference by similarity to ideal solution (TOPSIS), analytic network process (ANP), data envelopment analysis (DEA), fuzzy set theory, genetic algorithm (GA), mathematical programming, and their hybrids (see Ho et al. 2010 and Mafakheri et al. 2011). For the AHP approach to solving supplier selection, An interactive selection model with AHP is developed to facilitate decision-makers in selecting suppliers by Chan (2003). The model was so-called because it incorporates a method called chain of interaction, which is deployed to determine the relative importance of evaluating criteria without subjective human judgment. AHP was only applied to generate the overall score of alternative suppliers based on the relative importance ratings. An AHP-based decision making approach is developed to solve the supplier selection problem by Chan and Kumar (2007). A sensitivity analysis using expert choice is performed to examine the response of alternatives when the relative importance rating of each criterion is changed. A fuzzy analytic hierarchy process model is developed by Lee (2009a,b), which incorporates the benefits, opportunities, costs and risks concept, to evaluate various aspects of suppliers. Multiple factors positively or negatively affecting the success of the relationship are analyzed by considering experts' opinion on their importance, and a performance ranking of the suppliers is obtained. For the DEA approach to solving supplier selection, chance-constrained DEA approach is presented to evaluate the performance of suppliers in the presence of stochastic performance measures by Talluri et al. (2006). Price is considered an input, whereas quality and delivery are used as outputs. The model was compared with the deterministic DEA to highlight its usefulness. Imprecise DEA is presented to evaluate the performance of suppliers in the presence of both quantitative and qualitative data by Saen (2007). The author found that supplier reputation (SR), one of the output measures considered in the case study, could not be quantified legitimately. The proposed model enables decision-makers to provide a complete rank ordering of the suppliers on SR. Moreover, the proposed model can handle fuzzy data in the forms of bounded data. For the TOPSIS approach to solving suppliers selection, Integrated fuzzy TOPSIS and multi-choice goal programming approach are proposed to solve the supplier selection problem by Liao and Kao (2011). The advantage of this method is that it enables decision-makers to set multiple aspiration levels for supplier selection problems. A fuzzy quality function deployment (QFD) approach is proposed to support supplier selection by Bevilacqua et al. (2006). This approach uses both internal and external variables to rank the potential suppliers. The advantage of this method is in its ability to transforms decision-makers' verbal assessments to linguistic variables, which are more accurate than other non-fuzzy methods. However, it is used to rank potential suppliers, which is not the main objective in the early phase of supplier selection. The 14 most important evaluating factors are selected from 84 potential added-value attributes based on the questionnaire response from

US purchasing managers by Florez-Lopez (2007). To obtain a better representation of suppliers' ability to create value for the customers, a two-tuple fuzzy linguistic model was illustrated to combine both numerical and linguistic information. The proposed model can generate a graphical view showing the relative suitability of suppliers and identifying the strategic groups of suppliers.

For the mathematical programming and their hybrids approach to solving supplier selection, an optimum mathematical planning model is developed for green partner selection by Yeh and Chuang (2011), which involves four objectives such as cost, time, product quality, and green appraisal score. To solve these conflicting objectives, they adopted two multi-objective genetic algorithms to find the set of Pareto-optimal solutions, which utilized the weighted sum approach that can generate more number of solutions. A fuzzy multi-objective programming model is proposed to decide on supplier selection by Wu et al. (2010), taking risk factors into consideration. A supply chain was modeled, which consists of three levels and uses simulated historical quantitative and qualitative data to solve the fuzzy multi-objective programming model. A weighted max–min fuzzy model is developed to handle effectively the vagueness of input data and different weights of criteria by Amid et al. (2011). This model enabled matching of the achievement level of objective functions with the relative importance of the objective functions. In the current paper, an AHP was used to determine the weights of criteria. The proposed model can help the decision- maker determine the appropriate order to each supplier and enable the purchasing manager(s) to manage supply chain performance on cost, quality, and service. The hybrid methodology is developed for supplier selection and evaluation in a supply chain by Chen (2011). Based on the competitive strategy, the criteria and indicators of supplier selection are chosen to establish the supplier selection framework. Subsequently, potential suppliers are screened through DEA. TOPSIS, a multi-attribute decision-making method, was adapted to rank potential suppliers.

For supplier evaluation and selection, A stochastic efficiency analysis model ia developed to deal with supplier selection by Wu (2010). The model is a new methodological extension to DEA and is applicable to efficiency analysis for entities from different systems with imbedded uncertainty. The application of the proposed model to the international supplier evaluation was the first attempt to model suppler performance from different sub-systems with different environment factors and uncertainty using Stochastic DEA. The process of supplier selection and evaluation are designed based on the supplier integration approach for supply chain integration by Chen (2011). Subsequently, according to the characteristics of the supply chain commercial model, suitable analytical methods have been applied in conducting each activity involved in the supplier selection and evaluation process. The neural network-based is proposed to supplier selection and supplier performance evaluation systems by Aksoy and Öztürk (2011). The proposed approach is not limited to JIT supply. It can assist manufacturers in selecting the most appropriate suppliers and in evaluating supplier performance.

Cluster analysis is used for clustering a data set into groups of similar individuals (see Li and Fang 2009). It is one of the major techniques in pattern recognition. Since fuzzy sets is proposed that use the idea of partial membership described by a membership function by Zadeh (1965), many fuzzy clustering methods have been introduced. In popular fuzzy theories, the membership functions are often given subjectively by personal intuitions, and the logic operations are implemented by a kind of triangular norms or a short t-norm chosen in advance and independent of the distribution of the original data. However, in real-world applications, fuzzy phenomena exist throughout nature and extensively within the human society that defining the membership functions only by personal intuitions is impossible or difficult. In addition, different logic operator choices and membership function selections may lead to different results for the same data set. To cope with these issues, the authors in Liu et al. (2003, 2005, 2007), Xu et al. (2009), Liu (1998a,b) and Liu and Pedrycz (2009), proposed and developed the AFS theory, in which fuzzy sets (membership functions) and their logic operations are directly determined by a consistent algorithm according to the distributions of original data and the semantics of the fuzzy concepts.

All these approaches can deal with multiple criteria to suppliers' selection. In supplier evaluation, most methods conduct performance evaluation. However, only a few studies have taken into consideration supplier evaluation based on each criterion. In the current paper, this problem uses a two-stage solution methodology. In the first stage, the AFS is employed to evaluate and cluster potential suppliers based on fuzzy criteria. In the second stage, the FAHP is applied to calculate the weight of each criterion, and the final suppliers are established and ranked, considering both qualitative and quantitative criteria by TOPSIS.

2 Elementary AFS theory, FAHP, TOPSIS

2.1 AFS theory

A family of molecular lattices was defined in Liu (1998a): the AFS algebra as applied to study the semantics of natural language and the lattice-valued representations of fuzzy concepts. The following example serves as an introductory illustration of the AFS algebra.

Example 1 Let $X = \{x_1, x_2, \ldots, x_5\}$ be a set of five suppliers and their features (attributes), which are described by *quality, flexibility in service, profitability of supplier, sufficient delivery, and product/process flexibility* (Table 1).

Let $X = \{x_1; x_2; \ldots; x_5\}$ be a set of five suppliers and $M = \{m_1; m_2; \ldots; m_5\}$ be a set of fuzzy attributes on X, where m_1 is the "Attribute1 is good," m_2 is ""'Attribute2 is good," and m_5 is "Attribute5 is good." For each set of concepts $A \subseteq M$, $\Pi_{m \in A} m$ represents conjunction of the concepts in A. For instance, $A = \{m_1, m_5\} \subseteq M$, $\Pi_{m \in A} m = m_1 m_6$ representing a new fuzzy concept *"quality and product/process flexibility are good"*. For $\Sigma_{i \in I}(\Pi_{m \in A_i} m)$, which is a formal sum of $\Pi_{m \in Ai} m$, $A_i \subseteq M, i \in I$, is the disjunction of the conjunctions represented by $\Pi_{m \in Ai} m$'s (i.e., the disjunctive normal form of a formula representing a concept). For example, we may have $\gamma = m_1 m_5 + m_1 m_3 + m_2$ which translates as *"quality and product/process flexibility are good"* or *"quality and profitability of supplier are good"* or *flexibility in service is good"* (the "+" denotes a disjunction of concepts). Although M may be a set of fuzzy or Boolean (two-valued) concepts, every $\Sigma_{i \in I}(\Pi_{m \in Ai} m)$, $A_i \subseteq M, i \in I$, has a well-defined meaning

Table 1 Descriptions of the features

	Quality	Flexibility in service	Profitability of supplier	Sufficient delivery	Product/process flexibility
x_1	M	H	H	B.H&VH	VH
x_2	H	B.L&M	M	B.M&H	H
xx_3	M	H	B.M&H	H	M
xx_4	VL	M	H	B.VL&L	H
xx_5	L	L	B.H&VH	H	B.H&VH

where *VL* very low, *B.V L&L* between very low and low, *L* low, *B.L&M* between low and medium, *M* medium, *B.M&H* between medium and high, *H* high, *B.H&VH* between high and very high, and *VH* very high

such as the one previously discussed. Through a straightforward comparison of the expressions

$$m_3m_4 + m_1m_4 + m_1m_2m_5 + m_1m_4m_5 \quad \text{and} \quad m_3m_4 + m_1m_4 + m_1m_2m_5$$

we conclude that their left and right sides are equivalent. Considering the terms on the left side of the expression, for any x, the degree of x belonging to the fuzzy concept represented by $m_1m_4m_5$ is always less than or equal to the degree of x belonging to the fuzzy concept represented by m_1m_4. Therefore, the term $m_1m_4m_5$ is redundant when forming the left side of the fuzzy concept. Let us consider two expressions of the form $\alpha : m_1m_4 + m_2m_3m_5$ and $\nu : m_2m_4 + m_2m_5$. The semantic content of the fuzzy concepts "α or ν" and "α and ν" can be expressed as follows:

$$\text{“}\alpha \text{ or } \nu'' : m_1m_4 + m_2m_3m_5 + m_2m_4 + m_2m_5 \quad \text{equivalent to} \quad m_1m_4 + m_2m_4 + m_2m_5,$$

$$\text{“}\alpha \text{ and } \nu'' : m_1m_2m_4 + m_1m_2m_4m_5 + m_2m_3m_4m_5 + m_2m_3m_5 \quad \text{equivalent to} \quad m_1m_2m_4 + m_2m_3m_5$$

The semantics of the logic expressions, such as "*equivalent to*", "*or*," and "*and*" as expressed by $\Sigma_{i\in I}(\Pi_{m\in Ai}\ m)$, $A_i \subseteq M, i \in I$, can be formulated in terms of the AFS algebra in the following manner.

A lattice is a partially ordered set L in which any two elements $a, b \in L$ have a least upper-bound (i.e., $a \vee b$) and a greatest lower bound (i.e., $a \wedge b$). A partially ordered set L is called a complete lattice if every subset $A \subseteq L$ has a *sup* and an *inf*, denoted by $\vee_{a\in A}a$ and $\wedge_{a\in A}a$ respectively. A complete lattice is called a completely distributive lattices, if one of the conditions shown below ($CD1$ or $CD2$) holds

$$(CD1) \bigwedge_{i\in I}\left(\bigvee_{j\in J_I} a_{ij}\right) = \bigvee_{f\in\prod_{i\in I} J_i}\left(\bigwedge_{i\in I} a_{if(i)}\right),$$
$$(CD1) \bigvee_{i\in I}\left(\bigwedge_{j\in J_I} a_{ij}\right) = \bigwedge_{f\in\prod_{i\in I} J_i}\left(\bigvee_{i\in I} a_{if(i)}\right) \tag{1}$$

where $\forall\ i \in I, \forall\ j \in J_i, a_{ij} \in L$, and $f \in \Pi_{i\in I} J_i$ means that f is a mapping $f : I \to \cup_{i\in I} J_i$ such that $f(i) \in J_i$ for any $i \in I$.

Let M be a non-empty set. The set EM^* is defined by

$$EM^* = \{\Sigma_{i\in I}(\Pi_{m\in Ai}\, m) | A_i \subseteq M, i \in I, I \text{ is a non-empty indexing set}\}. \tag{2}$$

Definition 1 (Liu 1998a) Let M be a non-empty set. A binary relation R on EM^* is defined as follows. For $\Sigma_{i\in I}(\Pi_{m\in Ai}\ \ m), \Sigma_i \in J(\Pi_{m\in Bj} m) \in EM^*$, $[\Sigma_{i\in I}(\Pi_{m\in Ai} m)] R [\Sigma_{i\in J}(\Pi_{m\in Bj} m)] \Leftrightarrow$ (i) $\forall A_i (i \in I), \exists\ B_h (h \in J)$ such that $A_i \supseteq B_h$;(ii) $\forall B_j (j \in J), \exists\ A_k (k \in I)$ such that $B_j \supseteq A_k$.

Clearly, R is an equivalence relation. The quotient set EM^*/R is denoted by *EM*. The notation $\Sigma_{i\in I}(\Pi_{m\in Ai} m) = \Sigma_{i\in J}(\Pi_{m\in Bj} m)$ indicates that $\Sigma_{i\in I}(\Pi_{m\in Ai} m)$ and $\Sigma_{i\in J}(\Pi_{m\in Bj} m)$ are equivalent under equivalence relation R. Thus, the semantics they represent are equivalent. In Example 1, for $\xi = m_3 m_4 + m_1 m_4 + m_1 m_2 m_5 + m_1 m_4 m_5$, $\zeta = m_3 m_4 + m_1 m_4 + m_1 m_2 m_5 \in EM$, by Definition 1 we have $\xi = \zeta$. In what follows, each $\Sigma_{i\in I}(\Pi_{m\in Ai}\ m) \in EM$ is called a fuzzy concept.

Theorem 1 (Liu 1998a) *Let M be a non-empty set. Then, (EM, $\vee, \wedge$) forms a completely distributive lattice under the binary compositions $\vee$ and $\wedge$ defined as follows. For any $\Sigma_{i\in I}(\Pi_{m\in Ai} m), \Sigma_{i\in J}(\Pi_{m\in Bj} m) \in EM^*$*

$$[\Sigma_{i\in I}(\Pi_{m\in Ai} m)] \vee \left[\Sigma_{i\in J}(\Pi_{m\in Bj}\ m)\right] = \Sigma_{k\in I\cup J}(\Pi_{m\in Ck}\ m) \tag{3}$$
$$[\Sigma_{i\in I}(\Pi_{m\in Ai} m)] \wedge \left[\Sigma_{i\in J}(\Pi_{m\in Bj} m)\right] = \Sigma_{i\in I, j\in J}(\Pi_{m\in Ai\cup Bj}\ m) \tag{4}$$

where for any $k \in I|\_|J$ (the disjoint union of I and J, i.e., every element in I and every element in J are always regarded as different elements in $I|\_|J$), $C_k = A_k$, if $k \in I$, and $C_k = B_k$, if $k \in J$.

In Example 1, for $\gamma = m_1 m_5 + m_1 m_3 + m_2 \in EM$

$$\begin{aligned}\gamma\,' &= (m_1 m_5 + m_1 m_3 + m_2)' \\ &= (m_1' + m_5') \wedge (m_1' + m_3') \wedge m_2' \\ &= (m_1' + m_5' m_3') \wedge m_2' \\ &= m_1' m_2' + m_2' m_3' m_5'\end{aligned}$$

γ', which is the logical negation of $\gamma = m_1 m_5 + m_1 m_3 + m_2$, reads as *"quality and flexibility in service are not good"* or *"flexibility in service, profitability of supplier, and product/process flexibility are not good."* The authors proved that the operator " $'$ " is an order reversing involution of *EI* algebra *EM*, if for any $\Sigma_{i\in I}(\Pi_{m\in Ai} m) \in EM$,

$$(\Sigma_{i\in I}(\Pi_{m\in Ai} m))' = \wedge_{i\in I}(\vee_{m\in Ai} m') = \wedge_{i\in I}(\Sigma_{m\in Ai} m') \tag{5}$$
$$(\Sigma_{i\in I}(\Pi_{m\in Ai} m))' = \wedge_{i\in I}(\vee_{m\in Ai} m') = \wedge_{i\in I}(\Sigma_{m\in Ai} m') \tag{6}$$

If m' stands for the negation of the concept $m \in M$, then for any fuzzy concept $\zeta \in EM$, ζ' denotes the logical negation of ζ. In Example 1,

Definition 2 (Liu et al. 2005) Let X, M be the sets and 2^M be the power set of M. Let $\tau : X \times X \rightarrow 2^M$. (M, τ, X) is called an AFS structure if τ satisfies the following axioms:

$$\text{AX1}: \forall(x_1, x_2) \in X \times X, \tau(x_1, x_2) \subseteq \tau(x_1, x_1); \tag{7}$$

$$\text{AX2}: \forall(x_1, x_2), (x_2, x_3) \in X \times X, \tau(x_1, x_2) \cap \tau(x_2, x_3) \subseteq \tau(x_1, x_3). \tag{8}$$

X is universe of discourse. M is the concept set, and τ is a structure.

Let us continue with Example 1, in which $X = \{x_1, x_2, \ldots, x_5\}$ is the set of five suppliers and $M = \{m_1, m_1', m_2, m_2', \ldots, m_5, m_5'\}$, where m_1 is "Attribute1 is good," m_1' is "Attribute1 is not good," m_5 is "Attribute5 is good," and m_5' is "Attribute1 is not good." For the semantic meanings of the linguistic values, we have the following ordered relations: VH">"B.H&VH">"H"> "B.M&H">"M">"B.L&M">"L">"B.VL&L">"VL."

Through Table 1 and the semantic meanings of the attributes in M, we have

$$\begin{array}{ll} m_1 : x_4 < x_5 < x_1 = x_3 < x_2 & m_1' : x_4 > x_5 > x_1 = x_3 > x_2 \\ m_2 : x_5 < x_2 < x_4 < x_1 = x_3 & m_2' : x_5 > x_2 > x_4 > x_1 = x_3 \\ m_3 : x_2 < x_3 < x_4 = x_1 < x_5 & m_3' : x_2 > x_3 > x_4 = x_1 > x_5 \\ m_4 : x_4 < x_2 < x_5 = x_3 < x_1 & m_4' : x_4 > x_2 > x_5 = x_3 > x_1 \\ m_5 : x_3 < x_2 = x_4 < x_5 < x_1 & m_5' : x_3 > x_2 = x_4 > x_5 > x_1 \end{array}$$

Definition 3 (Liu 1998b) Let X and M be the sets, (M, τ, X) be an *AFS* structure, and (X, σ, m) be a measure space, where m is a finite and positive measure, $m(X) = 0$, $A_i^\tau \in \sigma, x \in X, i \in I$. For the fuzzy concept $\eta = \sum_{i \in I} (\prod_{m \in A_i} m) \in EM$, the membership function of η is defined as follows. For any $x \in X$,

$$\mu_\eta(x) = \sup_{i \in I} \frac{m\left(A_i^\tau(x)\right)}{m(X)} \tag{9}$$

where $A_i^\tau(x) = \{y \in X | x \geq_m y, \forall m \in A_i\}$, $A_i^\tau(x)$ is the set of all elements in X, whose degrees belonging to concept $\prod_{m \in A} m$ are less than or equal to that of x. $A_i^\tau(x)$ is determined by the semantics of the fuzzy concept.

In our study, let $o = 2^X$, for $W \in 2^X$, $m(W) = |W|(|W|$ is the cardinal number of the set $W)$. The equation can be stated as follows:

$$\mu_\eta(x) = \sup_{i \in I} \frac{\left|A_i^\tau(x)\right|}{|X|} \tag{10}$$

In Example 1, let $\eta_1 = m_1, \eta_2 = m_2, \eta_3 = m_3 m_4, \eta_4 = m_3 + m_4 \in EM$. Based on this equation, we have

$$For\ \eta_1, A = \{m_1\}, A^\tau(x_1) = \{x_4, x_5, x_1, x_3\}, \mu_{\eta_1}(x_1) = \frac{|A^\tau(x_1)|}{|X|} = 4/5 = 0.8$$

$$For\ \eta_2, A = \{m_2\}, A^\tau(x_1) = \{x_5, x_2, x_4, x_1, x_3\}, \mu_{\eta_2}(x_1) = \frac{|A^\tau(x_1)|}{|X|} = 5/5 = 1.0$$

$$For\ \eta_3, A = \{m_3, m_4\}, A^\tau(x_1) = \{x_2, x_3, x_4, x_1\}, \mu_{\eta_3}(x_1) = \frac{|A^\tau(x_1)|}{|X|} = 4/5 = 0.8$$

$$For\ \eta_4, A_1 = \{m_3\}, A_2 = \{m_4\}, \mu_{\eta_4}(x_1) = \sup_{i=1,2}\left(\frac{\left|A_i^\tau(x_1)\right|}{|X|}\right)$$

$$= \sup\{4/5, 5/5\} = 1.0$$

2.2 AFS clustering method

Cluster analysis is a very useful classification tool. It has been used frequently in product position, strategy formulation, market segmentation studies, and business system planning. Further, we can discriminate one or more strategies from the airfreight industry to understand the competitive situation better. The algorithm of AFS is as follows:

STEP 1: Find fuzzy set. $\vartheta = \vee_{b\in\wedge} b, x \in X, \mu_{\vee b\in\wedge} b(x)$ is the highest degree of x belonging to any cluster, as ϑ is the maximum element in $(\wedge)_{EI}$. To produce a well-defined clustering result, each x should belong to ϑ to the highest extent. Proposition 1 outlines the properties of the fuzzy set ϑ.

STEP 2: Find the fuzzy description of each object: for each $x \in X$, and the *fuzzy description* ξ_x of x, which is δ_x for the Boolean case. For fuzzy set $\xi_x \in (\wedge)_{EI}$, where $(\wedge)_{EI}$ is also the sub *EI* algebra generated by $\wedge$, not only is $\mu_{\xi x}(x)$ approaching $\mu_{\vee b\in\wedge} b(x)$, but $\mu_{\xi x}(y)$is also as small as possible for $y \in X, y \neq x$. In other words, x can be distinguished by ξ_x from other objects in X to the highest extent.

STEP 3: Evaluate the similarity between objects based on the fuzzy descriptions. Apply ξ_x, the fuzzy description of each $x \in X$, to establish the *fuzzy matrix* $M = (m_{ij})$ on $X = (x_1; x_2; \ldots; x_n)$, where m_{ij} is the *similarity degree between* x_i and x_j defined as follows: for any $x_i; x_j \in X, m_{ij} = \min\{\mu_{\xi_{xi}\wedge\xi_{xj}}(x_i), \mu_{\xi_{xi}\wedge\xi_{xj}}(x_j)\}$. Theorem 1 demonstrates that there exists an integer r such that $(M^r)^2 = M^r$; i.e., fuzzy matrix $Q = M^r$ can yield a partition tree with equivalence classes.

STEP 4: Cluster according to the determined similarity degrees. Let $Q = M^r = (q_{ij})$ and the Boolean matrix $Q_\alpha = (q_{ij}^\alpha)$, where $q_{ij}^\alpha = 1 \Leftrightarrow q_{ij} \geq \alpha$, and the threshold $\alpha \in [0, 1]$. For $\alpha \in [0, 1], x_i; x_j \in X; x_i; x_j$ are in the same cluster for a given threshold α if and only if $q_{ij}^\alpha = 1$. For some $x_i \in X$, if $q_{ij}^\alpha = 0$, the clustering label of x_i cannot be determined for fuzzy attributes in $\wedge$ under threshold α.

STEP 5: Select the well-delineated clustering results. For each cluster $C \subseteq X$ under the threshold α, the *fuzzy description of* C, ξ_C is defined as follows:

$$\xi_C = \bigvee_{x\in C} \xi_x \tag{11}$$

In the fuzzy description ξ_C of class C, its membership degree $\mu_{\xi_C}(x)$ is not only the most approachable $\mu_{\vee b\in\wedge} b(x)$ for each $x \in C$, but $\mu_{\xi_C}(y)$is also as small as possible for $y \in X$, $y \notin C$. In other words, the objects in cluster C can be distinguished from other objects in X to the highest possible extent. The *fuzzy description of the boundary among the clusters* $C_1; C_2; \ldots; C_l$ is a fuzzy set $\xi_{bou} \in EM$;

$$\xi_{bou} = \vee(\xi_{C_i} \wedge \xi_{C_j}) \tag{12}$$

where $\xi_{C_i}; i = 1; 2; \ldots; l$ is the fuzzy description for the ith cluster C_i. The clarity of the fuzzy clustering for some threshold α can be evaluated by I_α a *fuzzy cluster validity index* defined as follows. For any threshold $\alpha \in [0, 1]$,

$$I_\alpha = \frac{1}{\alpha^2} \times \frac{\sum_{Ci \in \cup_{1 \le k \le l}} C_k \mu_{bou}(ci)}{\sum_{Ci \in \cup_{1 \le k \le l}} C_k \mu_{total}(ci)} + \frac{|C|}{|X|} \tag{13}$$

where $\xi_{Total} = \vee_{1 \le k \le l} \xi_{C_k}, l > 2.|C|$ is the number of the clusters, and $|X|$ is the number of the objects. The less I_α there is, the better the clustering.

In STEP 1: For $x_1 : \mu_{m2}(x_1) = \mu_{m4}(x_1) = \mu_{m5}(x_1) = 1 = \mu_\vartheta(x_1)$, we have $B^0_{x1} = \{m_2, m_4, m_5\}$. $\mu_{m_2 m_4 m_5}(x_1) = 1 = \mu_\vartheta(x_1)$, with $m_2m_4m_5$ being the minimal element. $\Lambda^0_{x1} = \{m_2, m_4, m_5, m_2m_4, m_2m_5, m_4m_5, m_2m_4m_5\}$, thus, $\xi_{x1} = m_2m_4m_5$. We can also obtain the others in the same way: $B^0_{x2} = \{m_1, m'_3\}, \xi_{x2} = m_1m'_3$; $B^0_{x3} = \{m_2, m'_5\}, \xi_{x3} = m_2m'_5$; $B^0_{x4} = m'_1m'_4, \xi_{x4} = m'_1m'_{'4}$; $B^0_{x5} = m_3m'_2, and \xi_{x4} = m_3m'_2$

In STEP 2: The fuzzy relation matrix F is

$$F = \begin{bmatrix} 1.0 & 0 & 0 & 0 & 0.6 \\ & 1.0 & 0 & 0 & 0 \\ & & 1.0 & 0 & 0 \\ & & & 1.0 & 0 \\ 0.6 & & & & 1.0 \end{bmatrix}$$

$Q = F^3$ can yield a partition tree with equivalence classes.

In STEPS 3 and 4: The threshold can be $\alpha = 0.6, 1.0$. When the threshold $\alpha\ is\ 0.6$, four clusters can be obtained:

$$C_1 = \{x_2\}; C_2 = \{x_3\}; C_3 = \{x_4\}; C_4 = \{x_1, x_5\}.\ I_{0.6} = 1.8$$

When threshold $\alpha\ is$ 1.0, five clusters can be obtained:

$$C_1 = \{x_1\}; C_2 = \{x_2\}; C_3 = \{x_3\}; C_4 = \{x_4\}, C_5 = \{x_5\}. I_{0.6} = 1.64$$
$$\xi_{C_1} = \xi_{x1} = m_2m_4m_5, \quad \xi_{C_2} = \xi_{x2} = m_1m'_3, \xi_{C_3} = \xi_{x3} = m_2m'_5,$$
$$\xi_{C_4} = \xi_{x4} = m'_1m'_{'4}, \quad \xi_{C_5} = \xi_{x5} = m_3m'_2$$

$I_{1.0}$ is the smallest. The best cluster is $C_1 = \{x_1\}; C_2 = \{x_2\}; C_3 = \{x_3\}; C_4 = \{x_4\}, C_5 = \{x_5\}$.

The following is the description of each cluster:

The fuzzy description ξ_{C_1} of Cluster C_1 is $m_2m_4m_5$ with the following interpretation: *flexibility in service, sufficient delivery, and product/process flexibility are good*.
The fuzzy description ξ_{C_2} of Cluster C_2 is m_1m_3' with the following interpretation: *quality is good, but profitability of supplier is not good*.
The fuzzy description ξ_{C_3} of Cluster C_3 is m_2m_5' with the following interpretation: *flexibility in service is good, but product/process flexibility is not high*.
The fuzzy description ξ_{C_4} of Cluster C_4 is $m_1'm_4'$ with the following interpretation: *quality and sufficient delivery are not good*.
The fuzzy description ξ_{C_5} of Cluster C_5 is m_2' with the following interpretation: *flexibility in service is not good*.

2.3 FAHP

AHP was designed to solve complex problems involving multiple criteria by Saaty (1980). It enables decision-makers to specify their preferences using a verbal scale. This verbal scale can be very useful in helping a group or an individual make a fuzzy decision (see Finan and Hurley 1999). FAHP method is a systematic approach to the alternative selection and justification problem that uses the concepts of fuzzy set theory and hierarchical structure analysis. The decision maker can specify preferences in the form of natural language or numerical value about the importance of each performance attribute. The system combines these preferences with existing data using FAHP. In the FAHP method, the pair-wise comparisons in the judgment matrix are fuzzy numbers, and the fuzzy arithmetic and fuzzy aggregation operators. The procedure calculates a sequence of weight vectors that will be used to choose the main attribute.

The extent analysis method (EAM) is briefly discussed here (see Lee 2009a,b). Two triangular fuzzy numbers $S_1 = (s_1^+, s_1, s_1^-)$ and $S_2 = (s_2^+, s_2, s_2^-)$ are compared. When $s_1^- \geq s_2^-, s_1 \geq s_2, s_1^+ \geq s_2^+$, we define the degree of possibility $V(S_1 \geq S_2) = 1$. Otherwise, we can calculate the ordinate of the highest intersection point (see Chang 1996):

$$\mu(d) = (s_1^- - s_2^+)/((s_1^- - s_1) - (s_2^+ - s_2)) \leq 1 \tag{14}$$

The value of fuzzy synthetic extent with respect to the ith criterion for s goals, is defined as (see Chang 1996):

$$F_i = \sum_{j=1}^{m} S_{g_i}^j \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} S_{g_i}^j\right]^{-1} \quad \text{where } S_{g_i}^j = [S_{ij}^-, S_{ij}, S_{ij}^+] \tag{15}$$

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} S_{g_i}^j\right]^{-1} = \left(1/\sum_{i=1}^{n}\sum_{j=1}^{m} S_{ij}^+,\ 1/\sum_{i=1}^{n}\sum_{j=1}^{m} S_{ij},\ 1/\sum_{i=1}^{n}\sum_{j=1}^{m} S_{ij}^-\right) \tag{16}$$

$$\sum_{j=1}^{m} S_{g_i}^j = \left(\sum_{j=1}^{m} S_{ij}^-, \sum_{j=1}^{m} S_{ij}, \sum_{j=1}^{m} S_{ij}^+\right) \tag{17}$$

M as a convex fuzzy number can be defined by

$$
\begin{aligned}
&V(F \geq F_1, F_2, \ldots, F_k) \min V(F \geq F_i), \quad i = 1, 2, \ldots, k \\
&d(F_i) = \min V(F_i \geq F_k) = w_i', \quad k = 1, 2, \ldots, n \text{ and } k \neq i
\end{aligned} \tag{18}
$$

Based on this procedure, we can calculate the weight w_i' of the criteria,

$$W' = (w_1', w_2', \ldots, w_n')^T \tag{19}$$

After normalization, the priority weights are follows:

$$W = (w_1, w_2, \ldots, w_n)^T \tag{20}$$

2.4 TOPSIS

TOPSIS method was first developed by Hwang and Yoon (1981). TOPSIS is a multiple-criteria method to identify solutions from a finite set of alternatives. The basic principle is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution (Jahanshahloo et al. 2006; Shih et al. 2007). The positive-ideal solution is a solution that maximizes the benefit criteria and minimizes the cost criteria, where as the negative ideal solution maximizes the cost criteria and minimizes the benefit criteria. The TOPSIS procedure can be expressed in a series of steps (Gumus 2009; Yang and Hung 2007).

STEP 1: Calculate the normalized decision matrix. The normalized value r_{ij} is calculated as

$$r_{ij} = x_{ij} \Big/ \sqrt{\sum_{i=1}^{m} x_{ij}^2}, \; i = 1, \ldots, m; \; j = 1, \ldots, n \tag{21}$$

STEP 2: Calculate the weighted normalized decision matrix by multiplying the normalized decision matrix by its associated weights. The weighted normalized v_{ij} is calculated as

$$v_{ij} = w_j r_{ij}, \; i = 1, \ldots, m, \; j = 1, \ldots, n \tag{22}$$

STEP 3: Determine the positive ideal and negative ideal solutions

$$
\begin{aligned}
A^+ &= \{v_1^+, \ldots.., v_n^+\} = \{(\max_j v_{ij} \,|i \in I), \; (\min_j v_{ij} \,|i \in J)\}, \\
A^- &= \{v_1^-, \ldots.., v_n^-\} = \{(\min_j v_{ij} \,|i \in I), \; (\max_j v_{ij} \,|i \in J)\},
\end{aligned} \tag{23}
$$

where I is associated with the benefit criteria, and J is associated with the cost criteria.

STEP 4: Calculate the separation measures, using the n-dimensional Euclidean distance. The separation of each alternative from the positive ideal solution is given as

$$d_i^+ = \left\{ \sum_{j=1}^{n} (v_{ij} - v_j^+)^2 \right\}^{1/2}, i = 1, \ldots., m; j = 1, \ldots, n \tag{24}$$

Similarly, the separation from the negative ideal solution is given as

$$d_i^- = \left\{ \sum_{j=1}^{n} (v_{ij} - v_j^-)^2 \right\}^{1/2}, i = 1, \ldots., m; j = 1, \ldots, n \tag{25}$$

STEP 5: Calculate the relative closeness to the ideal solution. The relative closeness of the alternative A_i with respect to A^+ is defined as

$$C_i = d_i^- / (d_i^+ + d_i^-), i = 1, \ldots., m \tag{26}$$

As $d_i^- \geq 0$ and $d_i^+ \geq 0$, then, clearly, $C_i \in [0, 1]$

STEP 6: Rank the preference order. In ranking the alternatives using this index, we can rank the alternatives in decreasing order. The basic principle of the TOPSIS method is that the chosen alternative should have the "shortest distance" from the positive ideal solution and the "farthest distance" from the negative ideal

3 The proposed methods

The current paper presents a group of sustainable suppliers based on AFS, FAHP, and TOPSIS methodologies, and provides effective support for decision-making supplier selection and evaluation. The structure of the problem is shown in Fig. 1.

3.1 Application of AFS theory to supplier evaluation

There are six factors: flexibility in service, product/process technology, profitability of supplier, sufficient delivery, and conformance quality and relationship closeness.

AFS theory for backing up supplier evaluation and selection is proposed in this section. The steps are as follows:

- Express the six criteria (i.e., *flexibility in service, product/process technology, profitability of supplier, sufficient delivery, and conformance quality and relationship closeness)* by the defined trapezoidal fuzzy numbers; each supplier is sorted in each criterion.
- Obtain and evaluate the normalized value of each supplier.

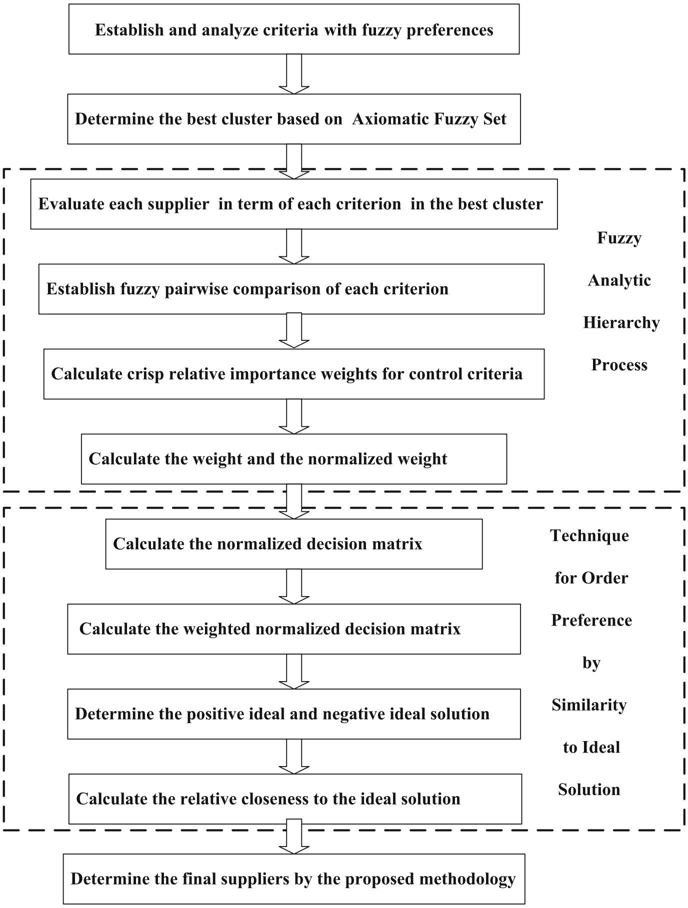

Fig. 1 The flow chart of this approach

- Find the fuzzy equivalence relation matrix based on the fuzzy compatibility relation matrix to cluster the next step.
- Obtain all feasible clusters based on the fuzzy equivalence relation matrix by Eq. (13).
- Determine minimal I_α, which is the best cluster.
- Evaluate each supplier and select the final supplier from backup suppliers in the best cluster.

In what follows, we apply data shown in Table 3 to demonstrate the proposed algorithm.

STEP 1: Let $X = \{c_1, c_2, \ldots, c_{30}\}$, $\varepsilon = 0$, $M = \{m_1, m'_1, \ldots, m_6, m'_6\}$, $\upsilon = m_1 + m'_1 + \cdots + m_6 + m'_6$; thus, we have $\mu_\upsilon(c_1) = \mu_\upsilon(c_2) = \cdots = \mu_\upsilon(c_{10}) = 1.0$. We only show c_1 as an example:

$$B^0_{c_1} = \{m'_4, m'_5\}, \xi_{C_1} = m'_4 m'_5$$

STEP 2: $Q = F^3$ can yield a partition tree with equivalence classes.
STEPS 3 and 4: When threshold $\alpha = 0.5667$, two clusters can be obtained.

$C_1 = \{c_2, c_3, c_6, c_8, c_9, c_{10}, c_{11}, c_{13}, c_{15}, c_{17}, c_{19}, c_{20}, c_{24}, c_{27}, c_{28}, c_{29}\}$
$C_2 = \{c_1, c_4, c_5, c_7, c_{12}, c_{14}, c_{16}, c_{18}, c_{21}, c_{22}, c_{23}, c_{25}, c_{26}, c_{30}\}$
$I_{0.5667} = 2.1263$

When threshold $\alpha = 0.6667$, three clusters can be obtained.

$C_1 = \{c_7\}$
$C_2 = \{c_2, c_3, c_6, c_8, c_9, c_{10}, c_{11}, c_{13}, c_{15}, c_{17}.c_{19}, c_{20}, c_{24}, c_{27}, c_{28}, c_{29}\}$
$C_3 = \{c_1, c_4, c_5, c_{12}, c_{14}, c_{16}, c_{18}, c_{21}, c_{22}, c_{23}, c_{25}, c_{26}, c_{30}\}$
$I_{0.6667} = 1.5990$

When threshold $\alpha = 0.7333$, six clusters can be obtained.

$C_1 = \{c_7\}$ $C_2 = \{c_{10}\}$ $C_3 = \{c_{18}\}$ $C_4 = \{c_{15}, c_{27}\}$
$C_5 = \{c_2, c_3, c_6, c_8, c_{11}, c_{13}, c_{17}.c_{19}, c_{20}, c_{24}, c_{28}, c_{29}\}$
$C_6 = \{c_1, c_4, c_5, c_{12}, c_{14}, c_{16}, c_{21}, c_{22}, c_{23}, c_{25}, c_{26}, c_{30}\}$
$I_{0.7333} = 1.7651$

When threshold $\alpha = 0.7667$, seven clusters can be obtained.

$C_1 = \{c_7\}$ $C_2 = \{c_{10}\}$ $C_3 = \{c_{15}\}$ $C_4 = \{c_{18}\}$ $C_5 = \{c_9, c_{27}\}$
$C_6 = \{c_2, c_3, c_6, c_8, c_{11}, c_{13}, c_{17}.c_{19}, c_{20}, c_{24}, c_{28}, c_{29}\}$
$C_7 = \{c_1, c_4, c_5, c_{12}, c_{14}, c_{16}, c_{21}, c_{22}, c_{23}, c_{25}, c_{26}, c_{30}\}$
$I_{0.7667} = 1.5681$

When threshold $\alpha = 0.8$, eleven clusters can be obtained.

$C_1 = \{c_2, c_6\}$ $C_2 = \{c_7\}$ $C_3 = \{c_{10}\}$ $C_4 = \{c_{15}\}$ $C_5 = \{c_{18}\}$
$C_6 = \{c_{17}, c_{20}\}$
$C_7 = \{c_{24}\}$ $C_8 = \{c_9, c_{27}\}$ $C_9 = \{c_3, c_{13}, c_{19}, c_{28}\}$ $C_{10} = \{c_8, c_{11}, c_{29}\}$
$C_{11} = \{c_1, c_4, c_5, c_{12}, c_{14}, c_{16}, c_{21}, c_{22}, c_{23}, c_{25}, c_{26}, c_{30}\}$
$I_{0.8} = 1.8554$

When threshold $\alpha = 0.8333$, twelve clusters can be obtained.

$C_1 = \{c_2\}$ $C_2 = \{c_6\}$ $C_3 = \{c_7\}$ $C_4 = \{c_{10}\}$ $C_5 = \{c_{15}\}$
$C_6 = \{c_{18}\}$
$C_7 = \{c_{17}, c_{20}\} C_8 = \{c_{24}\}$ $C_9 = \{c_9, c_{27}\}$ $C_{10} = \{c_3, c_{13}, c_{19}, c_{28}\}$
$C_{11} = \{c_8, c_{11}, c_{29}\}$
$C_{12} = \{c_1, c_4, c_5, c_{12}, c_{14}, c_{16}, c_{21}, c_{22}, c_{23}, c_{25}, c_{26}, c_{30}\}$
$I_{0.8333} = 1.7554$

When threshold $\alpha = 0.8667$, fourteen clusters can be obtained.

$C_1 = \{c_2\}$ $C_2 = \{c_6\}$ $C_3 = \{c_7\}$ $C_4 = \{c_{10}\}$ $C_5 = \{c_{15}\}$
$C_6 = \{c_{18}\}$ $C_7 = \{c_{19}\}$
$C_8 = \{c_{17}, c_{20}\}$ $C_9 = \{c_{24}\}$ $C_{10} = \{c_{12}, c_{21}.c_{23}, c_{25}\}$ $C_{11} = \{c_9, c_{27}\}$
$C_{12} = \{c_3, c_{13}, c_{28}\}$

$C_{13} = \{c_8, c_{11}, c_{29}\}$ $\quad C_{14} = \{c_1, c_4, c_5, c_{14}, c_{16}, c_{22}, c_{26}, c_{30}\}$
$I_{0.8667} = 1.7441$

When threshold $\alpha = 0.9$, fifteen clusters can be obtained.

$C_1 = \{c_2\}$ $\quad C_2 = \{c_6\}$ $\quad C_3 = \{c_7\}$ $\quad C_4 = \{c_{10}\}$ $\quad C_5 = \{c_{15}\}$ $\quad C_6 = \{c_{18}\}$
$C_7 = \{c_{19}\}$
$C_8 = \{c_{17}, c_{20}\}$ $\quad C_9 = \{c_{24}\}$ $\quad C_{10} = \{c_{12}, c_{21}.c_{23}, c_{25}\}$ $\quad C_{11} = \{c_{26}\}$
$C_{12} = \{c_9, c_{27}\}$ $\quad C_{13} = \{c_3, c_{13}, c_{28}\}$ $\quad C_{14} = \{c_8, c_{11}, c_{29}\}$
$C_{15} = \{c_1, c_4, c_5, c_{14}, c_{16}, c_{22}, c_{30}\}$
$I_{0.9} = 1.6860$

When threshold $\alpha = 0.9333$, seventeen clusters can be obtained.

$C_1 = \{c_1\}$ $\quad C_2 = \{c_2\}$ $\quad C_3 = \{c_6\}$ $\quad C_4 = \{c_7\}$ $\quad C_5 = \{c_{10}\}$
$C_6 = \{c_{12}\}$
$C_7 = \{c_{15}\}$ $\quad C_8 = \{c_{18}\}$ $\quad C_9 = \{c_{19}\}$ $\quad C_{10} = \{c_{17}, c_{20}\}$ $\quad C_{11} = \{c_{24}\}$
$C_{12} = \{c_{26}\}$
$C_{13} = \{c_{21}.c_{23}, c_{25}\}$ $\quad C_{14} = \{c_9.c_{27}\}$ $\quad C_{15} = \{c_3, c_{13}, c_{28}\}$
$C_{16} = \{c_8, c_{11}, c_{29}\}$
$C_{17} = \{c_4, c_5, c_{14}, c_{16}, c_{22}, c_{30}\}$
$I_{0.9333} = 1.6696$

When threshold $\alpha = 0.9667$, nineteen clusters can be obtained.

$C_1 = \{c_1\}$ $\quad C_2 = \{c_2\}$ $\quad C_3 = \{c_6\}$ $\quad C_4 = \{c_7\}$ $\quad C_5 = \{c_{10}\}$ $\quad C_6 = \{c_{11}\}$
$C_7 = \{c_{12}\}$ $\quad C_8 = \{c_{15}\}$ $\quad C_9 = \{c_{18}\}$ $\quad C_{10} = \{c_{19}\}$ $\quad C_{11} = \{c_{17}, c_{20}\}$
$C_{12} = \{c_{21}.c_{23}\}$
$C_{13} = \{c_{24}\}$ $\quad C_{14} = \{c_{25}\}$ $\quad C_{15} = \{c_{26}\}$ $\quad C_{16} = \{c_9.c_{27}\}$
$C_{17} = \{c_3, c_{13}, c_{28}\}$ $\quad C_{18} = \{c_8, c_{29}\}$ $\quad C_{16} = \{c_4, c_5, c_{14}, c_{16}, c_{22}, c_{30}\}$
$I_{0.9667} = 1.6845$

$I_{0.7667}$ is the smallest. The best cluster is $C_1 = \{c_7\}$, $C_2 = \{c_{10}\}$, $C_3 = \{c_{15}\}$, $C_4 = \{c_{18}\}$, $C_5 = \{c_{27}\}$, $C_6 = \{c_2, c_3, c_6, c_8, c_{11}, c_{13}, c_{17}.c_{19}, c_{20}, c_{24}, c_{28}, c_{29}\}$, and $C_7 = \{c_1, c_4, c_5, c_{12}, c_{14}, c_{16}, c_{21}, c_{22}, c_{23}, c_{25}, c_{26}, c_{30}\}$. The description of each supplier is obtained as followed:

The fuzzy description ξ_{C_1} of Cluster C_1 is supplier 7, which is $m_4 m_5' m_3'$ with the following interpretation: *sufficient delivery is good*, *but the profitability of supplier and conformance quality are not good*.

The fuzzy description ξ_{C_2} of Cluster C_2 is supplier 10, which is m_2 with the following interpretation: *product/process technology is good*.

The fuzzy description ξ_{C_3} of Cluster C_3 is supplier 15, which is m_6 with the following interpretation: *relationship closeness is good*.

The fuzzy description ξ_{C_4} of Cluster C_4 is supplier 18, which is $m_4 m_1' m_6'$ with the following interpretation: *sufficient delivery is good*, *but flexibility in service and relationship closeness are not good*.

The fuzzy description ξ_{C_5} of Cluster C_5 is suppliers 9 and 27, which is m_6 with the following interpretation: *relationship closeness is good*.

The fuzzy description ξ_{C_6} of Cluster C_6 is suppliers 2, 3, 6, 8, 11, 13, 17, 19, 20, 24, 28, and 29.

Supplier 2 is m_2 with the following interpretation: *product/process technology is good.*
Supplier 6 is $m'_4 m'_5$ with the following interpretation: *sufficient delivery and conformance quality are not good.*
Supplier 8 is $m'_3 m'_5$ with the following interpretation: *profitability of supplier and conformance quality are not good.*
Supplier 11 is $m'_1 m'_3 m'_5$ with the following interpretation: *flexibility in service, profitability of supplier*, *and conformance quality are not good.*
Supplier 17 is m'_3 with the following interpretation: *profitability of supplier is not good.*
Supplier 19 is m_5 with the following interpretation: *conformance quality is good.*
Supplier 20 is $m'_2 m'_3$ with the following interpretation: *product/process technology and profitability of supplier are not good.*
Supplier 24 is $m'_3 m'_4 m'_6$ with the following interpretation: *profitability of supplier, sufficient delivery and relationship closeness are not good.*
Supplier 29 is $m_3 m_5$ with the following interpretation: *profitability of supplier and conformance quality are good.*

The fuzzy description ξ_{C_7} of Cluster C_7 is suppliers 1, 4, 5, 12, 14, 16, 21, 22, 23, 25, 26, and 30.

Supplier 1 is $m_4 m_5$ with the following interpretation: *sufficient delivery and conformance quality are good.*
Supplier 4 is m_4 with the following interpretation: *sufficient delivery is good.*
Supplier 5 is m_4 with the following interpretation: *sufficient delivery is not good.*
Supplier 12 is $m_1 m_2 m_4 m_5$ with the following interpretation: *flexibility in service, product/process technology, sufficient delivery*, *and conformance quality are good.*
Supplier 14 is m_4 with the following interpretation: *sufficient delivery is good.*
Supplier 16 is m_4 with the following interpretation: *sufficient delivery is good.*
Supplier 21 is $m_2 m_3 m_4$ with the following interpretation: *product/process technology, profitability of supplier*, *and sufficient delivery are good.*
Supplier 22 is m_4 with the following interpretation: *sufficient delivery is good.*
Supplier 23 is $m_2 m_4$ with the following interpretation: *product/process technology and sufficient delivery are good.*
Supplier 25 is $m_1 m_2 m_4$ with the following interpretation: *flexibility in service, product/process technology*, *and sufficient delivery are good.*
Supplier 26 is $m_4 m'_2$ with the following interpretation: *sufficient delivery is good, but product/process technology is not good.*
Supplier 30 is m_4 with the following interpretation: *sufficient delivery is good.*

3.2 Application of FAHP and TOPSIS models to supplier selection

In this section, a FAHP model is proposed for the weight calculation of supplier selection. The steps are as follows:

- Form a committee of experts in the supply chain and define the qualitative criteria of supplier selection.
- Establish a fuzzy pairwise comparison of the control criteria according to the membership functions defined in Table 3.
- Calculate the crisp relative importance weights for control criteria to attain the goal using Eqs. (16) and (17).
- Calculate the values of the fuzzy synthetic extent with respect to the ith criterion for m goals using Eq. (15).
- Calculate the fuzzy synthetic degree values of the control criteria using Eq. (18).
- Calculate the weight and normalized weight into a number between zero and one using Eqs. (19) and (20). The calculated weight is for the application of TOPSIS algorithm.

3.2.1 Determination of the crisp relative importance weights

The goal of the fuzzy analytic hierarchy process is to calculate the relative importance of the six qualitative criteria based on the control criteria. This calculation is conducted through a pairwise comparison of the importance of control criteria to the goal. The fuzzy a pairwise comparison values are shown in Table 2.

The EAM is applied next to calculate the crisp relative importance weights for the control criteria. The control criteria data in Table 2, and the triangular fuzzy value are added by rows and coluumns, obtaining $\sum_{i=1}^{n}\sum_{j=1}^{m} S_{g_i}^{j}$. The calculations are as follows:

$$\sum_{i=1}^{n}\sum_{j=1}^{m} S_{g_i}^{j} = (1, 1, 1) + (1, 2, 3) + \cdots + (1, 1, 1) = (29.61, 48.53, 65.05)$$

Based on the result of $\sum_{i=1}^{n}\sum_{j=1}^{m} S_{g_i}^{j}$, $\left[\sum_{i=1}^{n}\sum_{j=1}^{m} S_{g_i}^{j}\right]^{-1}$ can be calculated by (16) as

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} S_{g_i}^{j}\right]^{-1} = (1/65.05, 1/48.53, 1/29.61) = (0.015, 0.021, 0.034)$$

The fuzzy pairwise comparison of each row control criteria is added by (17), and each $\sum_{j=1}^{m} S_{g_i}^{j}$ can be calculated as

Table 2 Fuzzy judgment of the suppliers (see Liang et al. 2005)

Supplier	Factor					
	Flexibility in service	Product/ process technology	Profitability of supplier	Sufficient delivery	Conformance quality	Relationship closeness
C_1	H	H	H	B.M&VH	VH	B.M&H
C_2	H	VH	B.M&H	B.M&H	H	B.M&H
C_3	H	H	B.M&H	H	H	B.M&H
C_4	H	B.M&VH	H	B.M&VH	H	B.M&VH
C_5	H	B.M&VH	H	B.M&VH	B.M&VH	B.M&VH
C_6	H	H	B.M&H	M	B.M&H	B.M&H
C_7	H	H	M	B.M&VH	B.M&H	B.M&H
C_8	B.M&H	H	M	B.M&H	B.M&H	H
C_9	B.M&H	H	H	H	B.M&VH	B.M&H
C_{10}	H	VH	H	B.M&H	B.M&VH	H
C_{11}	M	H	M	H	B.M&H	B.M&VH
C_{12}	VH	VH	H	B.M&VH	VH	H
C_{13}	B.M&H	H	B.M&H	H	B.M&VH	B.M&H
C_{14}	H	H	B.M&H	B.M&VH	B.M&VH	H
C_{15}	H	H	H	H	B.M&VH	VH
C_{16}	H	H	H	B.M&VH	B.M&VH	B.M&VH
C_{17}	B.M&H	H	M	H	B.M&VH	B.M&H
C_{18}	M	H	B.M&H	B.M&VH	B.M&VH	M
C_{19}	B.M&H	H	B.M&H	B.M&H	VH	B.M&H
C_{20}	B.M&H	M	M	H	B.M&VH	B.M&H
C_{21}	B.M&VH	VH	B.M&VH	B.M&VH	B.M&VH	B.M&VH
C_{22}	H	B.M&VH	B.M&H	B.M&VH	B.M&VH	H
C_{23}	B.M&VH	VH	H	B.M&VH	H	B.M&H
C_{24}	H	B.M&H	M	M	B.M&VH	M
C_{25}	VH	VH	H	B.M&VH	B.M&VH	B.M&VH
C_{26}	H	M	H	B.M&VH	B.M&VH	B.M&VH
C_{27}	B.M&H	B.M&H	H	H	B.M&VH	VH
C_{28}	B.M&VH	B.M&VH	B.M&H	H	B.M&VH	B.M&H
C_{29}	H	B.M&H	M	H	B.M&H	B.M&H
C_{30}	H	B.M&VH	H	B.M&VH	H	H

"VL": (0, 0, 0, 0), "B.VL&L": (0, 0, 0.1, 0.2), "L": (0, 0.2, 0.2, 0.2), "B.L&M": (0, 0.2, 0.4, 0.5), "M": (0, 0.3, 0.6, 0.7), "B.M&H": (0.3, 0.5, 0.8, 1), "H": (0.6, 0.8, 0.8, 1), "B.H&VH": (0.6,0.8, 0.9, 1), "VH":(1, 1, 1, 1), and $p = 3$ for the distance function $dp(.\ ,.\ )$

Table 3 Fuzzy pairwise comparison of control criteria

	Flexibility in service	Product/process technology	Profitability of supplier	Sufficient delivery	Conformance quality	Relationship closeness
Flexibility in service	(1,1,1)	(1,2,3)	(1,1.51,2.03)	(1.21,2.32,3.41)	(1.41,2.4,3.5)	(1.7,2.7,3.7)
Product/process technology	(0.33,0.5,1)	(1,1,1)	(0.28,0.32,0.9)	(1.13,2.03,2.54)	(1.24,2.32,2.93)	(1.27,2.45,3.18)
Profitability of supplier	(0.49,0.66,1)	(1.11,3.13,3.57)	(1,1,1)	(1.17,2.89,3.41)	(1.27,2.93,3.83)	(1.27,2.93,2.83)
Sufficient delivery	(0.29,0.43,0.83)	(0.39,0.49,0.88)	(0.29,0.35,0.85)	(1,1,1)	(1.34,1.89,2.32)	(1.47,2.55,3.43)
Conformance quality	(0.29,0.42,0.71)	(0.34,0.43,0.81)	(0.26,0.34,0.79)	(0.43,0.53,0.75)	(1,1,1)	(1,2,2)
Relationship closeness	(0.27,0.37,0.59)	(0.31,0.41,0.79)	(0.26,0.34,0.79)	(0.29,0.39,0.68)	(0.5,0.5,1)	(1,1,1)

$$\sum_{j=1}^{m} S_{g_1}^{j} = (1,1,1) + (1,2,3) + \cdots + (1.7, 2.7, 3.7) = (7.32, 11.93, 16.64)$$
$$\sum_{j=1}^{m} S_{g_2}^{j} = (5.25, 8.62, 11.55) \sum_{j=1}^{m} S_{g_3}^{j} = (6.31, 13.54, 16.04)$$
$$\sum_{j=1}^{m} S_{g_4}^{j} = (4.78, 6.71, 9.31)$$
$$\sum_{j=1}^{m} S_{g_5}^{j} = (3.32, 4.72, 6.06) \sum_{j=1}^{m} S_{g_6}^{j} = (2.63, 3.01, 4.85)$$

The fuzzy pairwise comparison of the control criteria is shown in Table 3.

Based on the $\sum_{i=1}^{n}\sum_{j=1}^{m} S_{g_i}^{j}$, $\left[\sum_{i=1}^{n}\sum_{j=1}^{m} S_{g_i}^{j}\right]^{-1}$, and each of the $\sum_{j=1}^{m} S_{g_i}^{j}$ values, the values of the fuzzy synthetic extent with respect to the ith criterion for m goals can be calculate by (15) as

$$\begin{aligned}
F_1 &= \sum_{j=1}^{m} S_{g_1}^{j} \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} S_{g_i}^{j}\right]^{-1} = (7.32 \times 0.015, 11.92 \times 0.021, 16.64 \times 0.034) \\
&= (0.11, 0.251, 0.566) \\
F_2 &= (0.079, 0.181, .0393) \qquad F_3 = (0.095, 0.284, 0.566) \\
F_4 &= (0.072, 0.141, 0.317) \\
F_5 &= (0.05, 0.099, 0.206) \qquad F_6 = (0.04, 0.063, 0.165)
\end{aligned}$$

Following a similar calculation, the fuzzy synthetic degree values of the other six control criteria are obtained by (18) as shown below:

$$\begin{aligned}
&V(F_1 \geq F_2) = 1 \quad V(F_1 \geq F_3) = 0.93 \quad V(F_1 \geq F_4) = 1 \quad V(F_1 \geq F_5) = 1 \\
&V(F_1 \geq F_6) = 1 \\
&V(F_2 \geq F_1) = 0.802 \quad V(F_2 \geq F_3) = 0.743 \quad V(F_2 \geq F_4) = 1 \quad V(F_2 \geq F_5) = 1 \\
&V(F_2 \geq F_6) = 1 \\
&V(F_3 \geq F_1) = 1 \quad V(F_3 \geq F_2) = 1 \quad V(F_3 \geq F_4) = 1 \quad V(F_3 \geq F_5) = 1 \\
&V(F_3 \geq F_6) = 1 \\
&V(F_4 \geq F_1) = 0.653 \quad V(F_4 \geq F_2) = 0.856 \quad V(F_4 \geq F_3) = 0.608 \\
&V(F_4 \geq F_5) = 1 \quad V(F_4 \geq F_6) = 1 \\
&V(F_5 \geq F_1) = 0.387 \quad V(F_5 \geq F_2) = 0.608 \quad V(F_5 \geq F_3) = 0.375 \\
&V(F_5 \geq F_4) = 0.761 \quad V(F_5 \geq F_6) = 1 \\
&V(F_6 \geq F_1) = 0.226 \quad V(F_6 \geq F_2) = 0.422 \quad V(F_6 \geq F_3) = 0.241 \\
&V(F_6 \geq F_4) = 0.544 \quad V(F_6 \geq F_5) = 0.762
\end{aligned}$$

The weight vectors are calculated as follows:

$$
\begin{aligned}
d(F_1) &= \min V(F_1 \geq F_2, F_1 \geq F_3, F_1 \geq F_4, F_1 \geq F_5, F_1 \geq F_6) \\
&= \min(1, 0.93, 1, 1, 1) = 0.93 \\
d(F_2) &= \min V(F_2 \geq F_1, F_2 \geq F_3, F_2 \geq F_4, F_2 \geq F_5, F_2 \geq F_6) = 0.743 \\
d(F_3) &= \min V(F_3 \geq F_1, F_3 \geq F_2, F_3 \geq F_4, F_3 \geq F_5, F_3 \geq F_6) = 1 \\
d(F_4) &= \min V(F_4 \geq F_1, F_4 \geq F_2, F_4 \geq F_3, F_4 \geq F_5, F_4 \geq F_6) = 0.608 \\
d(F_5) &= minV(F_5 \geq F_1, F_5 \geq F_2, F_5 \geq F_3, F_5 \geq F_5, F_5 \geq F_6) = 0.375 \\
d(F_6) &= \min V(F_6 \geq F_1, F_6 \geq F_2, F_6 \geq F_3, F_6 \geq F_4, F_6 \geq F_5) = 0.226
\end{aligned}
$$

Based on $d(F_i)$, we can calculate the weight, w_i' of the criteria by (19) as

$$
\begin{aligned}
W' &= (d(F_1), d(F_2), d(F_3), d(F_4), d(F_5), d(F_6)) \\
&= (0.93, 0.743, 1, 0.608, 0.375, 0.226)
\end{aligned}
$$

After normalization, the qualitative weights are as follows:

$$W = (0.24, 0.191, 0.258, 0.157, 0.097, 0.058)$$

3.2.2 TOPSIS model for supplier selection

In this section, the TOPSIS model is proposed for the final suppliers' selection. The steps are as follows:

- Calculate the normalized decision matrix. The normalized value is calculated using Eq. (21).
- Calculate the weighted normalized decision matrix by multiplying the normalized decision matrix using Eq. (22).
- Determine the positive ideal and negative ideal solution using Eq. (23).
- Calculation the separation of each alternative from the positive ideal solution and the negative ideal solution using Eqs. (24) and (25).
- Calculate the relative closeness to the ideal solution using Eq. (26).
- Rank the preference order.

STEP 1: For suppliers1, 2, 3, 4, 5, 6, 7, 9, and 10, normalization of original data is conducted. The result is as follows: $r_{ij} =$

	Felxibility in service 0.24	Product/ process Technology 0.191	Profitability of supplier 0.258	Sufficient delivery 0.157	Confor-mance quality 0.097	Relationship closness 0.058
Supplier 1	0.1844	0.1752	0.1996	0.1992	0.2112	0.1639
Supplier 2	0.1844	0.1752	0.1747	0.1549	0.1690	0.1639
Supplier 3	0.1844	0.1752	0.1747	0.1770	0.1690	0.1639
Supplier 4	0.1844	0.1971	0.1996	0.1992	0.1690	0.2107
Supplier 5	0.1844	0.1971	0.1996	0.1992	0.1901	0.2107
Supplier 6	0.1844	0.1752	0.1747	0.1328	0.1478	0.1639
Supplier 7	0.1844	0.1752	0.1497	0.1992	0.1478	0.1639
Supplier 8	0.1614	0.1752	0.1497	0.1549	0.1478	0.1873
Supplier 9	0.1614	0.1752	0.1996	0.1770	0.1901	0.1639
Supplier 10	0.1844	0.2191	0.1996	0.1549	0.1901	0.1873
Supplier 11	0.1383	0.1752	0.1497	0.1770	0.1478	0.2107
Supplier 12	0.2305	0.2191	0.1996	0.1992	0.2112	0.1873
Supplier 13	0.1614	0.1752	0.1747	0.1770	0.1901	0.1639
Supplier 14	0.1844	0.1752	0.1747	0.1992	0.1901	0.1873
Supplier 15	0.1844	0.1752	0.1996	0.1770	0.1901	0.2341
Supplier 16	0.1844	0.1752	0.1996	0.1992	0.1901	0.2107
Supplier 17	0.1614	0.1752	0.1497	0.1770	0.1901	0.1639
Supplier 18	0.1383	0.1752	0.1747	0.1992	0.1901	0.1405
Supplier 19	0.1614	0.1752	0.1747	0.1549	0.2112	0.1639
Supplier 20	0.1614	0.1314	0.1497	0.1770	0.1901	0.1639
Supplier 21	0.2075	0.2191	0.2246	0.1992	0.1901	0.2107
Supplier 22	0.1844	0.1971	0.1747	0.1992	0.1901	0.1873
Supplier 23	0.2075	0.2191	0.1996	0.1992	0.1690	0.1639
Supplier 24	0.1844	0.1533	0.1497	0.1328	0.1901	0.1405
Supplier 25	0.2305	0.2191	0.1996	0.1992	0.1901	0.2107
Supplier 26	0.1844	0.1314	0.1996	0.1992	0.1901	0.2107
Supplier 27	0.1614	0.1533	0.1996	0.1770	0.1901	0.1873
Supplier 28	0.2075	0.1971	0.1747	0.1770	0.1901	0.1639
Supplier 29	0.1844	0.1533	0.1497	0.1770	0.1478	0.1639
Supplier 30	0.1844	0.1971	0.1996	0.1992	0.1690	0.1873

STEP 2: The weight is $w_j = [0.240, 0.191, 0.258, 0.157, 0.097, 0.058]$. The weighted normalized matrix is as follows: $v_{ij}=$

	Felxibility in service	Product/ process technology	Profitability of supplier	Sufficient delivery	Confor-mance quality	Relationship closness
Supplier 1	0.0443	0.0335	0.0515	0.0313	0.0205	0.0095
Supplier 2	0.0443	0.0335	0.0451	0.0243	0.0164	0.0095
Supplier 3	0.0443	0.0335	0.0451	0.0278	0.0164	0.0095
Supplier 4	0.0443	0.0377	0.0515	0.0313	0.0164	0.0122
Supplier 5	0.0443	0.0377	0.0515	0.0313	0.0184	0.0122
Supplier 6	0.0443	0.0335	0.0451	0.0208	0.0143	0.0095
Supplier 7	0.0443	0.0335	0.0386	0.0313	0.0143	0.0095
Supplier 8	0.0387	0.0335	0.0386	0.0243	0.0143	0.0109
Supplier 9	0.0387	0.0335	0.0515	0.0278	0.0184	0.0095
Supplier 10	0.0443	0.0418	0.0515	0.0243	0.0184	0.0109
Supplier 11	0.0332	0.0335	0.0386	0.0278	0.0143	0.0122
Supplier 12	0.0553	0.0418	0.0515	0.0313	0.0205	0.0109
Supplier 13	0.0387	0.0335	0.0451	0.0278	0.0184	0.0095
Supplier 14	0.0443	0.0335	0.0451	0.0313	0.0184	0.0109
Supplier 15	0.0443	0.0335	0.0515	0.0278	0.0184	0.0136
Supplier 16	0.0443	0.0335	0.0515	0.0313	0.0184	0.0122
Supplier 17	0.0387	0.0335	0.0386	0.0278	0.0184	0.0095
Supplier 18	0.0332	0.0335	0.0451	0.0313	0.0184	0.0081
Supplier 19	0.0387	0.0335	0.0451	0.0243	0.0205	0.0095
Supplier 20	0.0387	0.0251	0.0386	0.0278	0.0184	0.0095
Supplier 21	0.0498	0.0418	0.0579	0.0313	0.0184	0.0122
Supplier 22	0.0443	0.0377	0.0451	0.0313	0.0184	0.0109
Supplier 23	0.0498	0.0418	0.0515	0.0313	0.0164	0.0095
Supplier 24	0.0443	0.0293	0.0386	0.0208	0.0184	0.0081
Supplier 25	0.0553	0.0418	0.0515	0.0313	0.0184	0.0122
Supplier 26	0.0443	0.0251	0.0515	0.0313	0.0184	0.0122
Supplier 27	0.0387	0.0293	0.0515	0.0278	0.0184	0.0109
Supplier 28	0.0498	0.0377	0.0451	0.0278	0.0184	0.0095
Supplier 29	0.0443	0.0293	0.0386	0.0278	0.0143	0.0095
Supplier 30	0.0443	0.0377	0.0515	0.0313	0.0164	0.0109

The positive ideal and negative ideal solutions can be obtained as follows:

STEPS 3 and 4: The separation of each alternative from the positive ideal solution is as follows:

	Supplier 1	Supplier 2	Supplier 3	Supplier 4	Supplier 5	Supplier 6	Supplier 7	Supplier 8	Supplier 9	Supplier 10
d_i^+	0.0154	0.0206	0.0197	0.0147	0.0142	0.0225	0.0246	0.0138	0.0201	0.0150
d_i^-	0.0228	0.0163	0.0174	0.0237	0.0239	0.0158	0.0178	0.0311	0.0187	0.0246
C_i	0.5979	0.4420	0.4688	0.6177	0.6274	0.4129	0.4202	0.6930	0.4811	0.6218
	Supplier 11	Supplier 12	Supplier 13	Supplier 14	Supplier 15	Supplier 16	Supplier 17	Supplier 18	Supplier 19	Supplier 20
d_i^+	0.0316	0.0070	0.0230	0.0192	0.0167	0.0160	0.0271	0.0270	0.0237	0.0308
d_i^-	0.0110	0.0330	0.0150	0.0191	0.0206	0.0220	0.0135	0.0163	0.0144	0.0106
C_i	0.2575	0.8253	0.3940	0.4989	0.5516	0.5800	0.3322	0.3766	0.3787	0.2562
	Supplier 21	Supplier 22	Supplier 23	Supplier 24	Supplier 25	Supplier 26	Supplier 27	Supplier 28	Supplier 29	Supplier 30
d_i^+	0.0072	0.0178	0.0095	0.0277	0.0079	0.0216	0.0223	0.0152	0.0265	0.0143
d_i^-	0.0325	0.0213	0.0292	0.0136	0.0326	0.0204	0.0169	0.0236	0.0143	0.0238
C_i	0.8192	0.5447	0.7539	0.3303	0.8052	0.4860	0.4312	0.6076	0.3503	0.6240

Table 4 Descriptions of features

	Profitability of supplier	Relationship closeness	Technological capability	Conformance quality	Conflict resolution
A_1	M	B.M&H	H	M	M
A_2	M	B.H&VH	B.H&VH	H	H
A_3	H	H	H	H	H
A_4	H	H	B.M&H	H	H
A_5	M	H	B.M&H	M	M

STEP 5: Based on this result, we can obtain $c_{12} \succ c_{21} \succ c_{25} \succ c_{23} \succ c_8 \succ c_5 \succ c_{30} \succ c_{10} \succ c_4 \succ c_{28} \succ c_1 \succ c_{16} \succ c_{15} \succ c_{22} \succ c_{14} \succ c_{26} \succ c_9 \succ c_3 \succ c_2 \succ c_{27} \succ c_7 \succ c_6 \succ c_{13} \succ c_{19} \succ c_{18} \succ c_{29} \succ c_{17} \succ c_{24} \succ c_{11} \succ c_{20}$.

Based on the supplier evaluation by AFS and ranking by TOPSIS, *suppliers 12, 21, 25, 23, 5, 30, 10, 4, 1, 16, 15, 22, 14, 2, 27, 19*, *and 29 are chosen as final suppliers.* Based on the AFS method, the profitability of supplier and conformance quality of supplier 8 are not good; product/process technology of supplier 26 is not good; sufficient delivery and conformance quality of supplier 6 are not good; profitability of supplier and conformance quality of supplier 7 are not good; flexibility in service and the relationship closeness of supplier 18 are not good; flexibility in service, profitability of supplier, and conformance quality of supplier 11 are not good; profitability of supplier of supplier 17 is not good; product/process technology and profitability of supplier of supplier 20 are not good; and profitability of supplier, sufficient delivery, and relationship closeness of supplier 24 are not good. Suppliers 3, 9, 13, and 28 are in the middle based on each criterion. Thus, *suppliers 3, 6, 7, 8, 9, 11, 13, 17, 18, 20, 24, 26, and 28 are rejected.*

4 The comparison of other supplier selection

Although many methods have been used for supplier evaluation, the AFS technique is a new approach for supplier evaluation. The FAHP and TOPSIS are used to rank suppliers, and the AFS is used to evaluate the supplier based on each criterion. The final suppliers are selected by above methods. In order to prove the selected suppliers are suitable in a supply chain. We demonstrate a comparison of supplier selection and evaluation between the proposed approach and other approach.

In Chena et al. (2006), let $A = \{A_1, A_2, \ldots, A_5\}$ be a set of five suppliers and their features (attributes) which are described by *profitability of supplier, relationship closeness, technological capability, conformance quality*, *and conflict resolution*, see Table 4.

In Table 1 and the semantic meanings of the attributes in C, we have

$$C_1 : A_1 = A_2 = A_5 < A_3 = A_4 \quad C_1' : A_1 = A_2 = A_5 > A_3 = A_4$$
$$C_2 : A_3 = A_4 = A_5 < A_1 = A_2 \quad C_2' : A_3 = A_4 = A_5 > A_1 = A_2$$

$$C_3 : A_4 = A_5 < A_1 = A_3 < A_2 \quad C_3' : A_4 = A_5 > A_1 = A_3 > A_2$$
$$C_4 : A_1 = A_5 < A_2 = A_3 = A_4 \quad C_4' : A_1 = A_5 > A_2 = A_3 = A_4$$
$$C_5 : A_1 = A_5 < A_2 = A_3 = A_4 \quad C_5' : A_1 = A_5 < A_2 = A_3 = A_4$$

In STEP 1: Let $X = \{A_1, A_2, \ldots, A_5\}$, $\varepsilon = 0$, $M = \{C_1, C_1', \ldots, C_5, C_5'\}$, $\upsilon = C_1 + C_1' + \cdots + C_5 + C_5'$, we have $\mu_\upsilon(A_1) = \mu_\upsilon(A_2) = \cdots = \mu_\upsilon(A_5) = 1.0$. We can obtain A_1:

$$B^0_{A_1} = \{C_2, C_1', C_4', C_5'\}., \xi_{A_1} = C_2C_1'C_4'C_5'$$

In STEP 2: The fuzzy relation matrix F is

$$F = \begin{bmatrix} 1.0 & 0.2 & 0.4 & 0.2 & 0.6 \\ 0.2 & 1.0 & 0.4 & 0.2 & 0.2 \\ 0.4 & 0.4 & 1.0 & 0.6 & 0.2 \\ 0.2 & 0.2 & 0.6 & 1.0 & 0.4 \\ 0.6 & 0.2 & 0.2 & 0.4 & 1.0 \end{bmatrix}$$

$Q = F3$ can yield a partition tree with equivalence classes.

In STEPS 3 and 4: The threshold can be $\alpha = 0.4, 0.6$. When the threshold $\alpha is 0.4$, five clusters can be obtained:

$$L_1 = \{A_2\}; L_2 = \{A_3\}; L_3 = \{A_4\}; L_4 = \{A_1\}; L_5 = \{A_5\}.$$
$$I_{0.4} = 6.45$$

When threshold $\alpha is 0.6$, three clusters can be obtained:

$$L_1 = \{A_2\}; L_2 = \{A_3, A_4\}; L_3 = \{A_1, A_5\} I_{0.6} = 1.8222$$
$$\xi_{L_1} = \xi_{A1} = C_2C_1'C_4'C_5', \quad \xi_{L_2} = \xi_{A2} = C_2C_3C_4C_5C_1',$$
$$\xi_{L_3} = \xi_{A3} = C_1C_4C_5C_2',$$
$$\xi_{L_4} = \xi_{A4} = C_1C_4C_5C_2'C_3', \quad \xi_{L_5} = \xi_{A5} = C_1'C_2'C_3'C_4'C_5'$$

$I_{0.6}$ is the smallest. The best cluster is $L_1 = \{A_2\}; L_2 = \{A_3, A_4\}; L_3 = \{A_1, A_5\}\}$.

The following is the description of each cluster:

The fuzzy description ξ_{L_1} of Cluster L_1 is $C_2C_1'C_4'C_5'$ with the following interpretation: *relationship closeness is good*, *but profitability of supplier, conformance quality and conflict resolution are not good*.
The fuzzy description ξ_{L_2} of Cluster L_2 is $C_2C_3C_4C_5C_1'$ with the following interpretation: *relationship closeness, technological capability, conformance quality and conflict resolution are good*, *but profitability of supplier is not good*.

The fuzzy description ξ_{L_3} of Cluster L_3 is $C_1C_4C_5C_2'$ with the following interpretation: *profitability of supplier, the conformance quality and conflict resolution are good, but relationship closeness is not good;*
The fuzzy description ξ_{L_4} of Cluster L_4 is $C_1C_4C_5C_2'C_3'$ with the following interpretation: *profitability of supplier, the conformance quality and conflict resolution are good, but relationship closeness and technological capability are not good;*
The fuzzy description ξ_{L_5} of Cluster L_5 is $C_1'C_2'C_3'C_4'C_5'$ with the following interpretation: *profitability of supplier, relationship closeness, technological capability, conformance quality, and conflict resolution are not good.*

Using TOPSIS method to rank the potential suppliers, and the step is as follows:
STEP 1: For supplier 1, 2, 3, 4 and 5, normalization of original data is conducted. The result is follows: r_{ij}=

	Profitability of supplier (0.7,0.8,0.8,0.9)	Relationship closeness (0.8,0.9,1.0,1.0)	Technological capability (0.7,0.87,0.93, 1.0)	Conformance quality (0.7,0.8,0.8,0.9)	Conflict resolution (0.7,0.8,0.8, 0.9)
Supplier 1	0.3906	0.3906	0.5208	0.5208	0.3906
Supplier 2	0.3901	0.5016	0.4458	0.4458	0.4458
Supplier 3	0.4566	0.5137	0.4566	0.3995	0.3995
Supplier 4	0.4581	0.4581	0.4008	0.4581	0.4581
Supplier 5	0.3693	0.4924	0.4924	0.4924	0.3693

STEP 2: The weighted normalized matrix is as follows: v_{ij}=

	Profitability of supplier (0.7,0.8,0.8, 0.9)	Relationship closeness (0.8,0.9,1.0, 1.0)	Technological capability (0.7,0.87,0.93, 1.0)	Conformance quality (0.7,0.8,0.8,0.9)	Conflict resolution (0.7,0.8,0.8, 0.9)	Profitability of supplier (0.7,0.8,0.8, 0.9)
Supplier 1	0.3125	0.3125	0.4166	0.4166	0.3125	0.3125
Supplier 2	0.3218	0.4138	0.3678	0.3678	0.3678	0.3218
Supplier 3	0.3995	0.4494	0.3995	0.3496	0.3496	0.3995
Supplier 4	0.3665	0.3665	0.3207	0.3665	0.3665	0.3665
Supplier 5	0.2954	0.3939	0.3939	0.3939	0.2954	0.2954

The positive ideal and negative ideal solutions can be obtained as follows:

STEPS 3 and 4: The separation of each alternative from the positive ideal solution is as follows:

	Supplier 1	Supplier 2	Supplier 3	Supplier 4	Supplier 5
d_i^+	0.1476	0.1433	0.1281	0.1476	0.1514
d_i^-	0.1195	0.1433	0.1243	0.1227	0.1179
CC_i	0.4474	0.4999	0.4926	0.4540	0.4378

STEP 5: Based on this result, we can obtain $A_2 \succ A_3 \succ A_4 \succ A_1 \succ A_4$

The ranking result is the same as the result of reference 37 that used a hierarchy multiple criteria decision-making model and TOPSIS method based on fuzzy-sets theory dealt with the supplier selection problems in the supply chain system. But we used the AFS to evaluate each potential supplier, supplier 2 is showed that relationship closeness, technological capability, conformance quality, and conflict resolution are good, but profitability of supplier is not good.

Supplier 3 is showed that profitability of supplier, conformance quality, and conflict resolution are good, but relationship closeness is not good.

Supplier 4 is showed that profitability of supplier, conformance quality, and conflict resolution are good, but relationship closeness and technological capability are not good. Based on the supplier evaluation by AFS and ranking by TOPSIS, s*upplier 2, 3, and 4 are selected according to different hobbies of decision-maker.* The supplier 1 is showed that relationship closeness is good, but profitability of supplier, conformance quality, and conflict resolution are not good. Supplier 5 is showed that profitability of supplier, relationship closeness, technological capability, conformance quality, and conflict resolution are not good. Thus *, supplier 1 and 5* are rejected.

5 Conclusion

Supplier selection is one of the most important activities in supply chain management. This importance is increased even more by new strategies in a supply chain, because of the key role suppliers perform in terms of flexibility in service, product/process technology, profitability of supplier, sufficient delivery, conformance quality and relationship closeness, which affect the outcome in the supply chain company. Supplier selection is a multiple-criteria decision-making problem in which the criteria are not equally important. In real cases, many input data are not known precisely for decision-making.

In the current paper, because of the decision-makers' experience, feel and subjective estimates often emerge in the process of supplier selection problem, and the AFS can effectively handle the vagueness and imprecision of input data. Although there are many supplier evaluation methods available, most methods usually conduct performance evaluation seldom perform each supplier's evaluation in terms of each criterion. AFS is proposed to deal with the qualitative and quantitative criteria and evaluate each factor; it can be used to select the suitable supplier effectively. To determine the relative importance weightings, FAHP is used to evaluate the importance of criteria in the supplier selection problem. The TOPSIS model is constructed to rank the order of suppliers for selection. Finally, we also compare our developed model with the models in the literature to highlight the relative advantages of our model. Our proposed methodology can reduce computational complexity and make the application of fuzzy methodology more understandable.

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